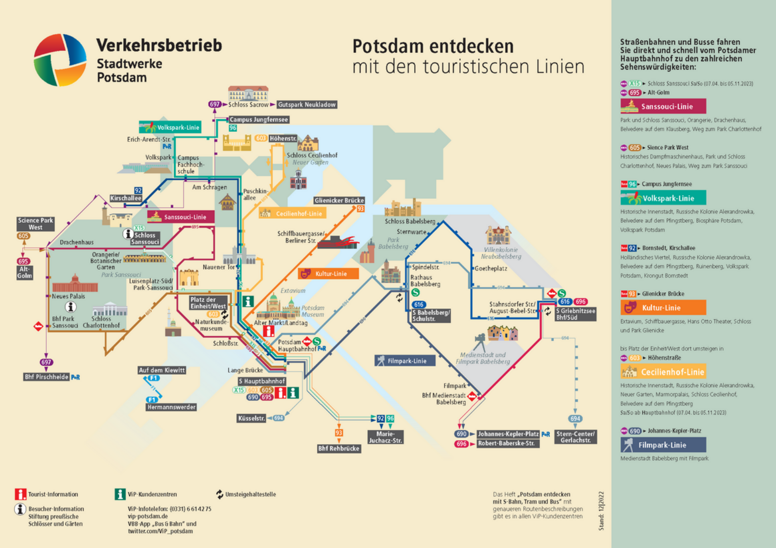


Source: `https://www.swp-potsdam.de/content/verkehr/bilder_6/liniennetz/touristischer_liniennetzplan_screenshot_1280_960.jpg`

Temporal Graphs

Daniel Cermann

January 28, 2025

Hasso Plattner Institute

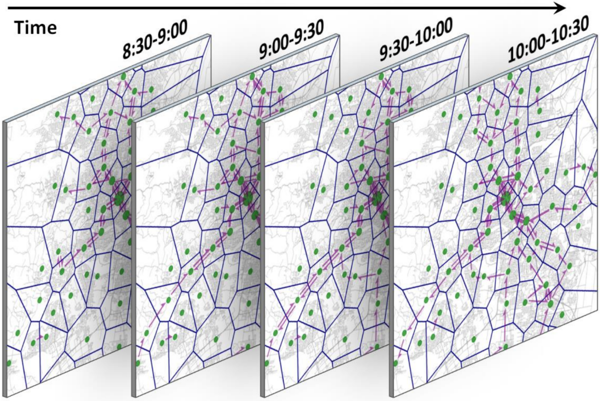
Motivation

Clip: School day

https://youtu.be/BSNJSUkc5-Q?t=996

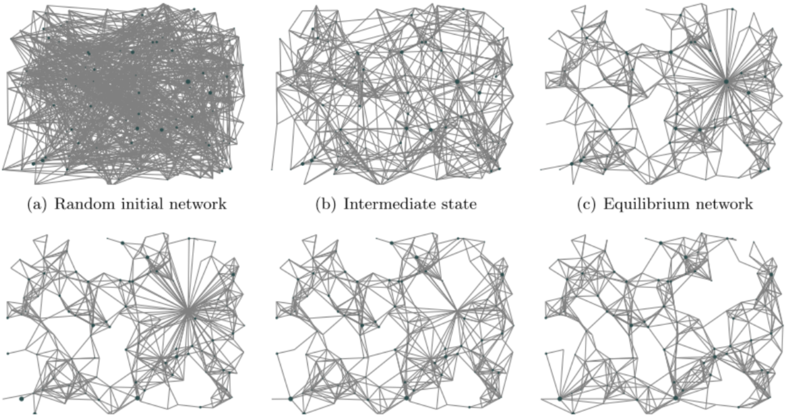
Google Maps

Figure 1: Illustration of a trajectory flow map, a dynamic graph of aggregated traffic flows constructed from trajectory data. The presented example is based on bus passenger trajectories obtained in Brisbane, Australia.



