Geological and Petrophysical Information in Geophysical Inversion Problems

by

Daniel Bild-Enkin

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Abstract

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Glossary

This glossary uses the handy `acroynym` package to automatically maintain the glossary. It uses the package's `printonlyused` option to include only those acronyms explicitly referenced in the LaTeX source.

MOF Model Objective Function

GIF Geophysical Inversion Facility

FCM Fuzzy C-Means

Acknowledgments

Thank those people who helped you.

Don't forget your parents or loved ones.

You may wish to acknowledge your funding sources.

Chapter 1

Introduction

1.1 Research Motivation

In mineral exploration there are many forms of information that can be used to determine the location of an economic deposit. These can be divided broadly into geological and geophysical data. Geological data refers to the study of the rocks in a region through surface samples, bore holes, and an understanding of how rock units interrelate under the surface. Geophysical data refers to recovered measurements of some field that is related to the physical properties of the rocks that will aid in the understanding of the deposit. For exploration to be as effective as possible, we need to find ways of integrating the geological and geophysical information that produce exploration vectors to the target. One of the major tools in using geophysical data to create geologically significant interpretations is inversion.

The overarching goal of geophysical inversion is to recover distributions of physical properties in the ground to aid in mineral exploration. To be useful to this end the spacial distribution of the physical property (the geophysical model) needs to both fit the geophysical data and match existing geological interpretations.

Since geophysical inversions are by their nature non-unique (because of data uncertainty and there typically being many more model parameters than data), *a priori* information needs to be added to provide a model that matches the geology of a deposit. Much work has been done to create a mathematical framework to allow the inclusion of geological and petrophysical information into geophysical inversions (for example Li and Oldenburg (1996)). However, an area where more work must be done is the creation of tools to take the petrophysical and geological data in the forms that are generally provided and create usable constraints that can be applied to inversions.

The research in this thesis will attempt to do exactly that: provide new tools in an integrated framework that will allow the incorporation of non-trivial *a priori* information into geophysical inversions. The inclusion of *a priori* information in inversions is not novel. Many researchers before me have used the mathematical framework to add geological and petrophysical information to inversions (for example Lelievre 2009,Phillips 2001, Farquharson et al. 2008).

In addition, in Williams (2008) develops a software package to create constraints for inversions from a wide array of data types. What is lacking in the previous research is the link between the creation of inversion constraints with the processing of data and the running of inversions. By integrating the tools I create in this thesis into the framework of GIFtools and Model Builder, I make the incorporation of *a priori* information into inversions much more expedient, thus encouraging greater uptake by industry. In total GIFtools and Model Builder allow users do quality control on data, create constraints, and run inversions within the same software framework. The interface by which *a priori* information can be incorporated has been much updated from Williams (2008), and tools to incorporate new forms of data into inversions have been added.

1.2 Regularized Inversion

In the general case, geophysical inversion involves the solving of a system that is defined by some forward operator that maps from a given model to predicted data,

$$\mathbf{d} = \mathbb{F}[\mathbf{m}], \tag{1.1}$$

where $\mathbf{d} \in \mathbb{R}^N$ is the geophysical data (and N is the number of data collected), $\mathbf{m} \in \mathbb{R}^M$ is a discretization of m which is the model that describes the distribution of some physical property (M is the number cells in the earth model), and $\mathbb{F}$ is the forward operator that mediates between them. In the context of this thesis, $\mathbf{d}$ is either magnetic or gravity data measured with a ground or aerial survey, $\mathbf{m}$ is either a susceptibility or density model, and $\mathbb{F}$ is the magnetic or gravitational forward operator which has the convenient property of being linear. Since we are interested in recovering the model $\mathbf{m}$, we are interested in finding the inverse of $\mathbb{F}$,

$$\mathbf{m} = \mathbb{F}^{-1}\mathbf{d}. \tag{1.2}$$

Unfortunately the inversion of $\mathbb{F}$ is far from trivial as the problem is ill-posed. Firstly since there are usually more model parameters than data ($M > N$) there are an infinite number of possible distributions of the physical property that will predict the observed data. Secondly the system is unstable, that is, a small amount of error in the measurements can lead to large changes in the recovered model. To recover a model despite these difficulties, the problem is regularized by adding *a prioi* information in the form of a Model Objective Function (MOF) or regularization functional. Once the problem is regularized, it is solved by minimizing the objective function,

$$\phi(\mathbf{m}) = \phi_d + \beta\phi_m \tag{1.3}$$

where ϕ is the objective function , ϕ_d and ϕ_m are the data and model objective functions respectively, and β is a tradoff parameter that scales between them. ϕ_d is defined as a least squares (or L_2) formulation,

$$\phi_d = \|\mathbf{W}_d\Big(\mathbb{F}[\mathbf{m}] - \mathbf{d}^{obs}\Big)\|^2 \tag{1.4}$$

