The motion of a pulse can be described by the hyperbolic PDE

$$\frac{\delta^2 y}{\delta t^2} = v^2 \frac{\delta^2 y}{\delta x^2}$$

Describing the transverse displacement, y of a wave at 450nm on which an impulse is travelling at c m/s. The wave occurs along x between 0 and 6.95cm. At y(0,t) = y(6.95,t) = 0. At time = 0 (nanoseconds), the elements of the wave have a transverse displacement of

$$y(x,0) = \frac{2}{1+x^4}$$

and a transverse velocity of

$$\frac{\delta y(x,t)}{\delta t} = \frac{16x^3}{(1+x^4)^2}$$

Use a spatial stepsize of 10nm and a time stepsize of 0.01 ns.

Solve the hyperbolic PDE using the implicit finite method.

Recall the equations for n = 0 and n > 0:

(n = 0): $$-py_{i-1}^1 + (2+2p)y_i^1 - py_{n+1}^1 = -p\Delta t g_{i-1} + (2+2p)\Delta t g_i - p\Delta t g_{i+1} + 2f_i$$

(n > 0): $$-py_{i-1}^{n+1} + (2+2p)y_i^{n+1} - py_{i+1}^{n+1} = py_{i-1}^{n-1} - (2+2p)y_{i-1}^n + py_{i+1}^{n-1} + 4y_i^n$$

Where p = $$(v * \frac{\Delta t}{\Delta x})^2$$

```matlab
%%  Initializers
xBounds = [0,6.95];                     % in centimeters
yBounds = [0,0];
tmax = 50;
dx = 10*10^-9;                               % in nanometers
dt = 0.01*10^-9;                              % in nanoseconds
v = 3*10^8;                                  % in meters per second
f = @(x) 2 ./ (1+x.^4);                 % y(x,t=0)
g = @(x) 16 * x.^3 ./ (1+x.^4).^2;   % dy(x,t=0)/dt

%%  Solve
x = [xBounds(1):dx:xBounds(2)]';
N = length(x)-2;
sigma = (v*dt/dx)^2;
invA = inv(diag(-sigma*ones(1,N-1),-1)+diag(-sigma*ones(1,N-
1),1)+diag((2+2*sigma)*ones(1,N)));
gb = [g(xBounds(1));zeros(N-2,1);g(xBounds(2))];
y0 = f(x(2:end-1));

%   At n = 0
y1 = dt*g(x(2:end-1)) + invA*(2*y0-sigma*dt*gb);

%  Plot the solution at every time step
for t = 2:tmax/dt

   y2 = 4*invA*y1 - y0;
   y0 = y1;
   y1 = y2;
   if ~mod(t,0.1/dt)
       plot(x,[yBounds(1);y2;yBounds(2)])
       ylim([-2.5,2.5])
       title(['Wave configuration after ',num2str(t*dt),'
nanoseconds'])
       drawnow
   end

end
```