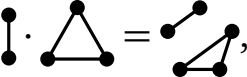


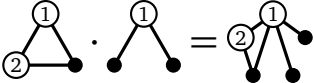
$$ex \left( \begin{array}{c} \bullet \\ \text{---} \\ \bullet \end{array} \right) := \max_c \left\{ \phi \left( \begin{array}{c} \bullet \\ \text{---} \\ \bullet \end{array} \right) : \phi \left( \begin{array}{c} \bullet \\ \text{---} \\ \bullet \end{array} \right) = 0 \right\}.$$

$$\text{ex}\left(\begin{array}{c} \text{orange path of length 2} \\ \text{green triangle} \end{array}\right) \geq \frac{1}{2},$$

$$\phi \left( \begin{array}{c} \bullet \\ | \\ \bullet \end{array} \right) \phi \left( \begin{array}{c} \bullet \\ / \quad \backslash \\ \bullet \quad \bullet \end{array} \right) = \phi \left( \begin{array}{c} \bullet \\ / \quad \backslash \\ \bullet \quad \bullet \\ | \quad | \\ \bullet \quad \bullet \end{array} \right) \cdot$$







$$\left(1\cancel{0}-2\begin{array}{c}\textcircled{1}\\\hline\bullet\end{array}\right)^2=\cancel{0}-4\begin{array}{c}\textcircled{1}\\\hline\bullet\end{array}+4\begin{array}{c}\textcircled{1}\\\hline\bullet\quad\bullet\end{array}\geq 0.$$

$$\left[ \left( 1\emptyset - 2 \begin{array}{c} \textcircled{1} \\ | \\ \bullet \end{array} \right)^2 \right] = \emptyset - 4 \begin{array}{c} \bullet \\ | \\ \bullet \end{array} + 4 \begin{array}{c} \bullet \\ / \quad \backslash \\ \bullet \quad \bullet \end{array} \geq 0.$$

$$\frac{1}{2} - \text{diagram} = \left[ \frac{1}{2} \left( 1 \text{diagram} - 2 \text{diagram} \right)^2 \right] + \left[ \left( \text{diagram} - \text{diagram} - \text{diagram} + \text{diagram} \right)^2 \right] - \text{diagram} \geq 0.$$

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$$H \text{ --- } \begin{array}{c} \bullet \\ | \\ \bullet \end{array} |E(H)| \geq 0.$$



$$\lim_{v \rightarrow 0} \frac{f(x + hv) - f(x)}{h} = f'(x)$$

$$\sigma_v^H := \lim_{i \rightarrow \infty} \frac{(H \text{ density in } (G_i - v)) - (H \text{ density in } G_i)}{i-1}$$

$$\partial_1 \text{ (edge)} = 2 \text{ (edge)} - 2 \text{ (edge with top vertex labeled 1)}$$

$$\partial_1 \text{ (triangle)} = 3 \text{ (triangle)} - 2 \text{ (triangle with bottom-left vertex labeled 1)} - \text{ (triangle with top vertex labeled 1)}$$

$$d_H^e := \lim_{i \rightarrow \infty} \frac{(H \text{ density in } (G_i - e)) - (H \text{ density in } G_i)}{i-2}$$

$$\partial_e \text{ (edge) } = -2 \text{ (edge with nodes 1 and 2)}$$

$$\partial_e \text{ (V-shape) } = -2 \text{ (V-shape with nodes 1 and 2 at top)} - 2 \text{ (V-shape with nodes 1 and 2 at bottom)}$$

$$\partial_e \text{ (triangle) } = -6 \text{ (triangle with nodes 1 and 2 at top and bottom-left vertices)}$$

$$\left[ \begin{array}{c} \textcircled{1} \\ \textcircled{3} \end{array} a_{\text{swap}}(P_3 - e^3) \right] = \text{graph 1} + \text{graph 2} - 2 \text{graph 3} \geq 0$$

The equation shows the difference of two graphs,  $P_3$  and  $e^3$ , multiplied by  $a_{\text{swap}}$ , and then the result is compared to zero. The graphs are represented by black dots (vertices) and black lines (edges).

Graph 1 (Left): A path of 3 vertices. The top vertex is labeled  $\textcircled{1}$  and the bottom vertex is labeled  $\textcircled{3}$ .

Graph 2 (Middle): A path of 3 vertices. The top vertex is labeled  $\textcircled{1}$  and the bottom vertex is labeled  $\textcircled{3}$ .

Graph 3 (Right): A path of 3 vertices. The top vertex is labeled  $\textcircled{1}$  and the bottom vertex is labeled  $\textcircled{3}$ .

But  $\begin{array}{c} \bullet \\ \diagup \quad \diagdown \\ \bullet \text{---} \bullet \\ \diagdown \quad \diagup \\ \bullet \end{array} - \begin{array}{c} \bullet \text{---} \bullet \\ \diagup \quad \diagdown \\ \bullet \quad \bullet \\ \diagdown \quad \diagup \\ \bullet \text{---} \bullet \end{array} = \text{SOS} \geq 0$  and  $\begin{array}{c} \bullet \\ \diagup \quad \diagdown \\ \bullet \text{---} \bullet \\ \diagdown \quad \diagup \\ \bullet \end{array} - \begin{array}{c} \bullet \text{---} \bullet \\ \diagup \quad \diagdown \\ \bullet \quad \bullet \\ \diagdown \quad \diagup \\ \bullet \text{---} \bullet \end{array} = \text{SOS} \geq 0.$

Thus





This proves

