Table 1: Model 1

Estimator	IMSE	IMSE   $Y < q_{.25}$	IMSE   $q_{.25} < Y < q_{.5}$	IMSE   $q_{.5} < Y < q_{.75}$	IMSE   $q_{.75} < Y$
$\hat{f}_{\hat{\mu},  ext{nonpar}}$	0.104(1)	0.0771(1)	0.00537(1)	0.00819(1)	0.0135(1)
$\hat{f}_{\mu,\xi}$	$0.0915 \ (0.878)$	0.0789(1.02)	$0.00373 \ (0.455)$	$0.00398 \ (0.294)$	$0.00491 \ (0.913)$
$\hat{f}_{\mu,\hat{\xi}}$	0.09 (0.864)	0.0787 (1.02)	$0.00303 \ (0.224)$	$0.00345 \ (0.642)$	$0.00489 \ (0.597)$
$\hat{f}_{\hat{\mu},\mathrm{par(rough)}}$	6.78(65.1)	$0.0635 \ (0.824)$	$0.00379 \ (0.281)$	$0.00354 \ (0.433)$	$6.71\ (1250)$
$\hat{f}_{\hat{\mu},\mathrm{par}}$	0.128(1.23)	0.058(4.29)	$0.0298 \; (0.386)$	0.019(3.54)	$0.0211\ (2.57)$
$ ilde{f}_{\hat{\mu},  ext{nonpar}}$	0.547 (5.25)	0.225 (41.8)	0.243 (3.15)	0.0704 (5.2)	$0.00868 \ (1.06)$
$ ilde{f}_{\mu,\xi}$	$0.0895 \ (0.859)$	0.0785 (1.02)	$0.00279 \ (0.206)$	$0.00348 \ (0.647)$	$0.00477 \ (0.583)$
$ ilde{f}_{\mu,\hat{\xi}}$	$0.0901 \ (0.865)$	0.0787 (5.82)	$0.00305 \ (0.567)$	$0.00347 \ (0.424)$	$0.00489 \ (0.0635)$
$\tilde{f}_{\hat{\mu}, \text{par(rough)}}$	6.78 (65.1)	0.0635 (7.76)	$0.00379 \ (0.0492)$	$0.00354 \ (0.262)$	$6.71\ (1250)$
$\widetilde{f}_{\hat{\mu},\mathrm{par}}$	$0.0993 \ (0.953)$	$0.0823\ (15.3)$	$0.00342 \ (0.418)$	$0.00343 \ (0.0445)$	$0.0101 \ (0.748)$
$f_{\hat{\mu},  ext{nonpar}}^{\dagger}$	0.0969 (0.93)	$0.0551 \ (0.715)$	0.0233 (2.85)	$0.00743 \ (0.549)$	0.011(2.04)
$f_{\mu,\varepsilon}^{\dagger}$	$0.0872 \ (0.837)$	$0.0763 \ (0.989)$	$0.00272\ (0.201)$	$0.00342 \ (0.637)$	$0.00476 \ (0.581)$
$f^{\dagger}_{\mu,\xi} \ f^{\dagger}_{\mu,\hat{\xi}}$	$0.0878 \ (0.843)$	0.0765 (5.66)	$0.00303 \; (0.0393)$	$0.00343 \ (0.639)$	$0.00488 \ (0.596)$
$f_{\hat{\mu}, \text{par(rough)}}^{\dagger}$	27.7(265)	1.77(22.9)	1.45 (107)	0.644 (120)	23.8(2910)
$f_{\hat{\mu},\mathrm{par}}^{\dagger}$	0.116(1.11)	0.0948 (1.23)	$0.00735 \ (0.544)$	$0.00492 \ (0.601)$	0.00886 (1.65)
$f_{\hat{\omega}, \text{nonpar}}^{\dagger}$	0.273(2.62)	0.148(11)	0.056 (10.4)	$0.0437 \ (0.567)$	0.0245 (2.99)
$f_{\hat{\omega},\mathrm{par}}^{\dagger}$	0.116(1.12)	0.0956 (17.8)	$0.00719 \ (0.0932)$	0.00472 (0.349)	0.00882(1.08)
