The Use of Geological and Petropghysical Information to refine the Model Objective Fuction of Geophysical Inversion Problems

by

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Abstract

This document provides brief instructions for using the `ubcdiss` class to write a University of British Columbias (UBCS)-conformant dissertation in LaTeX. This document is itself written using the `ubcdiss` class and is intended to serve as an example of writing a dissertation in LaTeX. This document has embedded Unique Resource Locators (URLS) and is intended to be viewed using a computer-based Portable Document Format (PDF) reader.

Note: Abstracts should generally try to avoid using acronyms.

Note: at UBC, both the Graduate and Postdoctoral Studies (GPS) Ph.D. defence programme and the Library's online submission system restricts abstracts to 350 words.

Preface

At UBC, a preface may be required. Be sure to check the GPS guidelines as they may have specific content to be included.

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Glossary

This glossary uses the handy `acroynym` package to automatically maintain the glossary. It uses the package's `printonlyused` option to include only those acronyms explicitly referenced in the LaTeX source.

GPS Graduate and Postdoctoral Studies

PDF Portable Document Format

URL Unique Resource Locator, used to describe a means for obtaining some resource on the world wide web

MOF Model Objective Function

NRM Natural Remanent Magnetization

TMI Total Magnetic Intensity

TMA Total Magnetic Anomaly

Acknowledgments

Thank those people who helped you.

Don’t forget your parents or loved ones.

You may wish to acknowledge your funding sources.

Chapter 1

Introduction

In all cases these are first guesses at what needs to be in each section more or less detail need to be added.

1.1 What problems

Geophysical inversions, specifically potential fields include formulation of non-regularized inverse problem

1.2 Difficulties with said problems

The standard way to fit a set of parameters to a set of data (especially when they are related by a linear operator) is least squares optimization. This is rendered problematic since, in general, geo-physical inversions are ill-conditioned (define) and undetermined (define) ([1] other sources I'm sure). In specific potential fields are particularly under-determined due to the lack of any depth information in the data.

show some form of problems with forward operator matrix in PF inversion

1.3 Solutions to said difficulties

To mitigate the difficulties presented above an extra term is added to the optimization.

$$\phi = \phi_d + \beta \phi_m \tag{1.1}$$

where ϕ_m is called the Model Objective Function (MOF) or model norm. This ϕ_m can be defined in many ways, following [1]

$$\phi_m(m) = \alpha_s \int (m - m_{ref})^2 dx + \alpha_x \int \left(\frac{d}{dx}(m - m_{ref}) \right)^2 dx \tag{1.2}$$

$$= \alpha_s \|\mathbf{W}_s(m - m_{ref})\|_2^2 + \alpha_x \|\mathbf{W}_x(m - m_{ref})\|_2^2 \tag{1.3}$$

in higher dimensions more smoothness terms can be added. The $\mathbf{W}$ terms contain both the operator (identity for $\mathbf{W}_s$ and derivative for $\mathbf{W}_x$ and other dimensions) and the relative weight each cell or face contributes to the MOF. This gives us several levers to add a-priori information into the inversion.

The MOF allows us to mathematically solve the problem by adding a priori information into the inversion. Namely we assume that the recovered model should be small and smooth. There are times when this is desired but often we have more specific information about the true model that needs to be inserted into the inversion. Luckily the various terms in the MOF allow us to add a significant amount of information is various ways to the inversion.