$$\mathbf{d}^{obs} = \mathbb{F}[\mathbf{m}^{true}] + \mathbf{e} \tag{1.5}$$

where $\mathbf{d}^{obs}$ is the observed data, that is the true data contaminated with noise $\mathbf{e}$, and $\mathbf{W}_d$ is the data weighting matrix with the diagonal entries equal to the reciprocal of each datum's standard deviation,

$$\mathbf{W}_d = \begin{bmatrix} \frac{1}{\sigma_1} & 0 & \cdots & 0 \\ 0 & \frac{1}{\sigma_2} & 0 & \vdots \\ \vdots & \ddots & & 0 \\ 0 & \cdots & 0 & \frac{1}{\sigma_N} \end{bmatrix}, \tag{1.6}$$

where each σ_i is that datum's assigned standard deviation. Meanwhile ϕ_m is can take many forms. In Li and Oldenburg (1996) it is defined in the continuous formulation as,

$$\begin{aligned} \phi_m = & \alpha_s \| \mathbf{w}_s \left[(\mathbf{m} - \mathbf{m}_{ref}) \right] \|^2 + \dots \\ & \alpha_x \| \mathbf{w}_x \mathbf{G}_x \left[(\mathbf{m} - \mathbf{m}_{ref}) \right] \|^2 + \dots \\ & \alpha_y \| \mathbf{w}_y \mathbf{G}_y \left[(\mathbf{m} - \mathbf{m}_{ref}) \right] \|^2 + \dots \\ & \alpha_z \| \mathbf{w}_z \mathbf{G}_z \left[(\mathbf{m} - \mathbf{m}_{ref}) \right] \|^2. \end{aligned} \tag{1.7}$$

The first term in Equation 1.7 promotes smallness, that is the model must be close in value to the reference model $\mathbf{m}_{ref}$ which is some first guess at the structure of the area being inverted. The next three terms promote smoothness by penalizing large derivatives in the model, determined by the discrete gradient operator $\mathbf{G}$ in each direction. There are many parameters in ϕ_m that the geophysicist can use to fine-tune the recovered model. α_s,α_x,α_y, and α_z, scale the contribution of smallness and smoothness in each spatial direction. α values can be used to broadly determine the length scales in each direction of a recovered model. The $\mathbf{w}_s$,$\mathbf{w}_x$,$\mathbf{w}_y$,$\mathbf{w}_z$ parameters allow finer control. They weight the model's smallness and smoothness variable across its extent allowing the user to specify certain regions as smoother than others or demanding that the recovered model more closely match the reference model in areas where the user is more certain of the reference model's validity.

In Equation 1.7 it is assumed the the norm of the regularization is L_2.

$$\|\mathbf{v}\|_2 = \left(\sum_{i=1}^{N} v_i^2 \right)^{\frac{1}{2}} \tag{1.8}$$

this formulation can be generalized to a L_p norm that takes the form

$$\|\mathbf{v}\|_p = \left(\sum_{i=1}^{N} v_i^p\right)^{\frac{1}{p}} \tag{1.9}$$

where p is some positive number. Lower values of p promote more sparsity in the vector $\mathbf{v}$

The last term in Equation 1.3 is β which determines the degree to which the model fits the data or obeys the regularization. Its value is determined iteratively. Assuming that the error in $\mathbf{d}^{obs}$ and Gaussian and independent with the standard deviations in $\mathbf{W}_d$, ϕ_d will be a random variable with a χ^2 distribution and an expected value of N, the number of data. Given this expected value, beta can be iteratively decreased until the misfit is sufficiently near N.

Now that I have given an overview of the structure of regularized inversion I will now discuss previous research on including geological and petrophysical information in inversion using regularization.

1.3 Literature Review

Firstly, there has been much research following the lines of Li and Oldenburg (1996) and Li and Oldenburg (1998). The methods used in these two papers are described above. The advantage of using least-squares methods for the regularization of inverse problems is that the objective function is convex, continuous, and differentiable, which greatly aids the implementation of the optimization. As mentioned above, reference models and weighting matrices allow for the incorporation of geological information, making smallness and smoothness more significant or less significant in different parts of the model. Li and Oldenburg (2003) extend the method by also implementing upper and lower bounds that can be specified for each model cell allowing the user to make hard constraints on the value of the model.

The above methods described in Li and Oldenburg (1996), Li and Oldenburg (1998), Li and Oldenburg (2003) recover smoothly varying models with broad distributions of the physical property. Sometimes the geological context indicates that the model should vary sharply and the anomalous body should be compact. In other words, either the model or its derivative should be sparse. There has been a great deal of research on using sparsity-promoting norms to achieve compact or sharply-varying models.

