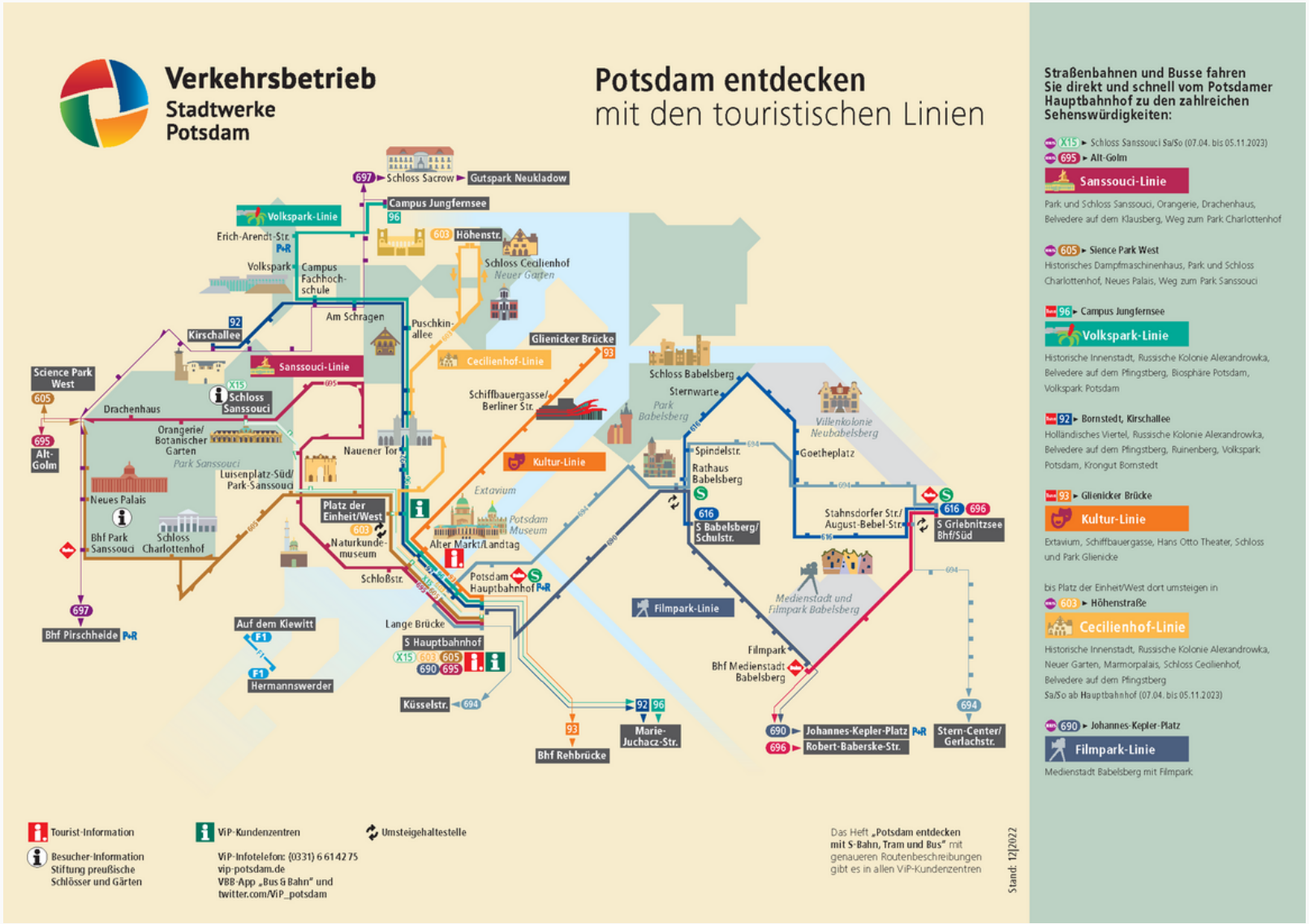

Source: https://www.swp-potsdam.de/content/verkehr/bilder_6/liniennetz/touristischer_liniennetzplan_screenshot_1280_960.jpg

Temporal Graphs

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Motivation

Clip: School day

Google Maps

How to represent time in graphs?

Time matters!