p	$0.0705 \ (0.677)$	$0.0506 \ (0.656)$	0.0108(0.799)	$0.00379 \ (0.705)$	$0.00534 \ (0.652)$
$\hat{f}_{\hat{\omega},  ext{nonpar}}$	1.83(17.6)	0.869(11.3)	0.581 (42.9)	0.304 (56.6)	0.0758 (9.26)
$\hat{f}_{\hat{\omega},\mathrm{par}}$	0.138(1.33)	0.0588(4.35)	0.0344 (6.41)	$0.0223 \ (0.289)$	0.0229(2.79)
$ ilde{f}_{\hat{\omega}, ext{nonpar}}$	2.22(21.3)	$1.43 \ (18.5)$	0.649(48)	$0.11\ (20.5)$	0.0326 (3.98)
$\widetilde{f}_{\hat{\omega},\mathrm{par}}$	$0.101\ (0.965)$	0.0835 (1.08)	$0.00345 \ (0.643)$	$0.00344 \ (0.421)$	$0.0101 \ (0.751)$

Table 2: Model 2

Estimator	IMSE	IMSE   $Y < q_{.25}$	IMSE   $q_{.25} < Y < q_{.5}$	IMSE   $q_{.5} < Y < q_{.75}$	IMSE   $q_{.75} < Y$
$\hat{f}_{\hat{\mu},  ext{nonpar}}$	0.00234(1)	0.00188 (1)	0.000235(1)	7.38e-05(1)	0.00015(1)
$\hat{f}_{\mu,\xi}$	0.00369 (1.58)	0.00343 (1.82)	0.000111 (1.5)	7.1e-05 (0.473)	$8.3e-05 \ (0.354)$
$\hat{f}_{\mu,\hat{oldsymbol{arxi}}}$	0.0124 (5.29)	0.00945 (5.03)	0.00218 (14.5)	$0.000169 \ (0.719)$	0.000572 (7.75)
$\hat{f}_{\hat{\mu}, \text{par(rough)}}$	$0.00391\ (1.67)$	0.00313(1.67)	0.000431 (2.87)	0.000118 (1.59)	$0.000224 \ (0.953)$
$\hat{f}_{\hat{\mu},\mathrm{par}}$	0.00764 (3.27)	0.00328(21.9)	$0.00101 \ (0.536)$	$0.00261\ (11.1)$	0.000741(10)
$ ilde{f}_{\hat{\mu},  ext{nonpar}}$	0.183 (78.1)	0.0272 (116)	0.00663 (3.53)	0.0158 (106)	$0.133\ (1800)$
$ ilde{f}_{\mu,\xi}$	$0.00361 \ (1.54)$	0.00335 (1.78)	$0.00011 \ (0.736)$	$7.17e-05 \ (0.306)$	7.99e-05 (1.08)
$ ilde{f}_{\mu,\hat{\xi}}$	0.012 (5.14)	0.00922 (61.4)	0.00203 (8.65)	0.000159 (2.15)	$0.000622 \ (0.331)$
$\tilde{f}_{\hat{\mu},\mathrm{par(rough)}}$	$0.00391\ (1.67)$	0.00313 (42.4)	$0.000431 \ (0.229)$	$0.000118 \ (0.784)$	$0.000224 \ (0.953)$
$ ilde{f}_{\hat{\mu},\mathrm{par}}$	0.00653 (2.79)	$0.00436 \ (18.6)$	0.000423 (5.73)	$0.00136 \ (0.723)$	0.000385 (2.57)
$f_{\hat{\mu},  ext{nonpar}}^{\dagger}$	0.0926 (39.6)	0.0429 (22.8)	0.0257 (348)	0.0037 (24.6)	0.0204 (86.9)
$f_{u,\varepsilon}^{\dagger}$	$0.000876 \ (0.375)$	$0.000475 \ (0.253)$	0.00021(1.4)	$0.000135 \ (0.575)$	5.62e-05 (0.762)
$f^\dagger_{\mu,\xi} \ f^\dagger_{\mu,\hat{\xi}}$	0.0116 (4.97)	0.0088 (58.6)	$0.00107 \ (0.568)$	$0.000451 \ (1.92)$	$0.00131\ (17.7)$
