

Grundlagen der Wissensverarbeitung

Blatt 8

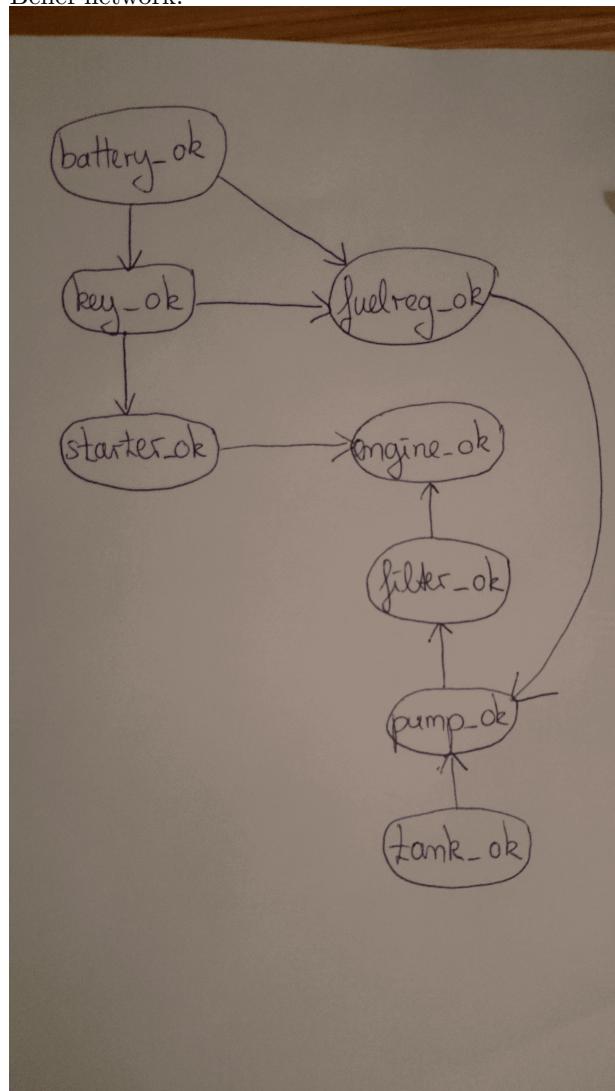
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Exercise 1.3: (Diagnosis (cont.))

Car engine.

Belief network:



$$\begin{aligned}
P(\text{battery_ok}) &= 0.9 \\
P(\text{key_ok} \mid \text{battery_ok}) &= 0.9 \cdot 0.9 = 0.81 \\
P(\text{fuelreg_ok} \mid \text{battery_ok}, \text{key_ok}) &= 0.9 \cdot 0.9 \cdot 0.81 = 0.6561 \\
P(\text{starter_ok} \mid \text{key_ok}) &= 0.9 \cdot 0.81 = 0.729 \\
P(\text{engine_ok} \mid \text{starter_ok}, \text{filter_ok}) &= 0.9 \cdot 0.729 \cdot 0.4782969 = 0.3138106 \\
P(\text{filter_ok} \mid \text{pump_ok}) &= 0.9 \cdot 0.531441 = 0.4782969 \\
P(\text{pump_ok} \mid \text{tank_ok}, \text{fuelreg_ok}) &= 0.9 \cdot 0.9 \cdot 0.6561 = 0.531441 \\
P(\text{tank_ok}) &= 0.9
\end{aligned}$$

Probability that the battery is working:

$$P(\text{battery_ok}) = 0.9$$

Probability that the starter is working:

$$P(\text{starter_ok} \mid \text{key_ok}) = 0.9 \cdot 0.81 = 0.729$$

Probability that the engine is working:

$$P(\text{engine_ok} \mid \text{starter_ok}, \text{filter_ok}) = 0.9 \cdot 0.729 \cdot 0.4782969 = 0.3138106$$

Probability that the engine is working after making the observation that the pump is working:

Following changes:

$$P(\text{pump_ok} \mid \text{tank_ok}, \text{fuelreg_ok}) = 1.0 \cdot 0.9 \cdot 0.6561 = 0.59049$$

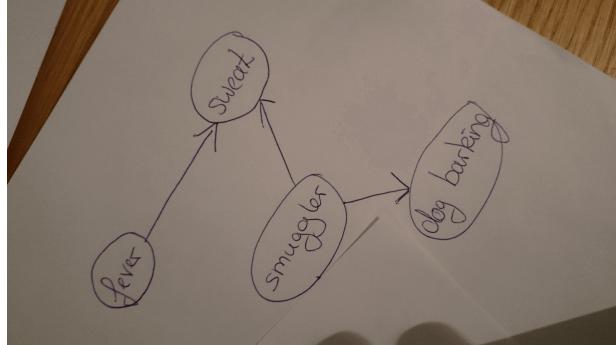
$$P(\text{filter_ok} \mid \text{pump_ok}) = 0.9 \cdot 0.59049 = 0.531441$$

$$P(\text{engine_ok} \mid \text{starter_ok}, \text{filter_ok}) = 0.9 \cdot 0.729 \cdot 0.531441 = 0.34867844$$

Exercise 1.4: (Bayesian Probabilities)

Smuggler case.

