3. Math primer and preface to Deep Learning

3.3 Matrix factorization and decomposition

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Introduction to Matrix Decomposition

Introduction to Matrix Decomposition

Matrix decomposition, also called matrix factorization, is the factorization of a matrix into the product of two or more matrices.

- This process helps to optimize solving systems of linear equations; and
- It helps finding eigenvalues and eigenvectors.

Common Types of Matrix Factorization

Common Types of Matrix Factorization

The LU (Lower, Upper) decomposition factors a matrix **A = LU** into a lower triangular **L** and an upper triangular **U** matrix.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \times \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

The factorization improves the process of repeatedly solving a system of linear equations.

Common Types of Matrix Factorization

The QR decomposition is the factorization of a matrix **A = QR**, where **Q** is an orthogonal matrix and **R** is an upper triangular matrix. **A** does not need to be a square matrix.

$$A = \begin{bmatrix} q_{11} & q_{12} & q_{13} & \cdots \\ q_{21} & q_{22} & q_{23} & \cdots \\ q_{31} & q_{32} & q_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \times \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots \\ 0 & a_{22} & a_{23} & \cdots \\ 0 & 0 & a_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

It's an important step for solving linear least squares problems, for computing the eigenvalues and eigenvectors of a matrix and for the SVD decomposition.

HUAWEI

Common Types of Matrix Factorization

The singular value decomposition, or SVD, is a generalization of the eigendecomposition for a **A** non-square matrix. **A** = **UΣV^t** where **U** and **V^t** are orthogonal matrices and called the left and right singular vectors of **A** and **Σ** is a diagonal matrix with positive real numbers on the diagonal known as the singular values of **A**.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \\ a_{51} & a_{52} & a_{53} \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} & 0 & 0 \\ u_{21} & u_{22} & u_{23} & 0 & 0 \\ u_{31} & u_{32} & u_{33} & 0 & 0 \\ u_{41} & u_{42} & u_{43} & 0 & 0 \\ u_{51} & u_{52} & u_{53} & 0 & 0 \end{bmatrix} \times \begin{bmatrix} \sigma_{11} & 0 & 0 \\ 0 & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} v_{11} & v_{21} & v_{31} \\ v_{12} & v_{22} & v_{32} \\ v_{13} & v_{23} & v_{33} \end{bmatrix}$$

$A_{mxn} = U_{mxm}\Sigma_{mxn}V^t_{nxn}$ where m > n

Common Types of Matrix Factorization

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$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{bmatrix} \times \begin{bmatrix} \sigma_{11} & 0 & 0 & 0 & 0 \\ 0 & \sigma_{22} & 0 & 0 & 0 \\ 0 & 0 & \sigma_{33} & 0 & 0 \end{bmatrix} \times \begin{bmatrix} v_{12} & v_{22} & v_{32} & v_{42} & v_{52} \\ v_{11} & v_{21} & v_{31} & v_{41} & v_{51} \\ v_{13} & v_{32} & v_{33} & v_{43} & v_{53} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_{mxn} = U_{mxm} \Sigma_{mxn} V^t_{nxn}$$ where m < n

