



## Original research article

## An improved method for registration of point cloud



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## ABSTRACT

A new hybrid least square method was proposed for registration of point cloud in this paper. The registration process was accomplished through two steps: the coarse registration and the accurate registration. The point cloud was transformed to the vicinity of the 3-D shapes by using genetic algorithm during the coarse registration procedure and the accuracy of point cloud registration was strongly raised in the accurate registration stage with iterative closest point algorithm. Experimental results show that the registration rate, matching accuracy and convergence rate of the algorithm proposed here are all improved.

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## 1. Introduction

Registration technology is a key technology in the world of digital detection which is currently used in many fields such as nondestructive testing, reverse engineering, virtual reality, robot and flexible assembly areas. Model registration is used to realize the components testing, model evaluation, the premise of error analysis and data fitting. Many experts and scholars did a lot of work in this area. The iterative closest point algorithm (ICP) was put forward by Besl and McKay [1], which is widely used in matching algorithm, the kernel technology of the ICP is searching the corresponding relation between the two registration point sets to calculate the transformation matrix, but the algorithm needs a good initial value to ensure the convergence, otherwise it may be not convergence. Yan Sijie [2] applied the genetic algorithm to deal with point cloud registration of large location, complex curved surface, it need to know the angular point before the registration, so it is very difficult to achieve automatic point cloud registration. Dai Lanjing [3] use the principal direction of joint for initial registration and the iterative closest point algorithm (ICP) for accurate point cloud registration, which obtained a good registration precision of the point cloud. Sun Yuwen [4] realizes complex precision by using BFGS, but the complex is involved in solving nonlinear equation and derivation operation, such as solving Hessian matrix, it is very complicated to implement the algorithm. Yago Diez [5] put forward a new method called Hierarchical Normal Space Sampling (HNSS) for point cloud coarse matching problems. This new method extends Normal Space Sampling, finds results much faster, stops the search earlier and performs better than Normal Space Sampling. Zhang [6] proposed structured region signature method for coarse registration and the final registration was realized using the LM algorithm. Darion Grant [7] proposed a P2P (point-to-plane) method, but the initial registration parameters must be determined in advance. Chow [8] uses the improved genetic algorithm of dynamic genetic operators for surface registration. Jiang [9] uses an angular-invariant feature registration for 3-D point cloud.

In this paper, we propose a hybrid algorithm which integrated the GA algorithm and the ICP algorithm to automatically tackle the point cloud and CAD model registration problem. A given measurement point cloud and a theoretical CAD model

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which don't need to any initial parameter estimation, can realize the automatic registration of point cloud and CAD model. Registration is divided into two steps: rough registration and fine registration, rough registration is on base of the GA algorithm and fine registration is mainly dependent on the ICP algorithm, the rough registration provides a good initial value for the fine registration.

The remaining of this paper is organized as follows. Section 2 describes point cloud registration problem statement and Section 3 describes the rough algorithm and how to apply it to solve the coarse point cloud registration problem. The fine point cloud registration was described in Section 4. Section 5 describes how to calculate the points on the CAD model corresponding to the measurement points. In Section 6, simulated experiments are described to demonstrate the effectiveness of the proposed two steps strategy about point cloud and CAD model registration, and the results show that the method is not only suitable for the 2-D graphic registration but also for 3-D point clouds registration. Finally, the last section gives a conclusion.

## 2. Registration problem statement

Given a measurement point sets  $S = \{P_i\}$  and CAD model of stl format  $\Omega = \{[l^i, k^i, r^i]\}$ , among them  $[l^i, k^i, r^i]$  are three vertices of  $i$ -th triangles, point cloud registration is aimed at finding and determining the measurement points and the corresponding points on the model. The registration procedure makes the sum of the distance about the measurement points and the corresponding points on the CAD model minimum. The minimal distance between two point sets can be achieved as follows. Assuming that the measurement point  $P_i (i = 1, 2, \dots, n)$  and corresponding points  $Q_i (i = 1, 2, \dots, n)$ , the objective function of the point cloud registration based on the least squares principle is:

$$E = \frac{1}{n} \sum_{i=1}^n \|Q_i - (R \cdot P_i + T)\|^2 \quad (1)$$

Where  $\|\cdot\|$  denotes the Euclidean norm. Parameters  $R \in R^{3 \times 3}$  and  $T \in R^{3 \times 1}$  are respectively rotation matrix and translation matrix.  $R$  is a function of  $\alpha, \beta, \gamma$ .  $\alpha, \beta, \gamma$  respectively express the rotation angle around the x, y, z axis. Translation matrix  $T = [t_x \ t_y \ t_z]^T$  expresses translational distance along x, y and z axis. In order to solve the Eq. (1), make it achieve the minimum value, that is to say, calculate the optimal  $R$  and  $T$  to get the minimum value.

