Chapter 1

Glossary

This glossary uses the handy `acroynym` package to automatically maintain the glossary. It uses the package's `printonlyused` option to include only those acronyms explicitly referenced in the LaTeX source.

MOF Model Objective Function

NRM Natural Remanent Magnetization

TMI Total Magnetic Intensity

TMA Total Magnetic Anomaly

Chapter 2

Introduction

2.1 Forms of A Priori Information

2.1.1 The use of Koenigsberger Ratios to Correct Bore Hole Susceptibility Measurements

Bore holes provide physical property measurements at depth either by sending geophysical instruments down hole or by recovering a core and then measuring it subsequently in the lab. Bore holes can also provide qualitative rock unit information. Much work has been done on including physical property bore hole information([7] among others I'm sure). If we can convert the lithology information from bore holes into physical property information (using petrophysical measurements of reasonably similar rocks) we can use the information in the same fashion as a physical property bore hole logs.

In some contexts one physical property can be used as a proxy for others. In one case study, we have magnetic susceptibility measurements down hole but surface samples have been measured which reveal high magnetic remanence. The simple susceptibility measurement is drastically lower than the recovered effective susceptibility derived from the inversion of a magnetic survey over the area. This is due to the fact that the inversion is recovering effective susceptibility (induced magnetization plus Natural Remanent Magnetization (NRM) normalized by and assuming the direction of earth's field in the location). To describe the method we need to discuss the general derivation of the magnetostatic problem and then discuss the effects of NRM on measured data and the recovered inversion result.

The following derivation follows the one in [3]. We can derive the magnetostatic problem from Maxwell's equations. If we assume no free currents and no time varying electric field Maxwell's equations simplify to

$$\nabla \cdot \mathbf{B} = 0 \tag{2.1}$$

$$\nabla \times \mathbf{H} = 0 \tag{2.2}$$

$$\mathbf{B} = \mu \mathbf{H}, \tag{2.3}$$

where $\mathbf{B}$ is the magnetic flux density measured in Tesla (T), $\mathbf{H}$ is the magnetic field measured in amperes per meter (A/m), and μ is magnetic permeability which relates $\mathbf{B}$ to $\mathbf{H}$ in matter. We can rewrite μ to take the permeability of free space (μ_0) into account and state that

$$\mu = \mu_0(1+\kappa), \tag{2.4}$$

where μ_0 is the permeability of free space ($4\pi \times 10^{-7} \frac{Tm}{A}$), and κ is the magnetic susceptibility of a material. κ is dimensionless and describes the ability of a material to become magnetized under some field $\mathbf{H}$. The definition of κ in Equation 2.4 gives us a definition for induced magnetization

$$\mathbf{M}_I = \kappa \mathbf{H}. \tag{2.5}$$

Since there are no free currents and Equation 2.2 states that $\mathbf{H}$ has no curl, it can be written as the gradient of a potential field

$$\mathbf{H} = \nabla \phi. \tag{2.6}$$

Since, by Equation 2.1, we assume that there are no magnetic monopoles, we approximate ϕ in terms of a dipole moment $\mathbf{m}$. If we have a magnetic dipole with a moment of $\mathbf{m}$ at a location $\mathbf{r}_Q$, then the potential field ϕ as measured at some $\mathbf{r}_P$ is given by

$$\phi(r) = \frac{1}{4\pi}\mathbf{m}\cdot\nabla\left(\frac{1}{r}\right), \tag{2.7}$$

where

$$\mathbf{r} = \|\mathbf{r}_Q - \mathbf{r}_P\|_2. \tag{2.8}$$

We can generalize Equation 2.7 to a continuous form by replacing the discrete **m** with a continuous **M** and integrating

$$\phi(r) = \frac{1}{4\pi}\int_V \mathbf{M}\cdot\nabla\left(\frac{1}{r}\right)dv. \tag{2.9}$$

If we take the gradient of Equation 2.9 we find **B** the magnetic flux density,

$$\mathbf{B}(\mathbf{r}_P) = \frac{1}{4\pi}\int_V \mathbf{M}\cdot\nabla\nabla\left(\frac{1}{r}\right)dv. \tag{2.10}$$

In Equation 2.10 the dependency of **B** on $\mathbf{r}_P$ is due to the fact that r depends on $\mathbf{r}_P$ as in Equation 2.8. In most geophysical surveys, the full vector **B** is not collected, usually only its magnitude, or the Total Magnetic Intensity (TMI),

$$B_{TMI} = \|\mathbf{B}_0 + \mathbf{B}_A\|_2 \tag{2.11}$$

where $\mathbf{B}_0$ is the primary field (earth's field) and $\mathbf{B}_A$ is the anomalous local field due to magnetization in the ground. For the purposes of geophysical exploration we are only interested in $\mathbf{B}_A$. Since we are only interested in $\mathbf{B}_A$ a useful quantity is therefor the Total Magnetic Anomaly (TMA), defined as

$$B_{TMA} = \|\mathbf{B}_{Total} + \mathbf{B}_0\|_2 \tag{2.12}$$

TMA is difficult to measure directly but can be approximated assuming $\frac{\|\mathbf{B}_A\|}{\|\mathbf{B}_0\|} \ll 1$,

