Hawking Radiation

Introduction:

The study of black holes has long been a cornerstone in the exploration of theoretical physics. They possess properties unique to any other objects in the universe. Once considered regions of spacetime from which nothing could escape, black holes were thought to be entirely dark, static objects. This perception changed dramatically in 1974 when Stephen Hawking proposed that black holes are not completely black but instead emit radiation due to quantum effects near their event horizons. Hawking's ideas gave rise to a theory that bridges the behaviour of the two major frameworks governing our universe: Quantum Field Theory and General Relativity (more on these shortly).

In this paper, I will be investigating this phenomenon, from what causes it to the consequences it has had on our understanding of black holes and physics altogether. I am presenting my understanding of the phenomenon based on my research and will try to convey my understanding through conceptual explanations rather than the advanced mathematics that is often used to describe such phenomena.

The Theories that Describe our Universe

Modern understanding of our universe tells us that it can be modelled by two theories: The Theory of General Relativity and Quantum Field Theory. These theories describe the events in our universe at their respective scales. The "holy grail" of theoretical physics is to devise a universal theory: one that unites quantum field theory with general relativity and incorporates all the fundamental constants of nature; a 'theory of everything'.

General relativity:

The Theory of General Relativity focuses on physics at a large scale, describing the behaviour of planets and large masses in the universe. It can be thought of as describing the fabric of the universe as a surface, spacetime that can be bent by large masses, causing gravitation. But in reality, spacetime is a four-dimensional entity, three dimensions of space, and one dimension of time. Gravitation, in this framework, is not a force, but the result of the curvature of spacetime on the motion of bodies; this curvature is caused by large masses like planets. It can be visualised by imagining what would happen if you placed a heavy ball on a trampoline; the surface of the trampoline, representing spacetime, would sink and bend as a result of the ball, representing a planet.

The idea of spacetime curvature paves the way for another aspect of general relativity that is extremely important in the phenomenon of Hawking Radiation. This is the idea that measurements of space and time are not absolute. For all observers, the laws of physics never change, but their measurements of space and time do. This is called ‘Gravitational Time Dilation’. What these phenomena describe is how space and time can be measured differently depending on the observer. For example, time appears to move more slowly for an observer within a strong gravitational field than it does for one floating freely in space.

Quantum Field Theory:

Quantum Field Theory focuses on events and systems on a much smaller, sub-atomic level. One of the core elements of Quantum Field Theory is quantum mechanics, which states that, unlike normal objects, which have a definite position in space, according to quantum mechanics, particles do not have definite positions; quantum particles exist in a superposition of possible states. Their presence is distributed across space in all of their possible positions, each with varying levels of possibility of finding that particle in a particular location. This can be summarised by stating that quantum mechanics uses the wave function, a mathematical object that encodes the probability amplitudes for all possible positions of that particle, to describe particles, rather than bodies with definite positions.

There is another detail in Quantum Field Theory that is important to the understanding of Hawking Radiation. This is how it describes vacuums, and what we think of as ‘empty space’. In quantum field theory, Vacuum Fluctuations fill this ‘empty’ space. A vacuum fluctuation is where a particle-antiparticle pair comes into existence and then annihilates extremely soon after coming into existence. These particles are referred to as virtual particles. And when we revisit quantum mechanics and describe these particles as waves of probability, we find ourselves with a vacuum where a wave comes into existence at the same instant as one that is effectively the opposite. It is important to note that virtual particles are not physically real particles or waves, just mathematical tools which we use to represent fluctuations in the quantum field.

