



**Verkehrsbetrieb  
Stadtwerke  
Potsdam**



Straßenbahnen und Busse fahren Sie direkt und schnell vom Potsdamer Hauptbahnhof zu den zahlreichen Sehenswürdigkeiten:

- X15** ► Schloss Sonnenburg Sa/So (07.04. bis 05.11.2023)  
**695** ► Alt-Görlitz

Sanssouci-Linie

Park und Schloss Sanssouci, Orangerie, Drachenhaus,  
Belvedere auf dem Klausberg, Weg zum Park Charlottenhof

-  **605** • Silesie Park West  
Historisches Dampfmaschinenhaus, Park und Schloss  
Charlottenhof, Neues Palais, Weg zum Park Sanssouci

→ 96 → Campus Jungfernsee

Volkspark-Linie

Historische Innenstadt, Russische Kolonie Alexandrowka,  
Belvedere auf dem Pfingstberg, Biosphäre Potsdam,  
Volkspark Potsdam

- 92 → Bernsdorf, Kirschallee**  
Holländisches Viertel, Russische Kolonie Alexandrowka,  
Belvedere auf dem Pfingstberg, Ruinenberg, Volkspark  
Potsdam, Kronau Bernsdorf

— 93 → Gleicher Brücke

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Ertavium, Schiffbauergasse, Hans Otto Theater, Schloss und Park Gilenide

Ibis Platz der Einheit/Welt direkt umsteigen in  
→  **bus** → Höherstraße

Cecilienhof-Linie

Historische Innenstadt, Russische Kolonie Alexandrowka,  
Neuer Garten, Marmorpalais, Schloss Cecilienhof,  
Belvedere auf dem Hingstberg  
SiSo ab Hauptbahnhof 10.04. bis 05.11.2022

- 690 ► Johannes-Keppler-Platz

Microsoft Office 2003 mit SharePoint

 Tourist-Information  
 Besucher-Information  
Stiftung preußische  
Schlösser und Gärten

The logo consists of a green square containing a white stylized 'i' character.

#### **Plantain**

Das Heft „Potsdam entdecken mit S-Bahn, Tram und Bus“ mit genaueren Routenbeschreibungen gibt es in allen VIP-Kundenzentren

Stand 12/2022

Source: [https://www.swp-potsdam.de/content/verkehr/bilder\\_6/liniennetz/touristischer\\_liniennetzplan\\_screenshot\\_1280\\_960.jpg](https://www.swp-potsdam.de/content/verkehr/bilder_6/liniennetz/touristischer_liniennetzplan_screenshot_1280_960.jpg)

# Temporal Graphs

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Daniel Cermann

January 28, 2025



Hasso Plattner Institute

# Motivation

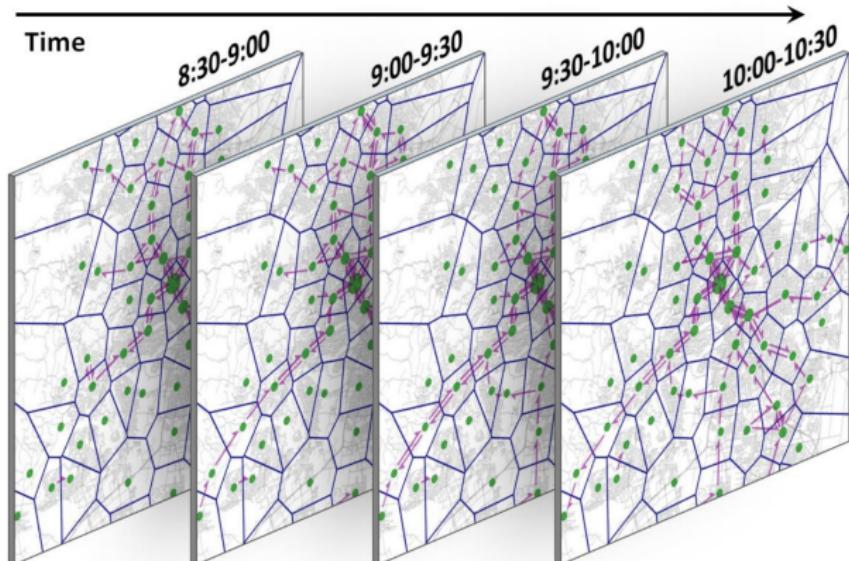
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## Clip: School day

<https://youtu.be/BSNJSUkc5-Q?t=996>

# Google Maps

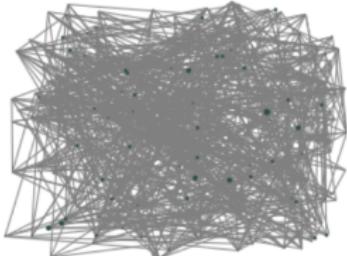
Figure 1: Illustration of a trajectory flow map, a dynamic graph of aggregated traffic flows constructed from trajectory data. The presented example is based on bus passenger trajectories obtained in Brisbane, Australia.



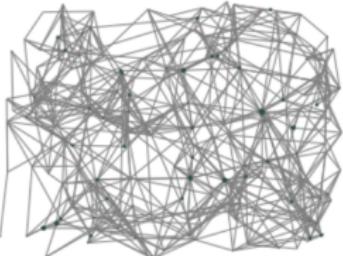
Source: [https://australiantransportresearchforum.org.au/wp-content/uploads/2022/03/ATRF2016\\_paper\\_166.pdf](https://australiantransportresearchforum.org.au/wp-content/uploads/2022/03/ATRF2016_paper_166.pdf)

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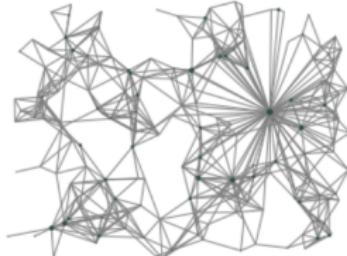
# Distributed systems



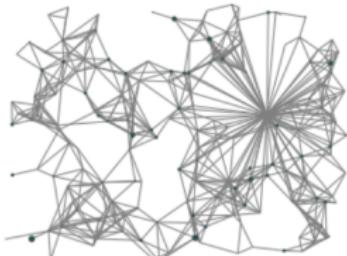
(a) Random initial network



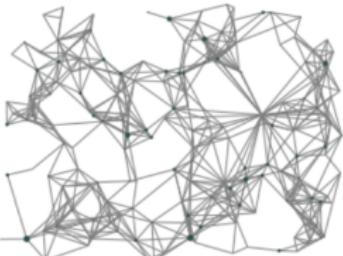
(b) Intermediate state



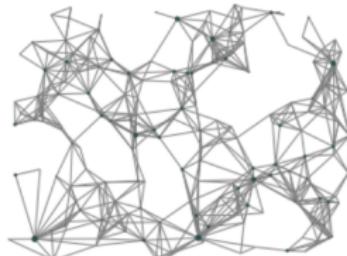
(c) Equilibrium network



(d) Change of node capacities



(e) Intermediate state



(f) Equilibrium network

Source: <https://www.sg.ethz.ch/publications/2012/scholtes2012organic-design-of/>

# Temporal graphs for physical/chemical models

DATA SCIENCE | May 9, 2023

## A Temporal Graph Model to Predict Chemical Transformations in Complex Dissolved Organic Matter

Philipp Plamper, Oliver J. Lechtenfeld\*, Peter Herzsprung, and Anika Groß\*

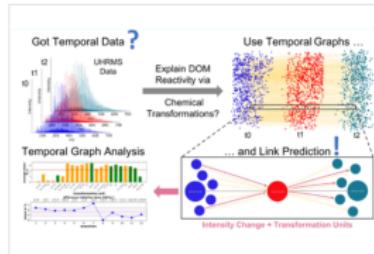
Open PDF

Supporting Information (1)

ACCESS

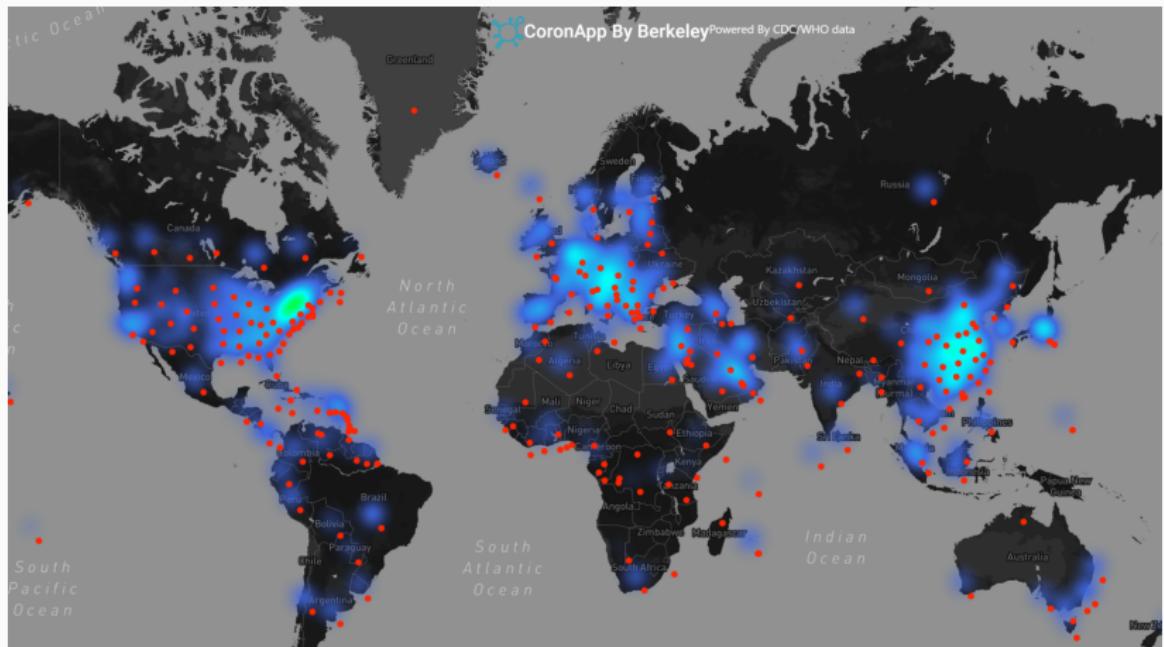
### Abstract

Dissolved organic matter (DOM) is a complex mixture of thousands of natural molecules that undergo constant transformation in the environment, such as sunlight induced photochemical reactions. Despite molecular level resolution from ultrahigh resolution mass spectrometry (UHRMS), trends of mass peak intensities are currently the only way to follow photochemically induced molecular changes in DOM. Many real-world relationships and temporal processes can be intuitively modeled using graph data structures (networks). Graphs enhance the potential and value of AI applications by adding context and interconnections allowing the uncovering of hidden or unknown relationships in data sets. We use a temporal graph model and link prediction to identify transformations of DOM molecules in a photo-oxidation experiment. Our link prediction algorithm simultaneously considers educt removal and product formation for molecules linked by predefined transformation units (oxidation, decarboxylation, etc.). The transformations are further weighted by the extent of intensity change and clustered on the graph structure to identify groups of similar reactivity. The temporal graph is capable of identifying relevant molecules subject to similar reactions and enabling to study their time course. Our approach overcomes previous data evaluation limitations for mechanistic studies of DOM and leverages the potential of temporal graphs to study DOM reactivity by UHRMS.



