

Daniel Brosch

Curriculum Vitae

Personal Details

Date of birth November 18, 1996

Place of birth Leverkusen, Germany

Citizenship German

PhD-Thesis

title Symmetry Reduction in Convex Optimization with Applications in Combinatorics

main supervisor *Etienne de Klerk*

co-supervisor *Monique Laurent*

summary We explore different approaches to and applications of symmetry reduction in convex optimization. Using tools from semidefinite programming, representation theory and algebraic combinatorics, we solve or bound hard problems coming from combinatorial optimization, energy minimization, queuing theory, and extremal combinatorics.

defended on October 19, 2022.

Employment

December 2022 — **Assistant Professor (non tenure track)**, *Department of Mathematics*,
today *University of Klagenfurt*, Austria, Limited to 6 years

September 2022 — **Senior Scientist**, *Department of Mathematics*,
November 2022 *University of Klagenfurt*, Austria

November 2018 — **PhD-student**, *Tilburg University*, the Netherlands
July 2022

January 2020 — **Researcher**, *Centrum Wiskunde & Informatica*, Amsterdam, the Netherlands
March 2020

October 2019 — **Researcher**, *Ortec*, Zoetermeer, the Netherlands
December 2019

Education

2018 — 2022 **PhD in Mathematics**, *Tilburg University*, the Netherlands
Under supervision of *Etienne de Klerk* and *Monique Laurent*, as early stage researcher
of the Marie-Curie innovative training network MINOA.

- 2017 — 2018 **Mathematics MSc**, University of Cologne, *cum laude*
 Thesis: *Semidefinite Bounds for Unequal Error Protection Codes*, under supervision of Frank Vallentin.
- 2015 — 2017 **Mathematics BSc**, University of Cologne, *cum laude*
 Thesis: *The Banach-Tarski Paradox*, under supervision of Alexander Lytchak.
- 2012 — 2015 **Project**, *Schülerinnen und Schüler an der Universität*, University of Cologne
 Project that allowed me to attend university early in parallel to high school.
- 2007 — 2015 **Abitur**, Otto-Hahn-Gymnasium, Monheim am Rhein
 Abitur in Mathematics, Physics, Latin, Philosophy
 Personnel Development Courses
- 2024 **Erfolgreich überzeugen, argumentieren und zielgerichtet Fragen stellen**,
Klagenfurt
- 2024 **Konferenzen stressfrei planen**, *Klagenfurt*
- 2024 **Online-Lehre interaktiv(er) gestalten mit H5P**, *Klagenfurt*
- 2024 **Was ist eigentlich dieses Gendern? Und wie schaut es im Uni-Alltag aus? - Ein Kurzworkshop zu gender- und diversitätssensibler Sprache**,
Klagenfurt
- 2024 **Wie aktiviere ich Studierende (online)? - Einsatz von Tweedback**,
Klagenfurt
- 2023 **Effective Presentation in Class and at Conferences**, *Klagenfurt*
- 2020 **Complementary Skills Session on intellectual property rights**, *Tilburg*

Papers

Preprints

- 2025 **Lower and Upper Bounds for Small Canonical and Ordered Ramsey Numbers**, joint work with Bernard Lidický, Sydney Miyasaki and Diane Puges, <https://arxiv.org/abs/2511.04364>
 We compute new bounds on three variants of Ramsey numbers of small graphs. They are generalizations for ordered graphs, graphs with infinitely many edge-colors, and both. We use integer programming for lower and upper bounds, flag algebras for upper bounds and obtaining information on potential constructions, and heuristic search algorithms to find them. We recovered many known Ramsey numbers, and found multiple new Ramsey numbers.
- 2025 **The Augmented Mixing Method: Computing High-Accuracy Primal-Dual Solutions to Large-Scale SDPs via Column Updates**, joint work with Jan Schwidder and Angelika Wiegele, <https://arxiv.org/abs/2507.20386>
 We developed a novel SDP-solver based the Burer-Monteiro factorization, modified to only update one column of the factors at a time. Through a novel update rule and direct handling of inequalities, we are able to solve SDPs with millions of inequalities to high precision, which is out of reach of any other solver in the literature.

