The Use of Geological and Petropghysical Information to refine the Model Objective Fuction of Geophysical Inversion Problems

by

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Abstract

This document provides brief instructions for using the `ubcdiss` class to write a University of British Columbias (UBCS)-conformant dissertation in LaTeX. This document is itself written using the `ubcdiss` class and is intended to serve as an example of writing a dissertation in LaTeX. This document has embedded Unique Resource Locators (URLS) and is intended to be viewed using a computer-based Portable Document Format (PDF) reader.

Note: Abstracts should generally try to avoid using acronyms.

Note: at UBC, both the Graduate and Postdoctoral Studies (GPS) Ph.D. defence programme and the Library's online submission system restricts abstracts to 350 words.

Preface

At UBC, a preface may be required. Be sure to check the GPS guidelines as they may have specific content to be included.

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Glossary

This glossary uses the handy `acroynym` package to automatically maintain the glossary. It uses the package's `printonlyused` option to include only those acronyms explicitly referenced in the LaTeX source.

GPS Graduate and Postdoctoral Studies

PDF Portable Document Format

URL Unique Resource Locator, used to describe a means for obtaining some resource on the world wide web

MOF Model Objective Function

NRM Natural Remanent Magnetization

TMI Total Magnetic Intensity

TMA Total Magnetic Anomaly

Acknowledgments

Thank those people who helped you.

Don't forget your parents or loved ones.

You may wish to acknowledge your funding sources.

Chapter 1

Introduction

The 'true' is only the expedient in our way of thinking, just as the 'right' is only the expedient in our way of behaving.
— William James (1909)

In all cases these are first guesses at what needs to be in each section more or less detail need to be added.

1.1 What problems

Geophysical inversions, specifically potential fields include formulation of non-regularized inverse problem

1.2 Difficulties with said problems

The standard way to fit a set of parameters to a set of data (especially when they are related by a linear operator) is least squares optimization. This is rendered problematic since, in general, geo-physical inversions are ill-conditioned (define) and undetermined (define) ([2] other sources I'm sure). In specific potential fields are particularly under-determined due to the lack of any depth information in the data.

show some form of problems with forward operator matrix in PF inversion

1.3 Solutions to said difficulties

To mitigate the difficulties presented above an extra term is added to the optimization.

$$\phi = \phi_d + \beta\phi_m \tag{1.1}$$

where ϕ_m is called the Model Objective Function (MOF) or model norm. This ϕ_m can be defined in many ways, following [2]

$$\phi_m(m) = \alpha_s \int (m - m_{ref})^2 dx + \alpha_x \int \left(\frac{d}{dx}(m - m_{ref})\right)^2 dx \tag{1.2}$$

$$= \alpha_s \|\mathbf{W}_s(m - m_{ref})\|_2^2 + \alpha_x \|\mathbf{W}_x(m - m_{ref})\|_2^2 \tag{1.3}$$

in higher dimensions more smoothness terms can be added. The $\mathbf{W}$ terms contain both the operator (identity for $\mathbf{W}_s$ and derivative for $\mathbf{W}_x$ and other dimensions) and the relative weight each cell or face contributes to the MOF. This gives us several levers to add a-priori information into the inversion.

The MOF allows us to mathematically solve the problem by adding a priori information into the inversion. Namely we assume that the recovered model should be small and smooth. There are times when this is desired but often we have more specific information about the true model that needs to be inserted into the inversion. Luckily the various terms in the MOF allow us to add a significant amount of information is various ways to the inversion. It must be said that all of the techniques listed below are not novel. Many researchers before me have used exactly these techniques to constrain inversions ([**?**],[**?**] among other (still need to add more)). What is novel in this thesis is the creation of a suit of tools (created by me and the rest of the GIF group) to make the incorporation of geological data into the MOF of inversions easy even in non-trivial cases.