The Popularised Simplification

The simplified explanation for Hawking radiation that has become the most popularly used is, in reality, quite misleading. This explanation defines Hawking radiation as a result of a pair of virtual particles coming into existence right next to the event horizon of a black hole. The event horizon is the point at which the gravitational pull of the black hole is so strong that nothing, not even light, can escape. When these particles come into existence, before they can annihilate each other, one falls beyond the event horizon and falls into the black hole. This means that its pair is not annihilated and escapes as a real particle; this escaped particle is the radiation that we measure. This simplification became widespread as a result of Hawking's own use of it in his 1988 book 'A Brief History of Time'. And in this context, it served its purpose; an easy-to-visualise and understand analogy of the phenomenon, which highlights some of the main characteristics of the more detailed theory.

However, this explanation is fundamentally flawed in a few ways. One of which is that this simplification leads us to believe that the radiation is released at the event horizon, when in reality it does not happen due to the event horizon, but due to the curvature of spacetime around the black hole. This explanation also implies that the escaped virtual particle is released as 'real' when in reality, virtual particles cannot be 'real' as such and are really just calculator tools used to describe quantum fluctuations.

The Quantum Vacuum Explanation

Now I will now try to break down what makes up the scientifically accurate version of Hawking's theory and explore the flaws of his popularised simplification.

Virtual and Real Particles:

We have already discussed that virtual particles are not physical objects that can be measured directly. They are fluctuations in the quantum field, mathematical tools that can be used to describe how real particles interact. They exist for an incredibly short period of time, meaning they cannot be directly observed.

Virtual particles are often explained in terms of Heisenberg's uncertainty principle, which gives intuition for how short-lived fluctuations can occur. One way to express this principle is in terms of energy and time:

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

Where ΔE is the uncertainty of energy in a system, Δt is the uncertainty in the time interval in which the system is observed or measured, and $\hbar$ is the reduced Planck's constant. What this principle tells us is that the product of the uncertainties in energy and time in a system is greater than or equal to half the reduced Planck's constant. In other words, a system that exists for a short time can not have an accurately defined energy, so to accurately define the energy, the system has to be observable for a longer period of time. Therefore, we cannot accurately define the energy of a system or particle that exists for a minimal period of time, meaning that a system's energy can fluctuate for a small period of time without violating the law of conservation of energy.

The final piece in the puzzle that is virtual particles is the 'Zero Point Energy'. The other form of Heisenberg's uncertainty principle will help explain:

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Where Δx is the uncertainty of the position of a particle, Δp is the uncertainty in the momentum of the particle, and $\frac{\hbar}{2}$ is half the reduced Planck's constant. This principle leads to a familiar conclusion: we cannot accurately measure or define the position and momentum of a particle simultaneously. Therefore, the quantum field can never have exactly zero energy, for if this were true, it would have to have zero kinetic energy, meaning the momentum of the modes of the quantum field is also zero (a quantum field is composed of modes, each of which acts like a harmonic oscillator). But if the momentum of all the modes were zero, the uncertainty in position would also be zero when multiplied, therefore defying the uncertainty principle. As a result of this, it is clear that there must be a minimum possible energy a quantum system can have that can exist in a vacuum, but not defy the uncertainty principle. This is known as the 'Zero-Point Energy' and is often referred to as "The Energy of the Vacuum".

And now, finally, we can dive into the manifestation of virtual particles. Virtual particles are simply mathematical tools used to represent quantum field fluctuations. I will now explore how the principles that we just covered explain these fluctuations. In quantum field theory, each type of particle exists due to oscillations of the modes in its corresponding field. These fields exist everywhere, even in a vacuum. And, as we just explored, the modes that make up these fields can never have zero energy. Because of the zero-point energy, each mode in the quantum field has a small amount of energy, leading to a constant "jitter" of the modes. In addition, the time-energy uncertainty we looked at shows us that energy fluctuations can happen in the quantum field as long as they exist only for a short period of time; the less energy, the longer they can exist and vice versa. These uncertainty principles mean that no mode in the quantum field is ever perfectly still; it is constantly exhibiting small fluctuations in position and momentum. And because a mode's energy is dependent on its position and momentum, fluctuations in these quantities lead to corresponding fluctuations in energy, only as allowed by the energy-time uncertainty. These fluctuations of energy are what we express mathematically as virtual particles; short-lived excitations of the quantum field that can only exist for a time short enough to obey the energy-time uncertainty principle.