Source: `https://australasiantransportresearchforum.org.au/wp-content/uploads/2022/03/ATRF2016_paper_166.pdf`

Distributed systems



(a) Random initial network (b) Intermediate state (c) Equilibrium network

(d) Change of node capacities (e) Intermediate state (f) Equilibrium network

Source: `https://www.sg.ethz.ch/publications/2012/scholtes2012organic-design-of/`

Temporal graphs for physical/chemical models

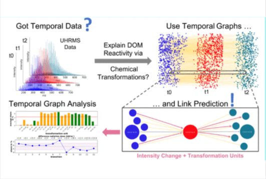
DATA SCIENCE | May 9, 2023

A Temporal Graph Model to Predict Chemical Transformations in Complex Dissolved Organic Matter

Philipp Plamper, Oliver J. Lechtenfeld*, Peter Herzsprung, and Anika Groß*

Open PDF | Supporting Information (1) | ACCESS

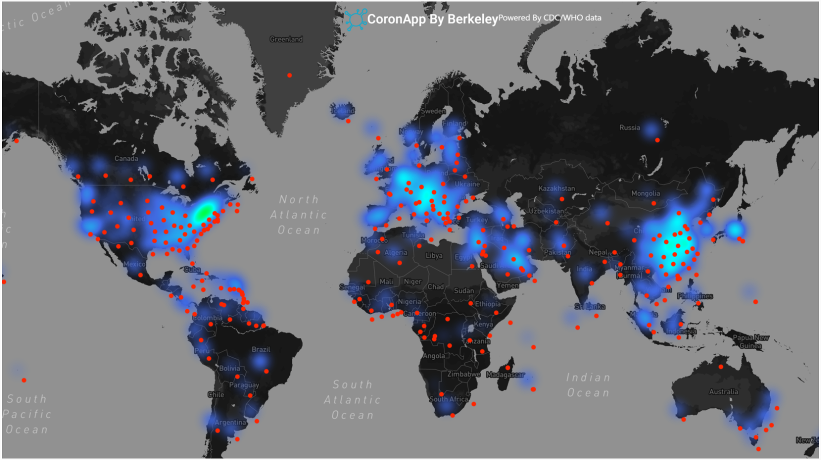
Abstract



Dissolved organic matter (DOM) is a complex mixture of thousands of natural molecules that undergo constant transformation in the environment, such as sunlight induced photochemical reactions. Despite molecular level resolution from ultrahigh resolution mass spectrometry (UHRMS), trends of mass peak intensities are currently the only way to follow photochemically induced molecular changes in DOM. Many real-world relationships and temporal processes can be intuitively modeled using graph data structures (networks). Graphs enhance the potential and value of AI applications by adding context and interconnections allowing the uncovering of hidden or unknown relationships in data sets. We use a temporal graph model and link prediction to identify transformations of DOM molecules in a photo-oxidation experiment. Our link prediction algorithm simultaneously considers educt removal and product formation for molecules linked by predefined transformation units (oxidation, decarboxylation, etc.). The transformations are further weighted by the extent of intensity change and clustered on the graph structure to identify groups of similar reactivity. The temporal graph is capable of identifying relevant molecules subject to similar reactions and enabling to study their time course. Our approach overcomes previous data evaluation limitations for mechanistic studies of DOM and leverages the potential of temporal graphs to study DOM reactivity by UHRMS.

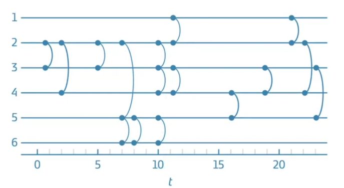
Source: `https://pubs.acs.org/doi/full/10.1021/acs.est.3c00351`

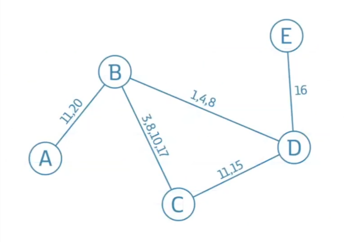
Dissemination processes

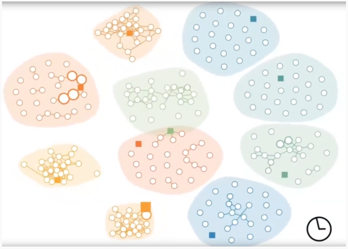


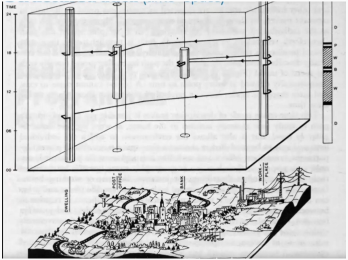
Source: `https://engineering.berkeley.edu/wp-content/uploads/2020/03/CoronApp.png`

How to represent time in graphs?