Source: <https://pubs.acs.org/doi/full/10.1021/acs.est.3c00351>

# Dissemination processes



Source: <https://engineering.berkeley.edu/wp-content/uploads/2020/03/CoronApp.png>

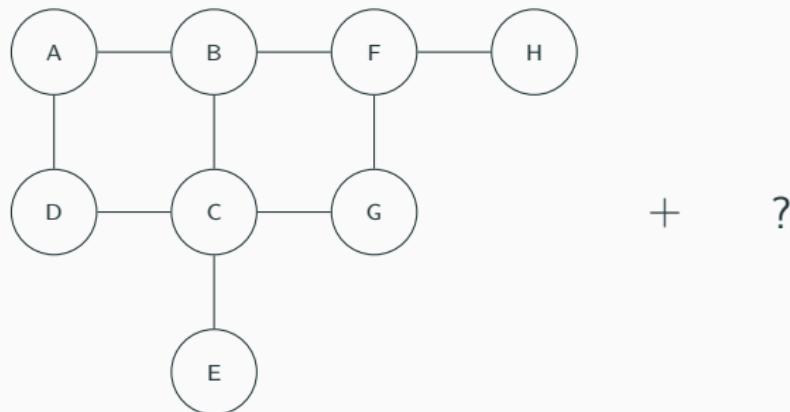
## How to model temporal graphs

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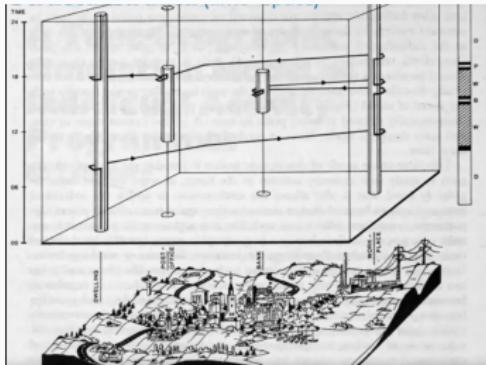
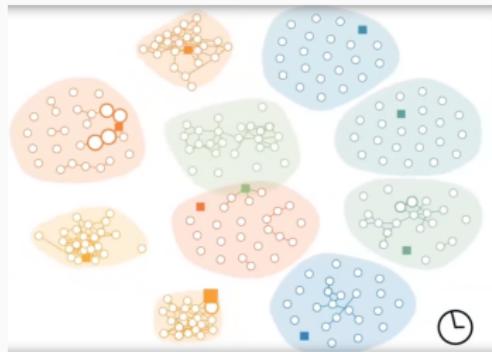
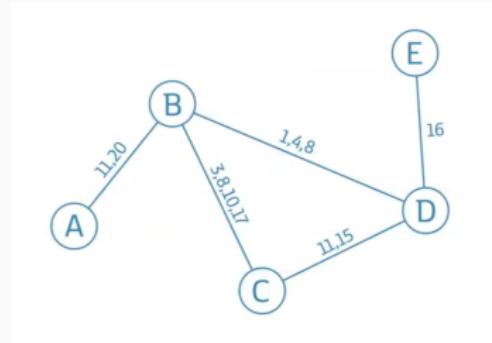
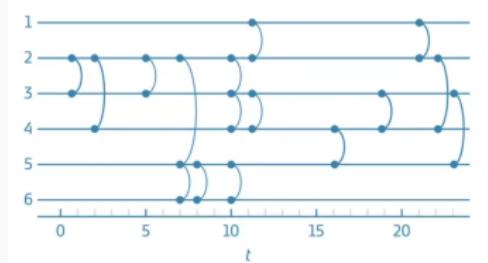
# How to represent time in graphs?

Brainstorm with your neighbor how time can be represented in graphs. Don't think of notation for now, just some visual ways to represent the additional dimension.

≈ 2 min

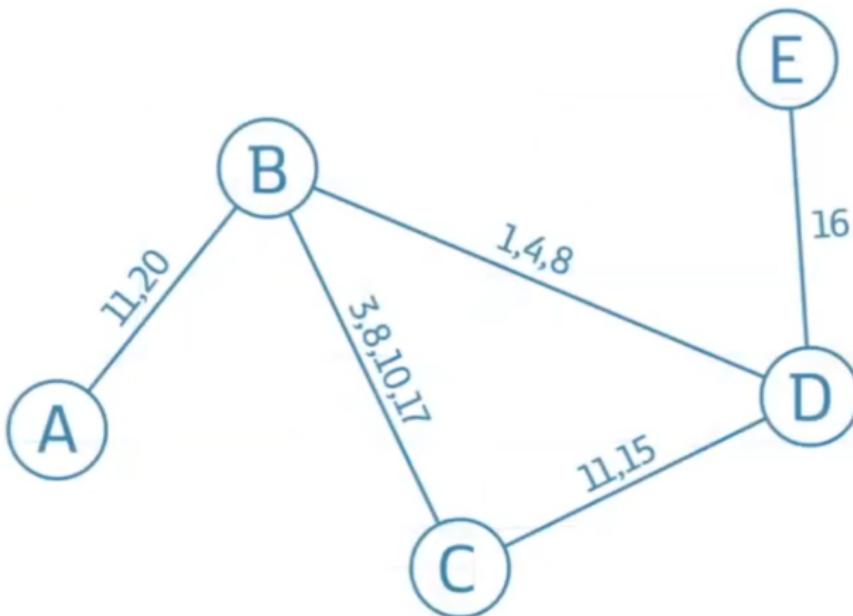


# How to represent time in graphs?



Source: <https://www.youtube.com/watch?v=BSNJSUkc5-Q>

# How to model temporal graphs



Source: <https://www.youtube.com/watch?v=BSNJSUkc5-Q>

# Definition labeled and temporal graphs

## Definition

A **labeled graph** [1, page 94] is a triple  $G = (V, E, \lambda)$  where:

- $(V, E)$  is a graph
- $\lambda : V \cup E \rightarrow Z$  is a mapping of nodes and edges to a set of labels  $Z$

# Definition labeled and temporal graphs

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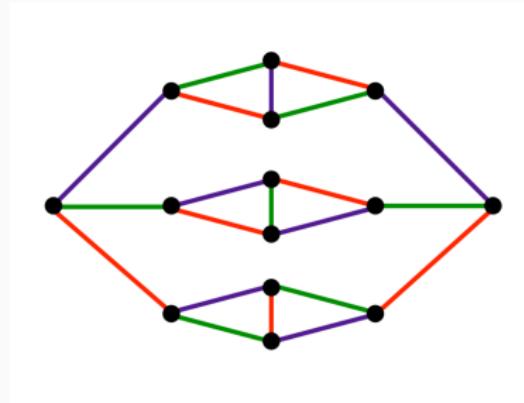
- $(V, E)$  is a graph
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## Definition

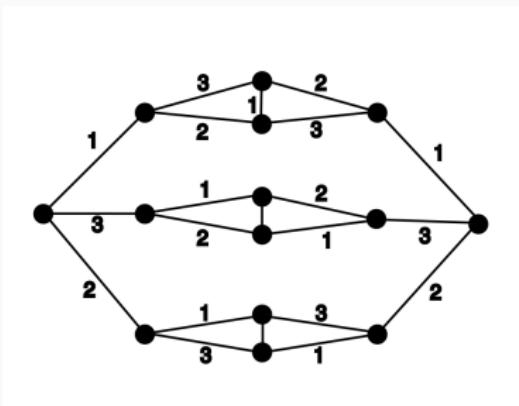
A **temporal graph** [4, page 243] is a triple  $G = (V, E, \lambda)$  where:

- $V, E$  is a graph
- $\lambda : E \rightarrow 2^{\mathbb{N}}$  is a mapping of edges to a set of natural numbers (time steps when this edge is active)

# Relationship labeled and temporal graphs



$\leftrightarrow$



a

a

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<sup>a</sup>Source: <https://www.algorist.com/images/figures/edge-coloring-R.png>

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<sup>a</sup>Source: me:)

## Exercise notation

Given the following temporal graph definition  $D = (V, E, \lambda)$   
draw the visual representation of the temporal graph on the  
template in the handout!

