



# Game Trees & Path Planning

*Group Members:*

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# Overview

Game Tree

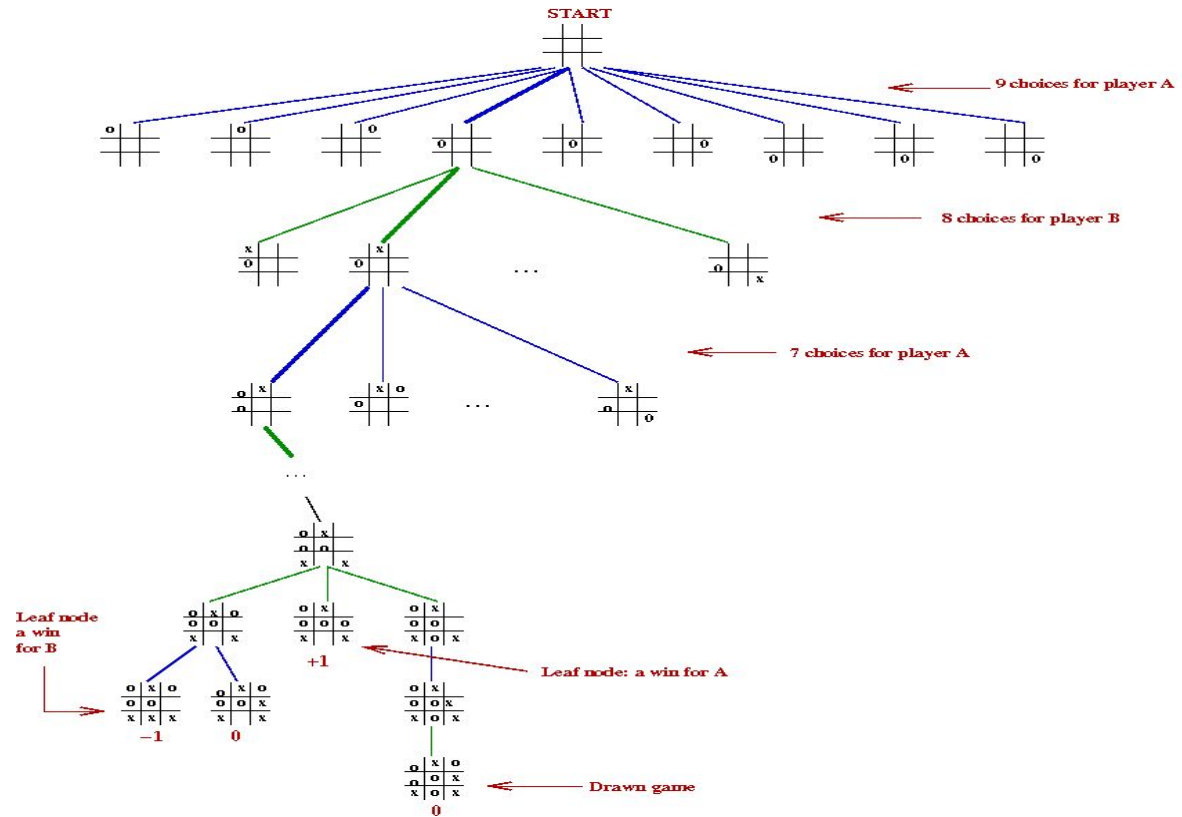
Minimax Algorithm

Connect 4

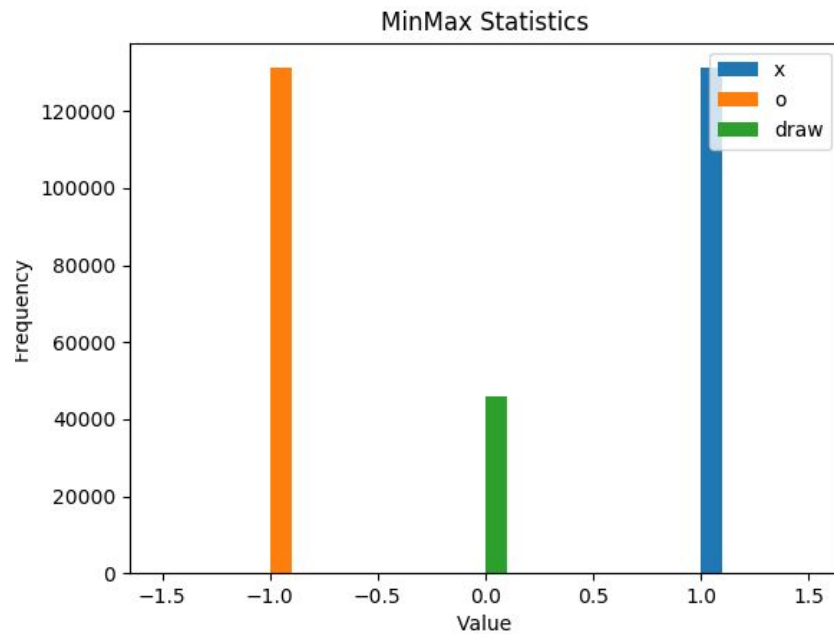
Breakout

Path Planning

# Game Tree



# Game Tree





# Game Tree

- Upper Bound on Number of Nodes: 255168.
- Wins of player X: 131184.
- Wins of player O: 77904.
- Draws: 46080.
- Number of Nodes: 549945.
- Number of Leaf Nodes: 255168.
- Number of Branches: 549936.
- Branching Factor: 0.99998