$f_{\hat{\mu}, \text{par(rough)}}^{\dagger}$	$0.00524 \ (2.24)$	0.00433 (2.3)	$0.000287\ (1.91)$	0.00026 (1.11)	0.000367 (4.97)
$f_{\hat{\mu},\mathrm{par}}^{\dagger}$	0.00779 (3.33)	0.00601 (3.19)	0.000349 (2.32)	$0.0011\ (14.9)$	$0.000342 \ (1.46)$
$f_{\hat{\omega}, \text{nonpar}}^{\dagger}$	0.0996 (42.6)	0.0456 (304)	0.0279 (119)	0.00403 (2.14)	0.022(298)
$f_{\hat{\omega},\mathrm{par}}^{\dagger}$	0.00905(3.87)	0.00652(27.8)	$0.000422 \ (0.224)$	0.00158(10.5)	0.000531(7.2)
p	0.0407 (17.4)	$0.0242\ (12.9)$	0.00814 (54.2)	0.000784(3.34)	0.00749 (101)
$\hat{f}_{\hat{\omega},  ext{nonpar}}$	0.153 (65.4)	0.0544 (28.9)	0.0526 (351)	0.0346(148)	0.0113(153)
$\hat{f}_{\hat{\omega},\mathrm{par}}$	$0.0102 \ (4.35)$	0.0035 (23.4)	$0.00146 \ (6.23)$	0.00402 (2.14)	$0.00119\ (16.1)$
$ ilde{f}_{\hat{\omega},  ext{nonpar}}$	0.378(161)	0.0372 (19.8)	0.00889 (59.3)	0.0445 (190)	0.287(3890)
$ ilde{f}_{\hat{\omega},\mathrm{par}}$	0.0134 (5.72)	0.00285 (1.52)	0.00275 (11.7)	0.00601 (81.4)	0.00177 (11.8)

Table 3: Model 3

Estimator	IMSE	IMSE   $Y < q_{.25}$	IMSE   $q_{.25} < Y < q_{.5}$	IMSE   $q_{.5} < Y < q_{.75}$	IMSE   $q_{.75} < Y$
$\hat{f}_{\hat{\mu},  ext{nonpar}}$	0.00213(1)	0.000524(1)	0.000738(1)	0.000653(1)	0.000219(1)
$\hat{f}_{\mu,\xi}$	0.12(56.4)	$0.0213 \ (40.6)$	0.0408 (62.5)	0.0393(180)	0.019(25.8)
$\hat{f}_{\mu,\hat{\xi}}$	0.12 (56.4)	0.0213 (40.6)	0.0408 (186)	0.0393 (53.3)	0.019(29.1)
$\hat{f}_{\hat{\mu}, \mathrm{par(rough)}}$	113 (52900)	113 (215000)	0.0243 (111)	$0.0165\ (25.3)$	0.00301 (4.08)
$\hat{f}_{\hat{\mu},\mathrm{par}}$	$0.0291\ (13.6)$	0.00376 (17.1)	0.0165 (31.5)	$0.00763\ (10.3)$	0.00125 (1.91)
$ ilde{f}_{\hat{\mu},  ext{nonpar}}$	1.09(509)	0.018(24.4)	0.0314(60)	$0.235\ (1070)$	0.802(1230)
$ ilde{f}_{\mu,\xi}$	$0.0591\ (27.7)$	0.016 (30.6)	0.0292 (133)	$0.0127\ (17.1)$	0.00124 (1.89)
$ ilde{f}_{\mu,\hat{\mathcal{E}}}$	$0.0591\ (27.7)$	0.016 (73.1)	0.0292 (39.5)	0.0127 (19.4)	0.00124 (2.36)
$\tilde{f}_{\hat{\mu}, \mathrm{par(rough)}}$	113 (52900)	113 (173000)	0.0243 (46.4)	0.0165 (75.4)	0.00301 (4.08)