$\approx 2$  min

$$V = \{C, H, B, P, M, G, W\}$$

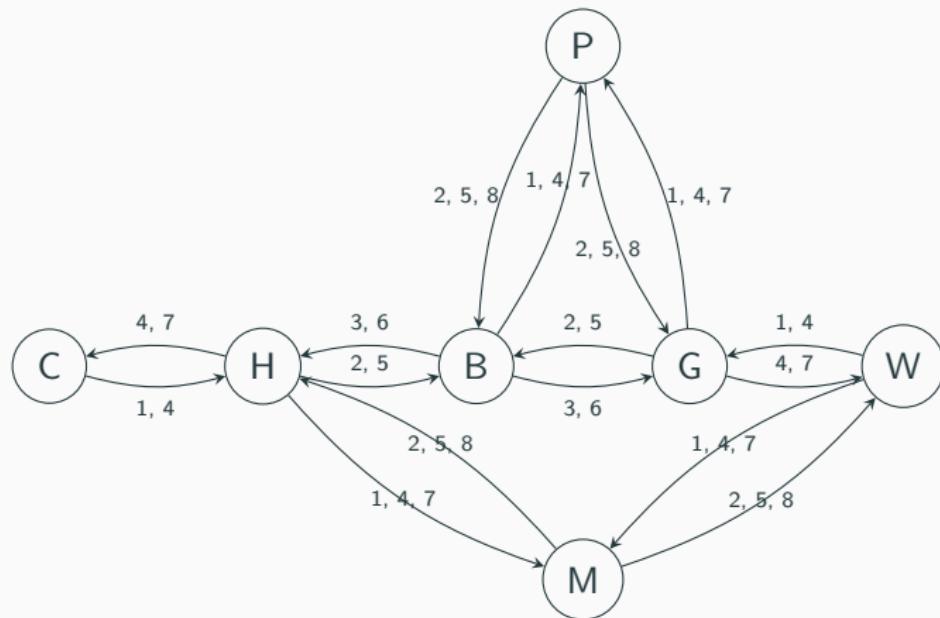
$$\begin{aligned}E = & \{(C, H), (H, B), (B, G), (G, W), (W, P), (H, P), (B, M), (M, G) \\& (H, C), (B, H), (G, H), (W, G), (P, W), (P, H), (M, B), (G, M)\}\end{aligned}$$

$$\lambda = \{$$

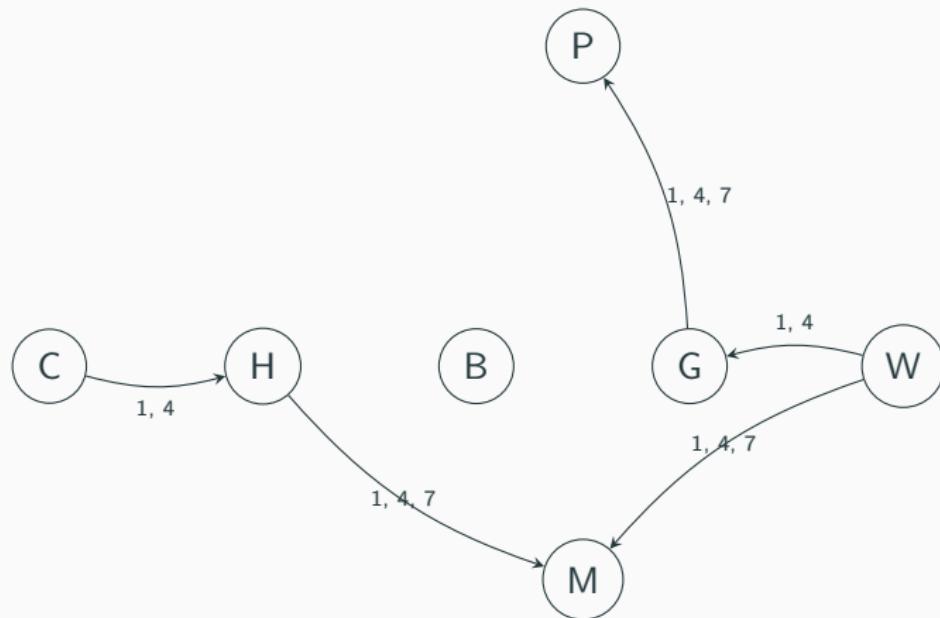
$$\begin{aligned}(C, H) &\mapsto \{1, 4\}, \\(H, C) &\mapsto \{4, 7\}, \\(H, B) &\mapsto \{2, 5\}, \\(B, H) &\mapsto \{3, 6\}, \\(B, G) &\mapsto \{3, 6\}, \\(G, B) &\mapsto \{2, 5\}, \\(G, W) &\mapsto \{4, 7\}, \\(W, G) &\mapsto \{1, 4\}, \\(H, M) &\mapsto \{2, 5\}, \\(M, H) &\mapsto \{3, 6\}, \\(M, W) &\mapsto \{3, 6\} \\(W, M) &\mapsto \{2, 5\} \\(B, P) &\mapsto \{1, 4, 7\} \\(P, B) &\mapsto \{2, 5, 8\} \\(G, P) &\mapsto \{2, 5, 8\} \\(P, G) &\mapsto \{1, 4, 7\}\end{aligned}$$