## 3. Implementation of coarse point cloud registration

The coarse point cloud registration was achieved on base of the genetic algorithm (GA) [10]. GA is the calculation model according to simulation of the natural biological evolution process. In recent years, in the field of engineering technology, GA has been widely used in the optimal design of complex nonlinear system with its strong robustness. Before using GA, the center alignment of the two points sets should be realized, that is to say, centers of two point sets are shifted to the same location. After that the optimal rotation matrix is found by the method of genetic algorithm (GA) and the result of the objective function (1) must be kept smallest. In the operating process, Genetic algorithm (GA) usually uses evolution algebra as a termination condition. The paper adopts the above criterion. The specific implementation plan of the genetic algorithm is as follows:

### 3.1. Define of searching space

After aligned two centriods of the measurement points set and CAD model, three translation axes  $\{t_x, t_y, t_z\}$  can be ignored in the following process. Then three angles  $\{\alpha, \beta, \gamma\}$  are used to rotate one of the points set to another and kept a minimum value about sum of Euclidean distances of the two sets, here the space constructed by three rotational angles  $\{\alpha, \beta, \gamma\}$  is defined as the searching space.

### 3.2. Chromosome encoding/decoding technology about searching space

Rotational angles  $\{\alpha, \beta, \gamma\}$  can be seen as three genomes respectively, then a genomes group is generated by combining three angle genomes randomly and every angle genome is encoded as a binary data with the same length. The encoding length of the binary data can be limited according to the practical circumstance. The chromosome decoding process is an inverse course of the encoding process. All of the above work is prepared for choosing the crossover and mutation genetic, setting the corresponding decoding variables and further finding the fitness function.

### 3.3. Determination of fitness function

Firstly, projected all measurement points to the CAD model, then a sum of distances from measurement points to corresponding points is obtained, as given by the Eq. (1). Result of the Eq. (1) is the minimum of the average distance and its value is always positive. Fitness function is defined as a reciprocal of the Eq. (1). Under the condition of choosing the maximum

value of the fitness function as an optimal value, then the corresponding angular binary chromosomes of  $\{\alpha, \beta, \gamma\}$  can be used to the next generation directly and the fitness function are also used to calculate the proportions of every genomes sample for the whole genome. The larger the value of the fitness function is, the greater the proportions of the corresponding genomes sample is.

### 3.4. Duplication, cross-over and mutation

Genetic operating course is very important in the genetic algorithm, which includes duplication, cross-over and mutation and so on. The process of the duplication is a genetic operation which chooses the optimal gene directly into the next generation and avoids the loss of good genes.

Cross-over operation used here is a crossing process of any two genomes groups from all randomly generated genomes groups. In course of executing this cross-over operation, one breakpoint separates the genome groups and each ones are divided into two segments respectively, then the segments are exchanged each other. The breakpoint of the cross-over operation is generated based on a random number. According to available tests by multiple iterations, the effective cross-over rate for each gene is 0.87 in all our experiments. Good cross-over operator can help to fine individual chromosome fragment passed to their children, at the same time cross-over operator generally acts as global search and can exploit unknown space. On the other hand, Mutation plays an important role in the optimization process and it can prevent premature convergence in the process of solving the local optimal solution rather than the overall optimal solution. Under mutation condition, each gene has a certain probability to change its value. In the experiment here, the available mutation operator is chose as 0.12.

## 4. Iterative closest point for the fine registration

After GA coarse registration, measurement points cloud is very close to the theoretical CAD model. In next step, the registration accuracy of the model is improved furtherly through the ICP iterative calculating method.