Instead of regularizing by smallness and smoothness, Last and Kubik (1983) regularize by compactness, essentially demanding that the anomaly be as small as possible while still fitting the data. They use an L_0 norm on the smallness component and do not use the smoothness constraints. Portniaguine and Zhdanov (1999) extend Last and Kubik (1983) by adding a minimum gradient support functional. The effect of this support functional is a L_0 norm on the smoothness terms instead of the smallness term as in Last and Kubik (1983).

Rudin et al. (1992) and Vogel and Oman (1998) propose total variation methods, in other words the use of L_1 norms to regularize, instead of L_2 norms as in Li and Oldenburg (1996) and Li and Oldenburg (1998). Since minimizing L_1 norms promotes sparsity, regularizing by them will have a comparable effect (blocky models with sharp boundaries) as the method used by Last and Kubik (1983) and Portniaguine and Zhdanov (1999). Total variation has been used more specifically in the context of geophysical inversion, such as with Guitton (2012).

Farquharson and Oldenburg (1998) also report ways of achieving sharp contrast by implementing non-L_2 norms such as Ekblom and Huber norms. Fournier (2015) implements a method of minimizing the general L_p (smallness) and L_q (smoothness) norms for any p and q (typically values between 0 and 2) allowing a user to specify the degree of compactness or blockiness of a recovered model in different spatial directions.

The formulation in Equation 1.7 allows a great deal of control of the way the model varies along the three cardinal directions. However, the geometry of a deposit does not always align with any of the cardinal directions and diagonal structures are preferred. Li and Oldenburg (2000) extend the method of Li and Oldenburg (1996) and Li and Oldenburg (1998) by rotating the model objective function to allow for linear features in the recovered model to be in a direction not in line with the mesh grid. Lelièvre and Oldenburg (2009) generalize the methods in Li and Oldenburg (2000) to the 3D case.

Guillen and Menichetti (1984) extend the method in Last and Kubik (1983). Instead of minimizing the volume of a deposit, the authors minimize its moment of inertia. By specifying an axis of rotation to determine the moment of inertia, they put dip information into the regularization in addition to having the sharp contrasts and compact models as in Last and Kubik (1983). Barbosa and Silva (1994) and Barbosa and Silva (2006) extend the method even further allowing multiple axes of rotation. The second paper also describes a GUI to interactively test the fit of various axes of rotation.

Chasseriau and Chouteau (2003) create a very general method of biasing the inversion algorithm towards anomalies of almost any shape by weighting the smallness term with a covariance matrix of the model, i.e., a matrix with the covariance of each cell versus every other cell in the model. The covariance matrix can be generated from bore hole or surface sample data or from a synthetic initial model.

In addition to the deterministic inversions described above, much research has been done on stochastic inversion. Bosch et al. (2001) directly invert for lithologies. The authors forward model physical properties by a probabilistic relation of the physical property to the lithology. New lithology distributions are created using a pseudo-random walk. *A priori* information is included partially in the probabilistic model that links the lithology to the physical properties but also in the initial probability distribution of the lithology model. Guillen et al. (2008) implement the method described in Bosch et al. (2001) in 3D.

Attempts have also been made at combining stochastic and deterministic methods. One particularly successful line of inquiry are Fuzzy C-Means (FCM). Paasche et al. (2006) uses FCM clustering of recovered models to derive membership functions of model cells in several clusters. The clusters are then used with *a priori* porosity data to create a likely porosity of each cluster and a porosity model is created from these results.

Instead of clustering after an inversion to achieve the effect of a cooperative inversion like Paasche et al. (2006), Sun and Li (2015) use the FCM function as an extra term in the model objective function. This allows them to simultaneously invert slowness and density (from travel time and gravity data) by linking them through the FCM clusters. It also allows them to guide the FCM cluster physical properties in a way that allows the integration of petrophysical measurements of geological units without directly forcing where the units are in space.

Finally, I will discuss implementations of constrained inversions. Phillips (2001) uses bore hole densities and susceptibilities to bound a gravity and a magnetic inversion. Farquharson et al. (2008) use density bore hole logs to create a reference model for a gravity inversion. They demonstrate the effect of having many bore holes versus only a few. Williams (2008) provides the most extensive review of this subject. He creates a software package to integrate a phenomenal number of types of geological and petrophysical data. Many of the tools that he created are integrated into the work that I present. He then uses these tools to make detailed susceptibility and density constraints. Finally, Lelievre (2009) discusses the use of surface samples and bore holes in constraining a synthetic example. He also shows the use of orientation information of linear features as a geological constraint.

As stated in the research motivation, what is lacking in previous implementations is an integrated environment where data, models, inversions, and constraints can be developed. By implementing the creation of constraints in such an environment, I make the creation of constraints faster even in non-trivial contexts with multiple sources of information and multiple forms of constraints.