$$\}$$

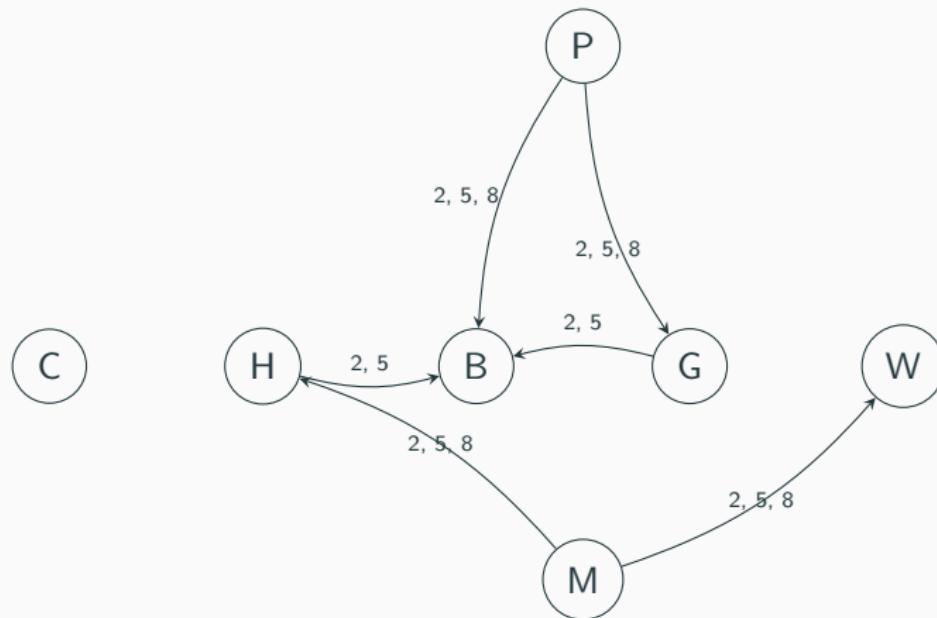
# Solution



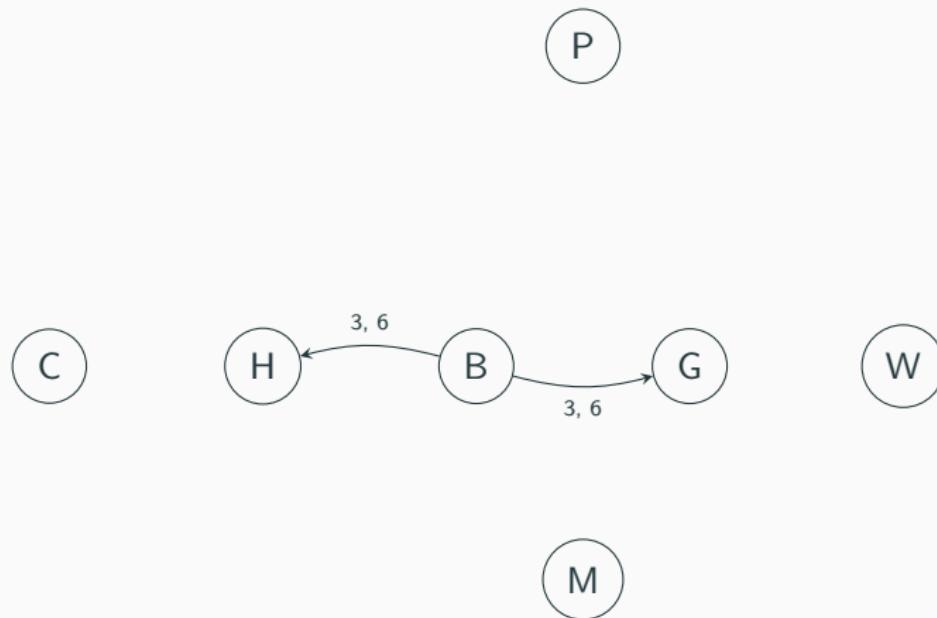
# Solution



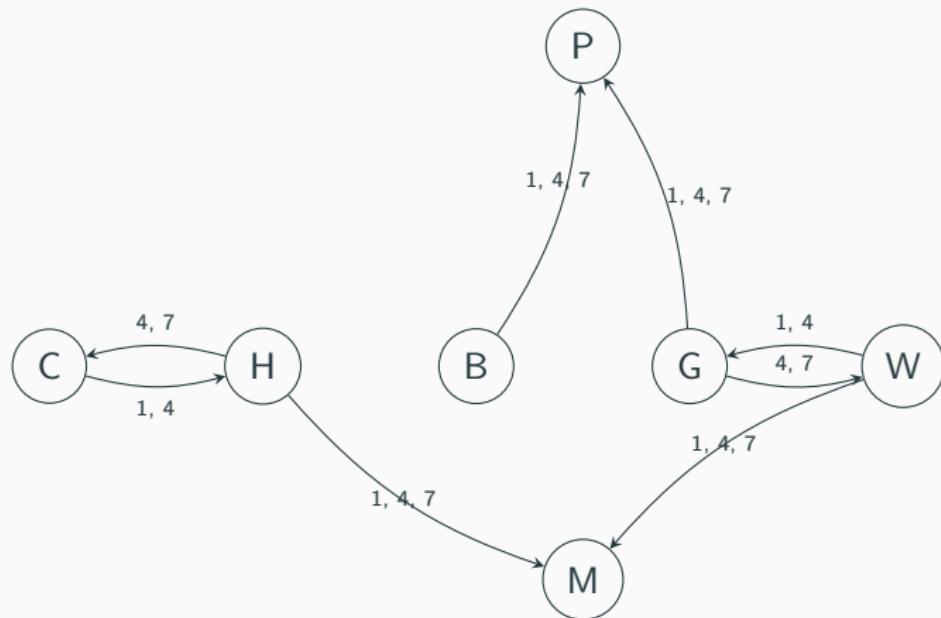
# Solution



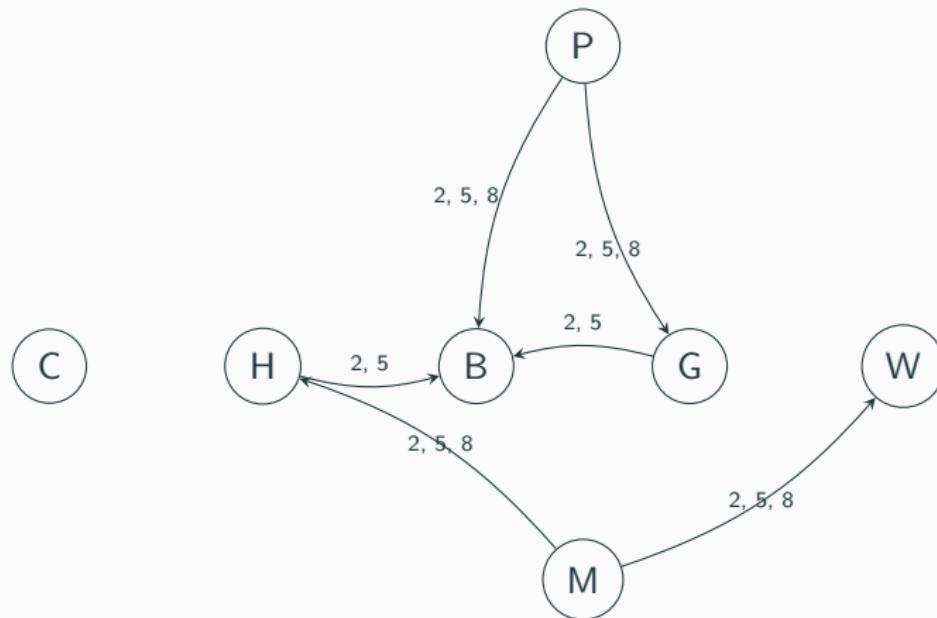
# Solution



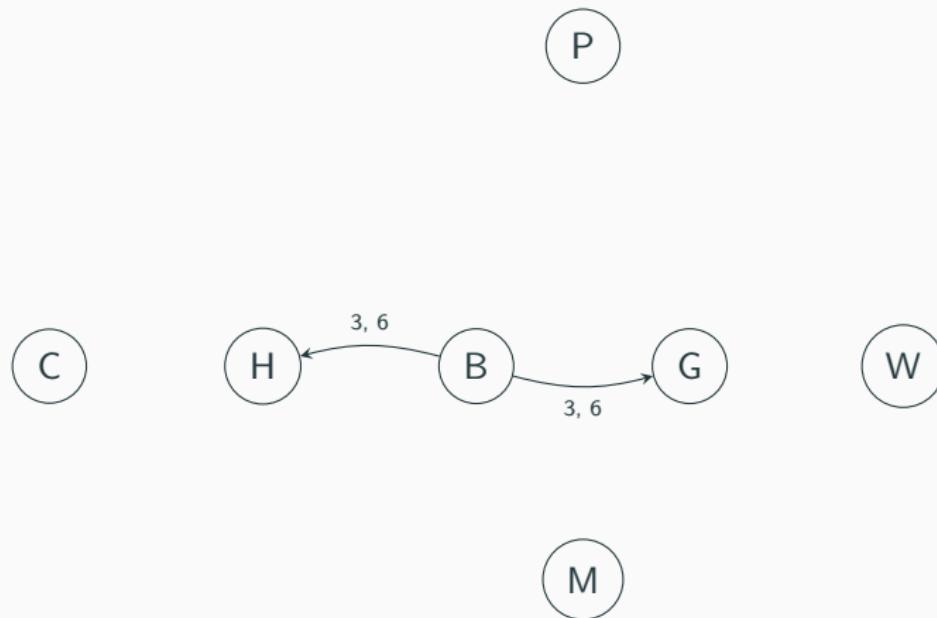
# Solution



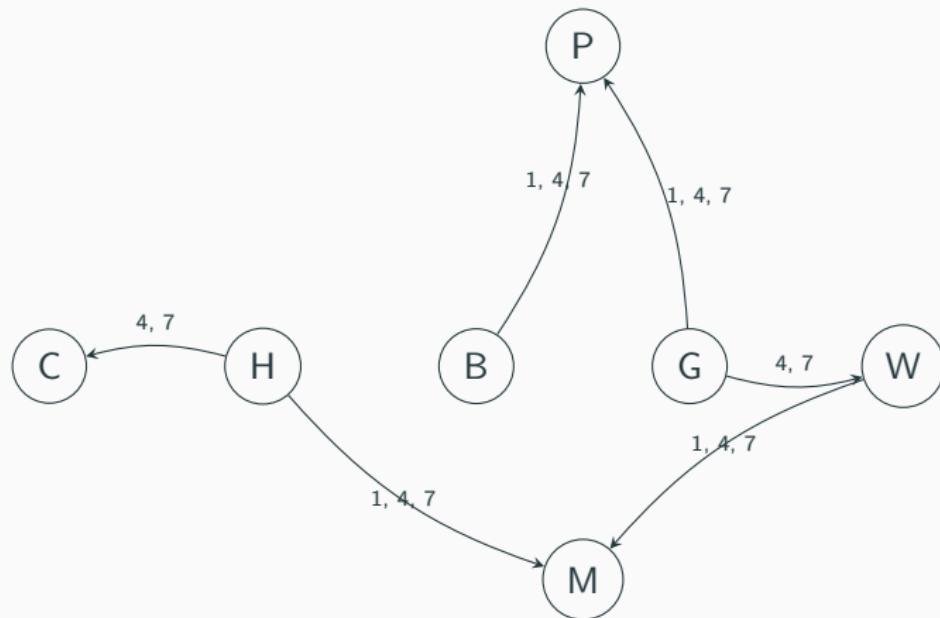
# Solution



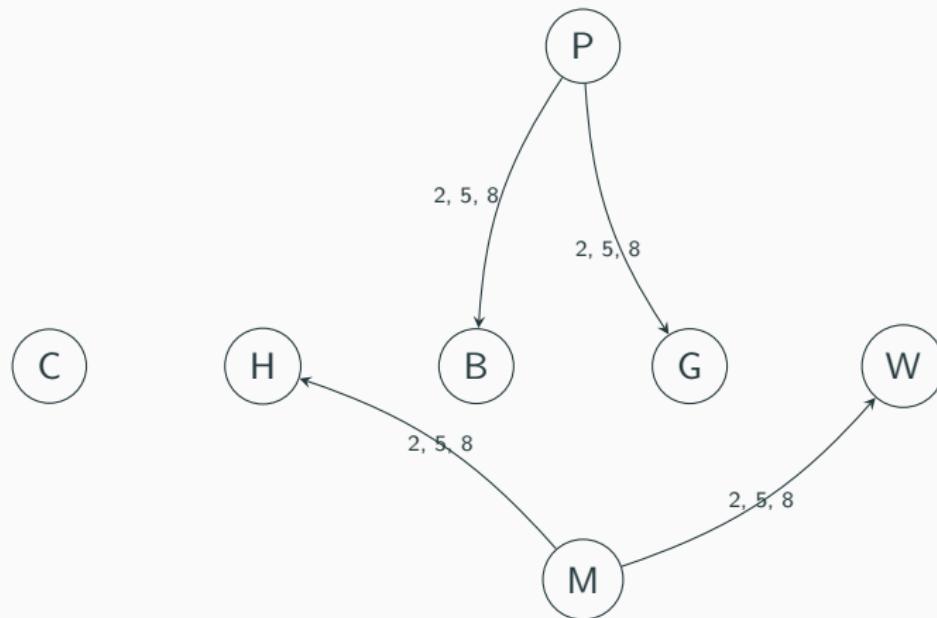
# Solution



# Solution

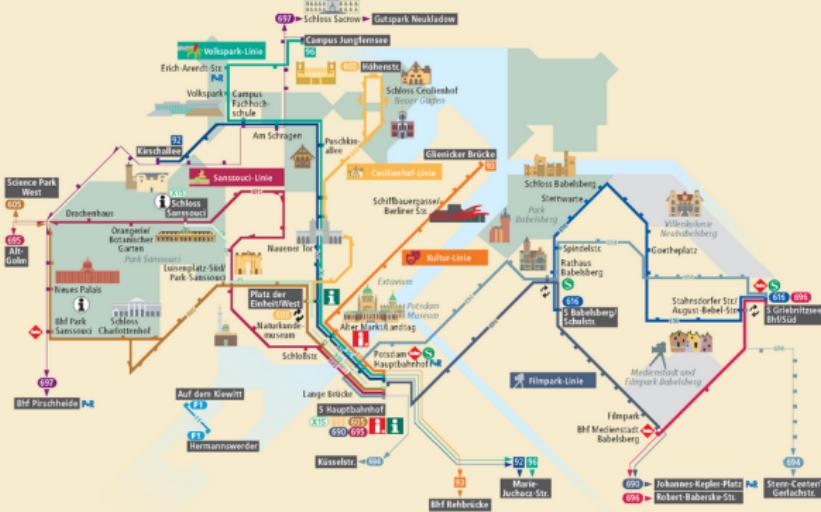


# Solution





## Verkehrsbetrieb Stadtwerke Potsdam



## Potsdam entdecken mit den touristischen Linien

Straßenbahnen und Busse fahren  
Sie direkt und schnell vom Potsdamer  
Hauptbahnhof zu den zahlreichen  
Sehenswürdigkeiten:

