Foundations of Machine Learning Assignment 2

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This is the given table in matrix form:

$$\begin{bmatrix} 2 & 6 & 5 \\ 4 & 5 & 3 \\ 6 & 8 & 6 \\ 5 & 7 & 8 \end{bmatrix}$$

The first thing we must do is find the mean of each column:

$$\text{Reading: } \frac{2+4+6+5}{4} = 4.25$$

$$\text{Videos: } \frac{6+5+8+7}{4} = 6.5$$

$$\text{Practice: } \frac{5+3+6+8}{4} = 5.5$$

Now, we subtract the value of each number in the matrix by the mean of its column.

This gives us the Centered Data Matrix:

$$\begin{bmatrix} 2-4.25 & 6-6.5 & 5-5.5 \\ 4-4.25 & 5-6.5 & 3-5.5 \\ 6-4.25 & 8-6.5 & 6-5.5 \\ 5-4.25 & 7-6.5 & 8-5.5 \end{bmatrix} = \begin{bmatrix} -2.25 & -0.5 & -0.5 \\ -0.25 & -1.5 & -2.5 \\ 1.75 & 1.5 & 0.5 \\ 0.75 & 0.5 & 2.5 \end{bmatrix}$$

Now that we have the Centered Data Matrix, we can compute the covariance matrix using the formula:

$$\hat{\Sigma} = [q_{j,k}] \in \mathbb{R}^{D \times D}, \quad \text{where} \quad q_{j,k} = \frac{1}{N-1} \sum_{i=1}^{N} (x_{i,j} - \bar{x}_j)(x_{i,k} - \bar{x}_k)$$

$$\text{Cov(x):} \quad \frac{1}{4-1} \quad * \quad \begin{bmatrix} -2.25 & -0.25 & 1.75 & 0.75 \\ -0. & -1.5 & 1.5 & 0.5 \\ -0.5 & -2.5 & 0.5 & 2.5 \end{bmatrix} \quad * \quad \begin{bmatrix} -2.25 & -0.5 & -0.5 \\ -0.25 & -1.5 & -2.5 \\ 1.75 & 1.5 & 0.5 \\ 0.75 & 0.5 & 2.5 \end{bmatrix}$$

This is equal to

$$\begin{bmatrix} 2.9167 & 1.5 & 1.5 \\ 1.5 & 1.667 & 2 \\ 1.5 & 2 & 4.333 \end{bmatrix}$$

We can then use the numpy library to find the eigenvectors and eigenvalues:

Eigenvalues: [6.556, 1.984, 0.377]

Eigenvectors:

$$\begin{bmatrix} -0.4931 & -0.8035 & 0.3335 \\ -0.4547 & -0.0888 & -0.8862 \\ -0.7417 & 0.5886 & 0.3216 \end{bmatrix}$$

Subtask 2: The eigenvalues represent the variance captured by each component. The larger the eigenvalue, the more variance is explained by that principal component (PC). In order to reduce dimensionality, we choose the principal components with the highest variances (¿70%).

Finding the variance of each PC

Total eigenvalue sum: $6.556 + 1.984 + 0.377 = 8.917$

$$\text{PC1: } \frac{6.556}{8.917} = 74\%$$

$$\text{PC2 : } \frac{1.984}{8.917} = 22\%$$

$$\text{PC3: } \frac{0.377}{8.917} = 4\%$$

PC1 explain 74% of the variance, so its provides a good summary of the students study patterns. This is the only principal component we need!

Subtask 4:

Projected values on the first principal component:

Student A: 1.7077

Student B: 2.6596

Student C: -1.9158

Student D: -2.4514

Student A's study behavior is somewhat aligned with the dominant study pattern captured by PC1.

Student B's study habits align most closely with the main trend captured by the first principal component.

Student C's study behavior diverges from the dominant pattern. Their study habits are less aligned with the general trend.

Student D's negative value indicates their study pattern is opposite to the dominant trend. The magnitude of the value suggests a strong divergence from the main pattern.