- 2025 **Lattice paths with dynamic boundary**, joint work with *Andrei Asinowski* and *Sarah Selkirk*, *submitted* (delayed preprint due to personal reasons)
 We consider directed lattice paths where the occurrence of a pattern shifts the boundary upwards or downwards by a certain quantity. We show that their generating functions are algebraic by constructing one-counter automata which describe them, and that they are ultimately equivalent to coloured pattern-avoiding directed lattice paths. We further demonstrate that lattice paths with dynamic boundary exhibit interesting links with other combinatorial structures. As an example, we establish a connection between one such model and shifted Young tableaux: we provide a bijection, and find a bivariate generating function by the vectorial kernel method.
- 2024 **Getting to the Root of the Problem: Sums of Squares for Infinite Trees**, joint work with *Diane Puges*, <https://arxiv.org/abs/2404.12838> (undergoing a minor revision at Mathematical Programming)
 Trees can be considered dense objects when we only consider the leaves of the trees to be its vertices. This leads to a natural theory of limits of trees, first considered by Czabarga, Székely and Wagner, allowing us to ask questions of the form "How many copies of a small tree can an infinite tree contain?". We define the flag algebra of binary trees in this setting, and use it to recover all existing bounds on the inducibilities of trees with up to 10 leaves, and compute hundreds of completely new bounds, including new sharp bounds. Finally, we compute the first outer approximations of profiles of trees, which encode the possible simultaneous densities of multiple small trees in an infinite tree.

In Journals

- 2023 **New lower bounds on crossing numbers of $K_{m,n}$ from semidefinite programming**, *Mathematical Programming*, joint work with *Sven Polak*, <https://doi.org/10.1007/s10107-023-02028-1>
 We develop representation theoretic tools to simplify SDP bounds for the crossing number of complete bipartite graphs, improving bounds both in the finite case and in the limit. We also introduce a new, slightly weaker, but computationally more efficient bound for the crossing number of $K_{m,n}$, allowing us to compute bounds for bigger parameters m and n .
- 2021 **Jordan symmetry reduction for conic optimization over the doubly non-negative cone: theory and software**, *Optimization Methods and Software*, joint work with *Etienne de Klerk*, <https://doi.org/10.1080/10556788.2021.2022146>
 We extend the Jordan Reduction, a method to exploit symmetries based on Jordan algebras, to the doubly nonnegative cone, and provide a Julia implementation of the algorithm.
- 2021 **Optimizing hypergraph-based polynomials modeling job-occupancy in queueing with redundancy scheduling**, *SIAM Journal on Optimization*, joint work with *Monique Laurent* and *Andries Steenkamp*, <https://doi.org/10.1137/20M1369592>
 We show that a family of highly symmetric polynomials is convex, thus (partially) solving a conjecture from queueing theory with redundancy scheduling. To do so, we exploit the symmetries of the Hessians of the polynomials algebraically.

- 2020 **Minimum energy configurations on a toric lattice as a quadratic assignment problem**, *Discrete Optimization*, joint work with Etienne de Klerk, <https://doi.org/10.1016/j.disopt.2020.100612>

We bound the potential energy of charged particles on an infinite, periodic grid from below, using semidefinite programming and symmetry reduction based on the Jordan Reduction method.