**697 Schloss Sanssouci S-Bahn** (07.04. bis 05.11.2022)

**695 Alt-Görlitz**

**Sanssouci-Linie**

Park und Schloss Sanssouci, Orangerie, Drachenhaus,  
Belvedere auf dem Kranzberg, Weg zum Park Sanssouci

**697 Glienicker Park**  
Historisches Dampfmaschinenhaus, Park und Schloss  
Charlottenhof, Neues Palais, Weg zum Park Sanssouci

**Campus Jungfernsee**

**Volkspark-Linie**

Historische Innstadt; Russische Kolonie Alexandrowka,  
Belvedere auf dem Pfingstberg, Biographie Potsdam,  
Volkspark Potsdam

**697 Jernstedt, Kirchholz**

Holländisches Viertel, Russische Kolonie Alexandrowka,  
Belvedere auf dem Pfingstberg, Ruinenberg, Volkspark  
Potsdam, Königin Boronat

**Glienicker Brücke**

**Kulturlinie**

Entwurf, Schiffbauerpasse, Hans Otto Theater, Schloss  
und Park Glienicke

Bei Platz der Einheit/West dort umsteigen in:

**Höherstrasse**

**Ceciliengärten-Linie**

Historische Innstadt; Russische Kolonie Alexandrowka,  
Neuer Garten, Marmorpalais, Schloss Glienicke,  
Belvedere auf dem Pfingstberg  
Südost ab Hauptbahnhof (07.04. bis 05.11.2022)

**697 Johannes-Kepler-Platz**

**Filmpark-Linie**

Mesleinstadt Babenberg im Filmpark

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**VIP-Kundenzentren**  
VIP-Hotline: (0331) 6 6142 75  
VIP-potsdam.de  
VBB App „Bus & Bahn“ und  
twitter.com/VIP\_potsdam

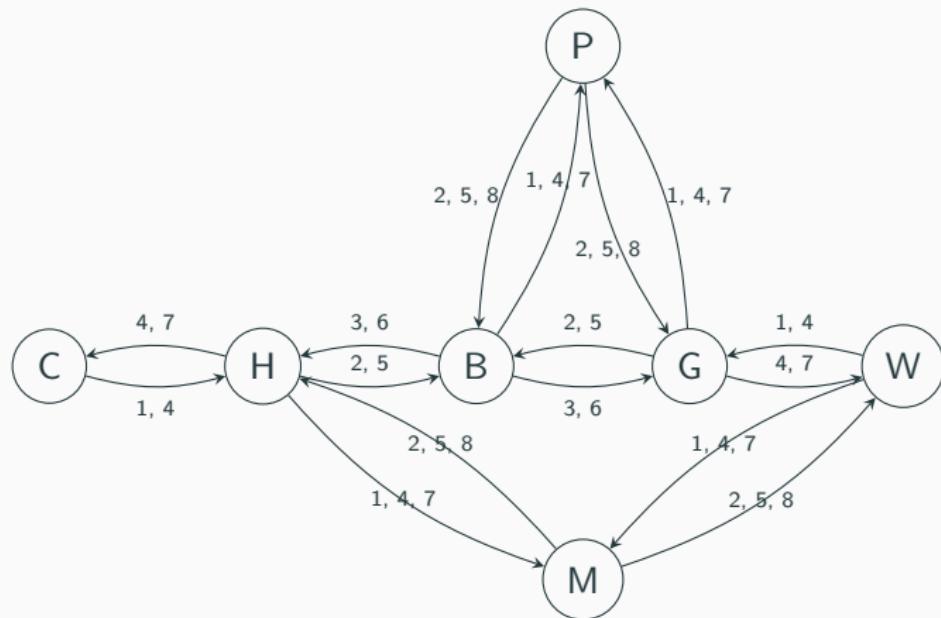
**Umsteigehaltestelle**

Das Heft „Potsdam entdecken  
mit S-Bahn, Tram und Bus“ mit  
genauerer Routendeskriptionen  
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Stand 12/2022

Source: [https://www.swp-potsdam.de/content/verkehr/bilder\\_6/liniennetz/touristischer\\_liniennetzplan\\_screenshot\\_1280\\_960.jpg](https://www.swp-potsdam.de/content/verkehr/bilder_6/liniennetz/touristischer_liniennetzplan_screenshot_1280_960.jpg)

# Solution



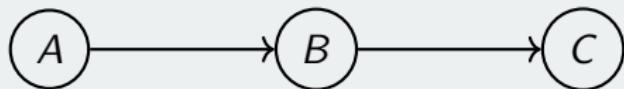
## Notation for convenience → [4, p. 243ff]

- $\lambda(G)$  - temporal graph with respect to  $G$
- $\lambda(E)$  - multiset of all labels
- $|\lambda| = \sum_{e \in E} |\lambda(e)|$
- $\lambda_{min} = \min\{l \in \lambda(E)\}$
- $\lambda_{max} = \max\{l \in \lambda(E)\}$
- $\alpha(\lambda) = \lambda_{max} - \lambda_{min} + 1$  - lifetime of a temporal graph  $\lambda(G)$

# Transitivity of reachability in static graphs

## Reachability in a static graph is transitive

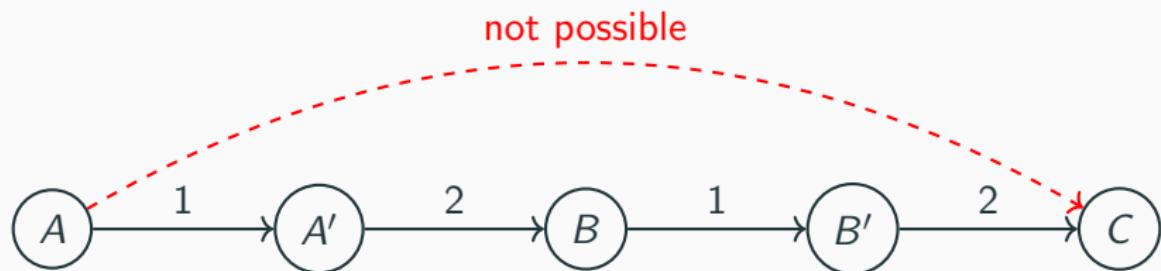
Given A static graph  $G = (V, E)$ , for all nodes  $A, B, C \in V$  we have: If B is reachable by A and C is reachable by B, then C is reachable by A.



## Transitivity of reachability in static graphs

Is reachability in a temporal graph transitive?

# Time matters!

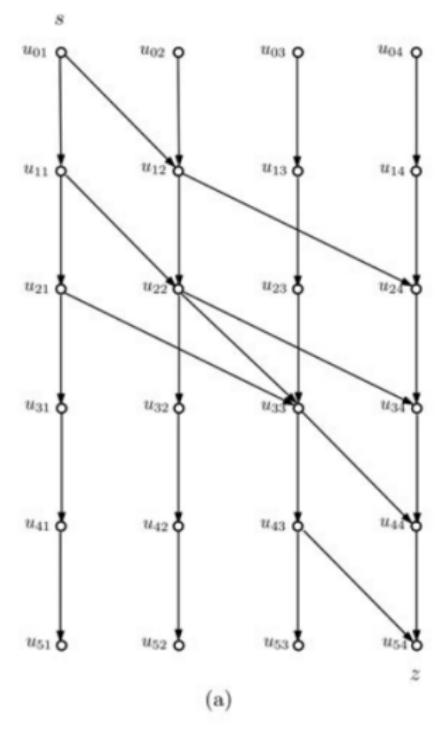


⇒ Deep implications for complexity of temporal graphs

## Notation #2

- A temporal graph  $D$  is an ordered set of disjoint sets  $(V, A)$
- $A \subseteq V^2 \times \mathbb{N}$  - 'time edges'
- $A(t) = \{e | (e, t) \in A\}$  - set of edges at time  $t$
- $D(t) = (V, A(t))$  - snapshot of graph  $D$  at time  $t$

# Static expansion of a temporal graph



[4, page 318]

## Static expansion of a temporal graph

### Definition: static expansion of a graph

The static expansion of a temporal graph  $D = (V, A)$  with  $V = \{u_1, u_2, \dots, u_n\}$  is a DAG  $H = (S, E)$  with:

$$S = \{u_{ij} \mid \lambda_{min} - 1 \leq i \leq \lambda_{max}, 1 \leq j \leq n\}$$

and

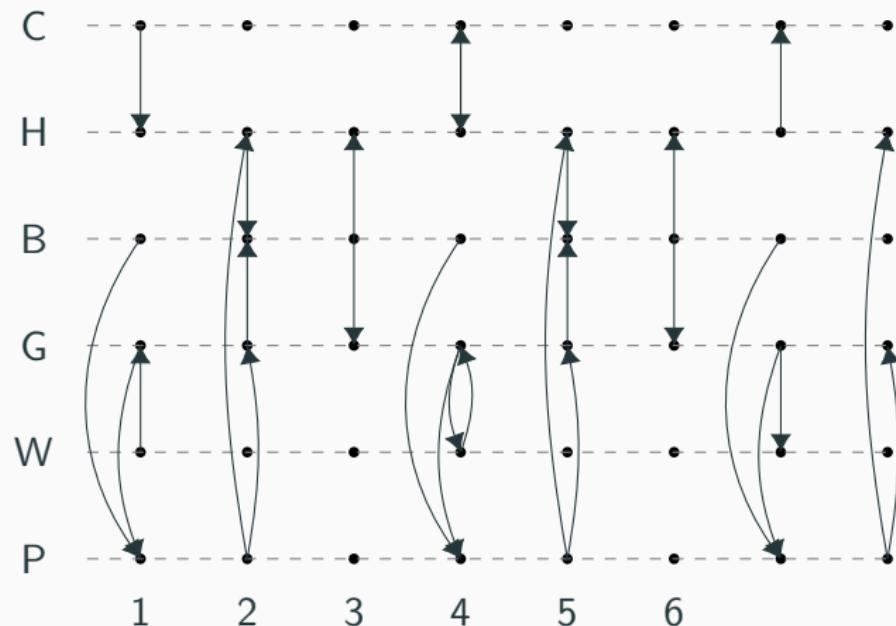
$$\begin{aligned} E = \{ & (u_{(i-1)j}, u_{ij'}) \mid \lambda_{min} \leq i \leq \lambda_{max} \wedge \\ & 1 \leq j, j' \leq n \wedge (j = j' \vee (u_j, u_{j'}) \in A(i)) \} \end{aligned}$$

## Exercise: Static expansion of a temporal graph

Turn the Potsdam-Map temporal graph into its static expansion using the template given on the handout (we leave out M(edienstadt Babelsberg) for simplicity sake here) !