Quantum Fields in Curved Spacetime:

We have already discussed in plenty of detail what a vacuum looks like in flat spacetime. An ocean of virtual particles, effectively coming in and out of existence as a result of the uncertainty principles and the zero-point energy.

But this all changes dramatically when we are near a black hole, where an object of unimaginable density and mass curves spacetime to a point where not even light can escape. It is the behavioural changes in quantum fields when in such curved spacetime that are the key to unlocking the workings of Hawking Radiation.

In flat spacetime, where quantum field theory is nearly always modelled, the concept of a particle is well defined. It is a positive frequency mode in the quantum field, which relies on a consistent definition of time because frequency is 'oscillations per unit time'. And this is simple in flat space-time because there is a 'Global Time

Coordinate', meaning time flows uniformly and consistently for all observers, allowing for a unique definition of a positive frequency.

But when in curved spacetime, due to General Relativity, we no longer have a global time coordinate because the geometry of the spacetime does not have the symmetry of flat spacetime. Time can only be defined locally, and these local definitions in close regions will not agree on a time coordinate. This means that the positive and negative frequency modes become observer-dependent. These differing definitions of 'time' mean that in curved spacetime, different observers cannot agree on the frequencies of field modes; where one observer may see positive frequencies, so real particles; another might see a mixture of positive and negative modes, so virtual particles or a vacuum.

Observer-dependence:

We have now laid down almost all of the necessary foundation to construct the final explanation of the actual mechanisms of Hawking Radiation. To start, we are going to picture three observers: one 'Far', one 'Stationary', and one 'Falling'. The Far observer is an infinite distance away from our black hole and is stationary in perfectly flat spacetime; the Stationary observer is a set distance outside of the black hole, accelerating uniformly to stay stationary and not fall in. While our Falling observer is freely falling into the black hole due to its gravity. Hawking Radiation stems from the different perceptions that these observers have of the vacuum around the black hole.

Falling Observer:

Our Falling observer will feel nothing unusual. As they fall into the black hole, they will observe a perfectly normal vacuum, full of only virtual particles. This is because the observer is not resisting gravity and is therefore in a local frame of free fall, a frame with no acceleration relative to the curved spacetime around them. Although the black hole strongly curves the overall spacetime, any small region surrounding our freely falling observer behaves as if it were flat, gravity-free space. The reason for this lies in Einstein's Equivalence Principle, which states that within a small region of spacetime, the effects of gravity are locally indistinguishable from those of acceleration. In other words, for our freely falling observer, being pulled toward the black hole feels exactly the same as floating in empty space far from any gravitational field. To them, the laws of physics appear completely normal, and no radiation is detected.

Stationary Observer:

The Stationary observer is a set distance away from the event horizon, but still close enough to be strongly affected by the curved spacetime. This means they must be accelerating at a constant rate to remain stationary and not fall into the black hole. As a result of Einstein's Equivalence Principle, this observer's acceleration to stay stationary is indistinguishable from uniform acceleration in flat spacetime. Meaning they experience what's called the Unruh Effect, which states that any observer accelerating uniformly through a vacuum will perceive a thermal bath and the temperature of this bath (Unruh Temperature) is given as:

$$T = \frac{\hbar a}{2\pi c k_B}$$

Where T is the temperature, $\hbar$ is the reduced Planck constant, a is the observer's acceleration, c is the speed of light, and k_B is the Boltzmann constant. This effect happens as a result of the observer's different notions of time and energy, so what a free-falling observer would perceive as vacuum modes, an accelerating observer perceives as excited modes. The closer this observer is to the event horizon, the more they have to accelerate, so the hotter the thermal bath.

