

# Daniel Brosch

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## *Curriculum Vitae*

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### Personal Details

Date of birth November 18, 1996  
Place of birth Leverkusen, Germany  
Citizenship German

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### PhD-Thesis

title Symmetry Reduction in Convex Optimization with Applications in Combinatorics  
main supervisor *Etienne de Klerk*  
co-supervisor *Monique Laurent*  
summary We explore different approaches to and applications of symmetry reduction in convex optimization. Using tools from semidefinite programming, representation theory and algebraic combinatorics, we solve or bound hard problems coming from combinatorial optimization, energy minimization, queuing theory, and extremal combinatorics.  
defended on October 19, 2022.

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### Employment

December 2022 — **Postdoc Assistant**, *Department of Mathematics*,  
today *University of Klagenfurt*, Austria, Limited to 6 years  
September 2022 — **Senior Scientist**, *Department of Mathematics*,  
November 2022 *University of Klagenfurt*, Austria  
November 2018 — **PhD-student**, *Tilburg University*, the Netherlands  
July 2022  
January 2020 — **Researcher**, *Centrum Wiskunde & Informatica*, Amsterdam, the  
March 2020 Netherlands  
October 2019 — **Researcher**, *Ortec*, Zoetermeer, the Netherlands  
December 2019

*University of Klagenfurt, Department of Mathematics, room N.2.26  
Universitätsstraße 65–67, 9020 Klagenfurt am Wörthersee, Austria*

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## Education

- 2018 — 2022 **PhD in Mathematics**, *Tilburg University*, the Netherlands  
Under supervision of *Etienne de Klerk* and *Monique Laurent*, as early stage researcher of the Marie-Curie innovative training network MINOA.
- 2017 — 2018 **Mathematics MSc**, *University of Cologne*, *cum laude*  
Thesis: *Semidefinite Bounds for Unequal Error Protection Codes*, under supervision of *Frank Vallentin*.
- 2015 — 2017 **Mathematics BSc**, *University of Cologne*, *cum laude*  
Thesis: *The Banach-Tarski Paradox*, under supervision of *Alexander Lytchak*.
- 2012 — 2015 **Project**, *Schülerinnen und Schüler an der Universität*, University of Cologne  
Project that allowed me to attend university early in parallel to high school.
- 2008 — 2015 **Abitur**, *Otto-Hahn-Gymnasium*, Monheim am Rhein  
Abitur in Mathematics, Physics, Latin, Philosophy

## Personnel Development Courses

- 2024 **Erfolgreich überzeugen, argumentieren und zielgerichtet Fragen stellen**, *Klagenfurt*
- 2024 **Konferenzen stressfrei planen**, *Klagenfurt*
- 2024 **Online-Lehre interaktiv(er) gestalten mit H5P**, *Klagenfurt*
- 2024 **Was ist eigentlich dieses Gendern? Und wie schaut es im Uni-Alltag aus? - Ein Kurzworkshop zu gender- und diversitätssensibler Sprache**, *Klagenfurt*
- 2024 **Wie aktiviere ich Studierende (online)? - Einsatz von Tweedback**, *Klagenfurt*
- 2023 **Effective Presentation in Class and at Conferences**, *Klagenfurt*
- 2020 **Complementary Skills Session on intellectual property rights**, *Tilburg*

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## Papers

### Preprints

- 2024 **Getting to the Root of the Problem: Sums of Squares for Infinite Trees**, joint work with *Diane Puges*, <https://arxiv.org/abs/2404.12838>

Trees can be considered dense objects when we only consider the leaves of the trees to be its vertices. This leads to a natural theory of limits of trees, first considered by Czaparg, Székely and Wagner, allowing us to ask questions of the form "How many copies of a small tree can an infinite tree contain?". We define the flag algebra of binary trees in this setting, and use it to recover all existing bounds on the inducibilities of trees with up to 10 leaves, and compute hundreds of completely new bounds. Finally, we compute first outer approximations of profiles of trees, which encode the possible simultaneous densities of multiple small trees in an infinite tree.

### In Journals

- 2023 **New lower bounds on crossing numbers of  $K_{m,n}$  from semidefinite programming**, *Mathematical Programming*, joint work with *Sven Polak*, <https://doi.org/10.1007/s10107-023-02028-1>

We develop tools based on representation theory to simplify SDP-based bounds for the crossing number of complete bipartite graphs, and improve bounds both in the finite case and in the limit. We also introduce a new, slightly weaker, but computationally more efficient bound for the crossing number of  $K_{m,n}$ , allowing us to compute bounds for bigger parameters  $m$  and  $n$ .