$ ilde{f}_{\hat{\mu},\mathrm{par}}$	0.0429 (20.1)	0.0065 (8.8)	0.0215(33)	0.0122 (23.2)	0.00274 (12.5)
$f_{\hat{\mu}, \mathrm{nonpar}}^{\dagger}$	0.187 (87.8)	0.0198(37.8)	0.0373 (57.2)	0.0226 (103)	0.108(146)
$f_{\mu,\varepsilon}^{\dagger}$	1210000 (5.69e+08)	10600 (20200000)	$27200 \ (1.24e+08)$	372000 (5.03e+08)	805000 (1.23e+09
$f^\dagger_{\mu,\xi} \ f^\dagger_{\mu,\hat{\xi}}$	1210000 (5.69e+08)	10600 (48300000)	27200 (51900000)	372000 (5.03e+08)	805000 (1.23e+09)
$f_{\hat{\mu}, \text{par(rough)}}^{\dagger}$	161 (75200)	139 (266000)	2.68 (12200)	8.28 (11200)	$10.1\ (15400)$
$f_{\hat{\mu},\mathrm{par}}^{\dagger}$	$0.0227\ (10.6)$	0.00146 (2.79)	0.00727 (33.2)	$0.00674\ (10.3)$	0.00721 (9.77)
$f_{\hat{\omega}, \text{nonpar}}^{\dagger}$	0.211 (98.9)	0.0193 (88.1)	$0.0364 \ (49.3)$	0.0206 (39.3)	0.135 (206)
$f_{\hat{\omega},\mathrm{par}}^{\dagger}$	0.0236(11)	0.00129(1.75)	$0.00721\ (13.8)$	0.00712(32.5)	0.00795 (12.2)
p	$0.0591\ (27.7)$	0.016 (30.6)	0.0292 (133)	0.0127(17.1)	0.00124 (1.89)
$\hat{f}_{\hat{\omega},  ext{nonpar}}$	0.126 (58.9)	0.0216 (41.2)	0.0415 (189)	0.0414 (56.1)	0.0213 (32.6)
$\hat{f}_{\hat{\omega},\mathrm{par}}$	0.0285(13.4)	0.00381 (17.4)	$0.0162\ (21.9)$	0.00727 (13.9)	0.00126 (1.93)
$ ilde{f}_{\hat{\omega}_2 ext{nonpar}}$	0.772(362)	0.0174(33.2)	0.0322(147)	0.0207 (28.1)	0.702 (1070)
$\widetilde{f}_{\hat{\omega},\mathrm{par}}$	0.0429 (20.1)	$0.00676 \ (12.9)$	0.0216 (29.2)	0.0119 (18.3)	0.00265 (12.1)

Table 4: Model 4

Estimator	IMSE	$\mathrm{IMSE} \mid Y {<} q_{.25}$	IMSE   $q_{.25} < Y < q_{.5}$	IMSE   $q_{.5} < Y < q_{.75}$	IMSE   $q_{.7}$
(12) $\hat{f}_{\hat{\mu},\text{nonpar}}$	0.0028 (1)	0.00046 (1)	0.00057 (1)	0.00058 (1)	0.0012 (
$(13) \hat{f}_{\mu,\xi}$	8.5e+45 (3e+48)	9.4e+44 (2e+48)	$2.6e+45 \ (4.6e+48)$	2.9e+45 (5e+48)	2.1e+45 (1.7)
(14) $\hat{f}_{\mu,\hat{\xi}}$	$3.1e+46 \ (1.1e+49)$	2.7e+45 (5.8e+48)	8.8e+45 (1.5e+49)	$1.1e+46 \ (1.8e+49)$	8.9e + 45 (7.4)
(15) $\hat{f}_{\hat{\mu},\mathrm{par}}$	0.0032(1.2)	0.00049(1.1)	0.00068(1.2)	0.00053 (0.91)	0.0016 (1
(20) $f_{\hat{\mu},\text{nonpar}}^{\dagger}$	0.003 (1.1)	0.00068 (1.5)	$0.00052 \ (0.92)$	0.00067(1.2)	0.0011 (0.