$$B_{TMA} \simeq \mathbf{B}_A\cdot\hat{\mathbf{B}}_0 \tag{2.13}$$

$$= \|\mathbf{B}_0 + \mathbf{B}_A\| - \|\mathbf{B}_0\| \tag{2.14}$$

We now have a formulation of **B** that depends on magnetization **M**, the vector field of magnetization in the ground and the quantities collected in geophysical magnetics surveys. We will now take a more specific look at magnetization and its effects both on $\mathbf{B}$ and B_{TMA}. If we assume no self-demagnetization (which is resonable for susceptabilities below 1^{-2} [4]) the inducing magnetic field is constant over the volume to be inverted and total magnetization can be characterized by the following

$$\mathbf{M} = \mathbf{M}_I + \mathbf{M}_{NRM} \tag{2.15}$$

$$\mathbf{M} = \kappa \mathbf{H} + \mathbf{M}_{NRM} \tag{2.16}$$

Here $\mathbf{M}$ is the total magnetization, $\mathbf{M}_I$ is the induced magnetization as in Equation 2.5 , κ is susceptibility as defined in Equation 2.4, $\mathbf{H}$ is the inducing field (in this case of the geomagnetic field) and $\mathbf{M}_{NRM}$ is the NRM. $\mathbf{M}_{NRM}$ is also characterized by what is called Koenigsberger ratio

$$Q = \frac{\mathbf{M}_{NRM}}{\kappa \mathbf{H}} = \frac{\text{remanent magnetization}}{\text{induced magnetization}} \tag{2.17}$$

In the case where NRM is negligible, the direction of $\mathbf{M}$ isthe same as $\mathbf{H}$, meaning that $\mathbf{B}_A$ and $\mathbf{B}_0$ are in the same direction and making the approximation in Equation 2.13 exact. The methods outlined in [5] and [6] assume that not only is there no self-demagnetization but also that there is no NRM. A slight generalization from assuming no NRM is that the anomalous magnetization is entirely in one direction [5]. The recovered quantity is effective susceptibility, or magnetization normalized by the earth's field,

$$\kappa_{eff} = \frac{||\mathbf{M}||}{||\mathbf{H}||}. \tag{2.18}$$

In the case that the magnitude of NRM is negligible, effective susceptibility and the true susceptibility are equal (i.e. $Q \ll 1$).

In the context of high remanent magnetization, the assumption that there is no NRM is by definition clearly false and NRM affects the inversion results. In the case that the NRM is in a similar direction as the earth's field, the measured field B_{TMA} will be higher than expected, given only susceptibility, and thus the recovered κ_{eff} will be higher than the true κ. Similarly, in the case that NRM is in a direction nearly anti-parallel to the earth's field, the measured field B_{TMA} will be lower than expected, given only susceptibility, and thus the recovered κ_{eff} will also be lower than the true κ.

Understanding the difference between κ and κ_{eff} is very important with respect to inserting magnetic petrophysical measurements into an inversion's Model Objective Function (MOF). It is an unfortunate truth that susceptibility is significantly easier to measure than NRM. As stated above in Equation 2.4 the permeability of an object is related to its susceptibility. In addition the inductance of a coil is proportional to the permeability inside and around it and thus is dependent on the susceptibility of the material. This change in inductance of a coil allows the precise measurement of susceptibility without contamination by NRM [2],[1]. The measurement of the inductance of a coil also allows the measurement of susceptibility of a rock without reorienting the sample and without shielding from ambient magnetic fields [2].

On the other hand, NRM as opposed to susceptibility is a vector quantity and thus the sample must be reoriented to measure it from different directions, even when only the magnitude of the NRM is required. In addition, unless the sample is very highly magnetized and stable, measurement will require shielding from ambient fields.

As can be seen from the above, it is not surprising that n the case of El Poma we have many susceptibility measurements including bore-holes and very few (only two within the areas of interest) measurements of NRM. However, if susceptibility is used to constrain an inversion in an area of strong NRM the inversion could be constrained to a value much higher or lower than the true effective susceptibility.

At first order, a potential correction given NRM measurements is to use the Koenigsberger ratio of a sample and assume that other samples will have a similar Koenigsberger ratio. It is recognized that this assumption will not be true. That said it is a closer approximation than would otherwise be possible without the sample. Once we have a Koenigsberger ratio and a susceptibility we can determine the magnetization of the sample using Equation 2.15 and Equation 2.17 in the form

$$\mathbf{M}_{eff} = \kappa\mathbf{H} + \|Q\kappa\mathbf{H}\|\hat{\mathbf{M}}_{NRM}, \tag{2.19}$$

where Q is the Koenigsberger ratio used and $\hat{\mathbf{M}}_{NRM}$ is the magnetization direction of the sample used. It is important to note that Equation 2.19 is a vector sum and the direction of $\mathbf{M}_{eff}$ will not be in the direction of either $\mathbf{H}$ or $\mathbf{M}_{NRM}$. It is also interesting to note that Equation 2.19 is more generally true if more samples have NRM measurements. If, instead of being a single measurement, we have a more detailed estimate of each sample's Koenigsberger ratio and magnetization direction, we can get a better estimate of $\mathbf{M}_{eff}$.

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