≈ 3 min

## Solution: Static expansion of a temporal graph



## Repetition - walks and paths in static graphs

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- A **walk** is a finite or infinite sequence of edges which joins a sequence of vertices.

## Repetition - walks and paths in static graphs

- A **walk** is a finite or infinite sequence of edges which joins a sequence of vertices.
- A **path** is a walk where all vertices are distinct.

## Definition: temporal/time respecting walk

A **temporal** or **time-respecting walk**  $W$  of a temporal graph  $D = (V, A)$  is an alternating sequence of nodes and times  $(u_1, t_1, u_2, t_2, \dots, u_{k-1}, t_{k-1}, u_k)$  where

- $\forall 1 \leq i \leq k - 1 : ((u_i, u_{i+1}), t_i) \in A$  and
- $1 \leq i \leq k - 2 : t_i < t_{i+1}$

- $t_1$  - departure time
- $t_{k-1}$  arrival time
- $t_{k-1} - t_1 + 1$  - duration/temporal length

## Definition: Journey

A **journey** is a temporal walk with pairwise distinct nodes  
 $\triangleq$  a journey of  $D$  is a path of the underlying static graph of  $D$  that uses strictly increasing edge-labels.

# Journeys

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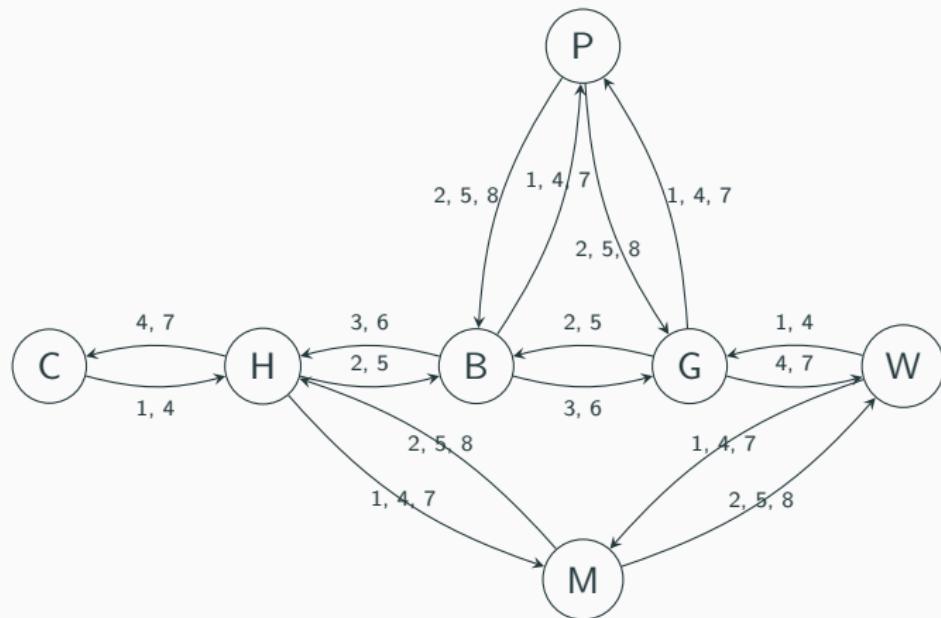
## Definition: Foremost Journey

A  $u$ - $v$  journey  $J$  is called foremost from time  $t \in \mathbb{N}$  if it departs after time  $t$  and its arrival time is minimized.

## Exercise: Journeys

What is the foremost journey from C to P in the  
Potsdam-Map temporal graph from time 2?

# Solution



## Definition: Temporal distance

The **temporal distance** from a node  $u$  to at time  $t$  to a node  $v$  is defined as the duration of a foremost journey from  $u$  to  $v$  that departs at time  $t$ .

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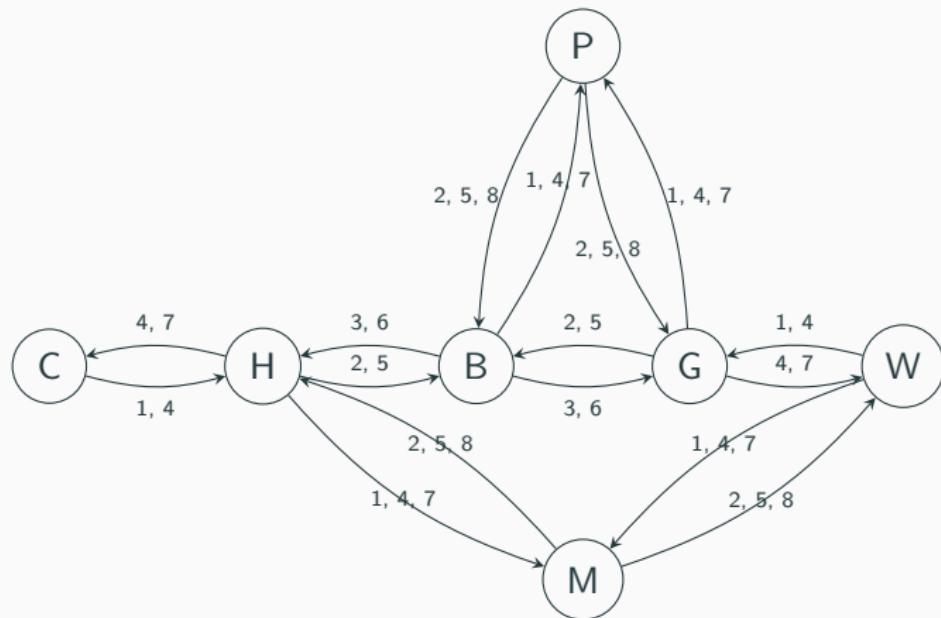
## Definition: Temporal diameter $d$

The minimum integer  $d$  such that there exists a foremost journey from every node  $(u, t) \in V \times \{0, 1, \dots, \alpha - d\}$  to every node  $v \in V$  with duration at most  $d$ .

## Exercise: Journeys

Find the temporal diameter of the Potsdam-Map temporal graph.

# Solution



## Computing foremost journeys - Problem formulation

Given a source node  $s \in V$  and a start time  $t_{start}$  compute the foremost  $s - w$  journey for all  $w \in V \setminus \{s\}$ .

Brainstorm ideas for an algorithm to solve this problem with a partner.

## Sidenote - offline vs. online algorithms

### offline algorithms

*takes whole temporal graph  $D$  as input*



### online algorithms

*temporal graph is revealed to algorithm over time*



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Source: <https://encrypted-tbn0.gstatic.com/images?q=tbn:ANd9GcQbpwyZnBM48yDEHLT9Sww1J8AJrgs4lj-VLi2EkxpIoKMeI6-stF-R9uAsLl5K4gXFByts&usqp=CAU>

ANd9GcQbpwyZnBM48yDEHLT9Sww1J8AJrgs4lj-VLi2EkxpIoKMeI6-stF-R9uAsLl5K4gXFByts&usqp=CAU

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# Computing foremost journeys - Algorithm

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## Algorithm 1 Computing earliest-arrival time

---

**Require:** A temporal graph  $G = (V, E)$  in its edge stream representation, source vertex  $x$ , time interval  $[t_\alpha, t_\omega]$

**Ensure:** The earliest-arrival time from  $x$  to every vertex  $v \in V$  within  $[t_\alpha, t_\omega]$

```
1: Initialize  $t[x] = t_\alpha$ , and  $t[v] = \infty$  for all  $v \in V \setminus \{x\}$ 
2: for all incoming edge  $e = (u, v, t)$  in the edge stream do
3:   if  $t + 1 \leq t_\omega$  and  $t \geq t[u]$  then
4:     if  $t + 1 < t[v]$  then
5:        $t[v] \leftarrow t + 1$ 
6:     end if
7:   else if  $t \geq t_\omega$  then
8:     break                                ▷ Go to Line 11
9:   end if
10:  end for
11:  return  $t[v]$  for each  $v \in V$ 
```

---

## Computing foremost journeys - Proof of correctness

*Let  $\mathbb{P}$  be the set of earliest-arrival paths from  $x$  to a vertex  $v_k$  within the time interval  $[t_\alpha, t_\omega]$ . If  $\mathbb{P} \neq \emptyset$  then there exists  $P = (x, v_1, v_2, \dots, v_k) \in \mathbb{P}$  such that every prefix-subpath,  $P_i = (x, v_1, v_2, \dots, v_i)$ , is an earliest-arrival path from  $x$  to  $v_i$  within  $[t_\alpha, t_\omega]$ , for  $1 \leq i \leq k$ .*

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*For any vertex  $v \in V$ , if the earliest-arrival path from  $x$  to  $v$  within the time interval  $[t_\alpha, t_\omega]$  exists, then  $t[v]$  returned by the Algorithm is the corresponding earliest-arrival time; otherwise,  $t[v] = \infty$ .*

## Other metrics to optimize

- latest departure time
- fastest path
- shortest path
- ...

# Reachability

## Definition: Reachability

A vertex  $v$  is **reachable** from a vertex  $u$  at time  $t$  if there exists a foremost journey from  $u$  to  $v$  that departs at time  $t$ .