1.4 Thesis Organization

In this thesis I describe the methods used to create the tools I contributed to GIFtools and give examples of their use. Chapter 2 will discuss the tools I have created. It describes the types of information that GIFtools and Model Builder can integrate into an inversion and discuss how they can be used to constrain an inversion result. I discuss how sample information (bore hole and surface sample data) can be used to set reference models and bounds. I also discuss geological maps and how Model Builder incorporates information from both cross section and plan view maps into the regularization of inversions. Finally in Chapter 2 I discuss the use of clustering multiple inversion results to create non-trivial face weights in addition to reference models and bounds.

In Chapter 3 I show the use of GIFtools and Model Builder in the creation of regularizations for a magnetic inversion in El Poma. El Poma is a porphyry deposit in Colombia that has magnetic properties measured from bore holes, surface samples, in addition to a geological map over the region. The region is also interesting due the large effect of remanent magnetization. I discuss a synthetic case matching the magnetic survey, bore holes, surface samples and map to recover the anomaly. I also show the result of the inversion of the actual field data.

Finally in Chapter 4 I show an example GIFtools and Model Builder in the context of the Tli Kwi Cho Kimberlite complex in the North West territories. In this case there have been several surveys flown over the region for electromagnetic (of which I use only the magnetic data) and gravity gradiometry data. In addition to the geophysical data there has been extensive drilling and cross section maps have been created. I show a synthetic example incorporating the *a priori* information as well as using clustering of the magnetic and gravity inversion to create further constraints.

Chapter 2

Including Geological Maps in Model Objective Functions

It is often the case that geological information is provided in the form of geological maps in either cross section or plan view. Such maps are particularly useful since they provide a great deal of information over their entire surface. Cross sections can provide information at depth and constrain a whole region often within the center of a target of interest. Plan view maps do not provide information at depth, but they do constrain the entire surface of the region being inverted. Constraining the surface of an inversion is of interest since the sensitivity of the data to the top cells is particularly high, which can lead to artifacts on the surface. For the next section the plan view model is from the El Poma case study and the cross section model is from TKC, specifically the map from (Harder et al., 2006)

Below in point form is the method I have developed to incorporate pixel maps.

2.1 Preprocessing images

Often a geological map image will not be immediately suitable to the methods used below and some prepossessing is required. The most notable features that are undesirable in a map are geological units that are not only one or two colours and text or other annotations that could be interpreted by the program as geological information. Also map images may be of too high resolution to be efficiently used in these methods and must also be down-sampled to save computer processing time and memory.

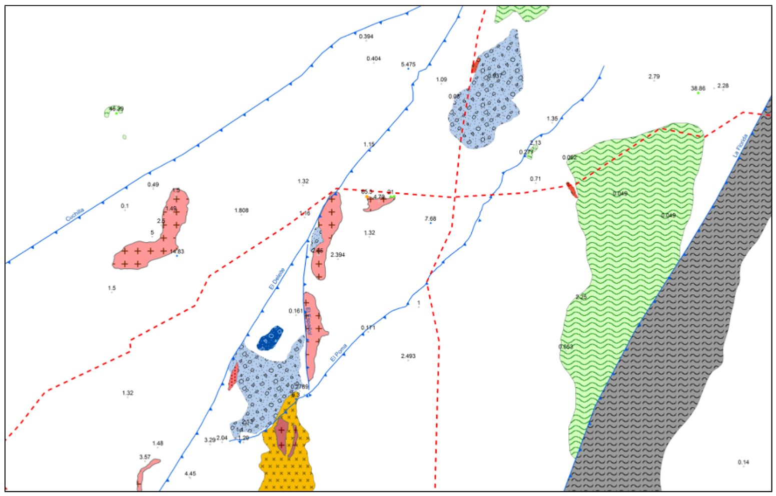


Figure 2.1: The El Poma map with fault lines (blue lines with barbs) included

For example Figure 2.1 has great deal of information, (faults, magnetic susceptibility surface samples, etc.) that are not information about geological units. In addition the geological units are not a single colour polygon. The image has been edited in the GNU Image Manipulation Program (GIMP), a free image editing program, to produce Figure 2.2.