- not transitive

How to model temporal graphs

Definition labeled graphs

Definition

A **labeled graph** [1, page 94] is a triple $G = (V, E, \lambda)$ where:

- V, E is a graph
- $\lambda : V \cup E \to Z$ is a mapping of nodes and edges to a set of labels Z

Definition temporal graphs

Definition

A **temporal graph** [2, page 243] is is triple $G = (V, E, \lambda)$ where:

- V, E is a graph
- $\lambda : E \to 2^{\mathbb{N}}$ is a mapping edges to a set natural numbers (time steps when this edge is active)

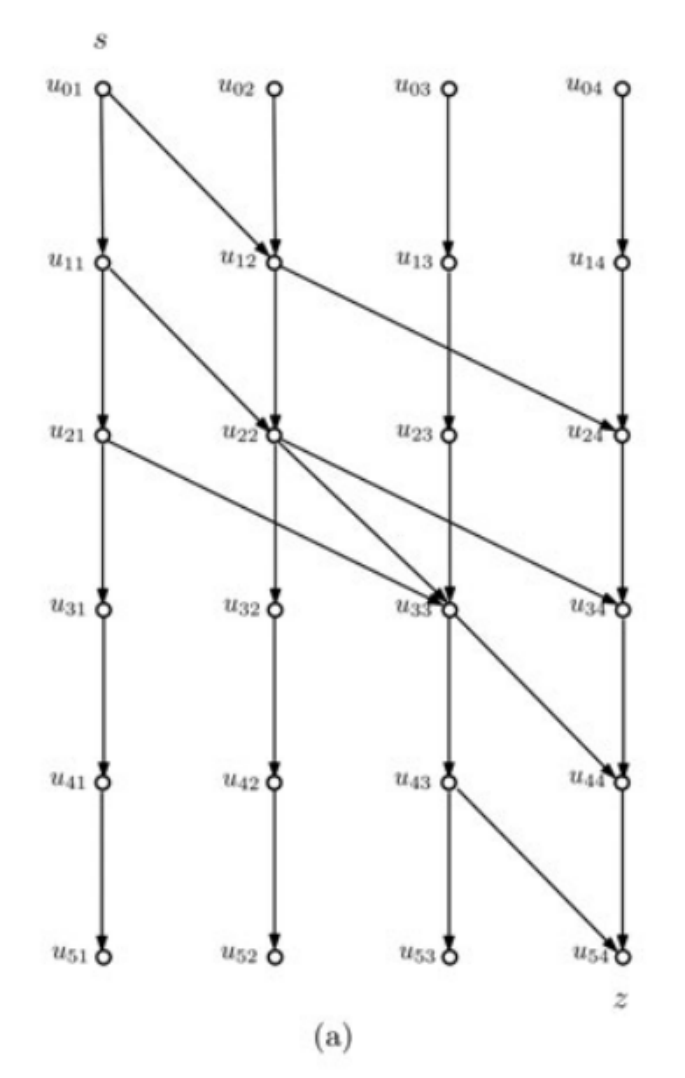
Notation for convenience $\rightarrow$ [2, p. 243ff]

- $\lambda(G)$ - temporal graph with respect to G
- $\lambda(E)$ - multiset of all labels
- $|\lambda| = \sum_{e \in E} |\lambda(e)|$
- $\lambda_{min} = min\{l \in \lambda(E)\}$
- $\lambda_{max} = max\{l \in \lambda(E)\}$
- $\alpha(\lambda) = \lambda_{max} - \lambda_{min} + 1$ - lifetime of a temporal graph $\lambda(G)$

Notation 2

- A temporal graph D is a an ordered set of disjoint sets (V, A)
- $A \subseteq V^2 \times \mathbb{N}$ - 'time edges'
- $A(t) = \{e|(e, t) \in A\}$ - set of edges at time t
- $D(t) = (V, A(t))$ - snapshot of graph D at time t

Static expansion of a temporal graph



[2, page 318]

Static expansion of a temporal graph

Definition: static expansion of a graph

The static expansion of a temporal graph $D = (V, A)$ with $V = \{u_1, u_2, ..., u_n\}$ is a DAG $H = (S, E)$ with:

$$S = \{u_{ij} | \lambda_{min} - 1 \leq i \leq \lambda_{max}, 1 \leq j \leq n\}$$

and

$$E = \{(u_{(i-1)j}, u_{ij'}) | \lambda_{min} \leq i \leq \lambda_{max} \wedge$$

$$1 \leq j, j' \leq n \wedge (j = j' \vee (u_j, u_{j'}) \in A(i)))\}$$

Journeys

Teasers

Graph Neural Networks

References

[1] Swarnendu Ghosh, Nibaran Das, Teresa Gonçalves, Paulo Quaresma, and Mahantapas Kundu. The journey of graph kernels through two decades. *Computer Science Review*, 27:88–111, 2018.

[2] Othon Michail. *An Introduction to Temporal Graphs: An Algorithmic Perspective*, pages 308–343. Springer International Publishing, Cham, 2015.