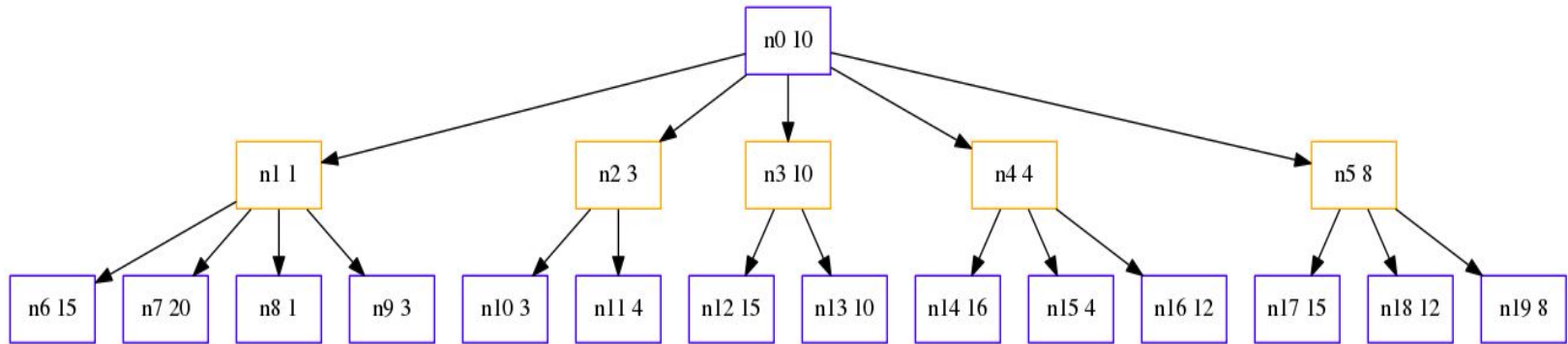
# Game Tree

## Upper Bound Combinatorial Argument

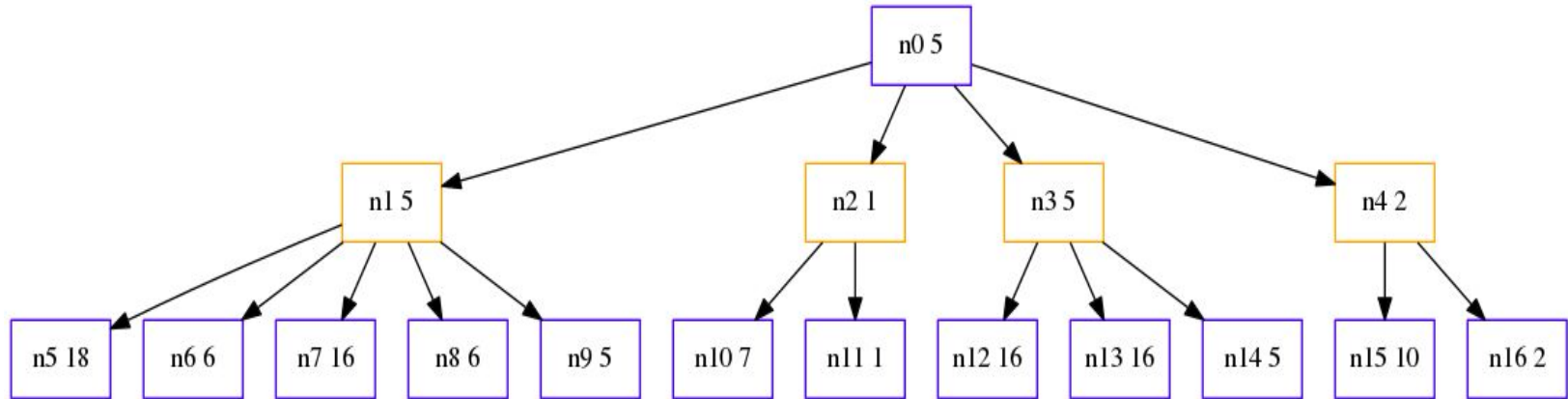
- Games ending on 5th Move:  $8 \cdot 3! \cdot 6 \cdot 5$  = 1440.
- Games ending on 6th Move:  $8 \cdot 3! \cdot 6 \cdot 5 \cdot 4 - 6 \cdot 3! \cdot 2 \cdot 3!$  = 5328.
- Games ending on 7th Move:  $8 \cdot 3 \cdot 6 \cdot 3! \cdot 5 \cdot 4 \cdot 3 - 6 \cdot 3 \cdot 6 \cdot 3! \cdot 3!$  = 47952.
- Games ending on 8th Move:  $8 \cdot 3 \cdot 6 \cdot 3! \cdot 5 \cdot 4 \cdot 3 \cdot 2 - 6 \cdot 3 \cdot 6 \cdot 3! \cdot 2 \cdot 4!$  = 72576.
- Games ending on 9th Move:  $2 \cdot 3 \cdot 8 \cdot 4! \cdot 4! + 6 \cdot 3 \cdot 4 \cdot 4! \cdot 4! + 22 \cdot 1 \cdot 4! \cdot 4! + 16 \cdot 5! \cdot 4!$  = 127872.

**Total** **255158**

# MiniMax



# MiniMax







# Tie-breaking Heuristics

- How to resolve the tie between  $n1$  and  $n3$ ?
  - Randomly
  - Recursive arg-max to pick the node having a child with highest score (i.e.  $n1$ )
- Outcome does not change assuming both players are rational (make optimal moves at each step)



# Connect 4

Recursive MiniMax Algorithm

DFS Game Tree traversal

Game Tree not explicitly but still traversed

Required:

- Reward Function for terminal states

- Evaluation Function for non-terminal states



## Game State Representation

						X