1.3.1 α coefficients

broad strokes weights the relative importance of the smallness and smoothness in the various directions. can also be thought of as length scales

1.3.2 Reference Models

we don't always want a model to be close to zero. Sometimes it should be close to another constant sometimes we have guesses of the property in some places and want the inversion result to be close to that value

1.3.3 Weighting matrices

much more precise. Can put interfaces in precise locations. Can also force a model towards the reference model where we are more sure

along with the terms in the MOF other parts of the optimization algorithm (may need more info in the optimization) can be used to add information into the inversion

1.3.4 Initial Model

In the optimization we assume that the initial guess is near enough to the truth that the problem is locally convex. The initial model is important in that way. In an under determined system it also provides a way to push the inversion towards a given result. Often the initial model is simply the reference model, or the reference model shifted slightly to keep it within the bounds

1.3.5 Bounds

we can also set values that each cell of the final model must lie between. This allows for a hard setting of confidence intervals in the physical properties

1.3.6 L_pL_q weights

Finally we can generalize the MOF somewhat. In Equation 1.3 we used L_2 norms as this is a natural norm that promotes a smoothly varying model that is close to the reference model. We do not always want such a model and can change the norm used in the MOF. Lower norms promote more sparsity in whatever measure they are being applied to. This leads to models being more compact (should lowever norms be applied to the smallness term) or more blocky with greater discontinuities (should lower norms be applied to one or more smoothness terms). Non L_2 norms can be applied across the whole MOF or can be applied variably across the model. This allows for placing discontinuities in a given direction but not perfectly placing the location allowing the inversion algorithm more freedom to chose the location itself.

1.4 Forms of A Priori Information

1.4.1 Bore Hole Data and the Use of Koenigsberger Ratios to Correct Bore Hole Susceptibility Measurements

Bore holes provide physical property measurements at depth either by sending geophysical instruments down hole or by recovering a core and then measuring it subsequently in the lab. Bore holes can also provide qualitative rock unit information. Much work has been done on including physical property bore hole information([**?**] among others I'm sure). If we can convert the lithology information from bore holes into physical property information (using petrophysical measurements of reasonably similar rocks) we can use the information in the same fashion as a physical property bore hole logs.

In some contexts one physical property can be used as a proxy for others. In one case study, we have magnetic susceptibility measurements down hole but surface samples have been measured which reveal high magnetic remanence. The simple susceptibility measurement is drastically lower than the recovered effective susceptibility derived from the inversion of a magnetic survey over the area. This is due to the fact that the inversion is recovering effective susceptibility (induced magnetization plus Natural Remanent Magnetization (NRM) normalized by and assuming the direction of earth's field in the location). To describe the method we need to discuss the general derivation of the magnetostatic problem and then discuss the effects of NRM on measured data and the recovered inversion result.

The following derivation follows the one in [?]. We can derive the magnetostatic problem from Maxwell's equations. If we assume no free currents and no time varying electric field Maxwell's equations simplify to

$$\nabla \cdot \mathbf{B} = 0 \tag{1.4}$$

$$\nabla \times \mathbf{H} = 0 \tag{1.5}$$

$$\mathbf{B} = \mu \mathbf{H}, \tag{1.6}$$

where $\mathbf{B}$ is the magnetic flux density measured in Tesla (T), $\mathbf{H}$ is the magnetic field measured in amperes per meter (A/m), and μ is magnetic permeability which relates $\mathbf{B}$ to $\mathbf{H}$ in matter. We can rewrite μ to take the permeability of free space (μ_0) into account and state that

$$\mu = \mu_0(1+\kappa), \tag{1.7}$$

where μ_0 is the permeability of free space ($4\pi \times 10^{-7}\frac{Tm}{A}$), and κ is the magnetic susceptibility of a material. κ is dimensionless and describes the ability of a material to become magnetized under some field $\mathbf{H}$. The definition of κ in Equation 1.7 gives us a definition for induced magnetization

$$\mathbf{M}_I = \kappa \mathbf{H}. \tag{1.8}$$

Since there are no free currents and Equation 1.5 states that $\mathbf{H}$ has no curl, it can be written as the gradient of a potential field