1.3.1 α coefficients

broad strokes weights the relative importance of the smallness and smoothness in the various directions. can also be thought of as length scales

1.3.2 Reference Models

we don't always want a model to be close to zero. Sometimes it should be close to another constant sometimes we have guesses of the property in some places and want the inversion result to be close to that value

1.3.3 Weighting matrices

much more precise. Can put interfaces in precise locations. Can also force a model towards the reference model where we are more sure

along with the terms in the MOF other parts of the optimization algorithm (may need more info in the optimization) can be used to add information into the inversion

1.3.4 Initial Model

In the optimization we assume that the initial guesse is near enough to the truth that the problem is locally convex. The initial model is important in that way. In an under determined system it also provides a way to push the invesion towards a given result. Often the initial model is simply the reference model, or the reference model shifted slightly to keep it within the bounds

1.3.5 Bounds

we can also set values that each cell of the final model must lie between. This allows for a hard setting of confidence intervals in the physical properties

1.3.6 L_pL_q weights

Finally we can generalize the MOF somewhat. In Equation 1.3 we used L_2 norms as this is a natural norm that promotes a smoothly varying model that is close to the reference model. We do not always want such a model and can change the norm used in the MOF. Lower norms promote more sparsity in whatever measure they are being applied to. This leads to models being more compact (should lowever norms be applied to the smallness term) or more blocky with greater discontinuities (should lower norms be applied to one or more smoothness terms). Non L_2 norms can be applied across the whole MOF or can be applied variably across the model. This allows for placing discontinuities in a given direction but not perfectly placing the location allowing the inversion algorithm more freedom to chose the location itself.

1.4 Forms of A Priori Information

1.4.1 Bore Hole Data

Bore holes can either give physical property measurements down hole or provide qualitative rock unit information. Much work has been done on inserting physical property bore hole information (NC Williams 2008). If we can convert the rock property information from bore holes into physical property information (using petrophysical measurements of reasonably similar rocks) we can use the information in the same fashion as a physical property bore hole

Secondly in some contexts one physical property can be used as a proxy for others. In one case study we have magnetic susceptibility measurements down hole but the context is sufficiently remanent that the simple susceptibility measurement drastically underestimates the recovered property from the magnetic inversion. This is due to the fact that the inversion is recovering effective susceptibility (susceptive magnetization plus Natural Remanent Magnetization (NRM) normalized by and assuming the direction of earth's field in the location). To describe the method I will use I will need to discuss the general derivation of the magnetostatic problem. (I don't think this is how the argument will be structured in the final thesis)

The following derivation follows that in (Fournier 2015). We can derive the magnetostatic problem from Maxwell's equations. If we assume no free currents and no time varying electric field Maxwell's equations simplify to

$$\nabla \cdot \mathbf{B} = 0 \tag{1.4}$$

$$\nabla \times \mathbf{H} = 0 \tag{1.5}$$

$$\mathbf{B} = \mu \mathbf{H}, \tag{1.6}$$

where $\mathbf{B}$ is the magnetic flux density measured in Tesla (T), $\mathbf{H}$ is the magnetic field measured in amperes per meter (A/m), and μ is magnetic permeability which relates $\mathbf{B}$ to $\mathbf{H}$ in matter. We can rewrite μ to take the permeability of free space (μ_0) into account and state that

$$\mu = \mu_0(1+\kappa), \tag{1.7}$$

where μ_0 is (as already stated) the permeability of free space, and κ is the susceptibility of a material. κ is dimensionless and describes the ability of a material to become magnetized under some field $\mathbf{H}$. Since Equation 1.5 state that $\mathbf{H}$ has no curl is can be written as the gradient of a potential field

$$\mathbf{H} = \nabla \phi. \tag{1.8}$$

Since, by Equation 1.4, we assume that there are no magnetic dipoles, we approximate ϕ in terms of a dipole moment $\mathbf{m}$. If we have a magnetic dipole with a moment of $\mathbf{m}$ at a location $\mathbf{r}_Q$, then the potential field ϕ as measured at some $\mathbf{r}_P$ is given by

$$\phi(r) = \frac{1}{4\pi}\mathbf{m} \cdot \nabla \left(\frac{1}{r}\right), \tag{1.9}$$

where

$$\mathbf{r} = \|\mathbf{r}_Q - \mathbf{r}_P\|_2. \tag{1.10}$$

We can generalize Equation 1.9 to a continuous form by replacing the discrete $\mathbf{m}$ with a continuous $\mathbf{M}$ and integrating

$$\phi(r) = \frac{1}{4\pi}\int_V \mathbf{M} \cdot \nabla \left(\frac{1}{r}\right) dv. \tag{1.11}$$