		O		X		O
	O	X	O	O	O	X
X	X	O	O	X	X	O

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	1
0	0	-1	0	1	0	-1
0	-1	1	-1	-1	-1	1
1	1	-1	-1	1	1	-1



# Evaluation Function

Define a function on the Game State:

$f(A, \text{GameState})$  = number of possible 4-in-a-row configurations left for player A in GameState.

$\text{eval}(A, \text{GameState}) =$   
 $f(A, \text{GameState}) - f(B, \text{GameState})$

Intuition:

It's easier for A to win, if there are more possibilities left for A to win than for B.

Example for vertical lines. Also count horizontal and diagonal ones to add them all up.

						X
		O		X		O
	O	X	O	O	O	X
X	X	O	O	X	X	O

$$f(X, \text{GameState}) = 3 + 1 + 0 + 1 + 1 + 1 + 0 = 7$$



# Reward for Terminal States

Win: 1000

Draw: 0

Loss: -1000

Evaluation of non terminal states will always lie between -1000 and 1000, so, so that information about terminal stats weights heavier.



# Make it more human-like

AI is predictable / makes same moves every time:

The Fix:

At the root, pick the Max move out out all maximum moves at random.



# AI doesn't win as fast as possible

This is NOT a bug, but it would look more human, to:

- Win as fast as possible.

- Delay losing as long as possible.

The Fix: In the MinMax-Tree:

- Pick the shortest path to a **win**:

  - Because it looks more human.

