# Temporal Graphs

Algorithmic Gems for Graphs, Probabilities and Processes
Winter semester 24/25

# 1    Motivation

- Route finding in transportation networks

- distributed networks $\rightarrow$ analysis of various properties

- dissemination processes

# 2    Basic Terms and definitions

> **Definition**
>
> A **labeled graph** [1, page 94] is a triple $G = (V, E, \lambda)$ where:
>
> - $V, E$ is a graph
>
> - $\lambda : V \cup E \rightarrow Z$ is a mapping of nodes and edges to a set of labels $Z$

> **Definition**
>
> A **temporal graph** [2, page 243] is is triple $G = (V, E, \lambda)$ where:
>
> - $V, E$ is a graph
>
> - $\lambda : E \rightarrow 2^N$ is a mapping edges to a set natural numbers (time steps when this edge is active)

## 2.1    Notation for convenience $\rightarrow$ [2, p. 243ff]

- $\lambda(G)$ - temporal graph with respect to $G$

- $\lambda(E)$ - multiset of all labels

- $|\lambda| = \sum_{e \in E} |\lambda(e)|$

- $\lambda_{min} = min\{l \in \lambda(E)\}$

- $\lambda_{max} = max\{l \in \lambda(E)\}$

- $\alpha(\lambda) = \lambda_{max} - \lambda_{min} + 1$ - lifetime of a temporal graph $\lambda(G)$

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**Definition: static expansion of a graph**

The static expansion of a temporal graph $D = (V, A)$ with $V = \{u_1, u_2, ..., u_n\}$ is a DAG $H = (S, E)$ with:

$$S = \{u_{ij} | \lambda_{min} - 1 \leq i \leq \lambda_{max}, 1 \leq j \leq n\}$$

and

$$E = \{(u_{(i-1)j}, u_{ij'}) | \lambda_{min} \leq i \leq \lambda_{max} \wedge$$
$$1 \leq j, j' \leq n \wedge (j = j' \vee (u_j, u_{j'}) \in A(i)))\}$$

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# 3   Journeys

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**Definition: temporal/time respecting walk**

A **temporal** or **time-respecting walk** $W$ of a temporal graph $D = (V, A)$ is an alternating sequence of of nodes and times $(u_1, t_1, u_2, t_2, ..., u_{k-1}, t_{k-1}, u_k)$ where

- $\forall 1 \leq i \leq k - 1 : ((u_i, u_{i+1}), t_i) \in A$ and

- $1 \leq i \leq k - 2 : t_i < t_{i+1}$

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- $t_1$ - departure time

- $t_{k-1}$ arrival time

- $t_{k-1} - t_1 + 1$ - duration/temporal length

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**Definition: Journey**

A **journey** is a is a temporal walk with pairwise distinct nodes $\hat{=}$ a journey of D is a path of the underlying static graph of D that uses strictly increasing edge-labels.

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**Definition: Foremost Journey**

A u-v journey J is called foremost from time $t \in N$ if it departs after time t and its arrival time is minimized.

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**Definition: Temporal distance**

The **temporal distance** from a node $u$ to at time $t$ to a node $v$ is defined as the duration of a foremost journey from $u$ to $v$ that departs at time $t$.

> **Definition: Temporal diameter $d$**
>
> The minimum integer $d$ such that there exists a foremost journey from every node $(u, t) \in V \times \{0, 1, ..., \alpha - d\}$ to every node $v \in V$ with duration at most $d$.

# 4 Sources

# References

[1] Swarnendu Ghosh, Nibaran Das, Teresa Gonçalves, Paulo Quaresma, and Mahantapas Kundu. The journey of graph kernels through two decades. *Computer Science Review*, 27:88–111, 2018.

[2] Othon Michail. *An Introduction to Temporal Graphs: An Algorithmic Perspective*, pages 308–343. Springer International Publishing, Cham, 2015.