Work in progress

- 2020– **The symmetries of the gluing algebra of graphs**
We exploit the symmetries of the SOS and moment hierarchies fully for the class of S_n -invariant polynomials over the k -subset-hypercube. This leads to computationally more efficient hierarchies equivalent to Razborov's Flag-SOS hierarchies, and extends their use case to finite and degenerate problems.
- 2021– **An efficient decomposition algorithm for quotients of permutation modules**
- 2021– **Combinatoric derivations: characterizing local and global minimizers in extremal combinatorics**
- 2022– **Möbius-transform based bounds for error correcting codes**, joint work with Sven Polak
- 2023– **Improved bounds for the Grothendieck constants**, joint work with Nando Leijenhorst, Fernando Oliveira, Frank Vallentin, Angelika Wiegele
- 2023– **The graph profile of even cycles**, joint work with Greg Blekherman
- 2024– **Lower and Upper Bounds for Small Canonical and Ordered Ramsey Numbers**, joint work with Bernard Lidický, Sydney Miyasaki and Diane Puges

Academic Service

Reviews for Journals, *Mathematical Programming*, *Electronic Journal of Combinatorics*, *Journal of Optimization Theory and Applications*, *INFORMS Journal on Computing*

Reviews for Conferences, *IPCO*

Sessions Organized, *EUROPT*, *EUCO*

Local Organizer Committee, *EUCO*

Software

2021– **FlagSOS.jl**

Extendable Julia package for solving fully symmetry-reduced Flag-SOS problems for a variety of combinatorial objects. Available at <https://github.com/DanielBrosch/FlagSOS.jl>

2021 **SDPSymmetryReduction.jl**

Julia package for automatic symmetry reduction of SDPs using the Jordan Reduction method. Available at <https://github.com/DanielBrosch/SDPSymmetryReduction.jl>

Programming Knowledge

Well familiar with Julia, C/C++, Python, Java and Matlab. Some experience with SageMath, Javascript and C#.

Supervision

PhD students

2025– **Johannes Schmucker**, *SynMaxCut: Synergies for Exact Solutions to the Maximum Cut Problem*, Klagenfurt

2022–2025 **Jan Schwiddessen**, *Semidefinite Programming for Integer Quadratic Problems*, Klagenfurt

Master students

2024–2025 **Johannes Schmucker**, *Symmetric Term Sparsity*, Klagenfurt

Funding

2025 **ÖFG Forschungsförderprogramm „Internationale Kommunikation“**, ÖFG, 350€

2025 **Lorentz Center travel grant**, Lorentz Center, Leiden, 600€

2023 **Young Scientist Mentoring**, University of Klagenfurt, 3100€

Grant for high-performing early career researchers (competitive selection procedure) to visit or invite their mentor. Mentor: *Greg Blekherman, Georgia Tech*

2021 **Talent Grant**, Tilburg University

Internal grant for a 9-month contract extension during my PhD (competitive selection procedure).

Research Visits

2025 **UiT The Arctic University of Norway**, Tromsø, USA, 5 days

Visited *Cordian Riener*

2024 **Iowa State University**, Ames, USA, 5 days

Visited *Bernard Lidicky*

2024 **Georgia Tech**, Atlanta, USA, 1 month

Visited *Greg Blekherman*

University of Klagenfurt, Department of Mathematics, room N.2.19

Universitätsstraße 65–67, 9020 Klagenfurt am Wörthersee, Austria

 daniel.brosch@aau.at •  danielbrosch.com

2024 **University of Iowa, Iowa City, USA**, 2 days

Visited *Kurt Anstreicher*

2023 **Georgia Tech, Atlanta, USA**, 10 days

Visited *Greg Blekherman*

Teaching

University of Klagenfurt

WS 25 **Proseminar Diskrete Mathematik**

WS 25 **Diskrete Mathematik**

Lecture and exercise classes

SS 25 **Kombinatorische Strukturen**

Lecture and exercise classes

WS 24 **Diskrete Mathematik**

Lecture and exercise classes

SS 24 **Selected Topics in Optimization: Symmetries and Semidefinite Programming**