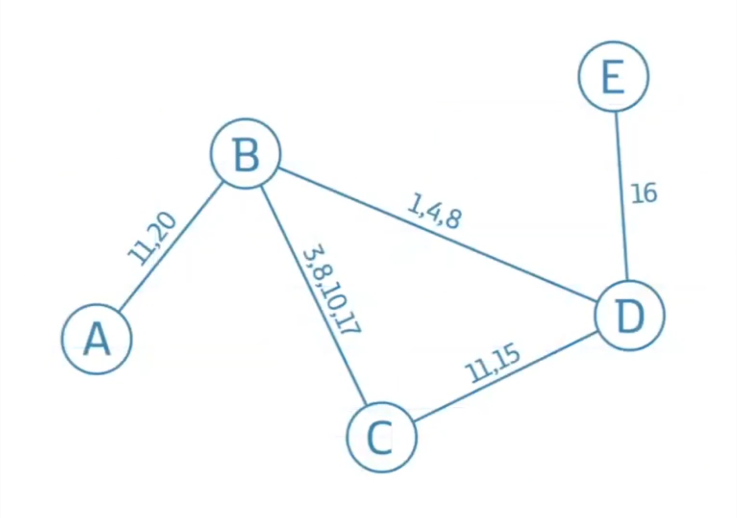




Source: `https://www.youtube.com/watch?v=BSNJSUkc5-Q`

How to model temporal graphs

How to model temporal graphs



Source: `https://www.youtube.com/watch?v=BSNJSUkc5-Q`

Definition labeled and temporal graphs

Definition

A **labeled graph** [1, page 94] is a triple $G = (V, E, \lambda)$ where:

- V, E is a graph
- $\lambda : V \cup E \to Z$ is a mapping of nodes and edges to a set of labels Z

Definition labeled and temporal graphs

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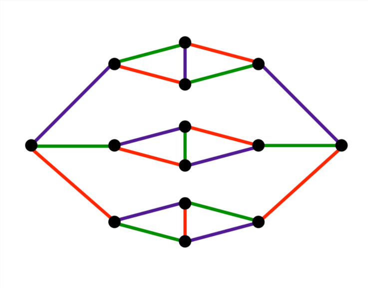
Definition

A **temporal graph** [2, page 243] is is triple $G = (V, E, \lambda)$ where:

- V, E is a graph
- $\lambda : E \to 2^{\mathbb{N}}$ is a mapping edges to a set natural numbers (time steps when this edge is active)

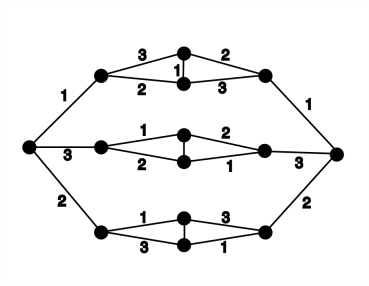
Relationship labeled and temporal graphs



[a]

[a] Source: `https://www.algorist.com/images/figures/edge-coloring-R.png`

$\leftrightarrow$



[a]

[a] Source: `me:)`

Notation for convenience $\rightarrow$ [2, p. 243ff]

- $\lambda(G)$ - temporal graph with respect to G
- $\lambda(E)$ - multiset of all labels
- $|\lambda| = \sum_{e \in E} |\lambda(e)|$
- $\lambda_{min} = min\{l \in \lambda(E)\}$
- $\lambda_{max} = max\{l \in \lambda(E)\}$
- $\alpha(\lambda) = \lambda_{max} - \lambda_{min} + 1$ - lifetime of a temporal graph $\lambda(G)$