Assuming measurement points set is  $\{P_i\}$ , after the GA algorithm registration transformation, points set is become set  $\{G_i\}$ , points set  $\{Q_i\}$  is the corresponding points set of  $\{G_i\}$  in the CAD model, The center of the measured points set  $\{G_i\}$  is  $\bar{G}$  and the center of the corresponding points set  $\{Q_i\}$  is  $\bar{Q}$ , the calculating expression of  $\bar{G}$  and  $\bar{Q}$  will be given by Eqs. (2) and (3).

$$\bar{G} = \frac{1}{n} \sum_{i=1}^n G_i \quad (2)$$

$$\bar{Q} = \frac{1}{n} \sum_{i=1}^n Q_i \quad (3)$$

then the cross-covariance matrix  $\Sigma_{GQ}$  of the sets G and Q is given by

$$\Sigma_{GQ} = \frac{1}{n} \sum_{i=1}^n G_i \cdot Q_i^T - \bar{G} \cdot \bar{Q}^T \quad (4)$$

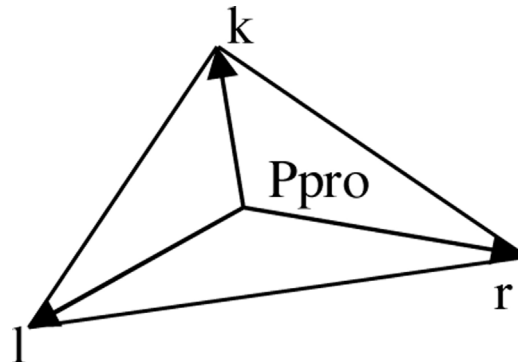
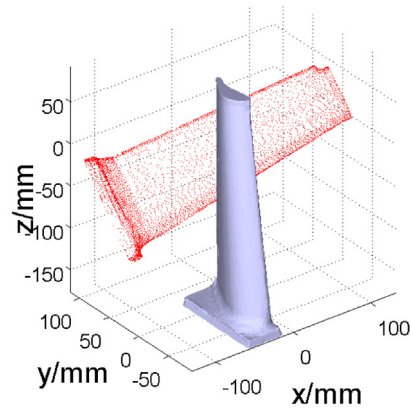
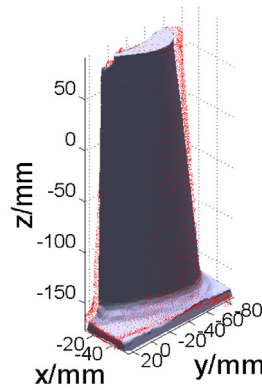


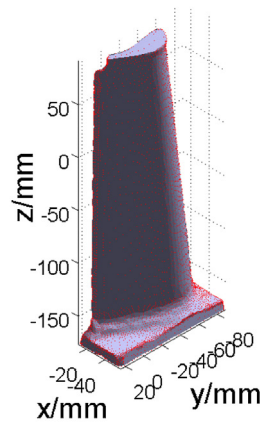
Fig. 1. Cross product judge schematic diagram.



(a) The initial position graphics



(b) The registration result of the GA



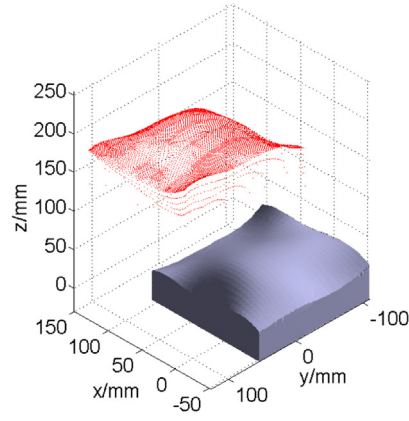
(c) The registration result of the new method

**Fig. 2.** The registration of the single blade.

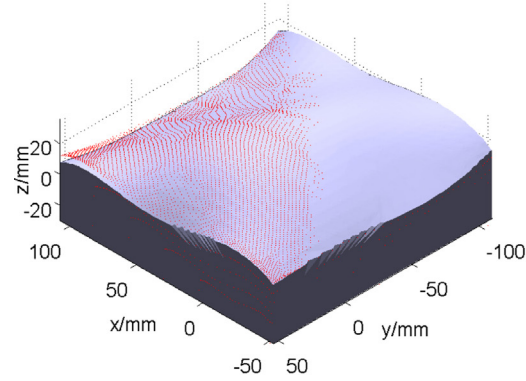
The cyclic components of the anti-symmetric matrix  $A_{ij} = (\Sigma_{GQ} - \Sigma_{GQ}^T)_{ij}$  is used to form a vector  $\Delta = [A_{23} \ A_{31} \ A_{12}]$ . Then this vector is used to constructed an symmetric matrix as

$$Q(\Sigma_{GQ}) = \begin{bmatrix} \text{tr}(\Sigma_{GQ}) & \Delta^T \\ \Delta & \Sigma_{GQ} + \Sigma_{GQ}^T - \text{tr}(\Sigma_{GQ})I_3 \end{bmatrix} \quad (5)$$