# The government has been lying to us

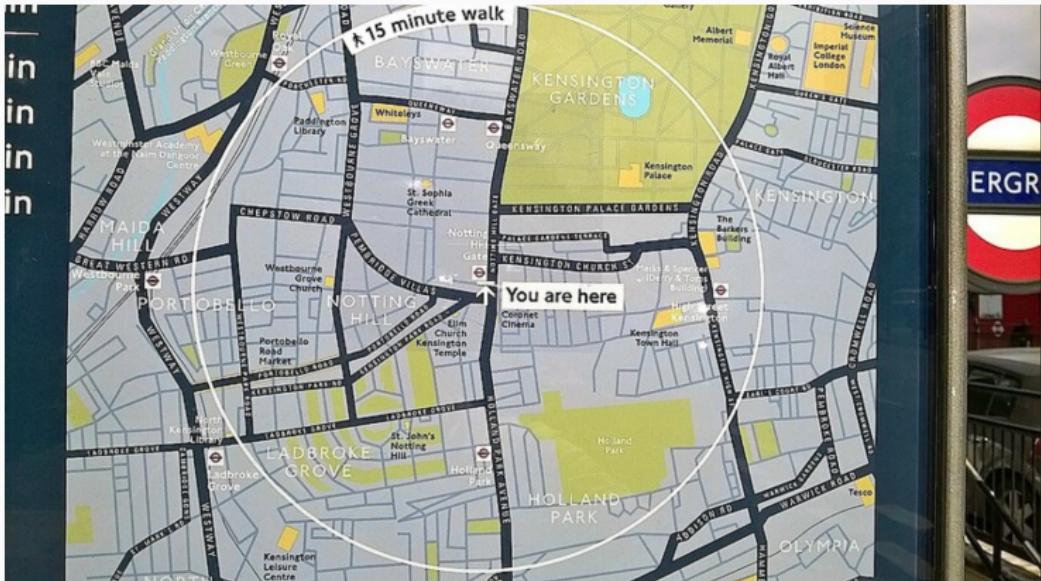
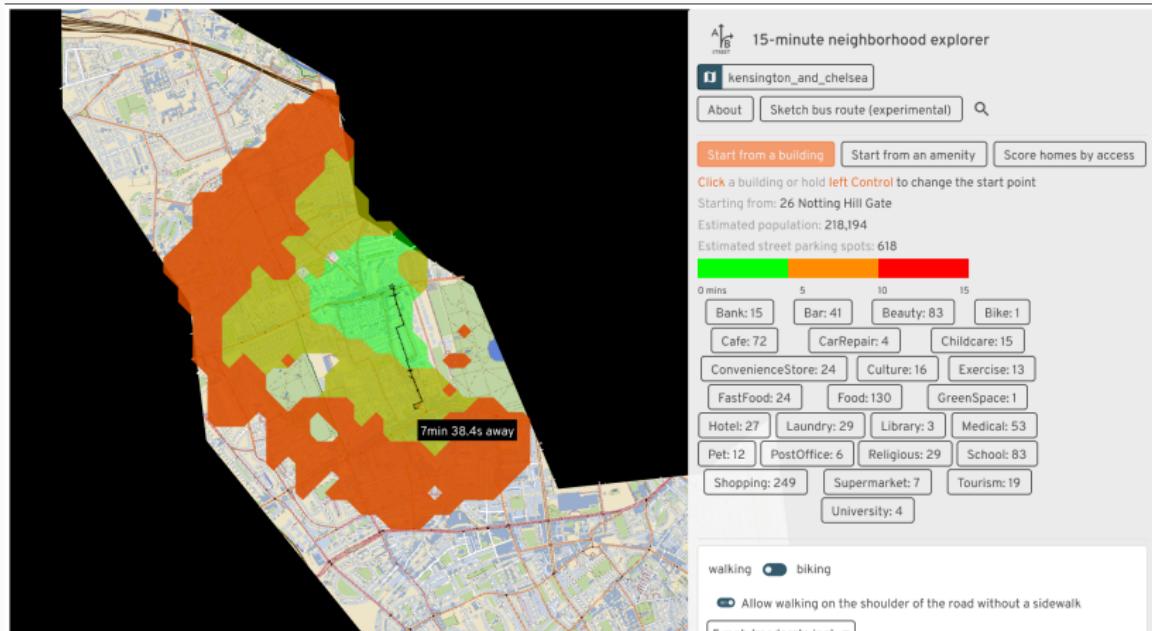


Figure 2: You-are-here-maps are wrong!

Source: [https://commons.wikimedia.org/wiki/File:Notting\\_Hill\\_Royal\\_Borough\\_of\\_K%26C\\_Council\\_Map\\_Outlining\\_the\\_Official\\_Area\\_of\\_Notting\\_Hill\\_and\\_the\\_Surrounding\\_Areas\\_2018.jpg](https://commons.wikimedia.org/wiki/File:Notting_Hill_Royal_Borough_of_K%26C_Council_Map_Outlining_the_Official_Area_of_Notting_Hill_and_the_Surrounding_Areas_2018.jpg)

# 15 min walk



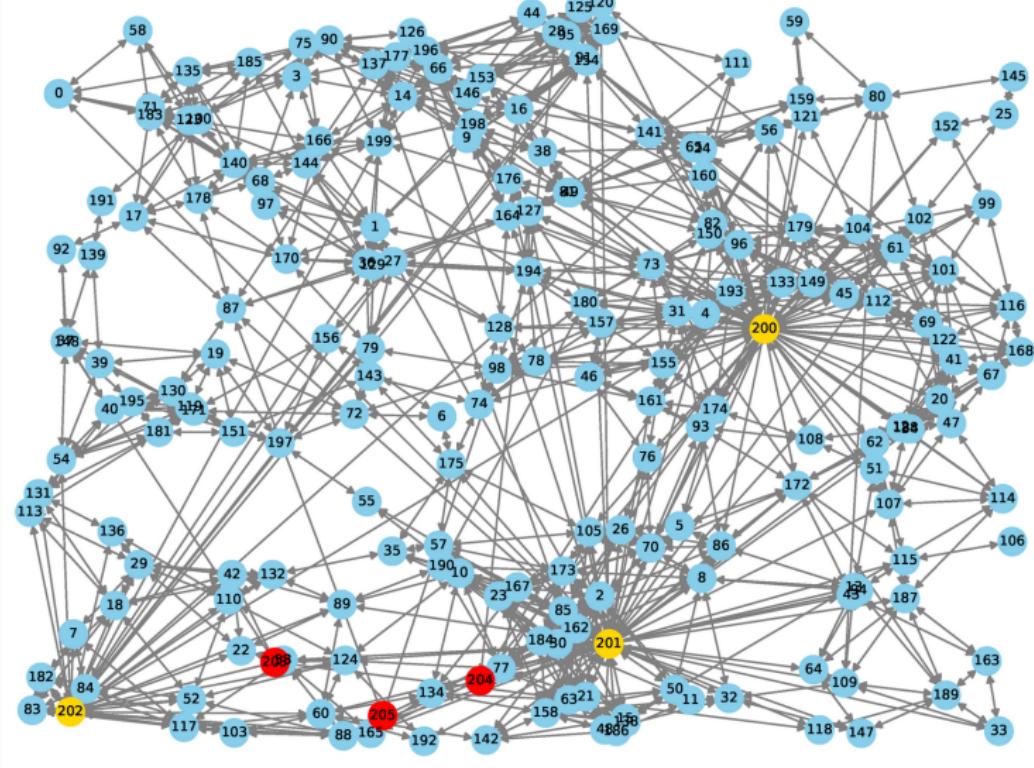
Source: [https://play.abstreet.org/0.3.49/fifteen\\_min.html](https://play.abstreet.org/0.3.49/fifteen_min.html)

# **Temporal graphs for modeling dissemination processes**

---

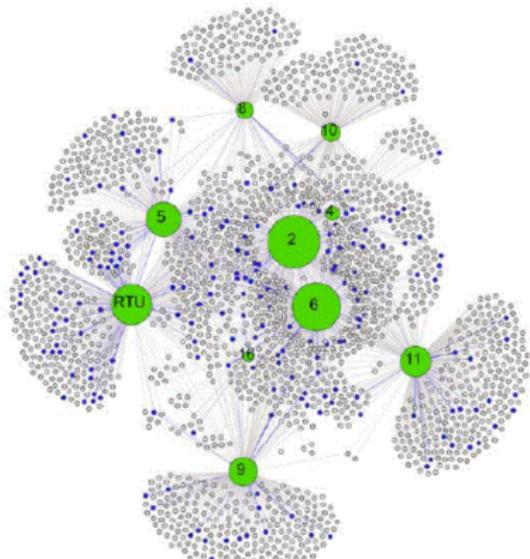
A natural application domain of temporal graphs is that of *gossiping*  
~ [4]

# What are dissemination processes?

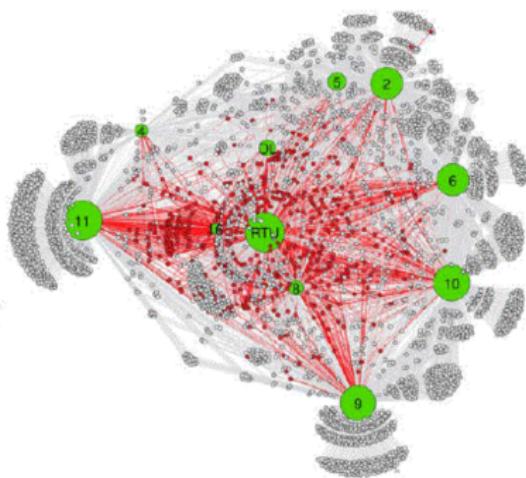


# What are dissemination processes?

A. Staff members



B. Detained persons



● Staff members with Covid-19  
○ Staff members without Covid-19

● Detained persons with Covid-19  
○ Detained persons without Covid-19

Source: [https://www.cdc.gov/mmwr/volumes/69/wr/figures/mm6944a3-F1.gif?\\_=06726](https://www.cdc.gov/mmwr/volumes/69/wr/figures/mm6944a3-F1.gif?_=06726)

# Vaccination Problem

**Definition:** The vaccination problem involves optimizing the allocation and timing of vaccines to control the spread of infectious diseases.

## Key Questions:

- Who should be vaccinated first?
- How can we minimize the spread of the disease?
- What role does timing play in vaccination strategies?