$$\mathbf{H} = \nabla \phi. \tag{1.9}$$

Since, by Equation 1.4, we assume that there are no magnetic monopoles, we approximate ϕ in terms of a dipole moment $\mathbf{m}$. If we have a magnetic dipole with a moment of $\mathbf{m}$ at a location $\mathbf{r}_Q$, then the potential field ϕ as measured at some $\mathbf{r}_P$ is given by

$$\phi(r) = \frac{1}{4\pi}\mathbf{m} \cdot \nabla\left(\frac{1}{r}\right), \tag{1.10}$$

where

$$\mathbf{r} = \|\mathbf{r}_Q - \mathbf{r}_P\|_2. \tag{1.11}$$

We can generalize Equation 1.10 to a continuous form by replacing the discrete $\mathbf{m}$ with a continuous $\mathbf{M}$ and integrating

$$\phi(r) = \frac{1}{4\pi}\int_V \mathbf{M}\cdot\nabla\left(\frac{1}{r}\right)dv. \tag{1.12}$$

If we take the gradient of Equation 1.12 we find $\mathbf{B}$ the magnetic flux density,

$$\mathbf{B}(\mathbf{r}_P) = \frac{1}{4\pi}\int_V \mathbf{M}\cdot\nabla\nabla\left(\frac{1}{r}\right)dv. \tag{1.13}$$

In Equation 1.13 the dependency of $\mathbf{B}$ on $\mathbf{r}_P$ is due to the fact that r depends on $\mathbf{r}_P$ as in Equation 1.11. In most geophysical surveys, the full vector $\mathbf{B}$ is not collected, usually only its magnitude, or the Total Magnetic Intensity (TMI),

$$B_{TMI} = \|\mathbf{B}_0 + \mathbf{B}_A\|_2 \tag{1.14}$$

where $\mathbf{B}_0$ is the primary field (earth's field) and $\mathbf{B}_A$ is the anomalous local field due to magnetization in the ground. For the purposes of geophysical exploration we are only interested in $\mathbf{B}_A$. Since we are only interested in $\mathbf{B}_A$ a useful quantity is therefor the Total Magnetic Anomaly (TMA), defined as

$$B_{TMA} = \|\mathbf{B}_{Total} + \mathbf{B}_0\|_2 \tag{1.15}$$

TMA is difficult to measure directly but can be approximated assuming $\frac{\|\mathbf{B}_A\|}{\|\mathbf{B}_0\|} \ll 1$,

$$\begin{aligned} B_{TMA} &\simeq \mathbf{B}_A\cdot\hat{\mathbf{B}}_0 && (1.16)\\ &= \|\mathbf{B}_0 + \mathbf{B}_A\| - \|\mathbf{B}_0\| && (1.17)\end{aligned}$$

We now have a formulation of $\mathbf{B}$ that depends on magnetization $\mathbf{M}$, the vector field of magnetization in the ground and the quantities collected in geophysical magnetics surveys. We will now take a more specific look at magnetization and its effects both on $\mathbf{B}$ and B_{TMA}. If we assume no self-demagnetization (which is reasonable for susceptibilities below 1^{-2} [**?**]) the inducing magnetic field is constant over the volume to be inverted and total magnetization can be characterized by the following

$$\mathbf{M} = \mathbf{M}_I + \mathbf{M}_{NRM} \tag{1.18}$$

$$\mathbf{M} = \kappa\mathbf{H} + \mathbf{M}_{NRM} \tag{1.19}$$

Here $\mathbf{M}$ is the total magnetization, $\mathbf{M}_I$ is the induced magnetization as in Equation 1.8 , κ is susceptibility as defined in Equation 1.7, $\mathbf{H}$ is the inducing field (in this case of the geomagnetic field) and $\mathbf{M}_{NRM}$ is the NRM. $\mathbf{M}_{NRM}$ is also characterized by what is called Koenigsberger ratio

$$Q = \frac{\mathbf{M}_{NRM}}{\kappa\mathbf{H}} = \frac{\text{remanent magnetization}}{\text{induced magnetization}} \tag{1.20}$$

In the case where NRM is negligible, the direction of $\mathbf{M}$ isthe same as $\mathbf{H}$, meaning that $\mathbf{B}_A$ and $\mathbf{B}_0$ are in the same direction and making the approximation in Equation 1.16 exact. The methods outlined in [1] and [3] assume that not only is there no self-demagnetization but also that there is no NRM. A slight generalization from assuming no NRM is that the anomalous magnetization is entirely in one direction [1]. The recovered quantity is effective susceptibility, or magnetization normalized by the earth's field,

$$\kappa_{eff} = \frac{||\mathbf{M}||}{||\mathbf{H}||}. \tag{1.21}$$

In the case that the magnitude of NRM is negligible, effective susceptibility and the true susceptibility are equal (i.e. $Q \ll 1$).

