Odporna wycena instrumentów pochodnych wypłacających zrealizowaną zmienność

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5 kwietnia 2025

$$dS_t = (r - q)S_t dt + \sigma_t(t, \omega)S_t dW_t \tag{1}$$

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zrealizowana wariancja

$$\int_{T}^{T'} \sigma_t^2 dt \tag{2}$$

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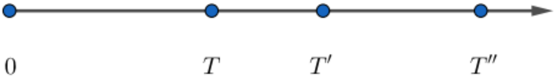
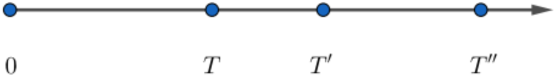
$$\int_T^{T'} \sigma_t^2 dt \tag{2}$$

$F_t = S_t e^{(r-q)(T''-t)}$ - kurs kontraktu forward zapadającego w T'', w chwili t

$$dF_t = \sigma_t S_t dWt \tag{3}$$



Zakładamy istnienie doskonale płynnego rynku opcji na kurs kontraktu forward zapadającego w chwili T''

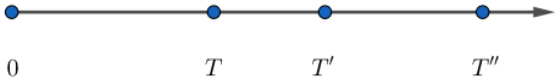


Co się stanie jeśli zastosujemy delta hedging błędnie założywszy model Blacka?

σ_B - stała zmienność w modelu Blacka

$f(F_{T'})$ - wypłata instrumentu w chwili T'

$V(F_t, t, \sigma_B)$ - wartość instrumentu wyliczona według modelu Blacka w chwili t



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W chwili T kupujemy instrument o wypłacie $f(F_{T'})$ w chwili T' oraz stosujemy delta hedging, zakładając stałą zmienność σ_B

Zastosujmy lemat Ito dla funkcji

$$g(F_t, t) = V(F_t, t, \sigma_B)e^{r(T' - t)} \tag{4}$$

Zastosujmy lemat Ito dla funkcji

$$g(F_t, t) = V(F_t, t, \sigma_B)e^{r(T'-t)} \tag{4}$$

$$\begin{aligned}
\frac{\partial g}{\partial F_t} &= e^{r(T'-t)} \frac{\partial V}{\partial F_t}(F_t, t, \sigma_B) \\
\frac{\partial^2 g}{\partial F_t^2} &= e^{r(T'-t)} \frac{\partial^2 V}{\partial F_t^2}(F_t, t, \sigma_B) \\
\frac{\partial g}{\partial t} &= e^{r(T'-t)} \frac{\partial V}{\partial t}(F_t, t, \sigma_B) - r \cdot e^{r(T'-t)} V(F_t, t, \sigma_B)
\end{aligned}$$

$$\begin{aligned}
d\, g(F_t, t) &= \left\{ e^{r(T'-t)} \frac{\partial V}{\partial t}(F_t, t, \sigma_B) - r \cdot e^{r(T'-t)} V(F_t, t, \sigma_B) + \frac{F_t^2}{2} \sigma_t^2 e^{r(T'-t)} \frac{\partial^2 V}{\partial F_t^2}(F_t, t, \sigma_B) \right\} dt \\
&\quad + \sigma_t F_t e^{r(T'-t)} \frac{\partial V}{\partial F_t}(F_t, t, \sigma_B) dW_t \\
&= \left\{ e^{r(T'-t)} \frac{\partial V}{\partial t}(F_t, t, \sigma_B) - r \cdot e^{r(T'-t)} V(F_t, t, \sigma_B) + \frac{F_t^2}{2} \sigma_t^2 e^{r(T'-t)} \frac{\partial^2 V}{\partial F_t^2}(F_t, t, \sigma_B) \right\} dt \\
&\quad + e^{r(T'-t)} \frac{\partial V}{\partial F_t}(F_t, t, \sigma_B) dF_t
\end{aligned}$$

$$g(F_{T'}, T') = V(F_{T'}, T', \sigma_B) = e^{r(T'-T)} V(F_T, t, \sigma_B) + \int_T^{T'} e^{r(T'-t)} \frac{\partial V}{\partial F_t}(F_t, t, \sigma_B) dF_t$$

$$+ \int_T^{T'} e^{r(T'-t)} \left(-rV(F_t, t, \sigma_B) + \frac{\partial V}{\partial t}(F_t, t, \sigma_B) \right) + e^{r(T'-t)} \frac{F_t^2}{2} \sigma_t^2 \frac{\partial^2 V}{\partial F_t^2}(F_t, t, \sigma_B) dt$$

