Lab 5

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1 Mean Value Monte Carlo Integration

Recall the mean value theorem:

$$\int_a^b f(x)dx = (b-a) < f > \tag{1}$$

The value of the integral of some function f(x) between a and b equals the length of the interval (b-a) multiplied by the average value of the function over that interval ¡f¿. The integration algorithm uses Monte Carlo techniques to evaluate the mean of f. With a sequence xi of N uniform random number $\epsilon[a, b]$, we wish to determine the sample mean by sampling the function f(x) at the following points:

$$< f > \ \frac{1}{N}\sum_{i=1}^{N} f(xi) \tag{2}$$

giving the integration rule:

$$\int_a^b f(x)dx = \frac{(b-a)}{N}\sum_{i=1}^{N} f(xi) \tag{3}$$

As N $\rightarrow \infty$ or (b - a) gets infinitely smaller, we likely converge to the correct answer.
The uncertainty in the value obtained for the integral I after N samples of f(x) is measured by the standard deviation, which is the square root of the difference between squared function divided by the sample size.
If N is very large and we have a noisy function, then we have to consider another method.

2 Integration by mean value with variance reduction

Can we remove the variance of a function to make it smoother? We can do this by constructing a flatter function on which to apply Monte Carlo Integration by mean value. Suppose we construct a function g(x) such that:

$$J + \frac{(b-a)}{N}\sum_{i=1}^{N}[f(x) - g(x)] \tag{4}$$

3 Integration by mean value with importance sampling

How about a smoothing technique that divides the noisy function by some other function? We call this importance sampling because it lets us sample the integrand in the most important region. This takes the form of:

$$I = \int_a^b f(x)dx = \int_a^b \frac{f(x)}{w(x)} w(x)dx \tag{5}$$

Now the trick is finding the right choice of w(x). Think of w(x) as increasing the density of points sampled where f(x) is large because dy $\rightarrow$ w(x)dx. Moreover, w(x) attenuates the important areas of the function. What we are doing is evaluating the integral $\frac{f(x)}{w(x)}$ with x distributed following the pdf w(x) for x $\epsilon[a, b]$. The requirements on w(x) are such that:

$$\int_a^b w(x)dx = 1 \tag{6}$$

w(x) also needs to be analytically integratable and the result of its integrand needs to be an invertible function.

Quiz: Define 3 functions you will need (f(x), w(x), and x(y)) in order to compute

$$I = \int_{-0.33}^{1.33} \frac{dx}{1+x^2} dx \tag{7}$$

with w(x) = A(4-2x) as the weight wherein A is the normalization constant.

4 Questions

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