Punto 2)

a) Sea $x_n = \bar{x}_n + \varepsilon_n$ la solucion exacta. sustituyendo en el algoritmo de verlet.

$a_n' = \frac{\partial a}{\partial x}$

$$\bar{x}_{n+1} + \varepsilon_{n+1} = 2\bar{x}_n + 2\varepsilon_n - \bar{x}_{n-1} - \varepsilon_{n-1} + a_n' \varepsilon_n h^2$$

$$\varepsilon_{n+1} - 2\varepsilon_n + \varepsilon_{n-1} - a_n'\varepsilon_n h^2 = \cancel{-\bar{x}_{n+1} + 2\bar{x}_n - \bar{x}_{n-1}} \to 0$$

$$\varepsilon_{n+1} - (2 + h^2 a_n')\varepsilon_n + \varepsilon_{n-1} = 0$$

b) Caso oscilador armonico

$$a(x) = -\omega^2 x \qquad a'(x) = -\omega^2$$

$$\Rightarrow \varepsilon_{n+1} - (2 - h^2\omega^2)\varepsilon_n + \varepsilon_{n-1} = 0$$

$$\varepsilon_{n+1} - (2 - 2R)\varepsilon_n + \varepsilon_{n-1} = 0$$

$$\varepsilon_{n+1} - 2(1-R)\varepsilon_n + \varepsilon_{n-1} = 0$$

c) $\varepsilon_n = \varepsilon_0 \lambda^n$

$$\varepsilon_0 \lambda^{n+1} - 2(1-R)\varepsilon_0\lambda^n + \varepsilon_0 \lambda^{n-1} = 0$$

$$\varepsilon_0 \lambda^{n-1}(\lambda^2 - 2(1-R)\lambda + 1) = 0$$

$$\lambda^2 - 2(1-R)\lambda + 1 = 0$$

c)

$$\Rightarrow \lambda_\pm = \frac{-(-2(1-R)) \pm \sqrt{4(1-R)^2 - 4}}{2}$$

$$= 1 - R \pm \sqrt{R^2 - 2R}$$

d) si $R \leq 2$ se tiene que $|\lambda_\pm| = 1$

$$R = \frac{h^2 w^2}{2} \leq 2$$

$$h^2 \leq \frac{4}{w^2}$$

$$h \leq \frac{2}{w}$$

Punto 4)

$$\frac{du}{dt} = u^q$$

Si $q = 1$

$$\int \frac{du}{u} = \int dt$$

$$\ln(u) = t + c$$

$$u = e^{t+c}$$

Si $1 < q < 1$

$$\int \frac{du}{u^q} = \int dt$$

$$\int u^{-q} du = \int dt$$

$$\frac{u^{-q+1}}{-q+1} = t + c$$

$$u^{1-q} = (1-q)(t+c)$$

$$u^{1-q} = t + c - qt - qc$$

$$u = (t + c - qt - qc)^{\frac{1}{1-q}}$$

$$u = (t(1-q) + c - qc)^{\frac{1}{1-q}}$$

Si $c = (1-q)^{-1}$

$$u = \left(t(1-q) + \frac{1}{1-q}(1-q)\right)^{\frac{1}{1-q}}$$

$$u = (t(1-q) + 1)^{\frac{1}{1-q}}$$

Punto 6)

→ Expandiendo en serie de Taylor

$$\frac{\partial^2 x}{\partial t^2}$$

$$① \quad \vec{x}(t+\Delta t) = \vec{x}(t) + \vec{v}(t)\Delta t + \frac{1}{2}\Delta t^2 \vec{a}(t)$$

→ Para hallar $\vec{v}$:

$$\vec{x}(t+\Delta t) = \vec{x}(t) + \vec{v}(t)\Delta t + \vec{a}(t)\frac{\Delta t^2}{2}$$

Restando $(-)$

$$\vec{x}(t-\Delta t) = \vec{x}(t) - \vec{v}(t)\Delta t + \vec{a}(t)\frac{\Delta t^2}{2}$$

$$\Rightarrow \quad \vec{x}(t+\Delta t) - \vec{x}(t-\Delta t) = 2\vec{v}(t)\Delta t$$

$$\vec{v}(t) = \left(\vec{x}(t+\Delta t) - \vec{x}(t-\Delta t)\right) / 2\Delta t$$