- 2021 **Jordan symmetry reduction for conic optimization over the doubly nonnegative cone: theory and software**, *Optimization Methods and Software*, joint work with *Etienne de Klerk*, <https://doi.org/10.1080/10556788.2021.2022146>

We extend the Jordan Reduction method to the doubly nonnegative cone, and describe a Julia software package implementing it.

- 2021 **Optimizing hypergraph-based polynomials modeling job-occupancy in queueing with redundancy scheduling**, *SIAM Journal on Optimization*, joint work with *Monique Laurent* and *Andries Steenkamp*, <https://doi.org/10.1137/20M1369592>

We show that a family of highly symmetric polynomials is convex, thus (partially) solving a problem coming from queueing with redundancy scheduling. To do this, we exploit the symmetries of the Hessians of the polynomials algebraically.

- 2020 **Minimum energy configurations on a toric lattice as a quadratic assignment problem**, *Discrete Optimization*, joint work with *Etienne de Klerk*, <https://doi.org/10.1016/j.disopt.2020.100612>  
We bound the potential energy of charged particles on an infinite, periodic grid from below, using semidefinite programming and symmetry reduction based on the Jordan Reduction method.

### Work in progress

- 2020– **The symmetries of the gluing algebra of graphs**  
We exploit the symmetries of the SOS and moment hierarchies fully for the class of  $S_n$ -invariant polynomials over the  $k$ -subset-hypercube. This leads to computationally more efficient hierarchies equivalent to Razborov's Flag-SOS hierarchies, and extends their use case to finite and degenerate problems.
- 2021– **An efficient decomposition algorithm for quotients of permutation modules**
- 2021– **Combinatoric derivations: characterizing local and global minimizers in extremal combinatorics**
- 2022– **Möbius-transform based bounds for error correcting codes**, joint work with *Sven Polak*
- 2022– **Generalizing the mixing method**, joint work with *Jan Schwiddessen* and *Angelika Wiegele*
- 2023– **Improved bounds for the Grothendieck constants**, joint work with *Nando Leijenhorst*, *Fernando Oliveira*, *Frank Vallentin*, *Angelika Wiegele*
- 2023– **The graph profile of even cycles**, joint work with *Greg Blekherman*
- 2023– **Lattice paths with moving boundaries**, joint work with *Sarah Selkirk* and *Andrei Asinowski*
- 2024– **New bounds for canonical Ramsey numbers**, joint work with *Bernard Lidický* and *Diane Puges*

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### Academic Service

**Reviews for Journals**, *Mathematical Programming*, *Electronic Journal of Combinatorics*, *Journal of Optimization Theory and Applications*  
**Reviews for Conferences**, *IPCO*  
**Sessions Organized**, *EUROPT*

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## Software

2021– **FlagSOS.jl**

Extendable Julia package for solving fully symmetry-reduced Flag-SOS problems for a variety of combinatorial objects. Available at <https://github.com/DanielBrosch/FlagSOS.jl>

2021 **SDPSymmetryReduction.jl**

Julia package for automatic symmetry reduction of SDPs using the Jordan Reduction method. Available at <https://github.com/DanielBrosch/SDPSymmetryReduction.jl>

## Programming Knowledge

Well familiar with Julia, C/C++, Python, Java and Matlab. Some experience with SageMath, Javascript and C#.

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## Supervision

### PhD-students

2022– **Jan Schwiddessen**, *Semidefinite Programming for Integer Quadratic Problems*, Klagenfurt

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## Funding

2023 **Young Scientist Mentoring**, *University of Klagenfurt*, 3100€

Grant for high-performing early career researchers (competitive selection procedure) to visit or invite their mentor. Mentor: *Greg Blekherman*, *Georgia Tech*

2021 **Talent Grant**, *Tilburg University*

Internal grant for a 9-month contract extension during my PhD (competitive selection procedure).

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## Research Visits

2024 **Iowa State University**, *Ames, USA*, 5 days

Visited *Bernard Lidicky*

2024 **Georgia Tech**, *Atlanta, USA*, 1 month

Visited *Greg Blekherman*

2024 **University of Iowa**, *Iowa City, USA*, 2 days

Visited *Kurt Anstreicher*

2023 **Georgia Tech**, *Atlanta, USA*, 10 days

Visited *Greg Blekherman*

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## Teaching

### University of Klagenfurt

SS 24 **Selected Topics in Optimization: Symmetries and Semidefinite Programming**

Lecture and exercise classes

WS 23 **Algebraische Strukturen**

Lecture and exercise classes

WS 23 **Lineare Algebra 1**

Exercise classes

WS 23 **Computermathematik für das Lehramt**

WS 23 **Preparatory Course for Mathematics**

SS 23 **Lineare Algebra 2**

Exercise classes

SS 23 **Linear Algebra for Engineers**

Exercise classes

WS 22 **Computermathematik für das Lehramt**

WS 22 **Proseminar Diskrete Mathematik**

### Tilburg University

SS 22 **Linear Algebra for Data Science**

Tutorials

WS 21 **Linear Optimization**

Tutorials and computer labs

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## Administrative Activities

2024 **Member of a hiring committee, *Professorship optimization***

2023 **Member of a hiring committee, *Postdoc optimization***

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## Outreach and PR

2024 **Ferialpraktikum**

High school students join us at university and work on projects together. I helped with the supervision.

