Lab 4

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Feb 2016

1 Brownian Motion

Brownian motion corresponds to a random walk in the limit where the step size, s, is infinitesimal. Say for example that steppers take steps every half unit of time. The root mean displacement of a random walk is sqrt(2t). If we reduce the number of steps that the steppers take by changing the step size factor k to 1/sqrt(k) we make the random walk continuous.
For each unit of time, there is a random variable that is normally distributed with mean 0 and variant t.
For each time increment [t1, t2], when t1¡t2, the difference between each coordinate pair [Bt1, Bt2] is independent of Bt1-Bt2.

Escape Times
Let a,b be positive integers and consider the time at which the random walk reaches the boundary of interval [-b,a]. The expected time is ab. The probability of escape at a is b/(a+b)

2 Radioactive Decay

The decay of radioactive particles is the best example of a random process. The particle decay proves is described by the analytical equation N(t) = N$e^{-(\lambda t)}$

3 Monte Carlo Integration

Very complicated integrals were solved by a rejection method. You had to draw your function very precisely on a piece of papery, weigh your piece of paper, use a planimeter to cut out the area under the curve, weigh the cut out of the area under the curve, and overall the area under the curve weight divided by the total area of the graph was your integral. For example, measuring the area of the creek bed behind your house. Draw a box surrounding the pond. Throw N number of stones making sure to evenly throw stones into the creek. Keep track of how many land in the pond versus how many miss the pond. As N approaches $\infty areapond/areaofshapeistheintegral$.

4 Questions

1/ The survival of patients living with cancer can be modelled as an exponential decay. The half-life of patients with leukemia is 1.5 years. Suppose that 500,000 leukemia patients are modeled for their survivability. Use a time step of 5 days and model the number of survivors until only 1 survivor remains. Plot the survivors as a function of time using a log plot. Overlay the theoretical plot N(t) relative to t. Display the time it took for only 100 survivors to remain.

2/ Use Monte Carlo Integration by rejection/stone throw to determine the following:

$$\int_{-1.9}^{1.3} [3x^3 - 8sin^3(2x) + 4x + 2]dx \tag{1}$$

Create a column vector that has 10E+06 that will give the estimate of the integral after N = 1 throw, N = 2 throws, etc. Solve the integral and then compare it against the estimate by stone throw.