Far Observer:

And finally, the Far observer. This observer is stationary at an infinite distance away from the black hole. But, unlike the stationary observer, they are in flat spacetime, so they do not need to accelerate to stay stationary. This observer detects what we define as Hawking Radiation, a steady stream of real particles emitted by the black hole. Unlike the stationary observer who perceives radiation as a result of their acceleration, the far observer perceives the radiation as actual outgoing quanta, carrying energy away from the black hole. This is because of the observer's different time coordinates, which are the result of the extreme spacetime curvature near the black hole. The far observer can describe the quantum field with the Schwarzschild Time, the time coordinate for an observer in perfectly flat spacetime. And the stationary observer describes time with a different set of coordinates, meaning their definition of positive frequency differs, and so does their definition of a real particle.

For the far observer in flat spacetime, positive frequency modes correspond to waves oscillating forward in time. However, in curved spacetime, the distortion of time near the event horizon mixes positive and negative frequency modes. Mathematically, the field modes that were just virtual particles to the freely falling observer become a mixture of virtual and real excitations when described using the far observer's time coordinate. To the far observer, this mixing appears as a continuous flux of outgoing quanta, real particles that carry energy away from the black hole. This radiation has a temperature proportional to the black hole's surface gravity, known as the Hawking Temperature. The key point is that this radiation does not arise from particles being created at the event horizon, but from the mismatch in how different observers define the vacuum due to the curvature of spacetime.

Observer-dependence Summary:

The picture that emerges from these three perspectives reveals the true origin of Hawking radiation. For the freely falling observer, the vacuum appears completely normal, full only of unmeasurable virtual fluctuations. For the stationary observer, acceleration through curved spacetime causes them to perceive a thermal bath through the Unruh effect. And for the far observer, the same vacuum modes appear as real, outgoing radiation because the curvature of spacetime alters how energy and time are defined across different regions.

In essence, Hawking radiation arises because "vacuum" is not an absolute concept. What one observer perceives as empty space, another perceives as filled with energy and particles. The curvature of spacetime around a black hole transforms the virtual excitations of the quantum vacuum into real particles as seen by distant observers. This observer-dependence, combined with the quantum uncertainty that allows fluctuations to exist at all, forms the bridge between general relativity and quantum field theory that gives rise to Hawking radiation.

The Consequences of Hawking Radiation:

As a result of Hawking Radiation, we had to completely reimagine how we thought of black holes, leading to many breakthroughs in our understanding, but also causing us to ask questions that had never previously been asked. Many of these questions remain unanswered. Now I will dive into the main effects that this phenomenon had on our understanding of black holes.

Hawking Temperature and Bridging the Theories:

Now that we have concluded that black holes are radiating, we can deduce that they must have a temperature because the radiation has energy. One of the most significant results of Hawking's findings is the equation he derived for the temperature of a black hole. This equation is given as:

$$T = \frac{\hbar c^3}{8\pi G M k_B}$$

Where T is the temperature. $\hbar$ is the reduced Planck constant. c is the speed of light. G is the gravitational constant. M is the mass of the black hole. And k_B is the Boltzmann constant. It is noticeable how similar this is to the previously looked at equation for Unruh temperature, further highlighting their connection. What this equation tells us, firstly, is that the temperature of a black hole is inversely proportional to its mass, so smaller black holes are hotter and therefore radiate more.

Most remarkably, this equation profoundly unifies constants that are the pillars of completely separate areas of physics. It is one of the only equations in science to unify each of these constants and their areas:

- The reduced Planck constant ($\hbar$) - quantum mechanics
- The speed of light (c) - special relativity
- The gravitational constant (G) - general relativity
- The Boltzmann constant (k_B) - thermodynamics

This equation is a window into a unified theory of everything. In this equation, Hawking demonstrates that the behaviour of black holes is not purely a gravitational phenomenon, as described by general relativity, nor purely a quantum phenomenon, as described by quantum field theory, but a combination of both. The temperature arises because quantum effects in curved spacetime allow energy to be radiated away, showing that spacetime itself has quantum properties.

