$$y' - x y^2 + \frac{2y}{x} + \frac{1}{x^3} = 0$$

$y_1 = x^{-2}$

$y_1' = -2x^{-3}$

① comprobar que $y_1 = x^{-2}$ es solución

$$x^3(-2x^{-3}) = x^4(x^{-2})^2 - 2x^2(x^{-2}) - 1$$

$$-2 = 1 - 2 - 1$$

$$-2 = -2$$

② $y = y_1 + u^{-1}$ $\Big\}$ $y = x^{-2} + u^{-1}$

$y' = -2x^{-3} - u^{-2}\, u'$

$x^3(-2x^{-3} - u^{-2}\, u') = x^4(x^{-2} + u^{-1})^2 - 2x^2(x^{-2} + u^{-1}) - 1$

$\Rightarrow$ $-2 - x^3 u^{-2} u' = 1 + 2x^2 u^{-1} + x^4 u^{-2} - 2 - 2x^2 u^{-1} - 1$

$\Rightarrow$ $-x^3 u^{-2} u' = x^4 u^{-2}$

$\Rightarrow$ $u' = \dfrac{x^4 u^{-2}}{x^3 u^{-2}} = x$

③ $u' = -x$ $\Big\}$ $\int du = \int -x\, dx$

$u = -x^2/2 + C$

④ $y = x^{-2} + \left(-\dfrac{x^2}{2} + C\right)^{-1}$