- Pick the longest path to a **loss**:

  - Because in games against humans or players that make random moves this might change the outcome of the game.

Implementation in Reward for terminal states:

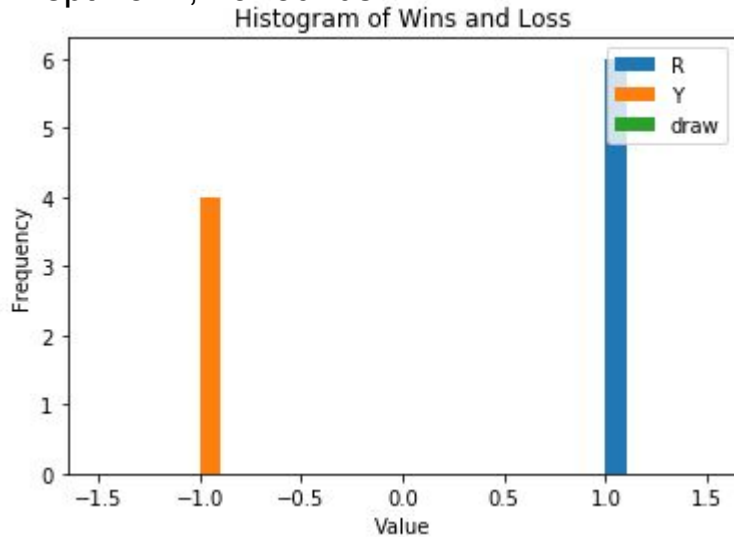
Win:  $1000 - \text{depth}$

Draw: 0

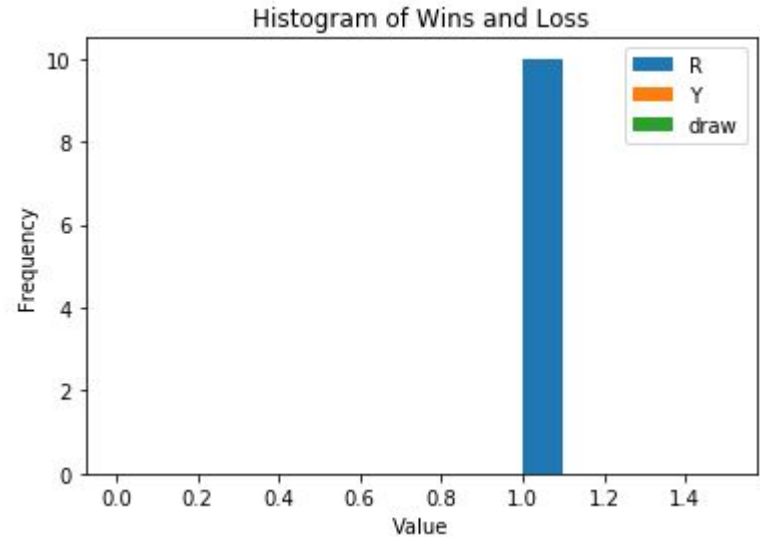
Loss:  $-1000 + \text{depth}$

# Statistics

Depth of 4, 10 rounds:



Both use MinMax



R use MinMax, Y moves a random





# Breakout

- Implemented breakout mechanics using PyGame. Strategy to move paddle is random.
- Increasing speed of paddle over time.