Black SDE dla procesu V

$$-rV(F_t, t, \sigma_B) + \frac{\partial V}{\partial t}(F_t, t, \sigma_B) = -\frac{F_t^2}{2} \sigma_B^2 \frac{\partial^2 V}{\partial F_t^2}(F_t, t, \sigma_B)$$

$$V(F_{T'}, T', \sigma_B) = f(F_{T'})$$

$$f(F_{T'}) = e^{r(T'-T)} V(F_T, t, \sigma_B) + \int_T^{T'} e^{r(T'-t)} \frac{\partial V}{\partial F_t}(F_t, t, \sigma_B) dF_t$$

$$+ \int_T^{T'} e^{r(T'-t)} \frac{F_t^2}{2} \frac{\partial^2 V}{\partial F_t^2}(F_t, t, \sigma_B)(\sigma_t^2 - \sigma_B^2) dt$$

$$f(F_{T'}) + \int\limits_{T}^{T'} e^{r(T'-t)} \frac{F_t^2}{2} \frac{\partial^2 V}{\partial F_t^2}(F_t, t, \sigma_B)(\sigma_B^2 - \sigma_t^2)dt$$

$$= e^{r(T'-T)} V(F_T, t, \sigma_B) + \int\limits_{T}^{T'} e^{r(T'-t)} \frac{\partial V}{\partial F_t}(F_t, t, \sigma_B) dF_t$$

Jeśli $\sigma_t \equiv \sigma_B$, otrzymujemy replikacje wypłaty $f(F_{T'})$

W przypadku, gdy $\sigma_t \not\equiv \sigma_B$, otrzymujemy jawny wzór na błąd replikacji

Możemy tak dobrać f oraz σ_B, aby zreplikować wypłatę zależną od zrealizowanej wariancji $\int\limits_{T}^{T'} h(F_t, t, \sigma_B)(\sigma_B^2 - \sigma_t^2)dt$

$$\int_{T}^{T'} e^{r(T'-t)} \frac{F_t^2}{2} \frac{\partial^2 V}{\partial F_t^2}(F_t, t, \sigma_B)(\sigma_B^2 - \sigma_t^2) dt$$

$$= e^{r(T'-T)} V(F_T, t, \sigma_B) - f(F_{T'}) + \int_{T}^{T'} e^{r(T'-t)} \frac{\partial V}{\partial F_t}(F_t, t, \sigma_B) dF_t$$

Załóżmy zerową zmienność w modelu Blacka ($\sigma_B = 0$)

$V(F_t, t, 0) = e^{r(t-T')} f(F_t)$

$\frac{\partial V}{\partial F_t}(F_t, t, 0) = e^{r(t-T')} f'(F_t)$

$\frac{\partial^2 V}{\partial F_t^2}(F_t, t, 0) = e^{r(t-T')} f''(F_t)$

$$\int_T^{T'} e^{r(T'-t)} \frac{F_t^2}{2} \frac{\partial^2 V}{\partial F_t^2}(F_t, t, \sigma_B)(\sigma_B^2 - \sigma_t^2) dt$$

$$= e^{r(T'-T)} V(F_T, t, \sigma_B) - f(F_{T'}) + \int_T^{T'} e^{r(T'-t)} \frac{\partial V}{\partial F_t}(F_t, t, \sigma_B) dF_t$$