Transitivity of reachability in static graphs

Reachability in a static graph is transitive

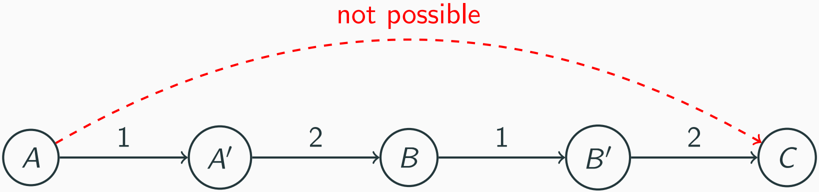
Given A static graph $G = (V, E)$, for all nodes $A, B, C \in V$ we have: If B is reachable by A and C is reachable by B, then C is reachable by A.

$$A \longrightarrow B \longrightarrow C$$

Transitivity of reachability in static graphs

Is reachability in a temporal graph transitive?

Time matters!

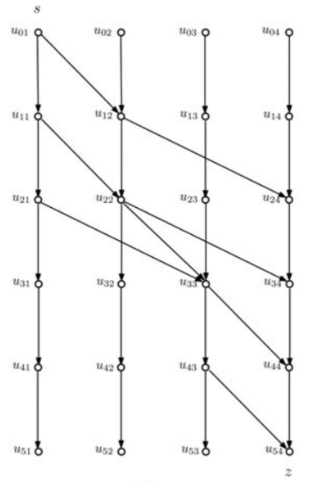


$\Longrightarrow$ Deep implications for complexity of temporal graphs

Notation #2

- A temporal graph D is an ordered set of disjoint sets (V, A)
- $A \subseteq V^2 \times \mathbb{N}$ - 'time edges'
- $A(t) = \{e | (e, t) \in A\}$ - set of edges at time t
- $D(t) = (V, A(t))$ - snapshot of graph D at time t

Static expansion of a temporal graph



(a)

[2, page 318]

Static expansion of a temporal graph

Definition: static expansion of a graph

The static expansion of a temporal graph $D = (V, A)$ with $V = \{u_1, u_2, ..., u_n\}$ is a DAG $H = (S, E)$ with:

$$S = \{u_{ij} | \lambda_{min} - 1 \leq i \leq \lambda_{max}, 1 \leq j \leq n\}$$

and

$$E = \{(u_{(i-1)j}, u_{ij'}) | \lambda_{min} \leq i \leq \lambda_{max} \wedge$$

$$1 \leq j, j' \leq n \wedge (j = j' \vee (u_j, u_{j'}) \in A(i)))\}$$

Repetition - walks and paths in static graphs

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- A **walk** is a finite or infinite sequence of edges which joins a sequence of vertices.

Repetition - walks and paths in static graphs

- A **walk** is a finite or infinite sequence of edges which joins a sequence of vertices.
- A **path** is a walk where all vertices are distinct.

Journeys

Definition: temporal/time respecting walk

A **temporal** or **time-respecting walk** W of a temporal graph $D = (V, A)$ is an alternating sequence of of nodes and times $(u_1, t_1, u_2, t_2, ..., u_{k-1}, t_{k-1}, u_k)$ where

- $\forall 1 \leq i \leq k-1 : ((u_i, u_{i+1}), t_i) \in A$ and
- $1 \leq i \leq k-2 : t_i < t_{i+1}$

- t_1 - departure time
- t_{k-1} arrival time
- $t_{k-1} - t_1 + 1$ - duration/temporal length

Journeys

Definition: Journey

A **journey** is a is a temporal walk with pairwise distinct nodes $\widehat{=}$ a journey of D is a path of the underlying static graph of D that uses strictly increasing edge-labels.

Journeys

Definition: Journey

A **journey** is a is a temporal walk with pairwise distinct nodes $\hat{=}$ a journey of D is a path of the underlying static graph of D that uses strictly increasing edge-labels.

Definition: Foremost Journey

A u-v journey J is called foremost from time $t \in \mathbb{N}$ if it departs after time t and its arrival time is minimized.

Journeys

Definition: Temporal distance

The **temporal distance** from a node u to at time t to a node v is defined as the duration of a foremost journey from u to v that departs at time t.