Uses for Matrix Decomposition

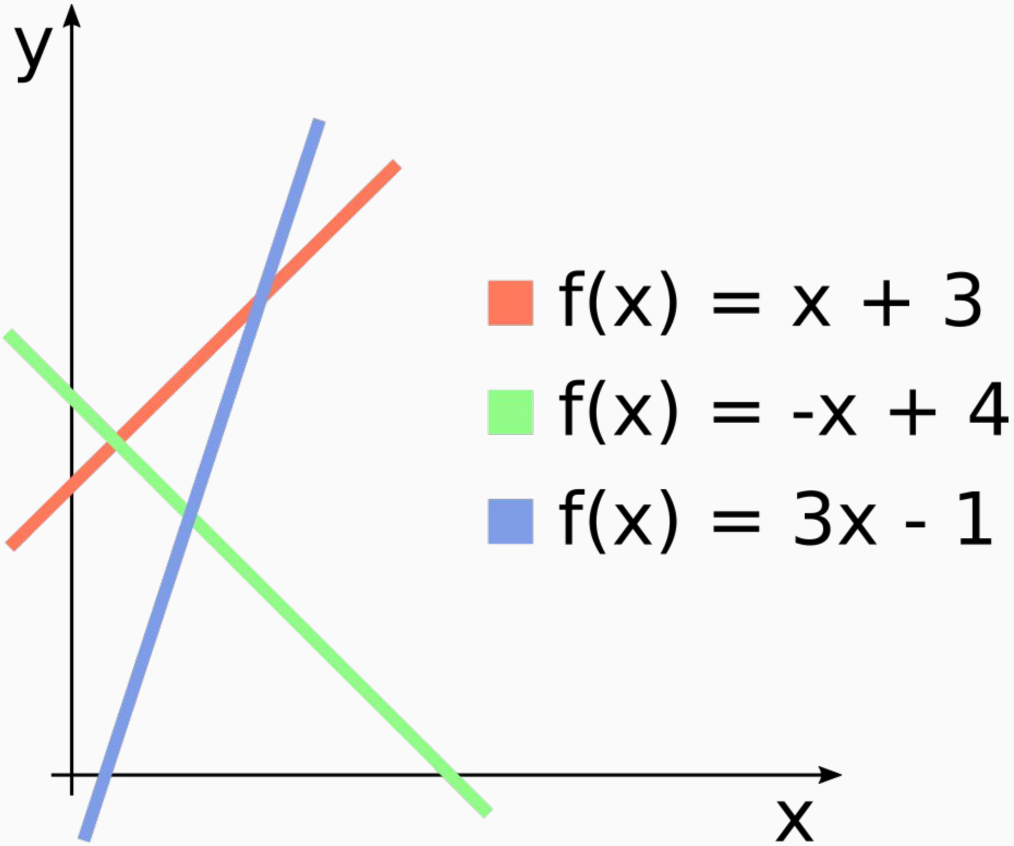
Uses for Matrix Decomposition

In an overdetermined linear system, a system with more equations than the number of variables, linear least squares (LLS) is a method for finding the best-fit solution which minimizes the difference between the data and the corresponding modeled values.