In the context of high remanent magnetization, the assumption that there is no NRM is by definition clearly false and NRM affects the inversion results. In the case that the NRM is in a similar direction as the earth's field, the measured field B_{TMA} will be higher than expected, given only susceptibility, and thus the recovered κ_{eff} will be higher than the true κ. Similarly, in the case that NRM is in a direction nearly anti-parallel to the earth's field, the measured field B_{TMA} will be lower than expected, given only susceptibility, and thus the recovered κ_{eff} will also be lower than the true κ.

Understanding the difference between κ and κ_{eff} is very important with respect to inserting magnetic petrophysical measurements into an inversion's MOF. It is an unfortunate truth that susceptibility is significantly easier to measure than NRM. As stated above in Equation 1.7 the permeability of an object is related to its susceptibility. In addition the inductance of a coil is proportional to the permeability inside and around it and thus is dependent on the susceptibility of the material. This change in inductance of a coil allows the precise measurement of susceptibility without contamination by NRM [**?**],[**?**]. The measurement of the inductance of a coil also allows the measurement of susceptibility of a rock without reorienting the sample and without shielding from ambient magnetic fields [**?**].

On the other hand, NRM as opposed to susceptibility is a vector quantity and thus the sample must be reoriented to measure it from different directions, even when only the magnitude of the NRM is required. In addition, unless the sample is very highly magnetized and stable, measurement will require shielding from ambient fields.

As can be seen from the above, it is not surprising that n the case of El Poma we have many susceptibility measurements including bore-holes and very few (only two within the areas of interest) measurements of NRM. However, if susceptibility is used to constrain an inversion in an area of strong NRM the inversion could be constrained to a value much higher or lower than the true effective susceptibility.

At first order, a potential correction given NRM measurements is to use the Koenigsberger ratio of a sample and assume that other samples will have a similar Koenigsberger ratio. It is recognized that this assumption will not be true. That said it is a closer approximation than would otherwise be possible without the sample. Once we have a Koenigsberger ratio and a susceptibility we can determine the magnetization of the sample using Equation 1.18 and Equation 1.20 in the form

$$\mathbf{M}_{eff} = \kappa\mathbf{H} + \|Q\kappa\mathbf{H}\|\hat{\mathbf{M}}_{NRM}, \tag{1.22}$$

where Q is the Koenigsberger ratio used and $\hat{\mathbf{M}}_{NRM}$ is the magnetization direction of the sample used. It is important to note that Equation 1.22 is a vector sum and the direction of $\mathbf{M}_{eff}$ will not be in the direction of either $\mathbf{H}$ or $\mathbf{M}_{NRM}$. It is also interesting to note that Equation 1.22 is more generally true if more samples have NRM measurements. If, instead of being a single measurement, we have a more detailed estimate of each sample's Koenigsberger ratio and magnetization direction, we can get a better estimate of $\mathbf{M}_{eff}$.

1.4.2 Surface Sample Data

1.4.3 Geological Maps

1.5 Using Multiple Data Types, with Clustering

Chapter 2

Solutions to Including Information Into MOF

As Far I as I can tell This section should organized by data type as well.

2.1 Forms of A Priori Information

2.1.1 Bore Hole Data and the Use of Koenigsberger Ratios to Correct Bore Hole Susceptibility Measurements

. I think there is some use in describing how GIFtools does it now.

