Hawking Radiation

The Theories that Describe our Universe

Modern understanding of our universe tells us that it can be modelled by two theories: The Theory of General Relativity and Quantum Field Theory. These theories describe the events in our universe at their respective scales. The holy grail of theoretical physics is to devise a universal theory: one that unites quantum field theory with general relativity and incorporates all the fundamental constants of nature; a “theory of everything.

General relativity:

The Theory of General Relativity focuses on physics at a large scale, describing the behaviour of planets and large masses in the universe. It can be thought of as describing the fabric of the universe as a surface, spacetime that can be bent by large masses, causing gravitation. But in reality, spacetime is a four-dimensional entity, three dimensions of space, and one dimension of time. Gravitation, in this framework, is not a force, but the result of the curvature of spacetime on the motion of bodies; this curvature is caused by large masses like planets. It can be visualised by imagining what would happen if you placed a heavy ball on a trampoline; the surface of the trampoline, representing spacetime, would sink and bend as a result of the ball, representing a planet.

The idea of spacetime curvature paves the way for another aspect of general relativity that is extremely important in the phenomenon of Hawking Radiation. This is the idea that measurements of space and time are not absolute. For all observers, the laws of physics never change, but their measurements of space and time do. This is called ‘Gravitational Time Dilation’. What these phenomena describe is how space and time can be measured differently depending on the observer. For example, time appears to move more slowly for an observer within a strong gravitational field than it does for one floating freely in space.

Quantum Field Theory:

Quantum Field Theory focuses on events and systems on a much smaller, sub-atomic level. One of the core elements of Quantum Field Theory is quantum mechanics, which states that, unlike normal objects, which have a definite position in space, according to quantum mechanics, particles do not have definite positions; quantum particles exist in a superposition of possible states. Their presence is distributed across space in all of their possible positions, each with varying levels of possibility of finding that particle in a particular location. This can be summarised by stating that quantum mechanics uses the wave function, a mathematical object that encodes the probability amplitudes for all possible positions of that particle, to describe particles, rather than bodies with definite positions.

There is another detail in Quantum Field Theory that is important to the understanding of Hawking Radiation. This is how it describes vacuums, and what we think of as ‘empty space’. In quantum field theory, Vacuum Fluctuations fill this ‘empty’ space. A vacuum fluctuation is where a particle-antiparticle pair comes into existence and then annihilates extremely soon after coming into existence. These particles are referred to as virtual particles. And when we revisit quantum mechanics and describe these particles as waves of probability, we find ourselves with a vacuum where a wave comes into existence at the same instant as one that is effectively the opposite. It is important to note that virtual particles are not physically real particles or waves, just mathematical tools which we use to represent fluctuations in the quantum field.

The Popularised Simplification

The simplified explanation for Hawking radiation that has become the most popularly used is, in reality, quite misleading. This explanation defines Hawking radiation as a result of a pair of virtual particles coming into existence right next to the event horizon of a black hole. The event horizon is the point at which the gravitational pull of the black hole is so strong that nothing, not even light, can escape. When these particles come into existence, before they can annihilate each other, one falls beyond the event horizon and falls into the black hole. This means that its pair is not annihilated and escapes as a real particle; this escaped particle is the radiation that we measure. This simplification became widespread as a result of Hawking’s own use of it in his 1988 book ‘A Brief History of Time’. And in this context, it served its purpose; an easy-to-visualise and understand analogy of the phenomenon, which highlights some of the main characteristics of the more detailed theory.

However, this explanation is fundamentally flawed in a few ways. One of which is that this simplification leads us to believe that the radiation is released at the event horizon, when in reality it does not happen due to the event horizon, but due to the curvature of spacetime around the black hole. This explanation also implies that the escaped virtual particle is released as ‘real’ when in reality, virtual particles cannot be ‘real’ as such and are really just calculator tools used to describe quantum fluctuations.

The Quantum Vacuum Explanation

Now I will now try to break down what makes up the scientifically accurate version of Hawking’s theory and explore the flaws of his popularised simplification.

Virtual and Real Particles:

We have already discussed that virtual particles are not physical objects that can be measured directly. They are fluctuations in the quantum field, mathematical tools that can be used to describe how real particles interact. They exist for an incredibly short period of time, meaning they cannot be directly observed.

