1. **Introduction**
   1. **Research Motivation**
      1. Improving geophysical inversion by easy incorporation of a priori data into inversion regularisations. In this thesis I only consider potential field data. However, the methods described only effect the model objective function and bounds and as such can be applied to any data type.
      2. **Inversion formulation**
         1. Model Objective Function in context to general Tikhonov inversion
         2. Constrained inversion
         3. Explain why I’m only considering L2 model norms
            * While LpLq model norms could greatly affect the results show in this thesis, the goal of this thesis is to show the inclusion of geological and petrophysical information in inversion model objective functions. All the techniques shown in this thesis can also be used on model objective functions that use different norms.
      3. **Data types exemplified in this document**
         1. Magnetics
            * Susceptibility
            * Remanence
         2. Gravity Gradiometry
         3. Note that both are linear systems and the forward model fits into the relevant part of section 1.1.2
      4. **Innovations and improvements upon Nick Williams’ thesis** 
         * + Maps (pixels versus shape files)  
             non-linearity of GUI
           + image analysis rather than only map polygons  
             cross section input
           + faults
           + ability to edit and QC data (geophysical, petrophysical, and geological) as you go through the inversion process
           + geological models
           + tools to help assign physical properties to geophysical units
           + tools to allow basic clustering of inversion results
           + voxel-parametric inversion
      5. **Motivating Synthetic Examples**
         1. El Poma style example
            * Two anomalies
            * Only susceptibility
            * North is near a outcropping unit in geo-map
            * South is bounded by the fault complex
            * Geo-map provided
            * Faults provided from geo map
            * Physical property bore hole (susceptibility)
         2. TKC style example
            * Only susceptibility
            * Gravity Gradiometry data and Magnetic data
            * Geologic bore holes
            * Physical properties measured from core
            * Cross section map
   2. **Literature Review**
      * + - Discussion of different ways that a priori information can be inserted into a model objective function
2. **Incorporation** **of prior information** 
   1. **Geology**
      1. **Plan-view map**Background: Geology map with colours -> voxel model constraints  
         New research:
         * + Can take a pixel map and discretize it into a voxel model
           + Assignment of geological unit to each pixel
           + Assignment of geological unit to each model cell
           + Determine topography (just on surface, extend to fixed depth, extend to variable depth below surface, extend to arbitrary surface)
           + Assignment of physical property and bounds to each geological unit to create constraints

Example: El Poma (synthetic)

* + - * + Show effect of several depths on inversion result
        + Show creation of reference model and bounds from geological unit
        + Show lowering of Ws in unit to bring anomaly to surface
        + Discuss best practices (some quantitative measure of recovered model’s fit to synthetic model)
    1. **Cross-section**Background: Geology cross section image with colours -> voxel model  
       New research:
       - * How to insert into 3D model -> 2D mesh to 3D mesh
         * Geo-rectify of 2D mesh onto 3D mesh
         * Geo-rectify of 2D image to view in 3D model

Example: TKC (synthetic)

* + - * + Show placement of cross section
        + Show width of cross section
        + Discuss assignments of physical properties to geological units
        + Discuss best practices (some quantitative measure of recovered model’s fit to synthetic model)
    1. **Faults maps**Background: Lines for faults -> allows discontinuity of physical properties by inputting low spacial weights where a fault would imply a discontinuity  
       New research:
       - * Assuming piece-wise continuous faults, can iteratively create fault-systems and curving faults
         * Need to have “water tight” surface that is only one face thick regardless of mesh and fault surface
         * can add dip information

Example: El Poma (synthetic)

* + - * + show placement of faults
        + discuss value of fault weights (degree of discontinuity across fault)
        + discuss dip
        + Discuss best practices(some quantitative measure of recovered model’s fit to synthetic model)
  1. **Discretization of point measurements** 
     1. **Borehole physical properties**Background information: bore hole point or interval data -> voxel model constraint  
        New research:
        + - Incorporation of Nick Williams’ tools for loading, and discretizing bore hole data
          - incorporation of bore hole data into GIFtools visualization and quality control tools
          - ability to edit data
          - incorporate Nick Williams’ tools for creating constraints from discretized bore hole data
          - tools to create discretization (different distributions, different methods of calculating bounds)
          - ability to edit discretization cell by cell

Example: El Poma (synthetic)

* + - * + discuss discretization and setting bounds from property data
        + discuss spreading weights
        + Discuss best practices(some quantitative measure of recovered model’s fit to synthetic model)
      1. **Using bore hole susceptibility in highly remanent environments**

Example: Remanent Block in a half space

* + - * + Block in a half space with MNRM in very different direction from Earth’s field and a much lower susceptibility
        + Synthetic bore hole through the block
        + Show result if only susceptibility is used as bore hole constraint
        + Show result if true effective susceptibility is used as bore hole constraint
        + Show method of determining a recovered Koenigsberger ratio from MVI result