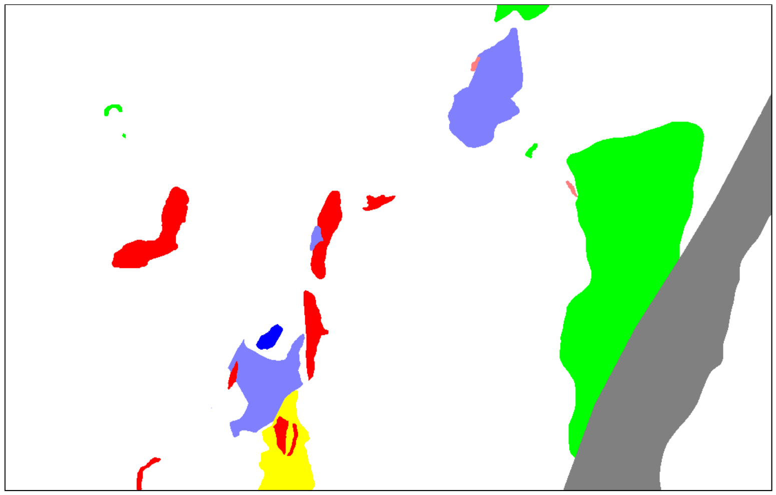


Figure 2.2: The El Poma map with extra information removed and geological units made a single colour

2.2 Loading Images into GIFtools

- load image into the GIFtools format (Figure 2.3)
  - Determine image format.
  - Load image using MATLAB utilities.
  - Convert image into .png style representation for faster computation.
  - Using .twf file (world file) assign location and spacial resolution to the image.
  - Assign a legend linking pixel RGB values to geological unit.
  - Assign topography (either number or GIFtools TOPOdata item) for visualization.
    * In the case of a cross section image, instead of topography, information for the location of the cross section in 3D or 2D space is required (Figure 2.4).

Storing a map as a GIFtools object allows its use in several ways. Notably it allows the integration of the map with models and data, allowing figures overlaying the

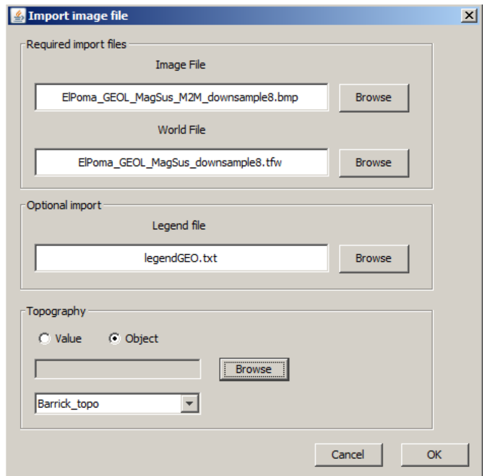


Figure 2.3: GUI for importing plan view image

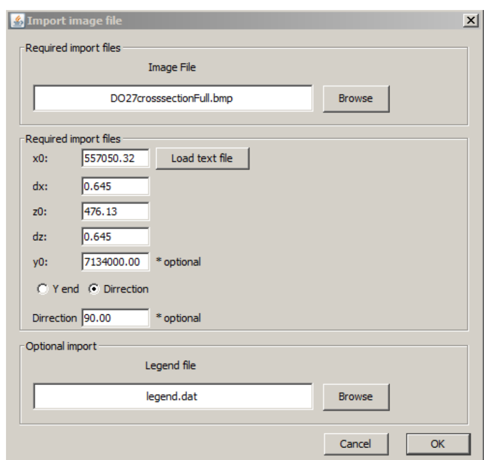


Figure 2.4: GUI for importing cross section image

map and data or model and allowing interpretation of the data or model with direct reference to the map (Figure 2.5).

2.3 Creating a Pixel Map Legend

Continuing on in the process of making a geological constraint.

- Find the geological unit represented of each pixel.

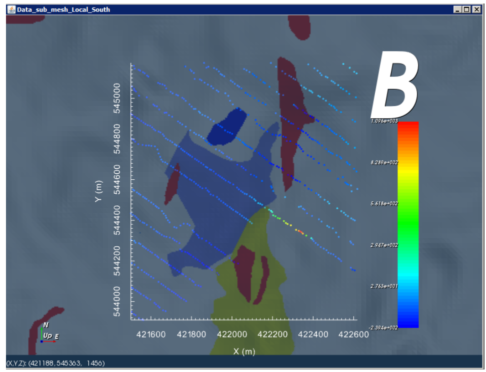


Figure 2.5: Example of magnetics data being viewed with a map overlaid

- In the .png style format as stored in MATLAB, an image consists of an "image" field, a matrix of integers, and a "map" field, which maps the image matrix to RBG value triplets.
- Each RGB triplet is compared to the legend that was provided when the image was loaded. A map field entry is considered to represent a geological unit if all three components of the RGB triplet are within a provided tolerance of any entry in the legend.
- Now that we have a relation of entries in the map field to geological units in the legend, we can assign a geological unit to each pixel in the original image simply by applying the new geological map to the image field.

2.4 Making a Geology Model from Map

2.4.1 Plan View

- Provide active model. For convenience this is usually an active model already associated with a Model Builder object.
  - The active model simultaneously provides a discretized topography for the map to lay along and also a mesh (Geophysical Inversion Facility (GIF) 3D tensor or OcTree).

