Temporal Graphs

Algorithmic Gems for Graphs, Probabilities and Processes
Winter semester 24/25

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1 Motivation

- Route finding in transportation networks
- distributed networks → analysis of various properties
- dissemination processes

2 Basic Terms and definitions

Definition

A **labeled graph** [1, page 94] is a triple $G = (V, E, \lambda)$ where:

- V, E is a graph
- $\lambda : V \cup E \to Z$ is a mapping of nodes and edges to a set of labels Z

Definition

A **temporal graph** [2, page 243] is is triple $G = (V, E, \lambda)$ where:

- V, E is a graph
- $\lambda : E \to 2^N$ is a mapping edges to a set natural numbers (time steps when this edge is active)

2.1 Notation for convenience → [2, p. 243ff]

- $\lambda(G)$ - temporal graph with respect to G
- $\lambda(E)$ - multiset of all labels
- $|\lambda| = \sum_{e \in E} |\lambda(e)|$
- $\lambda_{min} = min\{l \in \lambda(E)\}$
- $\lambda_{max} = max\{l \in \lambda(E)\}$

- $\alpha(\lambda) = \lambda_{max} - \lambda_{min} + 1$ - lifetime of a temporal graph $\lambda(G)$

Definition: static expansion of a graph

The static expansion of a temporal graph $D = (V, A)$ with $V = \{u_1, u_2, ..., u_n\}$ is a DAG $H = (S, E)$ with:

$$S = \{u_{ij} | \lambda_{min} - 1 \leq i \leq \lambda_{max}, 1 \leq j \leq n\}$$

and

$$E = \{(u_{(i-1)j}, u_{ij'}) | \lambda_{min} \leq i \leq \lambda_{max} \wedge$$
$$1 \leq j, j' \leq n \wedge (j = j' \vee (u_j, u_{j'}) \in A(i)))\}$$

3 Journeys

Definition: temporal/time respecting walk

A **temporal** or **time-respecting walk** W of a temporal graph $D = (V, A)$ is an alternating sequence of of nodes and times $(u_1, t_1, u_2, t_2, ..., u_{k-1}, t_{k-1}, u_k)$ where

- $\forall 1 \leq i \leq k - 1 : ((u_i, u_{i+1}), t_i) \in A$ and
- $1 \leq i \leq k - 2 : t_i < t_{i+1}$

- t_1 - departure time
- t_{k-1} arrival time
- $t_{k-1} - t_1 + 1$ - duration/temporal length

Definition: Journey

A **journey** is a is a temporal walk with pairwise distinct nodes $\hat{=}$ a journey of D is a path of the underlying static graph of D that uses strictly increasing edge-labels.

Definition: Foremost Journey

A u-v journey J is called foremost from time $t \in N$ if it departs after time t and its arrival time is minimized.

Definition: Temporal distance

The **temporal distance** from a node u to at time t to a node v is defined as the duration of a foremost journey from u to v that departs at time t.

Definition: Temporal diameter d

The minimum integer d such that there exists a foremost journey from every node $(u, t) \in V \times \{0, 1, ..., \alpha - d\}$ to every node $v \in V$ with duration at most d.

References

[1] Swarnendu Ghosh, Nibaran Das, Teresa Gonçalves, Paulo Quaresma, and Mahantapas Kundu. The journey of graph kernels through two decades. *Computer Science Review*, 27:88–111, 2018.

[2] Othon Michail. *An Introduction to Temporal Graphs: An Algorithmic Perspective*, pages 308–343. Springer International Publishing, Cham, 2015.