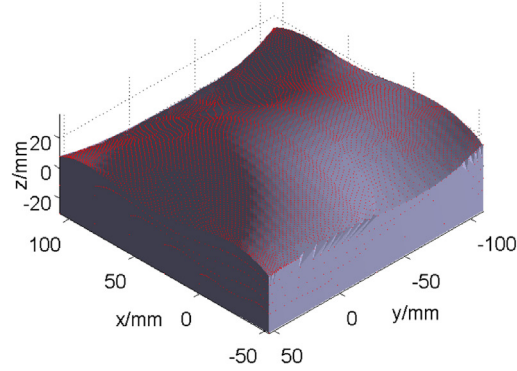
Here  $\text{tr}(\Sigma_{GQ})$  expression trace of the matrix  $\Sigma_{GQ}$ ,  $I_3$  expresses a  $3 \times 3$  unit matrix.



(a) The initial position graphics



(b) The registration result of the GA

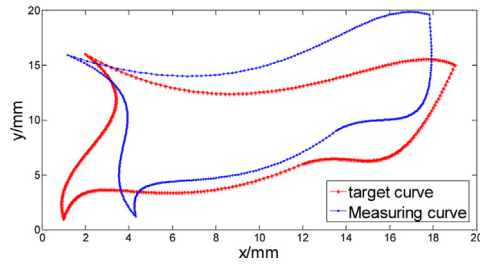


(c) The registration result of the new method

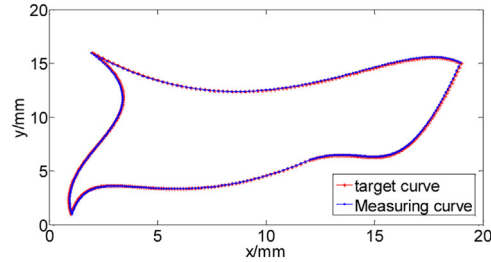
**Fig. 3.** The registration of the free surface.

The unit eigenvector  $q_R = [q_0 \ q_1 \ q_2 \ q_3]$  corresponding to the maximum eigenvalue of the matrix  $Q$  ( $\Sigma_{GQ}$ ) is selected as the optimal rotation matrix, this matrix can be expressed as

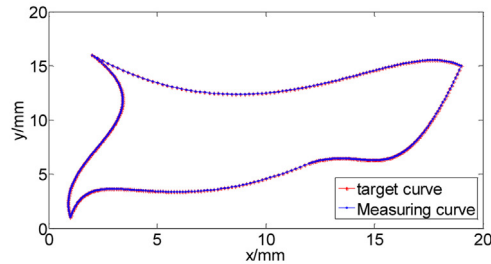
$$R(q_R) = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2q_1q_2 - 2q_0q_3 & 2q_1q_3 + 2q_0q_2 \\ 2q_1q_2 + 2q_0q_3 & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2q_2q_3 - 2q_0q_1 \\ 2q_1q_3 - 2q_0q_2 & 2q_2q_3 + 2q_0q_1 & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix} \quad (6)$$



(a) The initial position graphics



(b) The registration result of the GA



(c) The registration result of the new method

**Fig. 4.** The registration of the regular polygon.

Base on the above rotation matrix, then the translation vector can be obtained as

$$T = \bar{Q} - R \cdot \bar{G} \quad (7)$$

The rotation matrix  $R$  and translation vector  $T$  act on the point set  $\{G_i\}$ , then the mean square error can be calculated through the Eq. (1). The description above is a single step about the calculation of the mean square error. In practice, the former process is executed iteratively, at the  $k$ -th iterative execution, the mean square error  $E_k$  is obtained, the deviation  $\varepsilon$  is computed using  $E_k - E_{k+1}$ . The iteration is terminated when the deviation in mean square error falls below a preset threshold  $\varepsilon_T$ .

## 5. Corresponding points of measurement points to calculate

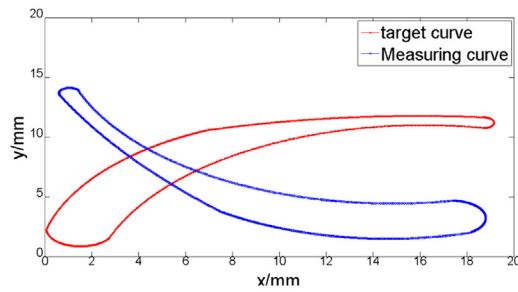
Corresponding points in the model of measurement point to calculate is the key step in the process of the registration, we adopt the following method:

Firstly, the CAD model is presented by STL format which express the theoretical model using a lists of triangles, their vertexes and normal. A k-d tree is built on base of vertexes of the triangular facets. For each measurement points, a nearest vertex on the CAD model to it can be obtained by tree traversal.

