Multidimensional Inventory Control

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We model the stochastic inventory problem with Markovian demand. The objective is to maximize expected profits by placing optimal order quantities x_{ti} for products $i \in I$ in periods $t \in T$. Demand is satisfied from on-hand inventory $I_{t-1,i}$ by selling s_{ti} products. Any excess demand is considered lost.

Inventory capacity: $C = 1000$; inventory holding cost: $h = 0.5$; sales price: $p = 5$; purchase cost: $c = 3$. Initial inventory: $I_{0,i} = 0$.

Demand is non-stationary and characterized by a discrete-time multi-dimensional lattice with T layers and $N = 100$ nodes per layer.

The optimization problem can be stated as a multistage stochastic linear optimization problem, which is given by

$$\max \operatorname{Exp}_\xi \left[\sum_t \sum_i (ps_{ti} - hI_{ti} - cx_{ti}) \right] \tag{1}$$

$$s.t.\ I_{ti} = I_{t-1,i} + x_{ti} - s_{ti}, \qquad t = 1, ...T,\ i = 1, ..., I \tag{2}$$

$$x_{ti} \leq I_{t-1,i}, \qquad t = 1, ...T,\ i = 1, ..., I \tag{3}$$

$$x_{ti} \leq D_{ti}(\xi), \qquad t = 1, ...T,\ i = 1, ..., I \tag{4}$$

$$I_{ti} \leq C, \qquad t = 1, ...T,\ i = 1, ..., I \tag{5}$$

$$I_{ti}, s_{ti}, x_{ti} \geq 0, \qquad t = 1, ...T,\ i = 1, ..., I \tag{6}$$