Solve the following elliptic PDE:

$$\frac{\delta^2 U}{\delta x^2} + \frac{\delta^2 U}{\delta y^2} = sin(\pi x)$$

For x ϵ [0, 1] and y ϵ [0, 1] with a stepsize of Δx = 0.2 and Δy = 0.25 with the following boundary conditions:

U(x,0) = x U(x,1) = 1 U(0,y) = y U(1,y) = 1

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% Bounds and step-size
xbnds = [0 1]; ybnds = [0 1]; dx = 0.2; dy = 0.25;

% Boundary conditions
lbc = @(y) y;
rbc = @(y) ones(size(y));
bbc = @(x) x;
tbc = @(x) ones(size(x));

% LHS of the Elliptic PDE, rho(x,y)
soln = [];
for y = ybnds(1)+dy:dy:ybnds(end)-dy
soln = [ soln; dx^2*sin(pi*[xbnds(1)+dx:dx:xbnds(end)-dx]') ];
end

% Computing sigma + dimensions (m columns, n rows)
sigma = (dx/dy)^2;
m = (xbnds(end)-xbnds(1))/dx-1;
n = (ybnds(end)-ybnds(1))/dy-1;

%% k indices become (i,j) for left, right, top, & bottom
neighbours
k2x = @(k) xbnds(1) + dx*(k-m*(ceil(k/m)-1));
k2y = @(k) ybnds(1) + dy*ceil(k/m);

% Left neighbour diagonal & bc
lvec = ones(m*n-1,1);
k = [1:m:(m*n)-m+1]';
lvec(k(2:end)-1) = zeros(n-1,1);
soln(k) = soln(k) - lbc( k2y(k) );

% Right neighbour diagonal & bc
rvec = ones(m*n-1,1);
k = [m:m:m*n]';
rvec(k(1:end-1)) = zeros(n-1,1);
soln(k) = soln(k) - rbc( k2y(k) );
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% Bottom neighbour diagonal & bc
bvec = sigma*ones(m*n-m,1);
k = [1:m]';
soln(k) = soln(k) - sigma*bbc( k2x(k) );

% Top neighbour diagonal & bc
tvec = sigma*ones(m*n-m,1);
k = [(n-1)*m+1:m*n]';
soln(k) = soln(k) - sigma*tbc( k2x(k) );

% Central diagonal & bc
cvec = -2*(1+sigma).*ones(m*n,1);

% Folding centre, left, right, bottom, & top into matrix
A = diag(cvec,0) + diag(lvec,-1) + diag(rvec,1) + diag(bvec,-m)
+ diag(tvec,m);

% Solution
res = A \ soln;


%% Plot Solution as a mesh %%
% Reshape into square matrix
res = transpose(reshape(res,m,n));

% Compute x and y vectors
xrange = [xbnds(1):dx:xbnds(end)];
yrange = [ybnds(1):dy:ybnds(end)]';

% Adding the BCs on each side
res = [ lbc(yrange(2:end-1)) res rbc(yrange(2:end-1)) ];
res = [ bbc(xrange); res; tbc(xrange) ];

% Plot
mesh(xrange,yrange,res); xlabel('x'); ylabel('y'); zlabel('U');
title(sprintf('Elliptic solution for dx = %g, dy = %g', dx,
dy));
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