2023–2024 **Modellierungstage**

Visited multiple schools in Kärnten for day-long workshops on various math related topics.

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## Talks

### Invited Talks

- March 14, 2024 **MoPAT-24: Moments and Polynomials: Applications and Theory**, Konstanz, Germany  
Combinatoric derivations in extremal graph theory and Sidorenko's conjecture.
- February 22, 2024 **Iowa State University**, Ames, USA  
New lower bounds on crossing numbers of  $K_{m,n}$ .
- February 20–21, 2024 **Iowa State University**, Ames, USA  
Mini Course: Symmetry reduction and semidefinite programming
- February 13, 2024 **GT Graph Theory/Combinatorics Seminar**, Georgia Tech, Atlanta, USA  
Combinatoric Derivations in Extremal Graph Theory and Sidorenko's Conjecture.
- February 9, 2024 **Tippie College of Business**, Iowa City, USA  
New lower bounds on crossing numbers of  $K_{m,n}$ .
- November 13, 2023 **Symmetry, Stability, and interactions with Computation**, CIRM, Luminy, France  
Flag Sums of Squares for Sidorenko's Conjecture.
- April 15, 2023 **Meeting on Applied Algebraic Geometry (MAAG) 2023**, Atlanta, USA  
The Flag Algebra of Rooted Binary Trees.
- August 17, 2023 **Mixed-integer Nonlinear Optimization: A Hatchery for Modern Mathematics**, Oberwolfach, Germany  
Is the set of trees convex?
- April 14, 2023 **Georgia Tech Graph Theory & Combinatorics Seminar**, Atlanta, USA  
New lower bounds on crossing numbers of  $K_{m,n}$ .
- February 22, 2023 **Semidefinite optimization approaches to classical and quantum combinatorial optimization**, Cologne, Germany  
SDPs for Extremal Combinatorics.
- December 1, 2022 **Three days of computational methods for extremal discrete geometry**, Cologne, Germany  
New lower bounds on crossing numbers of  $K_{m,n}$ .
- April 12, 2022 **Workshop on Conic Linear Optimization for Computer-Assisted Proofs**, Oberwolfach  
The Symmetries of Flag-Algebras.
- April 1, 2022 **Discrete Math Seminar**, University of Massachusetts Amherst  
Symmetry reduced Flag-hierarchies.

- January 2021 **Oberseminar Reelle Geometrie und Algebra**, Universität Konstanz  
More efficient and flexible flag algebras.
- Contributed Talks**
- July 25, 2024 **ISMP 2024**, Montreal, Canada  
Combinatoric Derivations and Sidorenko's Conjecture.
- June 6, 2024 **Klagenfurt-Berlin Workshop on Multiple Perspectives in Optimization**, Klagenfurt, Austria  
New lower bounds on crossing numbers of  $K_{m,n}$ .
- May 22, 2024 **ISCO 2024: International Symposium on Combinatorial Optimization**, Tenerife, Spain  
Flag Sums of Squares for Sidorenko's Conjecture.
- November 8, 2023 **Doctoral Seminar**, Klagenfurt, Austria  
Möbius Transform Based Bounds for Constant Weight Codes.
- September 19, 2023 **ÖMG Tagung 2023**, Graz, Austria  
Extremal Combinatorics in Julia.
- August 25, 2023 **Europt 2023**, Budapest, Hungary  
The flag algebra of rooted binary trees.
- July 10, 2023 **SIAM Conference on Applied Algebraic Geometry (AG23)**, Eindhoven, the Netherlands  
Möbius Transform Based Bounds for Constant Weight Codes.
- June 2, 2023 **SIAM Conference on Optimization (OP23)**, Seattle, USA  
Flag Sums of Squares for Sidorenko's Conjecture.
- October 23, 2022 **University of Klagenfurt**, Klagenfurt, Austria  
Derivatives in Continuous Combinatorics.
- July 26, 2022 **ICCOPT**, Betlehem, PA, USA  
Moebius-Transform Based Symmetry Reduction for Optimization in Binary Variables.
- March 23, 2022 **Polynomial optimization reading group**, CWI, Amsterdam  
Symmetry reduced Flag-hierarchies.
- August 20, 2021 **SIAM AG21**  
More efficient and flexible Flag-Algebras coming from polynomial optimization.
- July 20–23, 2021 **SIAM OP21**  
More efficient and flexible Flag-Algebras coming from polynomial optimization.
- February 2021 **Virtual OR seminar**, Tilburg University  
More efficient and flexible Flag-Algebras.
- January 2021 **Shared seminar Cologne Oberseminar/CWI reading group**  
More efficient and flexible Flag-Algebras.



