Lab 6

Daniel Molenhuis

Feb 2016

1 Solutions to Ordinary Differential Equations (ODEs)

Problems involving ODEs can always be reduced to the study of sets of first-order differntial equations. For example, the second-order equation

$$\frac{d^2y}{dx^2} + q(x)\frac{dy}{dx} = r(x) \tag{1}$$

can be written as two first-order equations
$\frac{dy}{dx} = z(x)$
$\frac{dz}{dx} = r(x) - q(x)z(x)$

And so the generic problem in ODE equations is reduced to the study of a set of N coupled first-order ODEs for the function y having the general form $\frac{dy(x)}{dx} = f(x, y0, y1, y2, ..., yn - 1$ where the functions of f are known.

2 Initial-value problems

These are problems where the value of y(t) at t = 0 is known, and solving the ODE consists in propagating the solution y(t), in time, yielding the evolution of the system in time.

3 Euler's Method

Based on the idea of using the Taylor series expansion of x around t. This is good to know but bad to implement as it is not very accurate compared to other methods. Furthermore, it is not very stable.
Example code:

```
x = initial value; y = initial value; h = step size;
for i in 1 to N do
y = y + f(x,y) * h;
x = x + h;
Print x y;
end
```

4 Runge-Kutta method

The problem with the Euler method is that it advances the solution through an interval h using the derivative information only at the beginning of that interval. This is corrected by taking a half step and use the slope to determine the next step. Code will be provided to explain this concept.