Probability network:



Complete the missing probabilities and draw the probability network:

$$P(\text{smuggler}) = 0.01 \quad / \text{given}$$

$$P(\overline{\text{smuggler}}) = 0.99 \quad / \text{calculated: } 1 - 0.01$$

$$P(\text{dog barking} \mid \text{smuggler}) = 0.8 \quad / \text{given}$$

$$P(\overline{\text{dog barking}} \mid \text{smuggler}) = 0.2 \quad / \text{calculated: } 1 - 0.8$$

$$P(\text{dog barking} \mid \overline{\text{smuggler}}) = 0.05 \quad / \text{given}$$

$$P(\overline{\text{dog barking}} \mid \overline{\text{smuggler}}) = 0.95 \quad / \text{calculated: } 1 - 0.05$$

$$\begin{aligned} P(\text{dog barking}) &= 0.85 && \text{/calculated: } 0.8 + 0.05 \\ P(\overline{\text{dog barking}}) &= 0.15 && \text{/calculated: } 1 - 0.85 \end{aligned}$$

$$\begin{aligned} P(\text{sweat} \mid \overline{\text{smuggler}}, \overline{\text{fever}}) &= 0 && \text{/given} \\ P(\text{sweat} \mid \text{smuggler}, \text{fever}) &= 0.4 && \text{/given} \\ P(\text{sweat} \mid \text{smuggler}, \text{fever}) &= 0.8 && \text{/given} \\ P(\text{sweat} \mid \overline{\text{smuggler}}, \text{fever}) &= 0.6 && \text{/given} \end{aligned}$$

$$\begin{aligned} P(\text{fever}) &= 0.013 && \text{/given} \\ P(\overline{\text{fever}}) &= 0.987 && \text{/calculated: } 1 - 0.013 \end{aligned}$$

Give an example of "explaining away" in the given network:

A status will get more likely or less likely dependent from other conditions.
Example: If somebody sweats, we have 2 explanations: fever and smuggler. If we also add the clue dog barking, then the explanation smuggler will get more likely than fever.

Probability that a person is a smuggler given the observation that the drug dog is barking:

$$P(\text{smuggler} \mid \text{dog barking}) = (P(\text{dog barking} \mid \text{smuggler}) \cdot P(\text{smuggler})) / P(\text{dog barking}) = (P(\text{dog barking} \mid \text{smuggler}) \cdot P(\text{smuggler})) / (P(\text{dog barking} \mid \text{smuggler}) \cdot P(\text{smuggler}) + P(\text{dog barking} \mid \overline{\text{smuggler}}) \cdot P(\overline{\text{smuggler}})) = (0.008 / 0.0575) = 0.14 \quad \text{/calculated with Bayes Law}$$

Probability that a suspect is sweating (without any prior observation):

$$\begin{aligned} P(\text{sweat}) &= \\ P(\text{sweat} \mid \overline{\text{smuggler}}, \overline{\text{fever}}) \cdot P(\overline{\text{smuggler}}, \overline{\text{fever}}) &+ \\ P(\text{sweat} \mid \text{smuggler}, \overline{\text{fever}}) \cdot P(\text{smuggler}, \overline{\text{fever}}) &+ \\ P(\text{sweat} \mid \overline{\text{smuggler}}, \text{fever}) \cdot P(\overline{\text{smuggler}}, \text{fever}) &+ \\ P(\text{sweat} \mid \text{smuggler}, \text{fever}) \cdot P(\text{smuggler}, \text{fever}) &= \\ (0 \cdot (0.99 \cdot 0.987)) + (0.4 \cdot (0.01 \cdot 0.987)) + (0.8 \cdot (0.01 \cdot 0.013)) + (0.6 \cdot (0.99 \cdot 0.013)) &= 0 + 0.003948 + 0.000104 + 0.007722 = 0.011774 \quad \text{/calculated with Law of Total Probability} \end{aligned}$$

Probability that a person is a smuggler given both the observations that that person is sweating and that the drug dog barked at him or her:

$$\begin{aligned} P(\text{smuggler} \mid \text{sweat}, \text{dog barking}) &= \\ P(\text{smuggler} \mid \overline{\text{sweat}}, \overline{\text{dog barking}}) \cdot P(\overline{\text{sweat}}, \overline{\text{dog barking}}) &+ \\ P(\text{smuggler} \mid \text{sweat}, \overline{\text{dog barking}}) \cdot P(\text{sweat}, \overline{\text{dog barking}}) &+ \\ P(\text{smuggler} \mid \overline{\text{sweat}}, \text{dog barking}) \cdot P(\overline{\text{sweat}}, \text{dog barking}) &+ \\ P(\text{smuggler} \mid \text{sweat}, \text{dog barking}) \cdot P(\text{sweat}, \text{dog barking}) &= \\ \dots \end{aligned}$$