- February 26, 2020 **Polynomial optimization reading group**, CWI, Amsterdam  
 and March 4, 2020 A two-part introduction to symmetry reduction for SDPs
- August 7, 2019 **ICCOPT**, Berlin  
 Minimum energy configurations on a toric lattice as a quadratic assignment problem.

## Conferences/Workshops/Summer Schools

- July 21–26, 2024 **ISMP 2024**, Montreal, Canada
- June 6–7, 2024 **Klagenfurt-Berlin Workshop on Multiple Perspectives in Optimization**, Klagenfurt, Austria
- May 22–24, 2024 **ISCO 2024: International Symposium on Combinatorial Optimization**, Tenerife, Spain
- March 11–14, 2024 **MoPAT-24: Moments and Polynomials: Applications and Theory**, Konstanz, Germany
- November 13–17, 2023 **Symmetry, Stability, and interactions with Computation**, CIRM, Luminy, France
- September 18–22, 2023 **ÖMG Tagung 2023**, Graz, Austria
- August 23–25, 2023 **Europt 2023**, Budapest, Hungary
- August 13–18, 2023 **Mixed-integer Nonlinear Optimization: A Hatchery for Modern Mathematics**, Oberwolfach, Germany
- July 10–14, 2023 **SIAM Conference on Applied Algebraic Geometry (AG23)**, Eindhoven, the Netherlands
- May 31–June 3, 2023 **SIAM Conference on Optimization (OP23)**, Seattle, USA
- April 15–16, 2023 **Meeting on Applied Algebraic Geometry (MAAG) 2023**, Atlanta, USA
- February 22–23, 2023 **Semidefinite optimization approaches to classical and quantum combinatorial optimization**, Cologne
- November 30–December 2 **Three days of computational methods for extremal discrete geometry**, Cologne
- September 5–9, 2022 **Final POEMA workshop**, Paris
- July 23–28, 2022 **ICCOPT**, Bethlehem, PA, USA
- June 7–9, 2022 **Nordic Combinatorial Conference (NORCOM)**, Tromsø
- April 10–15, 2022 **Workshop on Conic Linear Optimization for Computer-Assisted Proofs**, Oberwolfach
- June 21–29, 2021 **MINOA Doctoral School 2021**, Online

April 16, 2021 **General Julia training (POEMA)**, Online  
 March 4–5, 2021 **Second MINOA ESR days**, Online  
 March 1–3, 2021 **Annual MINOA Conference 2021**, Online  
 January—March 2021 **POEMA 3<sup>rd</sup> Workshop**, Online  
 November 23–24, 2020 **First MINOA ESR days**, Online  
 October–December 2020 **POEMA 2<sup>nd</sup> Workshop**, Online  
 May 27–September 16, 2020 **POEMA Online Learning Weeks**, Online  
 January 6–10, 2020 **2<sup>nd</sup> MINOA conference**, Aussois, France  
 January 6–10, 2020 **24<sup>th</sup> Workshop on Combinatorial Optimization**, Aussois, France  
 September 9–November 11, 2019 **Interior Point Methods**, *LNMB PhD Course*, Etienne de Klerk, Utrecht, the Netherlands  
 August 5–8, 2019 **6<sup>th</sup> International Conference on Continuous Optimization (IC-COPT)**, Berlin, Germany  
 June 24–28, 2019 **1<sup>st</sup> MINOA PhD school**, *Mixed-Integer Nonlinear Optimization meets Data Science*, Ischia, Italy  
 January 14–16, 2019 **1<sup>st</sup> MINOA conference**, Aussois, France  
 January 14–16, 2019 **23<sup>rd</sup> Workshop on Combinatorial Optimization**, Aussois, France  
 January 7–11, 2019 **44<sup>th</sup> conference on the mathematics of operations research**, Lunteren, the Netherlands  
 November 19–February 18, 2019 **Networks and Semidefinite Programming**, *LNMB PhD Course*, Monique Laurent, Utrecht, the Netherlands  
 2019–2022 **CWI reading group on polynomial optimization**, *hosted by Monique Laurent and Sven Polak*, CWI, Amsterdam  
 2020–2022 **Oberseminar**, *hosted by Frank Vallentin*, Cologne

Last updated on July 31, 2024.