Much of this work was done in [**?**]. Mostly what I have done is the inclusion of lithologies and the use of Koenigburgers

2.1.2 Surface Sample Data

Again [**?**] did much of this. I think the Surface Samples will be promary use in proving physical propertes for the map in El Poma

2.1.3 Geological Maps

This is novel as far as I can tell. I will describe the map to model sofware and I will descibe the use of the “add face weights along line” tool to add faults in

2.2 Using Multiple Data Types, with Clustering

There is interesting things to discuss in the storing of multiple inversion in GIFtools, and in the use of clustering algorythms used and then ability to take geological models and make reference models and non-trivial face weighting.

Chapter 3

Case Study #1 El Poma

In all cases these are first guesses at what needs to be in each section more or less detail need to be added.

3.1 General Overview of El Poma

Two anomalies. North and South.

Magnetic Anomaly. Remanent magnetization is clearly present

3.2 Overview of Deposits

3.3 Discussion of the Geophysical Data Given

Magnetics. Missing a corner over the southern anomaly

3.4 What Information is Available

Bore Hole -susceptibilities, much lower than the recovered model sue to remanent effects being present

Plan View Geological map -with susceptibility surface samples marked, in addition to surface samples and geological units, we also have a system of thrust faults over top of both anomalies.

Surface Samples -susceptibility, same as marked on map but includes many samples from outside map area as well -also have nine remanences measured with direction and K_n

3.5 Synthetic Model

TODO: Create Model - make iso-surface of Kris's result. Determine property from parametric inversion. show model discuss its creation - magnetization direction

show its fit to the field data

3.6 Blind Inversion of the Synthetic Model

Show results. Discuss how magnetization direction puts anomaly away from actual location

3.7 Determination of Magnetization Dirrection

Correlation of Vertical and Total Gradients of Half RTP field [**?**]

taking core direction from MVI result could also use parametric inversion and MVI sensitivities to provide more constraint.

apply recovered direction to the anomaly direction in MAG3D could apply anomalous dirrection locally to anomaly

3.8 Creation of Constraints

3.8.1 α coefficients

For El Españole (north south fault) we can lower the α_x to allow for greater discontinuity in general in that direction. Cannot account for other faults without rotation objective function.

(show result)

3.8.2 Reference Models

Most work to be done here.

Borehole: provides susceptibilities need to convert into effective susceptibilities. Assuming uniform magnetization direction this is not complicated. Choose a K_n and multiply susceptibility by that (maybe with +1). For MVI I need to apply the dirrection of magnetization as well. Either from the truth of the synth model from direction of the nearest remanent sample or from the bulk rem mag direction. (show reference model) (show result)

Map: geological units have susceptibilities attached. Have to convert into effective susceptibilities. Might extend the map cells down below surface to be less weighted (show reference model) (show result)

Surface Samples: used to make susc values for geological units. Can also be used for reference model directly but this provides less cover of the surface. Perhaps use surface samples in white region instead of just applying nothing. (show reference model) (show combined map and SS reference model) (show result)

(show Combined result)

3.8.3 Weighting matrices

smallness: using some measure of confidence in the measures decrease in cells with reference model specified to force the result to approximate the reference model. In case that map is extended down I will lower the W_s as model cells are further below the surface. (show smallness weight model) (show result, compare to result without)

smoothness: to spread the model values away from where they are specified I can increase the smoothness weights in the vicinity of cells with specified reference models. (show face weight model) (show result, compare to result without) The other application of smoothness weights is to allow discontinuities on the faults (perhaps with increased smoothing on either side of the fault). Need to experiment with orientations of the faults. (show result)

(show Combined result)

3.8.4 Bounds

Also useful for forcing model values to be near the specified reference model while allowing for uncertainty in our phys prop value (show result)

3.8.5 L_pL_q weights

allows the more fuzzy placement of faults. By rotating the MOF we can place them in arbitrary directions. The trouble is having more than one fault in more than one orientation. Can't currently apply to MVI inversions, can still apply on MAG3D inversions (show result)

Need to determine if showing the field example is worthwhile at this point and how to bring it into the narrative

Chapter 4

Case Study #2 TKC

In all cases these are first guesses at what needs to be in each section more or less detail need to be added.