Lecture and exercise classes

WS 23 **Algebraische Strukturen**

Lecture and exercise classes

WS 23 **Lineare Algebra 1**

Exercise classes

WS 23 **Computermathematik für das Lehramt**

WS 23 **Preparatory Course for Mathematics**

SS 23 **Lineare Algebra 2**

Exercise classes

SS 23 **Linear Algebra for Engineers**

Exercise classes

WS 22 **Computermathematik für das Lehramt**

WS 22 **Proseminar Diskrete Mathematik**

Tilburg University

SS 22 **Linear Algebra for Data Science**

Tutorials

WS 21 **Linear Optimization**

Tutorials and computer labs

Administrative Activities

2025 **Member of a hiring committee, PhD optimization**

2025 **Member of a hiring committee, Postdoc optimization**

2024 **Member of a hiring committee, Professorship optimization**

2023 **Member of a hiring committee, Postdoc optimization**

University of Klagenfurt, Department of Mathematics, room N.2.19

Universitätsstraße 65–67, 9020 Klagenfurt am Wörthersee, Austria

 daniel.brosch@aau.at •  danielbrosch.com

Outreach and PR

2024 **Ferialpraktikum**

High school students join us at university and work on projects together. I helped with the supervision.

2023–2024 **Modellierungstage**

Visited multiple schools in Kärnten for day-long workshops on various math related topics.

Talks

Invited Talks

- October 16, 2025 **Flag algebras and extremal combinatorics**, American Institute of Mathematics, Pasadena, USA
Derivatives of Flag Algebras (recording [here](#))
- July 11, 2025 **Semidefinite programming: applications & solution methods**, Lorentz Center, Leiden, the Netherlands
Optimality Conditions for Extremal Combinatorics
- June 4, 2025 **Algebra Seminar**, UiT, Tromsø, Norway
Combinatoric Derivations and Sidorenko's Conjecture.
- March 14, 2024 **MoPAT-24: Moments and Polynomials: Applications and Theory**, Konstanz, Germany
Combinatoric derivations in extremal graph theory and Sidorenko's conjecture.
- February 22, 2024 **Iowa State University**, Ames, USA
New lower bounds on crossing numbers of $K_{m,n}$.
- February 20–21, 2024 **Iowa State University**, Ames, USA
Mini Course: Symmetry reduction and semidefinite programming
- February 13, 2024 **GT Graph Theory/Combinatorics Seminar**, Georgia Tech, Atlanta, USA
Combinatoric Derivations in Extremal Graph Theory and Sidorenko's Conjecture.
- February 9, 2024 **Tippie College of Business**, Iowa City, USA
New lower bounds on crossing numbers of $K_{m,n}$.
- November 13, 2023 **Symmetry, Stability, and interactions with Computation**, CIRM, Luminy, France
Flag Sums of Squares for Sidorenko's Conjecture.
- April 15, 2023 **Meeting on Applied Algebraic Geometry (MAAG) 2023**, Atlanta, USA
The Flag Algebra of Rooted Binary Trees.
- August 17, 2023 **Mixed-integer Nonlinear Optimization: A Hatchery for Modern Mathematics**, Oberwolfach, Germany
Is the set of trees convex?
- April 14, 2023 **Georgia Tech Graph Theory & Combinatorics Seminar**, Atlanta, USA
New lower bounds on crossing numbers of $K_{m,n}$.
- February 22, 2023 **Semidefinite optimization approaches to classical and quantum combinatorial optimization**, Cologne, Germany
SDPs for Extremal Combinatorics.

- December 1, 2022 **Three days of computational methods for extremal discrete geometry**,
Cologne, Germany
New lower bounds on crossing numbers of $K_{m,n}$.
- April 12, 2022 **Workshop on Conic Linear Optimization for Computer-Assisted Proofs**,
Oberwolfach
The Symmetries of Flag-Algebras.
- April 1, 2022 **Discrete Math Seminar**, University of Massachusetts Amherst
Symmetry reduced Flag-hierarchies.
- January 2021 **Oberseminar Reelle Geometrie und Algebra**, Universität Konstanz
More efficient and flexible flag algebras.