- Provide some form of depth information.
  - Thickness, a certain amount of depth below topography at each point will be assigned the geological unit at each.
  - Depth, the map will be used to assign a geological unit down to a fixed depth across the whole model.
  - Surface, if you provide another surface below topography the cells between topography and the other surface will be assigned.
- Crop all pixels that extend outside of the mesh or that represent the background geological unit.
  - The cropping greatly speeds up the process and makes it require much less computer memory.
  - Furthermore, in the event of a mistake with coordinates the process ends almost instantly as there are few pixels to process.
- Finally the geological model is created.
  - We determine which cell of the mesh each pixel is in, including those cells below each pixel to account for thickness.
  - Each cell is assigned a geological unit based on the mode of the geological values of each pixel which colours that cell. In other words, each cell is identified with the geological unit which fills the greatest proportion of the cell.
    - The mode is used since each cell will be a particular unit. Since the property being mapped onto each cell by construction must represent a single geological unit, interpolation between the units will not provide the desired result.
  - The geology definition which will allow the assignment of physical properties to each geological unit. The result is shown in Figure 2.6, the continuous colour bar is not an indication of a continuous model. All model values are integers that represent geological units in the map.

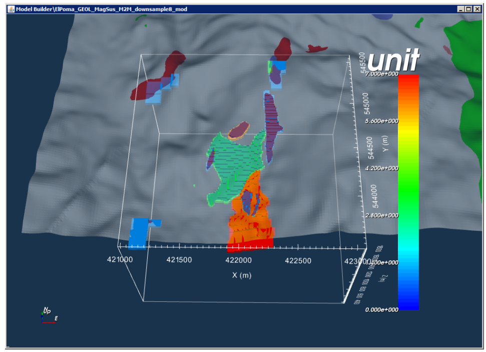


Figure 2.6: Example of a geology model created from a map with the map overlaid

2.4.2 Cross Section

The cross section case follows much the same procedure with a few exceptions. An imported cross section map is shown overlaid on a 2D mesh in Figure 2.7. Notably no parameter for the vertical extent is needed. The other notable exception is that mesh that is used is a GIF 2D mesh. The result is shown in Figure 2.8.

A 2D Geology model can be used to create constraints for a 2D inversion, it can also be used to add constraints to a 3D inversion as well. After the 2D geology model is created from the cross section map, it can be inserted into a 3D mesh (GIF 3D tensor or OcTree) given a starting and ending position or a starting position and a direction Figure 2.9,Figure 2.10.

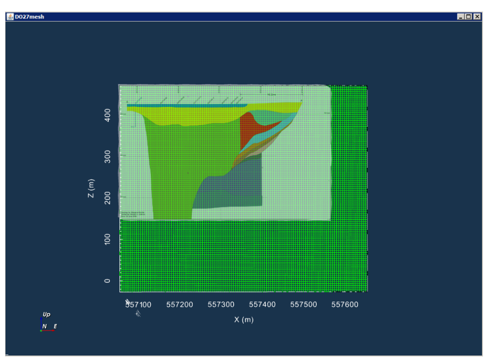


Figure 2.7: Example of a 2D mesh with the map overlaid

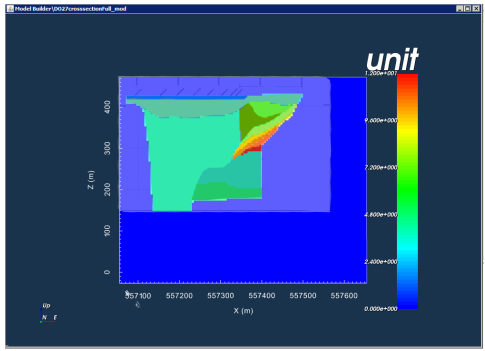


Figure 2.8: Example of a 2D geology model created from a cross section map with the map overlaid

