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Temporal Graphs

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Motivation

Clip: School day

Google Maps

How to represent time in graphs?

Time matters!

- not transitive

How to model temporal graphs

Definition labeled graphs

Definition

A **labeled graph** [1, page 94] is a triple $G = (V, E, \lambda)$ where:

- V, E is a graph
- $\lambda: V \cup E \rightarrow Z$ is a mapping of nodes and edges to a set of labels Z

Definition temporal graphs

Definition

A **temporal graph** [2, page 243] is is triple $G = (V, E, \lambda)$ where:

- V, E is a graph
- $\lambda: E \to 2^{\mathbb{N}}$ is a mapping edges to a set natural numbers (time steps when this edge is active)

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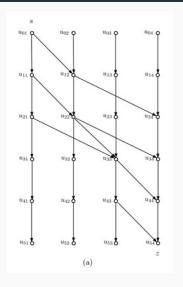
Notation for convenience \rightarrow [2, p. 243ff]

- $\lambda(G)$ temporal graph with respect to G
- $\lambda(E)$ multiset of all labels
- $|\lambda| = \sum_{e \in E} |\lambda(e)|$
- $\lambda_{min} = min\{I \in \lambda(E)\}$
- $\lambda_{max} = max\{I \in \lambda(E)\}$
- $\alpha(\lambda) = \lambda_{max} \lambda_{min} + 1$ lifetime of a temporal graph $\lambda(G)$

Notation 2

- A temporal graph D is a an ordered set of disjoint sets (V, A)
- $A \subseteq V^2 \times \mathbb{N}$ 'time edges'
- $A(t) = \{e | (e, t) \in A\}$ set of edges at time t
- D(t) = (V, A(t)) snapshot of graph D at time t

Static expansion of a temporal graph



[2, page 318] ₁₀

Static expansion of a temporal graph

Definition: static expansion of a graph

The static expansion of a temporal graph D=(V,A) with $V=\{u_1,u_2,...,u_n\}$ is a DAG H=(S,E) with:

$$S = \{u_{ij} | \lambda_{min} - 1 \le i \le \lambda_{max}, 1 \le j \le n\}$$

and

$$E = \{(u_{(i-1)j}, u_{ij'}) | \lambda_{min} \le i \le \lambda_{max} \land$$
$$1 \le j, j' \le n \land (j = j' \lor (u_j, u_{j'}) \in A(i)))\}$$

Journeys

Definition: temporal/time respecting walk

A **temporal** or **time-respecting walk** W of a temporal graph D = (V, A) is an alternating sequence of of nodes and times $(u_1, t_1, u_2, t_2, ..., u_{k1}, t_{k1}, u_k)$ where

- $\forall 1 \leq i \leq k-1 : ((u_i,u_{i+1}),t_i) \in A$ and
- $1 \le i \le k2 : t_i < t_{i+1}$
- t₁ departure time
- t_{k-1} arrival time
- $t_{k-1} t_1 + 1$ duration/temporal length

Journeys

Definition: Journey

A **journey** is a is a temporal walk with pairwise distinct nodes $\hat{}$ a journey of D is a path of the underlying static graph of D that uses strictly increasing edge-labels.

Definition: Foremost Journey

A u-v journey J is called foremost from time t IN if it departs after time t and its arrival time is minimized.

Teasers

Graph Neural Networks

References

- [1] Swarnendu Ghosh, Nibaran Das, Teresa Gonçalves, Paulo Quaresma, and Mahantapas Kundu. The journey of graph kernels through two decades. *Computer Science Review*, 27:88–111, 2018.
- [2] Othon Michail. An Introduction to Temporal Graphs: An Algorithmic Perspective, pages 308–343. Springer International Publishing, Cham, 2015.