If we take the gradient of Equation 1.11 we find $\mathbf{B}$ the magnetic flux density,

$$\mathbf{B}(\mathbf{r}_P) = \frac{1}{4\pi}\int_V \mathbf{M} \cdot \nabla\nabla \left(\frac{1}{r}\right) dv. \tag{1.12}$$

In Equation 1.12 the dependency of $\mathbf{B}$ on $\mathbf{P}$ is due to the fact that r depends on $\mathbf{r}_P$

as in Equation 1.10. In most geophysical surveys, the full vector **B** is not collected, usually only its magnitude, or the Total Magnetic Intensity (TMI),

$$b_{TMI} = \|\mathbf{B}_0 + \mathbf{B}_A\|_2 \tag{1.13}$$

where $\mathbf{B}_0$ is the primary field (earth's field) and $\mathbf{B}_A$ is the anomalous local field due to magnetization in the ground. For the purposes of geophysical exploration we are only interested in $\mathbf{B}_A$. Since we are only interested in $\mathbf{B}_A$ a useful quantity is therefor the Total Magnetic Anomaly (TMA), defined as

$$b_{TMA} = \|\mathbf{B}_{Total} + \mathbf{B}_0\|_2 \tag{1.14}$$

TMA is difficult to measure directly but can be approximated assuming $\frac{\|\mathbf{B}_A\|}{\|\mathbf{B}_0\|} \ll 1$,

$$\begin{aligned} b_{TMA} &\simeq \mathbf{B}_A \cdot \hat{\mathbf{B}}_0 && (1.15) \\ &= \|\mathbf{B}_0 + \mathbf{B}_A\| - \|\mathbf{B}_0\| && (1.16) \end{aligned}$$

We now have a formulation of **B** that depends on magnetization **M**. We also have looked into the quantities collected in geophysical magnetics surveys. We will now take a more specific look at magnetization and it's effects both on **B** and b_{TMA}. If we assume no self-demagnetization the magnetic field is constant over the region and magnetization can be characterized by the following

$$\mathbf{M} = \kappa\mathbf{H} + \mathbf{M}_{NRM} \tag{1.17}$$

Here **M** is once again the total magnetization, κ is susceptibility as defined in Equation 1.7, **H** is the inducing field (in this case of the earth) and $\mathbf{M}_{NRM}$ is the NRM. $\mathbf{M}_{NRM}$ is also characterized by what is called Koenigsberger ratio

$$Q = \frac{\mathbf{M}_{NRM}}{\kappa\mathbf{H}} \tag{1.18}$$

In the case where there is no NRM the direction of **M** is going to be the same as **H**, meaning that $\mathbf{B}_A$ and $\mathbf{B}_0$ will be in the same direction and making the approximation in Equation 1.15 exact. [**?**] and [**?**] assume that not only is there no self-demagnetization but also that there is no NRM. A slight generalization from assuming no NRM is that the anomalous magnetization is entirely in one direction, essentially implying that the Koenigberger ratio is constant.

Having the only direction of mag

The recovered property in a magnetic inversion is susceptibility as in Equation 1.17 except it is normalized by **H**. The inversion used assumes that all magnetization is due to susceptibility and that all magnetization is in the same direction. There is a parameter in the inversion that allows for an anomaly magnetization direction other than that of earth's field but all magnetization is still all in the same direction. If NRM

1.4.2 Surface Sample Data

In a addition to

Bibliography

[1] D. W. Oldenburg and Y. Li. Inversion for applied geophysics: A tutorial. *Investigations in geophysics*, 13:89–150, 2005. → pages 1, 2

Appendix A

Supporting Materials

This would be any supporting material not central to the dissertation. For example:

- additional details of methodology and/or data;
- diagrams of specialized equipment developed.;
- copies of questionnaires and survey instruments.