4.1 Overview of Deposits

Kimberlite Complex Two anomalies, focusing on the southern one (DO27)

Magnetic Anomaly. Remanent magnetization is likely present but largely in the dirrection of the earth's field Density Anomaly.

4.2 Discussion of the Geophysical Data Given

Magnetics: Three different surveys
Gravity: Ground mag (of usable but dubious quality), Gravity Gradiometry airborne data

much EM as well, outside the scope of this Master's Thesis

4.3 What Information is Available

Great deal of borehole data with facies at each depth We also have Phys Props at various points along the holes. We can either mean these across the facies or take the value of each facies that the specially nearest the the measured result.

From the borehole data we also have created a surface model of each of the units again from the borehole data, we have graphical cross section maps

4.4 Synthetic Model

TODO: Create Model - Already mostly done - we have a surface from the borehole data and we can use the parametric inversion to assign the properties. show model discuss its creation - magnetization direction

show its fit to the field data

4.5 Blind Inversion of the Synthetic Model

Show results. show how model is insufficiently compact and over estimates the amount of kimberlite

4.6 Determination of Magnetization Dirrection

Correlation of Vertical and Total Gradients of Half RTP field [**?**]

taking core direction from MVI result

apply recovered direction to the anomaly direction in MAG3D could also use parametric inversion and MVI sensitivities to provide more constraint.

could apply anomalous dirrection locally to anomaly

In any case the result will be very similar to the earth's field in the location

4.7 Creation of Constraints and Types of Data

Extensive Boreholes with rock units Multiple cross sections (crreated from said bore holes) Multiple data types to cluster

4.7.1 α coefficients

not much with alphas to be done here given that we don't expect discontinuity in any one particular dirrection.

4.7.2 Reference Models

Most work to be done here.

We can create a reference model from the phys prop results from the borehole data. Perhaps we should only use some of the boreholes so that we have a more realistic amount of information than in a fully drilled example. We have two ways of applying phys prop measures to inversion and might use both. Also using K_ns to improve degree of fit between phys prop measures and effective susc recovered properties (show reference model) (show result)

with sufficient boreholes we could make a incorrect surface that approximates the “true” model used. Use this with parametric inversion for reference model (show reference model) (show result)

using clustering between density and mag (and conductivity and chargeablity) to create clusters, populate each cluster with a value either the mean value of the cluster or a parametric inversion and use as reference (show reference model) (show result)

use cross section from [**?**] perhaps extend away from line and down weight (show reference model) (show result)

(show Combined result)

4.7.3 Weighting matrices

smallness: using some measure of confidence in the measures decrease in cells with reference model specified to force the result to approximate the reference model. In case the cross section is extended down I will lower the W_s as model cells are further away from the cross section (show smallness weight model) (show result, compare to result without)

smoothness: to spread the model values away from where they are specified I can increase the smoothness weights in the vicinity of cells with specified reference models. (show face weight model)

with sufficient boreholes we could make a incorrect surface that approximates the “true” model used. Use this with parametric inversion for reference mode,l put lower weights along this surface. (show face weight model)

using clustering between density and mag (and conductivity and chargeablity) to create clusters, populate each cluster with a value either the mean value of the cluster or a parametric inversion and use as reference (show result)

(show result, compare to result without)

(show Combined result)

4.7.4 Bounds

Also useful for forcing model values to be near the specified reference model while allowing for uncertainty in our phys prop value. Since we have more statistical info on the (show result)

Need to determine if showing the field example is worthwhile at this point and how to bring it into the narrative

Bibliography

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[2] D. W. Oldenburg and Y. Li. Inversion for applied geophysics: A tutorial. *Investigations in geophysics*, 13:89–150, 2005. → pages 1, 2

[3] M. Pilkington. 3-d magnetic imaging using conjugate gradients. *Geophysics*, 62(4):1132–1142, 1997. → pages 7

Appendix A

Supporting Materials

This would be any supporting material not central to the dissertation. For example:

- additional details of methodology and/or data;
- diagrams of specialized equipment developed.;
- copies of questionnaires and survey instruments.