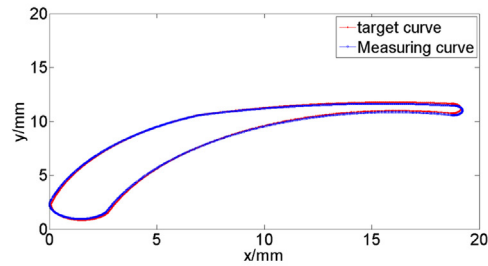
To find the nearest vertex distance measurement point, the measurement point to connect all triangles in the vertex projection and the projection distance of the shortest point to corresponding points of measurement point.

Calculate the spatial point to a triangles in the distance, there are two situations[11]: one kind is space point in the projection point on the triangles fall within the triangles, the other is a space point in the projection point on the triangles fall outside the triangles. According to the method of cross product[12] to determine whether falls within the triangle surface, see Fig. 1,  $P_{pro}$  is projection point on the triangles corresponding to the space point  $P$ ,  $k, l, r$  is three vertices of triangles. Make

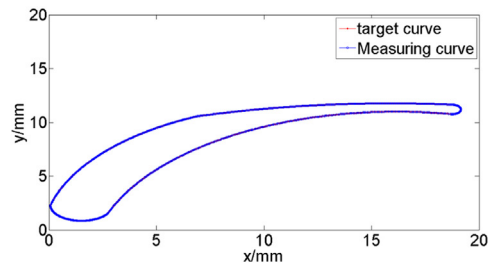
$$V_1 = k - P_{pro}, V_2 = l - P_{pro}, V_3 = r - P_{pro}, V_4 = V_1,$$



(a) The initial position graphics



(b) The registration result of the GA



(c) The registration result of the new method

**Fig. 5.** The registration of the blade section curve.**Table 1**

Registration error (mm).

	GA	the new method
single blade	2.720	$5.144 \times 10^{-6}$
free surface	2.032	$4.174 \times 10^{-5}$
Irregular polygon	$8.255 \times 10^{-4}$	$1.171 \times 10^{-7}$
Blade section	$6.100 \times 10^{-3}$	$1.112 \times 10^{-3}$

Projection point within the triangles in the necessary and sufficient conditions:  $V_i \times V_{i+1}$  ( $i = 1, 2, 3$ ) same sign. When the projection point is inside the triangles in the projection point is the point on the triangles in the projection of point, when the projection point on the outside of the triangles, find a point within the triangles to the closest point of measurement point as the projection point on the triangles.

## 6. The simulation results and discussion

To verify the validity of the algorithm, in this paper, under the Windows environment, we use the MATLAB to do the simulation. The evolution algebra of the genetic algorithm is 60, the size of the population is 50, cross-over operator  $p_m$  is 0.87, and mutation operator  $p_c$  is 0.12. The simulation results show that the genetic algorithm provides precise point cloud registration with a good initial position. The point cloud registration through the genetic algorithm is basically coincided, with a slight deviation. Further by the aid of the ICP algorithm, the new algorithm can achieve a higher accuracy and has universal practicability. Fig. 2 shows the simulation for a single blade model and Fig. 3 shows the simulation for a freeform

surface. At the same time, Figs. 4 and 5 are given matching results using the new method to deal with the 2-d graphics registration. Table 1 shows the average error square after the coarse registration and the accurate registration.

The experimental results show that the algorithm has universality. No matter registration of point cloud and 3d models, or 2-d curve registration, the algorithm has ideal registration effect and it has strong robustness.

## 7. Conclusion

In this paper the genetic algorithm and iterative closest point of two step matching strategy were put forward, in order to solve the registration of point cloud and 3d models. Conducting the initial registration by making use of the ability searching the global optimal solution that the genetic algorithm has, which provide exact registration with nice iterative initial value. After the registration through iterative closest point algorithm, a higher registration precision can be achieved. Also it can achieve nice registration effect for registration of point cloud and 3d models, which is superior to the traditional iterative closest point algorithm. Experiments prove that this method has strong commonality, and the algorithm is reliable and stable.

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