Hawking's derivation thus bridges the macroscopic and microscopic realms. It connects the curvature of spacetime, the uncertainty of quantum fields, and the statistical behaviour of energy, suggesting that at a fundamental level, the universe operates according to principles that unify quantum mechanics, gravity, and thermodynamics. In this sense, the Hawking Temperature is more than a property of black holes; it is a concrete step toward the ultimate goal of physics. A single, unified theory of quantum field theory and general relativity.

The Fate of a Black Hole:

As demonstrated, black holes radiate, meaning that they release energy into the surroundings. Because of Einstein's incredibly famous mass-energy equivalence:

$$E = mc^2$$

We know that mass and energy are equivalent. So, if a black hole is emitting radiation, and therefore energy, it must be losing mass. And here we find another of the revelations that emerged from Hawking’s findings: black holes must eventually die. The equation tells us that they do this at an exponential rate. As the black hole’s mass decreases, according to the equation, its temperature rises. And a higher temperature means a higher rate of emission of radiation. This feedback loop causes the black hole's rate of evaporation to increase as the black hole shrinks, suggesting that black holes are all slowly evaporating entities. Eventually, if nothing stops the process, the black hole will evaporate completely in a final burst of high-energy radiation. This idea transforms our view of black holes from perfectly black, inescapable regions of spacetime, into dynamic, thermodynamic systems with entropy and temperature.

The Information Paradox:

Something else that we encounter when investigating Hawking Radiation is a paradox of which we are yet to answer. It relies on one simple question. If black holes eventually die, what happens to what fell in? This question is the origin of the information paradox.

To understand the black hole information paradox, it is important to first be aware of the concept of quantum information. Quantum information is defined as the data encoded in a quantum system that describes its quantum state, including the probabilities and correlations that define every particle’s properties. The key idea though is that according to quantum mechanics, quantum information can never be destroyed, only transformed, so must always be considered. To visualise this, picture a book, full of writing; information. If you decided to burn this book, obviously, the information would no longer be legible. But, in theory, if you collected all of the ash, all of the fumes from the fire, everything that the book became when it was burned, you would be able to reconstruct the information. And this is the key to quantum information, the laws of physics dictate that it can never be completely destroyed, only transformed.

Every time something falls into a black hole, so does the information about its energy and matter. But because of Hawking Radiation, we know that the black hole will eventually die out, so what happens to the information inside it? The obvious answer is that the information was released in the Hawking Radiation, but according to Hawking's original calculation, the radiation emitted is purely thermal, carrying no detailed information about the black hole's contents. This suggests that information could be lost forever, violating a fundamental principle of physics.

This contradiction is known as the information paradox. It highlights the clear disagreement between quantum field theory and general relativity under our current understanding. On the one hand, general relativity predicts smooth, continuous spacetime and allows black holes to evaporate without any trace. On the other hand, quantum mechanics insists that the evolution of quantum states must be unitary, meaning information cannot be destroyed.

Conclusion

Hawking Radiation is one of the most profound theoretical discoveries in physics. It is arguably as close as we have ever come to a unified theory of quantum field theory and general relativity and acts as both a signal that a unified theory is possible, and as a reminder of how deeply disconnected they currently are.

To summarise the process, Hawking radiation relies on how the extreme curvature around a black hole affects different observers' definitions of time, and how this means that they do not agree on what particles are virtual and real. So observers who are not affected by the curvature, like us on earth, see a thermal bath of radiation being emitted by the black hole.

Ultimately, Hawking radiation is not just a property of black holes, it is a window into the unification of physics. By revealing that quantum effects can alter the structure of spacetime itself, it brings us closer to the long-sought theory of quantum gravity, one that will reconcile the large with the small and complete our picture of the universe.