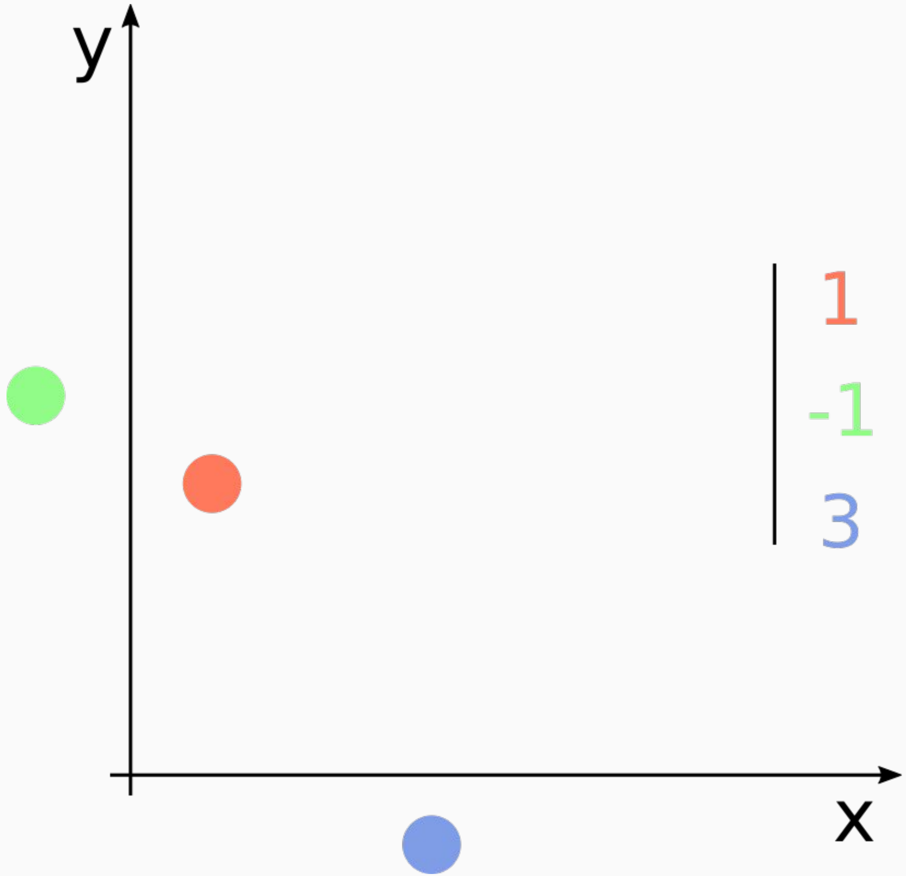
Uses for Matrix Decomposition

On its turn, principal component analysis (PCA) is used to find the best fitting line that minimizes the distance from all data points to a line going through these points.

Uses for Matrix Decomposition

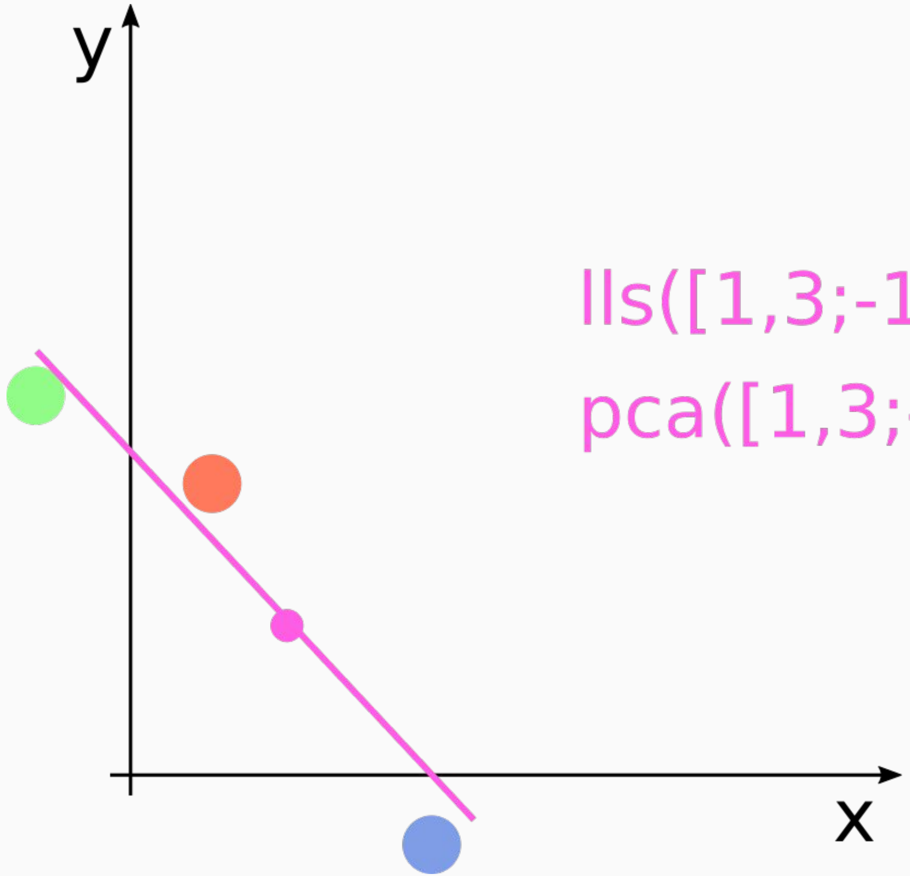


Uses for Matrix Decomposition



$$\begin{vmatrix} 1 & 3 \\ -1 & 4 \\ 3 & -1 \end{vmatrix}$$

Uses for Matrix Decomposition



lls([1,3;-1,4;3,-1],v)

pca([1,3;-1,4;3,-1])