Virtual particles are often explained in terms of Heisenberg’s uncertainty principle, which gives intuition for how short-lived fluctuations can occur. One way to express this principle is in terms of energy and time:

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

Where ΔE is the uncertainty of energy in a system, Δt is the uncertainty in the time interval in which the system is observed or measured, and $\hbar$ is the reduced Planck’s constant. What this principle tells us is that the product of the uncertainties in energy and time in a system is greater than or equal to half the reduced Planck’s constant. In other words, a system that exists for a short time can not have an accurately defined energy, so to accurately define the energy, the system has to be observable for a longer period of time. Therefore, we cannot accurately define the energy of a system or particle that exists for a very small period of time, meaning that a system's energy can fluctuate for a small period of time without violating the law of conservation of energy.

The final piece in the puzzle that is virtual particles is the 'Zero Point Energy'. The other form of Heisenberg's uncertainty principle will help explain:

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Where Δx is the uncertainty of the position of a particle, Δp is the uncertainty in the momentum of the particle, and $\frac{\hbar}{2}$ is half the reduced Planck's constant. This principle leads to a familiar conclusion: we cannot accurately measure or define the position and momentum of a particle simultaneously. Therefore, the quantum field can never have exactly zero energy, for if this were true, it would have to have zero kinetic energy, meaning the momentum of the modes of the quantum field is also zero (a quantum field is composed of modes, each of which acts like a harmonic oscillator). But if the momentum of all the modes were zero, the uncertainty in position would also be zero when multiplied, therefore defying the uncertainty principle. As a result of this, it is clear that there must be a minimum possible energy a quantum system can have that can exist in a vacuum, but not defy the uncertainty principle. This is known as the 'Zero-Point Energy' and is often referred to as "The Energy of the Vacuum".

And now, finally, we can dive into the manifestation of virtual particles. Virtual particles are simply mathematical tools used to represent quantum field fluctuations. I will now explore how the principles that we just covered explain these fluctuations. In quantum field theory, each type of particle exists due to oscillations of the modes in its corresponding field. These fields exist everywhere, even in a vacuum. And, as we just explored, the modes that make up these fields can never have zero energy. Because of the zero-point energy, each mode in the quantum field has a small amount of energy, leading to a constant "jitter" of the modes. In addition, the time-energy uncertainty we looked at shows us that energy fluctuations can happen in the quantum field as long as they exist only for a short period of time; the less energy, the longer they can exist and vice versa. These uncertainty principles mean that no mode in the quantum field is ever perfectly still; it is constantly exhibiting small fluctuations in position and momentum. And because a mode's energy is dependent on its position and momentum, fluctuations in these quantities lead to corresponding fluctuations in energy, only as allowed by the energy-time uncertainty. These fluctuations of energy are what we express mathematically as virtual particles; short-lived excitations of the quantum field that can only exist for a time short enough to obey the energy-time uncertainty principle.

Quantum Fields in Curved Spacetime:

We have already discussed in plenty of detail what a vacuum looks like in flat spacetime. An ocean of virtual particles, effectively coming in and out of existence as a result of the uncertainty principles and the zero-point energy.

But this all changes dramatically when we near a black hole, where an object of unimaginable density and mass curves spacetime to a point where not even light can escape. It is the behavioural changes in quantum fields when in such curved spacetime that are the key to unlocking the workings of Hawking Radiation.

In flat spacetime, where quantum field theory is nearly always modelled, the concept of a particle is well defined. It is a positive frequency mode in the quantum field, which relies on a consistent definition of time because frequency is ‘oscillations per unit time’. And this is simple in flat space-time because there is a ‘Global Time Coordinate’, meaning time flows uniformly and consistently for all observers, allowing for a unique definition of a positive frequency.

But when in curved spacetime, due to General Relativity, we no longer have a global time coordinate because the geometry of the spacetime does not have the symmetry of flat spacetime. Time can only be defined locally, and these local definitions in close regions will not agree on a time coordinate. This means that the positive and negative frequency modes become observer-dependent. These differing definitions of ‘time’ mean that in curved spacetime, different observers cannot agree on the frequencies of field modes; where one observer may see positive frequencies, so real particles; another might see a mixture of positive and negative modes, so virtual particles or a vacuum.

Observer-dependence:

We have now laid down almost all of the necessary foundation to construct the final explanation of the actual mechanisms of Hawking Radiation. To start, we are going to picture three observers: one ‘Far’, one ‘Stationary’, and one ‘Falling’. The Far observer is an infinite distance away from our black hole and is stationary in perfectly flat spacetime; the Stationary observer is a set distance outside of the black hole, accelerating uniformly to stay stationary and not fall in. While our Falling observer is freely falling into the black hole as a result of its gravity. Hawking Radiation stems from the different perceptions that these observers have of the vacuum around the black hole.