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The **temporal distance** from a node u to at time t to a node v is defined as the duration of a foremost journey from u to v that departs at time t.

Definition: Temporal diameter d

The minimum integer d such that there exists a foremost journey from every node $(u, t) \in V \times \{0, 1, ..., \alpha - d\}$ to every node $v \in V$ with duration at most d.

Computing foremost journeys - Problem formulation

Given a source node $s \in V$ and a start time t_{start} compute the foremost $s - w$ journey for all $w \in V \backslash \{s\}$

Sidenote - offline vs. online algorithms

offline algorithms

takes whole temporal graph D as input

online algorithms

temporal graph is revealed to algorithm over time

Source: `https://encrypted-tbn0.gstatic.com/images?q=tbn:ANd9GcQbpwyZnBM48yDEHLT9Sww1J8AJrgs4lj-VLi2EkxpIoKMeI6-stF-R9uAsLl5K4gXFBts&usqp=CAU`

Computing foremost journeys - Algorithm

Algorithm 1: Computing earliest-arrival time

Input : A temporal graph $G = (V, E)$ in its edge stream representation, source vertex x, time interval $[t_\alpha, t_\omega]$

Output : The earliest-arrival time from x to every vertex $v \in V$ within $[t_\alpha, t_\omega]$

1 Initialize $t[x] = t_\alpha$, and $t[v] = \infty$ for all $v \in V \setminus \{x\}$;
2 **foreach** *incoming edge* $e = (u, v, t, \lambda)$ *in the edge stream* **do**
3   **if** $t + \lambda \leq t_\omega$ *and* $t \geq t[u]$ **then**
4   **if** $t + \lambda < t[v]$ **then**
5             $t[v] \leftarrow t + \lambda$;
6   **else if** $t \geq t_\omega$ **then**
7   Break the for-loop and go to Line 8;
8 **return** $t[v]$ for each $v \in V$;

[3, page 724]

Computing foremost journeys - Proof of correctness

Computing foremost journeys - Running time

Reachability

Definition: Reachability

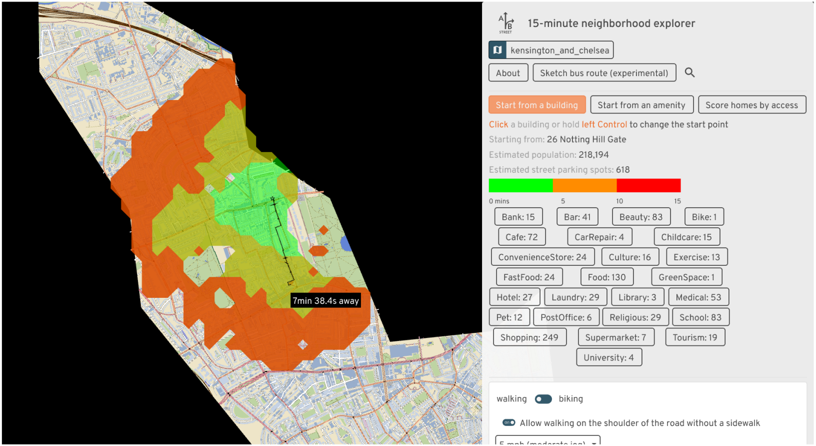
A vertex v is **reachable** from a vertex u at time t if there exists a foremost journey from u to v that departs at time t.

The government has been lying to us

Figure 2: You-are-here-maps are wrong!

Source: `https://commons.wikimedia.org/wiki/File:Notting_Hill_Royal_Borough_Of_K%26C_Council_Map_Outlining_the_Official_Area_of_Notting_Hill_and_the_Surrounding_Areas_2018.jpg`

15 min walk



Source: `https://play.abstreet.org/0.3.49/fifteen_min.html`

Temporal graphs for modeling dissemination processes

A natural application domain of temporal graphs is that of *gossiping*

~ [2]

What are dissemination processes?

- spread of rumors
- spread of fake news
- spread of diseases

Teasers

Centrality of nodes in a temporal graph

Temporal Graph Neural Networks

Sources

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