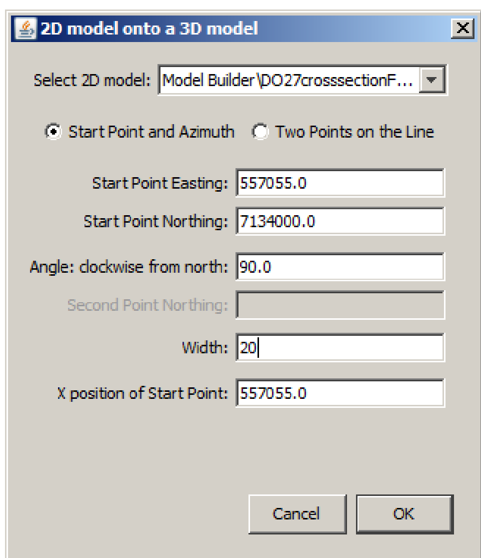


Figure 2.9: GUI for adding a 2D model to a 3D model

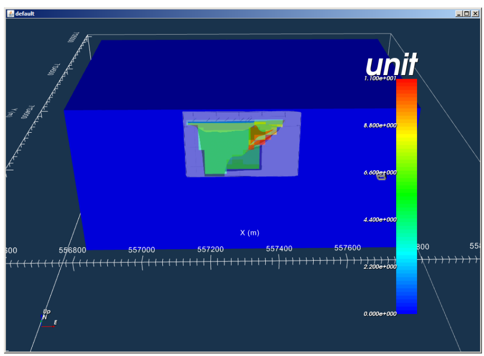


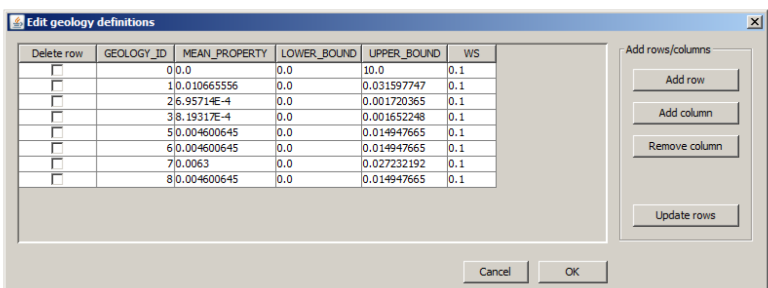
Figure 2.10: Example of a 2D geology inserted into a 3D model with the map overlaid

2.5 Making Constraints for an Inversion

The model that has been created is a geology model. That is, a model in which each cell represents a given geological unit. To be able to convert this model into a constraint for a geophysical inversion the link between between the geology and the petrophysics needs to be provided.

The link is stored in what is called a geology definition. In GIFtools this takes the form of a lookup table that contains information of each particular geological unit's property, lower and upper bounds, and optionally the smallness weight associated with each unit.

Using the geology definition we can convert a geology model that has information about the spatial distribution of geological units but not of their physical properties into constraints that are usable by an inversion. In the figures below the geological definition came from surface measurements of magnetic susceptibility within each geological unit. Figure 2.11 is an example of a geology definition in the GIFtools GUI.



Delete row	GEOLOGY_ID	MEAN_PROPERTY	LOWER_BOUND	UPPER_BOUND	WS
☐	0	0.0	0.0	10.0	0.1
☐	1	0.010665556	0.0	0.031597747	0.1
☐	2	6.95714E-4	0.0	0.001720365	0.1
☐	3	8.19317E-4	0.0	0.001652248	0.1
☐	5	0.004600645	0.0	0.014947665	0.1
☐	6	0.004600645	0.0	0.014947665	0.1
☐	7	0.0063	0.0	0.027232192	0.1
☐	8	0.004600645	0.0	0.014947665	0.1

Figure 2.11: Example of a geological definition as displayed in the GIFtools GUI

Once the geology definition is provided, we can use the Combine Model Dialog (Figure 2.12) in Model Builder to create a reference model and bounds. In this case the resolution of conflicts is trivial as there is a single source of information. Less trivial examples of the creation of reference models and bounds will be discussed later. The resulting reference model is shown in Figure 2.13.

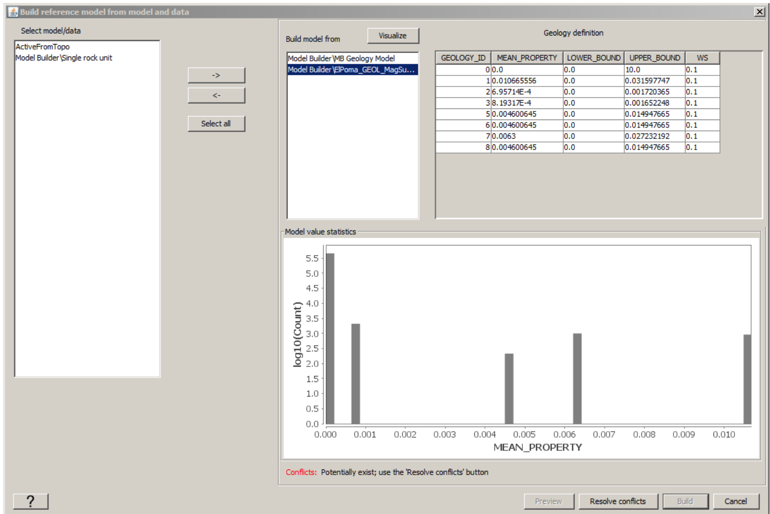


Figure 2.12: Example of a typical combine model dialog for a reference model

2.6 Inputing Fault information from Geological Maps

Another piece of information that can be in geological maps are fault locations. Again in the context of El Poma the map provided a whole complex of thrust faults as shown in the un-doctored map in Figure 2.1

The method used to insert faults into an inversion is as follows:

- Determine the end points of the fault.
  - GIFtools makes this easy by reporting the location of the cursor in the data viewer allowing you to find the location (including elevation) of a point along the fault.
- Using the locations provided create a box of a desired thickness (default value is based on the core mesh size) that includes the ends points of the fault and dips in the desired direction.
- faces within this box are assigned a new value that is provided in the GUI Figure 2.14.