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$$\int_T^{T'} \frac{F_t^2}{2} f''(F_t) \sigma_t^2 dt = f(F_{T'}) - f(F_T) - \int_T^{T'} f'(F_t) dF_t$$

$$\int_{T}^{T'} \frac{F_t^2}{2} f''(F_t)\sigma_t^2 dt = f(F_{T'}) - f(F_T) - \int_{T}^{T'} f'(F_t) dF_t$$

$f(x) = 2\left(\ln\left(\frac{\kappa}{x}\right) + \frac{x}{\kappa} - 1\right)$

$f'(x) = 2\left(\frac{1}{\kappa} - \frac{1}{x}\right)$

$f''(x) = \frac{2}{x^2}$

$$\int_{T}^{T'} \frac{F_t^2}{2} f''(F_t)\sigma_t^2 dt = f(F_{T'}) - f(F_T) - \int_{T}^{T'} f'(F_t) dF_t$$

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Dekompozycja Carra-Madana

$$f(x) = f(\kappa) + (x-a)f'(\kappa) + \int_{-\infty}^{\kappa} (u-x)^+ f''(u) du + \int_{\kappa}^{\infty} (x-u)^+ f''(u) du$$

$$f(x) = \int_{-\infty}^{\kappa} (u-x)^+ \frac{2}{u^2} du + \int_{\kappa}^{\infty} (x-u)^+ \frac{2}{u^2} du$$

$$\int_{T}^{T'} \sigma_t^2 dt = \int_{-\infty}^{\kappa} (u - F_{T'})^+ \frac{2}{u^2} du + \int_{\kappa}^{\infty} (F_{T'} - u)^+ \frac{2}{u^2} du$$

$$- \int_{-\infty}^{\kappa} (u - F_T)^+ \frac{2}{u^2} du - \int_{\kappa}^{\infty} (F_T - u)^+ \frac{2}{u^2} du - \int_{T}^{T'} 2 \left(\frac{1}{\kappa} - \frac{1}{F_t} \right) dF_t$$

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π_0 - operator wyceny w chwili 0

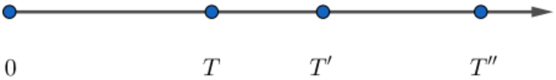
$$\pi_0 \left(\left(F_t - K \right)^+ \right) = C_0(t, K)$$

$$\pi_0 \left(\left(K - F_t \right)^+ \right) = P_0(t, K)$$

$$\pi_0 \left(\int_T^{T'} \sigma_t^2 dt \right) = \int_{-\infty}^{\kappa} P_0(T', u) \frac{2}{u^2} du + \int_{\kappa}^{\infty} C_0(T', u) \frac{2}{u^2} du$$

$$- e^{-r(T'-T)} \left(\int_{-\infty}^{\kappa} P_0(T, u) \frac{2}{u^2} du + \int_{\kappa}^{\infty} C_0(T, u) \frac{2}{u^2} du \right)$$

$$\int_T^{T'} \sigma_t^2 dt = \int_{-\infty}^{\kappa} (u - F_{T'})^+ \frac{2}{u^2} du + \int_{\kappa}^{\infty} (F_{T'} - u)^+ \frac{2}{u^2} du$$
$$- \int_{-\infty}^{\kappa} (u - F_T)^+ \frac{2}{u^2} du - \int_{\kappa}^{\infty} (F_T - u)^+ \frac{2}{u^2} du - \int_T^{T'} 2 \left(\frac{1}{\kappa} - \frac{1}{F_t} \right) dF_t$$



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Kalibracja lokalnej zmienności

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Założywszy, że $\sigma_t(\omega, t) = g(F_t, t)$, jak znaleźć funkcję g obserwując ceny opcji?

$$\int_{T}^{T'} \frac{F_t^2}{2} f''(F_t)\sigma_t^2 dt = f(F_{T'}) - f(F_T) - \int_{T}^{T'} f'(F_t) dF_t$$