Falling Observer:

Our Falling observer will feel nothing unusual. As they fall into the black hole, they will observe a perfectly normal vacuum, full of only virtual particles. This is because the observer is not resisting gravity and is therefore in a local frame of free fall, a frame with no acceleration relative to the curved spacetime around them. Although the overall spacetime is strongly curved by the black hole, any small region surrounding our freely falling observer behaves as if it were flat, gravity-free space. The reason for this lies in Einstein's Equivalence Principle, which states that within a small region of spacetime, the effects of gravity are locally indistinguishable from those of acceleration. In other words, for our freely falling observer, being pulled toward the black hole feels exactly the same as floating in empty space far from any gravitational field. To them, the laws of physics appear completely normal, and no radiation is detected.

Stationary Observer:

The Stationary observer is a set distance away from the event horizon, but still close enough to be strongly affected by the curved spacetime. This means that they have to be accelerating at a constant rate to remain stationary and not fall into the black hole. As a result of Einstein's Equivalence Principle, this observer's acceleration to stay stationary is indistinguishable from uniform acceleration in flat spacetime. Meaning they experience what's called the Unruh Effect, which states that any observer accelerating uniformly through a vacuum will perceive a thermal bath and the temperature of this bath (Unruh Temperature) is given as:

$$T = \frac{\hbar a}{2\pi c k_B}$$

Where T is the temperature, $\hbar$ is the reduced Planck constant, a is the observer's acceleration, c is the speed of light, and k_B is the Boltzmann constant.. This effect happens as a result of the observer's different notions of time and energy, so what a free-falling observer would perceive as vacuum modes, an accelerating observer perceives as excited modes. The closer this observer is to the event horizon, the more they have to accelerate, so the hotter the thermal bath.

Far Observer:

And finally, the Far observer. This observer is stationary at an infinite distance away from the black hole, but unlike the stationary observer, they are in flat spacetime, so do not need to accelerate to stay stationary. This observer detects what we define as Hawking Radiation, a steady stream of real particles coming out of the black hole. Unlike the stationary observer who perceives radiation as a result of their acceleration, the far observer perceives the radiation as actual outgoing quanta, carrying energy away from the black hole. This is again because of their different time coordinates; the far observer can describe the quantum field with Schwarzschild Time, the time coordinate for an observer in perfectly flat spacetime. Due to this, the far observer perceives real particles as ones with positive frequency modes. The stationary observer describes time with a different set of coordinates. The time coordinates of the two observers have a nonlinear relationship, and this relationship is key to the effect.

The Causes of Hawking Radiation:

Now we will imagine two observers, you could picture them as astronauts floating in space. One of them will be in the gravitational field, the well in spacetime, of the black hole near its event horizon, but he is accelerating at a constant rate away from the black hole so as not to fall in. The other is an infinite distance away, floating in flat spacetime. As a consequence of General Relativity, the observer in the gravitational field will experience time to be slower; the astronaut's watch is ticking more slowly than his friend's. For ease, we will call the observer close to the black hole the local observer, and the observer in flat spacetime the far observer.

Thinking back to the Heisenberg Uncertainty Principle, we see that for particles ot be virtual, the uncertainty in time multiplied by that of energy must be greater than or equal to a constant for the particle to be real. The Zero Point Energy is observed as the same value for any observer anywhere in the universe, no matter where they are located, so what happens when observers experiencing different flows of time look at the same area? For our local observer, time is moving more slowly, meaning that the value for the uncertainty of time (Δt) is low enough for the product of the uncertainties ($\Delta E \Delta t$) to be less than the constant ($\frac{h}{4\pi}$), meaning that the local observer does not perceive any real particles, only the unmeasurable virtual particles. However, when being observed by the far observer on flat spacetime, where time seems to be moving faster, the value for the uncertainty of time (Δt) is greater than that of the local observer. This means that the product of the uncertainties ($\Delta E \Delta t$) would be greater than or equal to the constant ($\frac{h}{4\pi}$), thus the particles obey the uncertainty and are observed as real. These observable, real particles are what we measure ot be Hawking Radiation.

To summarise Relativity's role in this phenomenon, the curvature of spacetime causes time to be perceived differently near the event horizon of the black hole. This means that, because the Zero Point Energy is constant, when observed locally, the virtual particles forming around the black hole don't obey Heisenberg's Uncertainty Principle, so they appear virtual. But to an observer further away in less curved spacetime, the particles seem to have a greater uncertainty in time, meaning that they obey the uncertainty principle and are real. Some of these real particles are then able to escape the black hole into space and are what we observe as Hawking Radiation.