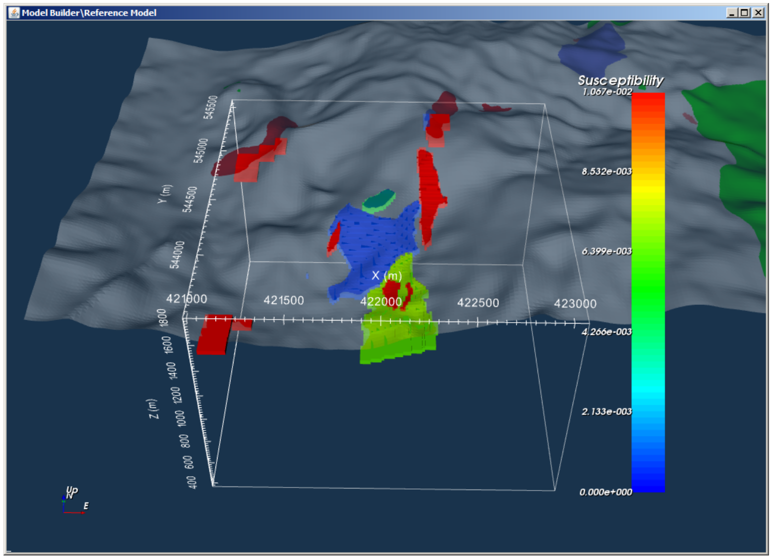


Figure 2.13: Example of a reference model created from a geological map

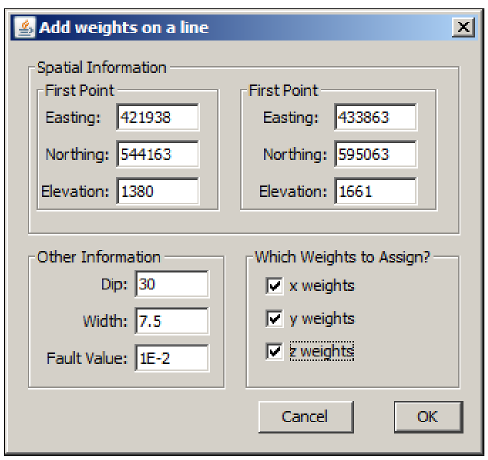


Figure 2.14: The GUI for the creation of fault weights

This process can be done multiple times to create non-trivial fault complexes as shown in Figure 2.15

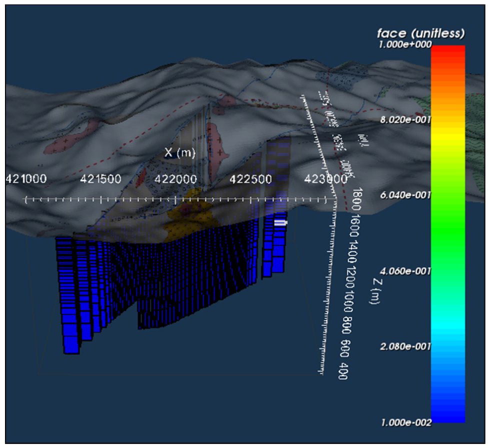


Figure 2.15: An example of fault weights that can be created with GIFtools

In this section I have shown the creation of constraints that are compatiple with GIF inversion codes. I have created these constraints from multiple types of map (Cross Section and Plan View) and have used different pieces of information from these maps (geological units and fault locations).

Chapter 3

Case Study #1 El Poma

3.1 General Overview of El Poma

3.2 Overview of Deposits

3.3 Discussion of the Geophysical Data Given

3.4 What Information is Available

3.5 Synthetic Model

3.6 Blind Inversion of the Synthetic Model

3.7 Determination of Magnetization Dirrection

3.8 Creation of Constraints

3.8.1 α coefficients

3.8.2 Reference Models

3.8.3 Weighting matrices

3.8.4 Bounds

3.8.5 L_pL_q weights

Chapter 4

Case Study #2 TKC

4.1 Overview of Deposits

4.2 Discussion of the Geophysical Data Given

4.3 What Information is Available

4.4 Synthetic Model

4.5 Blind Inversion of the Synthetic Model

4.6 Determination of Magnetization Dirrection

4.7 Creation of Constraints and Types of Data

4.7.1 α coefficients

4.7.2 Reference Models

4.7.3 Weighting matrices

4.7.4 Bounds

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Appendix A

Supporting Materials

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