Contributed Talks

- September 30, 2025 **EUCCO 2025**, Klagenfurt, Austria
Optimality Conditions for Extremal Combinatorics.
- July 23, 2025 **ICCOPT 2025**, Los Angeles, USA
Optimality Conditions for Extremal Combinatorics.
- March 17, 2025 **KoKoKO**, Graz, Austria
Combinatoric Derivations and Sidorenko's Conjecture.
- July 25, 2024 **ISMP 2024**, Montreal, Canada
Combinatoric Derivations and Sidorenko's Conjecture.
- June 6, 2024 **Klagenfurt-Berlin Workshop on Multiple Perspectives in Optimization**,
Klagenfurt, Austria
New lower bounds on crossing numbers of $K_{m,n}$.
- May 22, 2024 **ISCO 2024: International Symposium on Combinatorial Optimization**,
Tenerife, Spain
Flag Sums of Squares for Sidorenko's Conjecture.
- November 8, 2023 **Doctoral Seminar**, Klagenfurt, Austria
Möbius Transform Based Bounds for Constant Weight Codes.
- September 19, 2023 **ÖMG Tagung 2023**, Graz, Austria
Extremal Combinatorics in Julia.
- August 25, 2023 **Europt 2023**, Budapest, Hungary
The flag algebra of rooted binary trees.
- July 10, 2023 **SIAM Conference on Applied Algebraic Geometry (AG23)**, Eindhoven,
the Netherlands
Möbius Transform Based Bounds for Constant Weight Codes.
- June 2, 2023 **SIAM Conference on Optimization (OP23)**, Seattle, USA
Flag Sums of Squares for Sidorenko's Conjecture.
- October 23, 2022 **University of Klagenfurt**, Klagenfurt, Austria
Derivatives in Continuous Combinatorics.
- July 26, 2022 **ICCOPT**, Bethlehem, PA, USA
Moebius-Transform Based Symmetry Reduction for Optimization in Binary Variables.

*University of Klagenfurt, Department of Mathematics, room N.2.19
Universitätsstraße 65–67, 9020 Klagenfurt am Wörthersee, Austria*

✉ daniel.brosch@aau.at • danielbrosch.com

- March 23, 2022 **Polynomial optimization reading group**, CWI, Amsterdam
Symmetry reduced Flag-hierarchies.
- August 20, 2021 **SIAM AG21**
More efficient and flexible Flag-Algebras coming from polynomial optimization.
- July 20–23, 2021 **SIAM OP21**
More efficient and flexible Flag-Algebras coming from polynomial optimization.
- February 2021 **Virtual OR seminar**, Tilburg University
More efficient and flexible Flag-Algebras.
- January 2021 **Shared seminar Cologne Oberseminar/CWI reading group**
More efficient and flexible Flag-Algebras.
- February 26, 2020 and March 4, 2020 **Polynomial optimization reading group**, CWI, Amsterdam
A two-part introduction to symmetry reduction for SDPs
- August 7, 2019 **ICCOPT**, Berlin
Minimum energy configurations on a toric lattice as a quadratic assignment problem.