$f(x) = 2\left(\ln\left(\frac{\kappa}{\underline{x}}\right) + x\left(\frac{1}{\kappa} - \frac{1}{\underline{x}}\right)\right)$

$$\underline{x} = \max\left(\kappa - \Delta\kappa, \min\left(x, \kappa + \Delta\kappa\right)\right)$$

$$f'(x) = 2\left(\frac{1}{\kappa} - \frac{1}{\underline{x}}\right)$$

$$f''(x) = \frac{2}{x^2}\mathbb{1}_{\{x \in (\kappa - \Delta\kappa, \kappa + \Delta\kappa)\}}$$

Dekompozycja Carra-Madana

$$f(x) = f(\kappa) + (x - a)f'(\kappa) + \int_{-\infty}^{\kappa} (u - x)^+ f''(u) du + \int_{\kappa}^{\infty} (x - u)^+ f''(u) du$$

$$f(x) = \int_{-\infty}^{\kappa} (u - x)^+ \frac{2}{u^2} du + \int_{\kappa}^{\infty} (x - u)^+ \frac{2}{u^2} du$$

$$\pi_0\left(\int\limits_T^{T'} \mathbb{1}_{\{F_t\in(\kappa-\Delta\kappa,\kappa+\Delta\kappa)\}}\sigma_t^2 dt\right) = \int\limits_{\kappa-\Delta\kappa}^{\kappa} \frac{2}{K^2}P_0(K,T')dK + \int\limits_{\kappa}^{\kappa+\Delta\kappa} \frac{2}{K^2}C_0(K,T')dK$$

$$-e^{-r\left(T'-T\right)}\left(\int\limits_{\kappa-\Delta\kappa}^{\kappa} \frac{2}{K^2}P_0(K,T)dK + \int\limits_{\kappa}^{\kappa+\Delta\kappa} \frac{2}{K^2}C_0(K,T)dK\right)$$

$$\pi_0\left(\int_T^{T'} \mathbb{1}_{\{F_t\in(\kappa-\Delta\kappa,\kappa+\Delta\kappa)\}}\sigma_t^2 dt\right) = \int_{\kappa-\Delta\kappa}^{\kappa} \frac{2}{K^2}P_0(K,T')dK + \int_{\kappa}^{\kappa+\Delta\kappa}\frac{2}{K^2}C_0(K,T')dK$$
$$-e^{-r\left(T'-T\right)}\left(\int_{\kappa-\Delta\kappa}^{\kappa} \frac{2}{K^2}P_0(K,T)dK + \int_{\kappa}^{\kappa+\Delta\kappa}\frac{2}{K^2}C_0(K,T)dK\right)$$

$$\lim_{\Delta\kappa\to 0}\pi_0\left(\frac{1}{2\Delta\kappa}\int_T^{T'} \mathbb{1}_{\{F_t\in(\kappa-\Delta\kappa,\kappa+\Delta\kappa)\}}\sigma_t^2 dt\right) = \frac{1}{\kappa^2}\Big(P_0(\kappa,T')+C_0(\kappa,T')$$
$$-e^{-r(T'-T)}\left(P_0(\kappa,T)+C_0(\kappa,T)\right)\Big)$$

$$\pi_0\left(\int_T^{T'}\delta(F_t-\kappa)\sigma_t^2 dt\right) = \frac{1}{\kappa^2}\left(V_0(\kappa,T') - e^{-r(T'-T)}V_0(\kappa,T)\right) \tag{5}$$

$$\pi_0\left(\delta(F_T-\kappa)\sigma_T^2\right)=\lim_{T'\to T}\pi_0\left(\frac{1}{T'-T}\int_T^{T'}\delta(F_t-\kappa)\sigma_t^2dt\right)=\lim_{T'\to T}\pi_0\left(\frac{1}{\kappa^2}\frac{1}{T'-T}e^{-r(T'-T)}\right.$$

$$\left.\left(e^{r(T'-T)}V_0(\kappa,T')-e^{-r(T-T)}V_0(\kappa,T)\right)\right)=\frac{1}{\kappa^2}\left(\frac{\partial V_0}{\partial T}(\kappa,T)+rV_0(\kappa,T)\right)$$