$(21) f_{\mu,\xi}^{\dagger}$	$0.0026 \ (0.94)$	$0.00043 \ (0.93)$	$0.00054 \ (0.94)$	0.00055 (0.95)	0.0011 (0.
$(22) f^{\dagger}_{\mu,\hat{\xi}}$	0.0027 (0.96)	0.00046 (1)	$0.00054 \ (0.95)$	$0.00056 \ (0.96)$	0.0012 (0.
$(23) f_{\hat{\mu}, \text{par}}^{\dagger}$	$0.0027 \ (0.96)$	0.00046(1)	$0.00054 \ (0.95)$	$0.00056 \ (0.97)$	0.0012 (0.
(25) $f_{\hat{\omega},\text{nonpar}}^{\dagger}$	0.003(1.1)	7e-04 (1.5)	$0.00051 \ (0.89)$	0.00065(1.1)	0.0011 (0.
(26) $f_{\hat{\omega}, par}^{\dagger}$	0.0027 (0.96)	0.00046 (1)	$0.00054 \ (0.95)$	$0.00056 \ (0.96)$	0.0012 (0.
(4) p	0.016(5.8)	0.008 (17)	0.00077(1.4)	0.0035(6.2)	0.004 (3.
f	$0 \ (0)$	0 (0)	0 (0)	0 (0)	0(0)
(hat25) $\hat{f}_{\hat{\omega},\text{nonpar}}$	0.0027 (0.96)	0.00047(1)	$0.00054 \ (0.94)$	$0.00054 \ (0.95)$	0.0012 (0.
(hat26) $\hat{f}_{\hat{\omega}, par}$	0.0032(1.1)	0.00048 (1)	0.00067(1.2)	$0.00052 \ (0.91)$	0.0015(1
$\hat{\mu},  ext{nonpar}$	$0.00043 \ (0.15)$	$0.00041 \ (0.89)$	2.1e-06 (0.0037)	2e-06 (0.0035)	9.8e-06 (0.0
$\mu, \xi$	0 (0)	0 (0)	0 (0)	0 (0)	0(0)
$\mu,\hat{\xi}$	0.0012 (0.43)	0.0012(2.5)	3e-05 (0.053)	$1.4e-05 \ (0.024)$	1.1e-05 (0.0
$\hat{\mu}, \mathrm{par}$	0.00017(0.06)	0.00017(0.36)	7.4e-07 (0.0013)	$3.1e-07 \ (0.00054)$	3.1e-07 (0.0
$\hat{\omega},  ext{nonpar}$	$0.0012 \ (0.42)$	0.0012(2.5)	4e-06 (0.007)	8.1e-07 (0.0014)	6.3e-06 (0.0
$\hat{\omega}$ , par	$0.00016 \ (0.056)$	$0.00016 \ (0.34)$	7.7e-07 (0.0013)	$3.2e-07 \ (0.00055)$	3.1e-07 (0.0

Table 5: Model 5

Estimator	IMSE	IMSE   $Y < q_{.25}$	IMSE   $q_{.25} < Y < q_{.5}$	IMSE   $q_{.5} < Y < q_{.75}$	IMSE   $q_{.75} < Y$
(12) $\hat{f}_{\hat{\mu},\text{nonpar}}$	0.0011(1)	0.00012(1)	2e-04 (1)	0.00023(1)	0.00052(1)
(13) $\hat{f}_{\mu,\xi}$	0.0016(1.5)	0.00015(1.2)	0.00027(1.3)	9.1e-05 (0.39)	0.0011(2.1)
(14) $\hat{f}_{\mu,\hat{\xi}}$	0.0018(1.7)	0.00019(1.6)	0.00029(1.4)	$9.7e-05 \ (0.42)$	0.0012(2.3)