Conferences/Workshops/Summer Schools

- October 13–17, 2025 **AIM workshop: Flag algebras and extremal combinatorics**, Pasadena, USA
- September 29–October 1, 2025 **EUCO 2025**, Klagenfurt, Austria
- July 19–24, 2025 **ICCOPT 2025**, Los Angeles, USA
- July 7–11, 2025 **Semidefinite programming: applications & solution methods**, Lorentz Center, Leiden, the Netherlands
- March 17, 2025 **KoKoKo**, Graz, Austria
- July 21–26, 2024 **ISMP 2024**, Montreal, Canada
- June 6–7, 2024 **Klagenfurt-Berlin Workshop on Multiple Perspectives in Optimization**, Klagenfurt, Austria
- May 22–24, 2024 **ISCO 2024: International Symposium on Combinatorial Optimization**, Tenerife, Spain
- March 11–14, 2024 **MoPAT-24: Moments and Polynomials: Applications and Theory**, Konstanz, Germany
- November 13–17, 2023 **Symmetry, Stability, and interactions with Computation**, CIRM, Luminy, France
- September 18–22, 2023 **ÖMG Tagung 2023**, Graz, Austria
- August 23–25, 2023 **Europ 2023**, Budapest, Hungary
- August 13–18, 2023 **Mixed-integer Nonlinear Optimization: A Hatchery for Modern Mathematics**, Oberwolfach, Germany
- July 10–14, 2023 **SIAM Conference on Applied Algebraic Geometry (AG23)**, Eindhoven, the Netherlands

- May 31–June 3, 2023 **SIAM Conference on Optimization (OP23)**, Seattle, USA
- April 15–16, 2023 **Meeting on Applied Algebraic Geometry (MAAG) 2023**, Atlanta, USA
- February 22–23, 2023 **Semidefinite optimization approaches to classical and quantum combinatorial optimization**, Cologne
- November 30–December 2 **Three days of computational methods for extremal discrete geometry**, Cologne
- September 5–9, 2022 **Final POEMA workshop**, Paris
- July 23–28, 2022 **ICCOPT**, Bethlehem, PA, USA
- June 7–9, 2022 **Nordic Combinatorial Conference (NORCOM)**, Tromsø
- April 10–15, 2022 **Workshop on Conic Linear Optimization for Computer-Assisted Proofs**, Oberwolfach
- June 21–29, 2021 **MINOA Doctoral School 2021**, Online
- April 16, 2021 **General Julia training (POEMA)**, Online
- March 4–5, 2021 **Second MINOA ESR days**, Online
- March 1–3, 2021 **Annual MINOA Conference 2021**, Online
- January—March 2021 **POEMA 3rd Workshop**, Online
- November 23–24, 2020 **First MINOA ESR days**, Online
- October–December 2020 **POEMA 2nd Workshop**, Online
- May 27– September 16, 2020 **POEMA Online Learning Weeks**, Online
- January 6–10, 2020 **2nd MINOA conference**, Aussois, France
- January 6–10, 2020 **24th Workshop on Combinatorial Optimization**, Aussois, France
- September 9– November 11, 2019 **Interior Point Methods**, LNMB PhD Course, Etienne de Klerk, Utrecht, the Netherlands
- August 5–8, 2019 **6th International Conference on Continuous Optimization (ICCOPT)**, Berlin, Germany
- June 24–28, 2019 **1st MINOA PhD school**, *Mixed-Integer Nonlinear Optimization meets Data Science*, Ischia, Italy
- January 14–16, 2019 **1st MINOA conference**, Aussois, France
- January 14–16, 2019 **23rd Workshop on Combinatorial Optimization**, Aussois, France
- January 7–11, 2019 **44th conference on the mathematics of operations research**, Lunteren, the Netherlands
- November 19– February 18, 2019 **Networks and Semidefinite Programming**, LNMB PhD Course, Monique Laurent, Utrecht, the Netherlands
- 2019–2022 **CWI reading group on polynomial optimization**, *hosted by Monique Laurent and Sven Polak*, CWI, Amsterdam
- 2020–2022 **Oberseminar**, *hosted by Frank Vallentin*, Cologne

*University of Klagenfurt, Department of Mathematics, room N.2.19
Universitätsstraße 65–67, 9020 Klagenfurt am Wörthersee, Austria*

 daniel.brosch@aau.at •  danielbrosch.com

Last updated on November 7, 2025.

*University of Klagenfurt, Department of Mathematics, room N.2.19
Universitätsstraße 65–67, 9020 Klagenfurt am Wörthersee, Austria*

✉ daniel.brosch@aau.at •  danielbrosch.com

11/11