Divide measured susceptibility from bore hole by effective susceptibility result from MVI, take the average

* + - * + Show result if recovered Koenigsberger ratio is used to determine effective susceptibility as bore hole constraint
        + Discuss best practices(some quantitative measure of recovered model’s fit to synthetic model)
        + Show check of the same procedure in a case where there is no remanence to show that the recovered Koenigsberger ratio ends up being 1 anyway
    1. **Geology IDs**Background information: bore hole point or interval Geology data -> voxel model constraint  
       New research:
       - * Link petrophysical measurements to geological units in bore hole data

Example: TKC (Synthetic)

* + - * + Discuss setting properties to geological units
        + discuss discretization and setting bounds from units physical property data
        + Discuss best practices(some quantitative measure of recovered model’s fit to synthetic model)
  1. **Combining Constraints**
     1. **Multiple A Priori Data Types**Background information: multiple a priori data types have been discussed so far. Using all a priori data at ones disposal regardless of type is useful.  
        New research:
        + - Update Nick Williams’ tools to allow more flexibility and ease of use
          - Make resolution of constraints by rule and on a case by case basis

Example: TKC and El Poma (Synthetics)

* + - * + Show combined constraints of previous examples
        + Map + faults + bore hole for El Poma
        + Bore hole + Cross Section for TKC
  1. **Putting it all together**Background information: Instead of constraining an inversion by a model objective function it can be constrained by a fixed distribution of geological units. Once a geological interpretation is created through the methods above assigning property values to each unit is useful. Parametric inversion allows the setting of property values based the fit to geophysical data.  
     New research:
     + - * How to make geological units (iso surfaces and clustering)
         * Define voxel-parametric inversion
         * Create synthetic models using voxel-parametric inversion (point out that this is how the motivating examples where made)
         * improves assignment of properties to units (does not rely on regularization outside of the geological or pseudo-geological model provided)
         * Helps in the determination of anomalous magnetization direction (the direction can vary greatly between each unit since they will not have smoothness constraints)

Example: TKC and El Poma (Synthetics)

1. **Application to El Poma**
   * + - * Only consider northern anomaly
   1. **Information provided**
      * + - Mag data set (ground mag)
          - Surface susceptibility measurements
          - Surface sample oriented NRM measurement (including Koenigsburger ratio)
          - Bore hole susceptibility measurement
   2. **Blind**
      * + - Invert mag data with MAGinv3D blind, assuming the inducing field direction
          - Show that recovered distribution of susceptibility does not match that from the bore holes (evidence of there being remanence)
          - Invert with MVIinv (use MVI result to determine new magnetization direction)
          - Show that the recovered effective susceptibility is of much greater magnitude than the susceptibility measured
          - Using an iso surface (perhaps from clustering) on the MVI effective susceptibility result use a voxel parametric MVI inversion to find a new magnetization direction
   3. **Blind with new direction**
      * + - Repeat 3.3. with new magnetization direction
          - Show better fit to bore hole susceptibility distribution
   4. **Constrained inversion** 
      * + - Correct bore hole susceptibility to effective susceptibility for constraint
          - Multiple ways of determining the recovered Koenigsberger ratio

Use measured result from surface

Divide measured susceptibility from bore hole by effective susceptibility result from MVI, take the average

Determine the Koenigsberger from a voxel-parametric result

* + - * + Make reference model and bounds from corrected bore hole data   
          (section 2.2.1)
        + Make Constraint from outcrop unit in geological map (out crop of susceptible rock on surface, but the anomaly if deeper and further north than the outcrop)

Forward model the unit (thin unit of susceptibility) see if it has an effect on the data

Make reference model and bound from unit

Lower Ws in the unit

* 1. **Summary of results**

1. **Application to TKC**
   1. **Information provided**
      * + - Mag data set (VTEM mag)
          - Gravity Gradiometry
          - Bore hole with geological units (PK,HK,VK,background)
          - Physical property measurements from bore hole samples, categorised by geological unit (susceptibility, un-oriented NRM, Koenigsburger ratio, saturated density)
   2. **Blind inversion**
      * + - Invert gravity
          - Invert mag data blind using magnetization direction from Devriese et. al.
   3. **Constrained inversion**
      * + - Show setting of physical properties (density and effective susceptibility)to geological bore hole
          - Show creation of constraints from the bore hole data
          - Difference in inversion result from blind result
   4. **Summary of results**
2. **Conclusion**
3. **Appendix A: Application of research to industry**
   1. **Visualization**
      1. Show tools to visualize data and models as shown in the Thesis
   2. **User experience**
      1. Show tools and procedures to create the constraints shown in the thesis
         * + Maps
           + Faults
           + Bore hole data
           + Clustering