Lab 7

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1 Shooting Method

A boundary value problem is one where a set of N coupled first-order ODEs will be given $n_1 <$ N initial conditions at one boundary, and $n_2 < N - n_1$ initaial conditions at the other boundary.
We are always missing n_2 but this can be found through a trial and error technique known as the shooting method. The best explanation for this technique is through example.

The following differential

$$\frac{d^2y}{dx} = -k^2x$$

can be re-written as:

$$\frac{dy}{dx} = y'$$
$$\frac{dy'}{dx} = -k^2x$$

Suppose we are given y(0) = 0 and y(L) = 0.

To solve this problem:

1. Take a guess for y'(0) and let it equal 'A',
2. Solve the system of first-order differential equations as an initial-value problem using Euler's method, RK2, or RK4.
3. Once you get iterate along x to the maximum, determine how much the target has been missed. This involves root finding. You can use either the bisection or secant method to accomplish this task to find 'A'. The secant method is known to be a faster method however it may not converge to a solution.
4. Back to 2 until the target is reached.

2 The Relaxation Method

Express the ODE as a finite-difference, divide solution into N sub-steps, each corresponding to a given finite difference equation, solve the set of simultaneous equations. Approaches depend on whether the problem is linear or nonlinear. For example:

$$y'' = y + x(x-4), 0 \leq x \geq 4$$

We can take care of the second derivative by taking the central difference and then gather the terms, giving:

$$y_{i-1} - (2 + h^2)y_i + y_{i+1} = h^2 x_i(x_i - 4)$$

If we choose a step size of 1, and our domain is between 1 and 4, we have the following system in matrix for:

$$\begin{pmatrix} -3 & 1 & 0 \\ 1 & -3 & 1 \\ 0 & 1 & -3 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \\ -3 \end{pmatrix}$$

The solution is given by $\mathrm{A}^{-1}b = y$.