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$$\left(e^{r(T'-T)}V_0(\kappa,T')-e^{-r(T-T)}V_0(\kappa,T)\right)\Bigg)=\frac{1}{\kappa^2}\left(\frac{\partial V_0}{\partial T}(\kappa,T)+rV_0(\kappa,T)\right)$$

Założywszy lokalną postać zmienności $\sigma(\omega,t)=\sigma(F_t,t)$:

Związek między deltą Diraca i warunkową wartością oczekiwaną

$$e^{rT}\pi_0\left(\delta(F_T-\kappa)\sigma_T^2\right)=\mathbb{E}^{\mathbb{Q}}\left[\delta(F_T-\kappa)\sigma_T^2\right]=\mathbb{E}^{\mathbb{Q}}\left[\mathbb{E}^{\mathbb{Q}}\left[\delta(F_T-\kappa)\sigma_T^2|F_T\right]\right]=$$

$$\mathbb{E}^{\mathbb{Q}}\left[\delta(F_T-\kappa)\mathbb{E}^{\mathbb{Q}}\left[\sigma_T^2|F_T\right]\right]=\int_0^\infty\delta(x-\kappa)\cdot(\sigma_T^2|F_T=x)\cdot\varphi(x)dx=(\sigma_T^2|F_T=\kappa)\cdot\varphi(\kappa)$$

$$(\sigma_T^2|F_T=\kappa)\cdot\varphi(\kappa)=e^{rT}\frac{1}{\kappa^2}\left(\frac{\partial V_0}{\partial T}(\kappa,T)+rV_0(\kappa,T)\right)\tag{6}$$

$$(\sigma_T^2|F_T=\kappa)\cdot\varphi(\kappa)=e^{rT}\frac{1}{\kappa^2}\left(\frac{\partial V_0}{\partial T}(\kappa,T)+rV_0(\kappa,T)\right) \tag{7}$$

Gęstość jako pochodna ceny opcji

$$\varphi(\kappa)=\int_0^\infty \delta(x-\kappa)\cdot\varphi(x)dx=\mathbb{E}^{\mathbb{Q}}\left[\delta(F_T-\kappa)\right]=e^{rT}\cdot e^{-rT}\times$$

$$\times\,\mathbb{E}^{\mathbb{Q}}\left[\lim_{\Delta\kappa\to 0}\frac{1}{2}\frac{1}{\kappa^2}\left(\left|F_T-(\kappa-\Delta\kappa)\right|-2\left|F_T-\kappa\right|+\left|F_T-(\kappa+\Delta\kappa)\right|\right)\right]=e^{rT}\frac{1}{2}\frac{\partial^2V_0}{\partial K^2}(\kappa,T)$$

Kalibracja lokalnej zmienności:

$$(\sigma_T^2|F_T=\kappa)=\frac{\frac{\partial V_0}{\partial T}(\kappa,T)+rV_0(\kappa,T)}{\kappa^2\frac{1}{2}\frac{\partial V_0^2}{K^2}(\kappa,T)} \tag{8}$$

Kalibracja lokalnej zmienności:

$$(\sigma_T^2|F_T=\kappa)=\frac{\frac{\partial V_0}{\partial T}(\kappa,T)+rV_0(\kappa,T)}{\kappa^2\frac{1}{2}\frac{\partial V_0^2}{K^2}(\kappa,T)} \tag{9}$$

Po zmianie na cenę spot (gdzie V_0 to cena stelaża na cenę spot):

$$(\sigma_T^2|S_T=\kappa)=\frac{\frac{\partial V_0}{\partial T}(\kappa,T)+(r-q)\kappa\frac{\partial V_0}{\partial K}V_0(\kappa,T)+qV_0(\kappa,T)}{\frac{1}{2}\kappa^2\frac{\partial V_0^2}{K^2}(\kappa,T)} \tag{10}$$

Wzór Dupire (1994):

$$(\sigma_T^2|S_T=\kappa)=\frac{\frac{\partial C_0}{\partial T}(\kappa,T)+(r-q)\kappa\frac{\partial C_0}{\partial K}C_0(\kappa,T)+qC_0(\kappa,T)}{\frac{1}{2}\kappa^2\frac{\partial C_0^2}{K^2}(\kappa,T)} \tag{11}$$

Powyższy wzór pozwala na kalibrację modelu lokalnej zmienności
$dS_t=(r-q)S_t dt+\sigma(S_t,t)S_t dW_t$
Obserwując ceny opcji (C lub V) możemy aproksymować pochodne po czasie i kursie wykonania.