(15) $\hat{f}_{\hat{\mu},\mathrm{par}}$	0.0017(1.6)	0.00023(1.9)	4e-04 (1.9)	$0.00014 \ (0.59)$	0.00098(1.9)
(20) $f_{\hat{\mu},\text{nonpar}}^{\dagger}$	0.0015(1.4)	0.00043 (3.5)	$0.00015 \ (0.73)$	$0.00041\ (1.8)$	$0.00051 \ (0.99)$
$(21) f_{\mu,\xi}^{\dagger}$	$0.00094 \ (0.87)$	9.8e-05 (0.81)	$0.00018 \ (0.86)$	$0.00021\ (0.9)$	$0.00045 \ (0.87)$
(22) $f_{\mu,\hat{\xi}}^{\dagger,\hat{\zeta}}$	$0.001\ (0.96)$	$0.00012 \ (0.99)$	$0.00019 \ (0.94)$	$0.00021 \ (0.91)$	$0.00051 \ (0.98)$
(23) $f_{\hat{\mu}, \text{par}}^{\dagger}$	$0.001 \ (0.94)$	0.00013(1)	0.00019 (0.91)	$0.00022 \ (0.95)$	$0.00048 \ (0.93)$
(25) $f_{\hat{\omega},\text{nonpar}}^{\dagger}$	0.0015(1.4)	0.00046 (3.8)	$0.00014 \ (0.69)$	4e-04 (1.7)	5e-04 (0.97)
(26) $f_{\hat{\omega}, \text{par}}^{\dagger}$	$0.001\ (0.93)$	0.00012(1)	0.00018 (0.89)	0.00022 (0.94)	0.00048 (0.92)
(4) p	0.015(14)	0.0072(59)	0.00052(2.6)	0.0033(14)	0.0037(7.1)
f	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)
(hat25) $\hat{f}_{\hat{\omega},\text{nonpar}}$	0.001 (0.96)	0.00014(1.2)	$0.00019 \ (0.94)$	0.00022 (0.97)	$0.00048 \ (0.92)$
(hat26) $\hat{f}_{\hat{\omega},par}$	0.0017(1.6)	0.00023(1.9)	0.00039(1.9)	$0.00013 \ (0.57)$	0.00097(1.9)
$\hat{\mu}$ , nonpar	$0.00041 \ (0.38)$	0.00039(3.2)	$2.7e-06 \ (0.013)$	$2.8e-06 \ (0.012)$	1e-05 (0.02)
$\mu, \xi$	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)
$\mu,\hat{\xi}$	0.00089(0.83)	0.00086(7)	2.1e-05 (0.1)	$9.2e-06 \ (0.04)$	$7.1e-06 \ (0.014)$
$\hat{\mu}, \mathrm{par}$	0.00016 (0.15)	0.00016(1.3)	$7.7e-07 \ (0.0038)$	$3.3e-07 \ (0.0014)$	$3.2e-07 \ (0.00061)$
$\hat{\omega}$ , nonpar	0.0014(1.3)	0.0014(12)	$4.5e-06 \ (0.022)$	$8.1e-07 \ (0.0035)$	$5.4e-06 \ (0.01)$
$\hat{\omega}$ , par	$0.00015 \ (0.14)$	0.00015 (1.2)	8e-07 (0.0039)	$3.3e-07 \ (0.0014)$	3.1e-07 (6e-04)