

$$1) f(x) = \begin{cases} cx & 0 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

$$a) \int_0^4 cx \, dx = 1$$

$$\frac{1}{2} cx^2 \Big|_0^4 = 1$$

$$c \left[ \frac{1}{2} [4]^2 - \frac{1}{2} [0]^2 \right] = 1$$

$$c [8 - 0] = 1$$

$$8c = 1$$

$$\boxed{c = \frac{1}{8}}$$

$$b) \frac{1}{8} \int_{-1}^1 x \, dx$$

$$\frac{1}{8} \left( \frac{1}{2} x^2 \right) \Big|_{-1}^1$$

$$\frac{1}{16} \left[ (1)^2 - (-1)^2 \right]$$

$$\frac{1}{16} (1 - 1)$$

$$\frac{1}{16} (0)$$

$$\boxed{0}$$

$$c) \frac{1}{8} \int_2^4 x \, dx$$

$$\frac{1}{16} \left[ x^2 \right]_2^4$$

$$\frac{1}{16} ((4)^2 - (2)^2)$$

$$\frac{1}{16} (16 - 4)$$

$$\frac{12}{16} = \boxed{\frac{3}{4}}$$

$$d) \frac{1}{8} \int_0^3 x \, dx$$

$$\frac{1}{16} (x^2) \Big|_0^3$$

$$\frac{1}{16} (9 - 0)$$

$$\boxed{\frac{9}{16}}$$

$$\frac{1}{8} \int_1^4 x \, dx$$

$$\frac{1}{16} (x^2) \Big|_1^4$$

$$\frac{1}{16} (16 - 1)$$

$$\boxed{\frac{15}{16}}$$

$$\frac{P(x < 3 \cap x > 1)}{P(x > 1)}$$

$$\frac{1/2}{15/16} = \frac{1}{2} \cdot \frac{16}{15}$$

$$\boxed{\frac{8}{15}}$$

$$e) \frac{1}{8} \int_0^4 x^2 \, dx$$

$$\frac{1}{24} (x^3) \Big|_0^4$$

$$\frac{1}{24} ((4)^3 - 0)$$

$$\frac{1}{24} (64)$$

$$\frac{64}{24} = \boxed{\frac{8}{3}}$$

$$E(x)$$

$$f) \frac{1}{8} \int_0^4 x^3 \, dx$$

$$\frac{1}{32} (x^4) \Big|_0^4$$

$$\frac{1}{32} ((4)^4 - 0)$$

$$\frac{256}{32} = \boxed{8} = E(x^2)$$

$$\text{Var}(x) = E(x^2) - (E(x))^2$$

$$= 8 - \left(\frac{8}{3}\right)^2$$

$$8 - \frac{64}{9}$$

$$\frac{72}{9} - \frac{64}{9}$$

$$\boxed{= \frac{8}{9}}$$

$$\sigma_x = \boxed{\frac{\sqrt{8}}{3}}$$

$$g) \frac{1}{8} \int_0^4 x \, dx$$

$$\frac{1}{16} (x^2) \Big|_0^4$$

$$\frac{1}{16} (16 - 0)$$

$$\boxed{F(x) = \frac{1}{16} x^2}$$