Dziękuję za uwagę i zachęcam do zadawania pytań

Literatura

1. Carr Peter, Dilip Madan *Towards a theory of volatility trading. Option Pricing* (2001)
2. Dupiere B, *A Unified Theory of Volatility* , Paribas working paper (1996)

Dostęp do prezentacji

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Kontakt

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Dodatek: Przejście pomiędzy lokalną zmiennością dla ceny forward F_t i ceną spot S_t

$$F_t = e^{(r-q)(T''-t)} S_t \qquad \tilde{\kappa} := e^{(r-q)(T-T'')}\kappa \qquad \delta(F_T - K) = \delta(S_T - \tilde{\kappa})$$

$$V(\kappa, T) = |F_T - \kappa| = |e^{(r-q)(T''-T)} S_T - \kappa| = e^{(r-q)(T''-T)} |S_T - \tilde{\kappa}| = e^{(r-q)(T''-T)} \tilde{V}(\tilde{\kappa}, T)$$

$$\pi_0\left(\delta(S_T - \tilde{\kappa})\right) = \pi_0\left(\delta(F_T - K)\right) = \lim_{T' \to T} \frac{1}{T' - T}\left(V_0(\kappa, T') - e^{-r(T'-T)} V_0(\kappa, T)\right)$$

$$= \lim_{T' \to T} \frac{1}{T' - T} e^{-r(T'-T)}\left(e^{r(T'-T)+(r-q)(T''-T')} \tilde{V}_0(\tilde{\kappa}, T') - e^{r(T-T)+(r-q)(T''-T)} \tilde{V}_0(\tilde{\kappa}, T)\right)$$

$$= g'(T)$$

$$g(t) = e^{r(t-T)+(r-q)(T''-t)}\left(\tilde{V}_0(\tilde{K}(t), t)\right) \qquad \tilde{K}(t) = K e^{(r-q)(t-T'')} \tag{12}$$

$$\pi_0\left(\delta(S_T - \tilde{\kappa})\right) = e^{(r-q)(T''-T)}\left(\frac{\partial \tilde{V}_0}{T}(\tilde{K}, T) + (r-q)\frac{\partial \tilde{V}_0}{K}(\tilde{K}, T)\tilde{\kappa} + q\tilde{V}_0(\tilde{K}, T)\right) \tag{13}$$

$$\kappa^2 \frac{\partial^2 V_0}{K^2}(\kappa, T) = \kappa^2 \frac{\partial^2}{K^2}\left(e^{(r-q)(T''-T)} \tilde{V}_0(\tilde{\kappa}, T)\right) = e^{(r-q)(T''-T)} \kappa^2 e^{2(r-q)(T-T'')} \frac{\partial^2 \tilde{V}_0}{K^2}(\tilde{\kappa}, T)$$

$$= e^{(r-q)(T''-T)} \tilde{\kappa}^2 \frac{\partial^2 \tilde{V}_0}{K^2}(\tilde{\kappa}, T)$$

$$(\sigma_T^2 | S_T = \tilde{\kappa}) = \frac{\frac{\partial \tilde{V}_0}{T}(\tilde{K}, T) + (r-q)\frac{\partial \tilde{V}_0}{K}(\tilde{K}, T)\tilde{\kappa} + q\tilde{V}_0(\tilde{K}, T)}{\frac{1}{2}\tilde{\kappa}^2 \frac{\partial^2 \tilde{V}_0}{K^2}(\tilde{\kappa}, T)} \tag{14}$$

Dziękuję za uwagę!