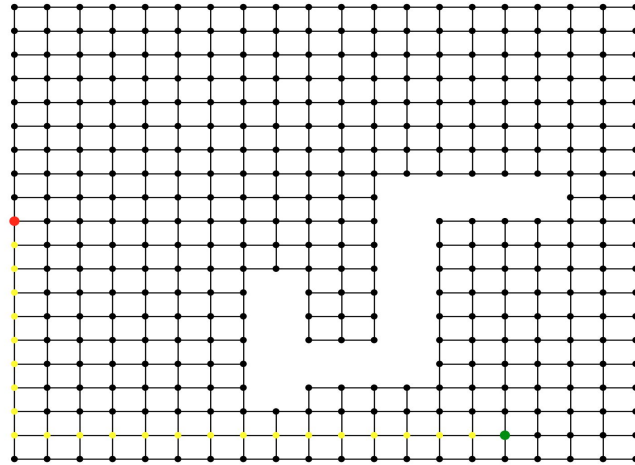


# Path Planning

## Graph Reading

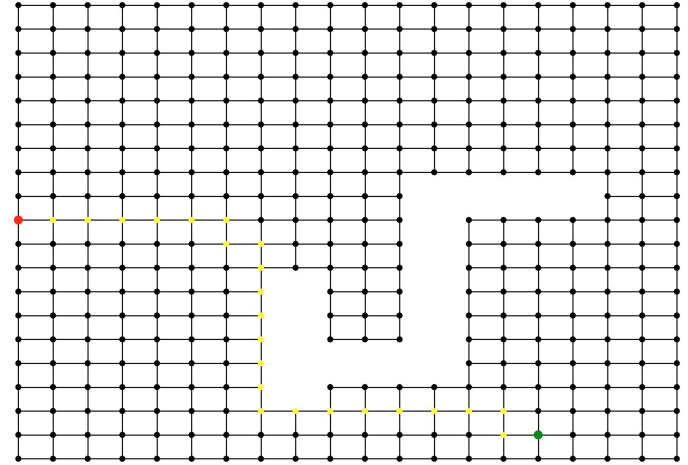
- Read data into a 2d array.
- Create a 2d grid graph using networkx.
- Create a mapping to relabel coordinates in the grid graph.
- Iteratively remove nodes with values “1”.

# Path Planning



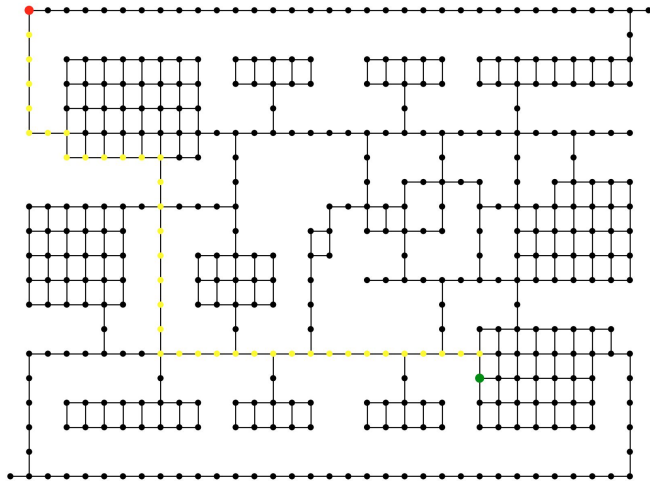
Dijkstra Path

(0,10) to (15, 1)



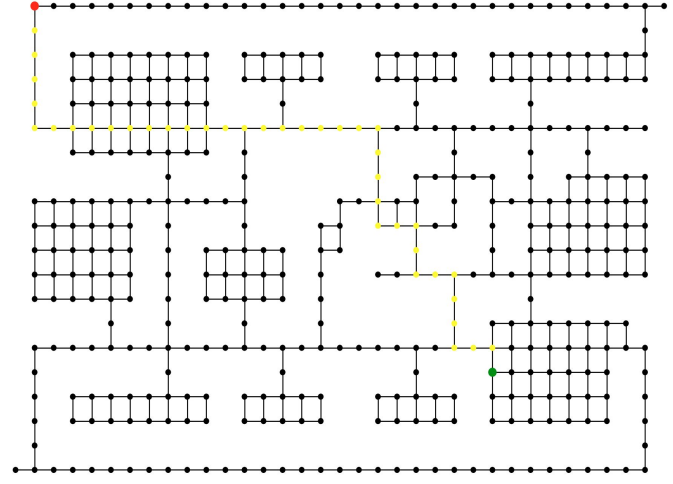
A\* Path

# Path Planning



Dijkstra Path

(1,20) to (25, 5)



A\* Path



# Conclusion

- **GameTree**
  - In Tic-Tac-Toe player 1 has an advantage over player 2.
  - However, if both player are the optimal game will always result in a draw.
- **MiniMax**
  - Exponential state space complexity.
- **Connect-4**
  - Player 1 has an advantage over player 2.
- **Path-Planning**
  - In an unweighted graph A\* and Dijkstra find a path with same cost.



# Questions.