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### Herd immunity

**Herd immunity** occurs when a large enough fraction  $f$  of a community is immune to a disease, thus limiting its ability to spread.

⇒ lower  $f$  by vaccinating people in risk

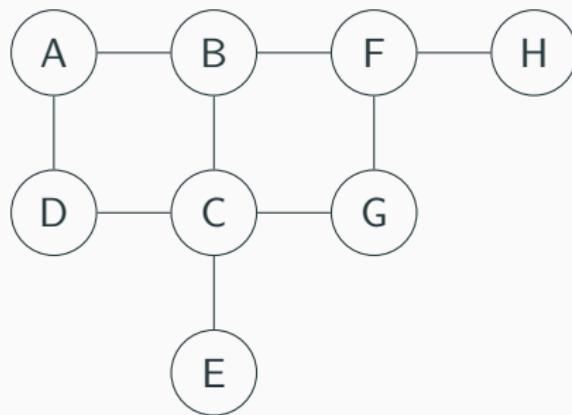
## Vaccination Problem - technical problem statement

How can we optimally choose a fraction  $f$  of a population to vaccinate using only local information?

# Vaccination Problem on static graphs

## Neighbourhood Vaccination protocol

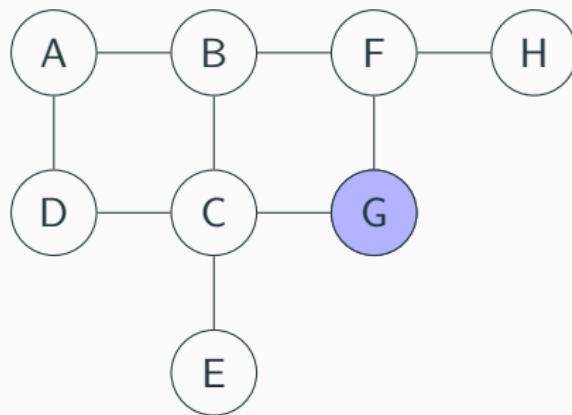
choose a person at random among all persons that have been involved in at least one contact at time  $t^*$ , ask her to name someone she met, vaccinate this other person, and repeat until a desired fraction of the vertices are vaccinated



# Vaccination Problem on static graphs

## Neighbourhood Vaccination protocol

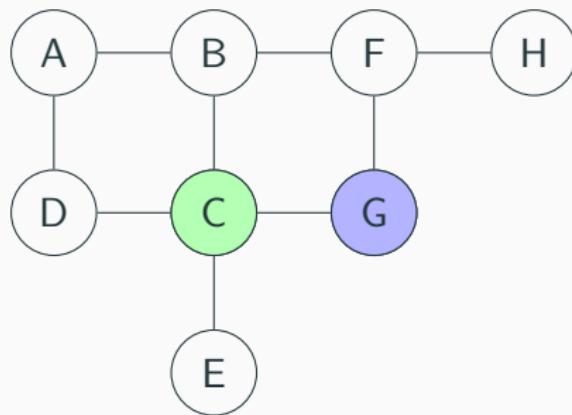
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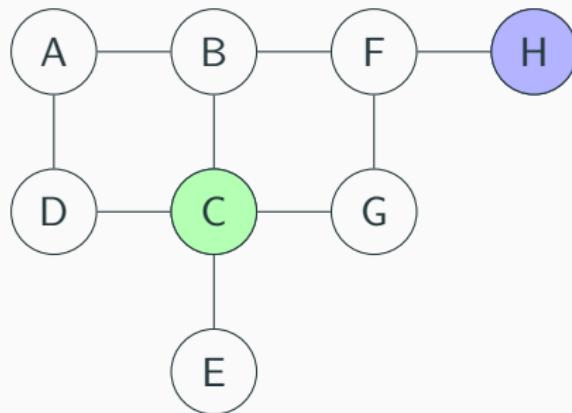
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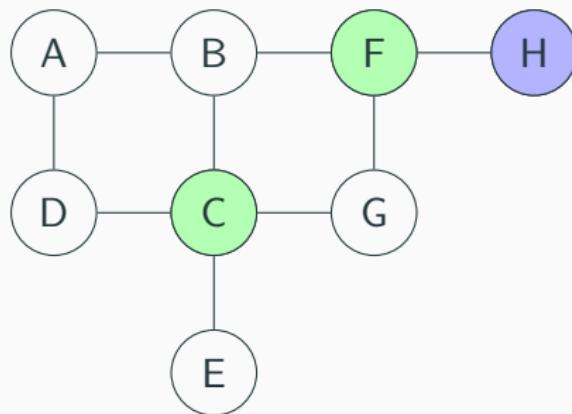
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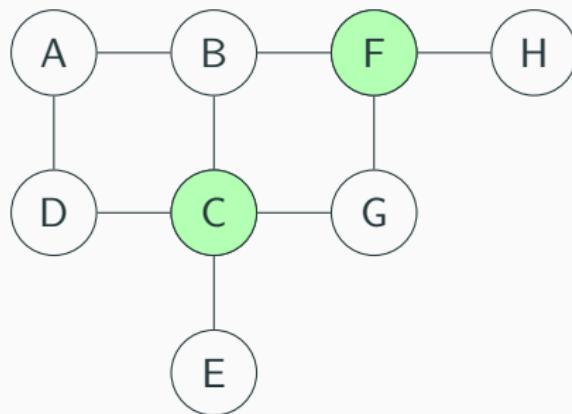
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choose a person at random among all persons that have been involved in at least one contact at time  $t^*$ , ask her to name someone she met, vaccinate this other person, and repeat until a desired fraction of the vertices are vaccinated



## Exercise

Discuss reasons why this strategy is so effective for eliminating the spread of a disease with a partner.

3 min

## Exercise

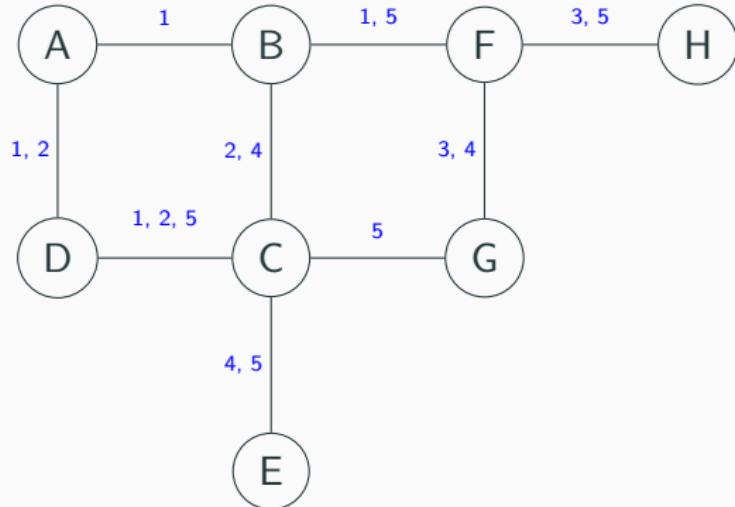
- probability  $p$  that node  $a$  is infected is roughly proportional to  $\deg(a)$
- number of infected nodes is roughly proportional to  $\deg(a)$   
→ 'risk' is roughly proportional to  $\deg^2(a)$

## Modelling dissemination processes

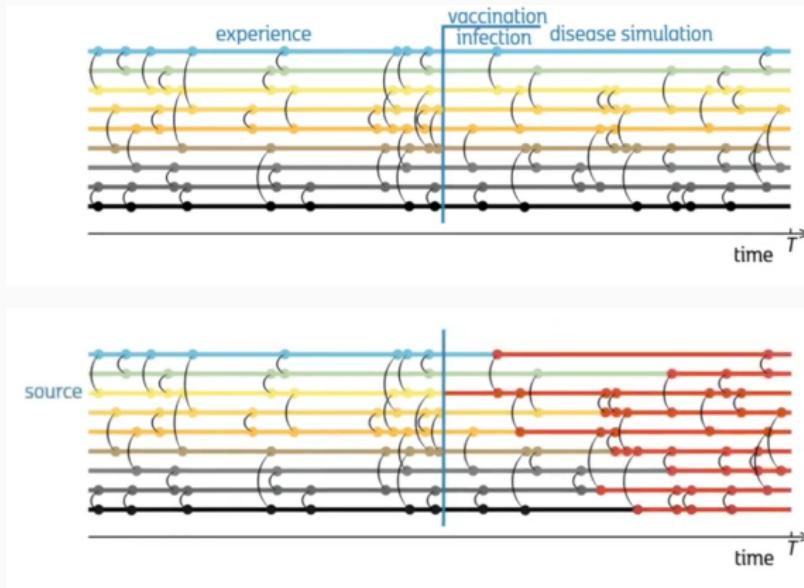
- Nodes represent individuals/groups/organizations
- Edges represent interactions
- Time labels represent the time of interaction

# Modelling dissemination processes

- Nodes represent individuals/groups/organizations
- Edges represent interactions
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# The infection process



[2]

## Protocols for vaccination in temporal graphs

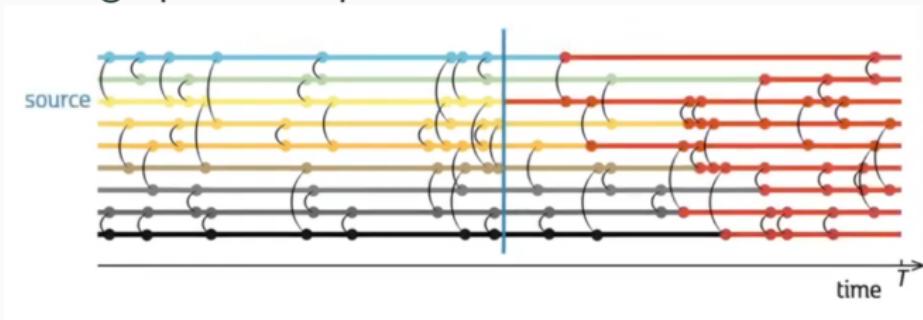
Brainstorm protocols that could be used to vaccinate individuals in a temporal graph.

Use the neighbourhood vaccination protocol from static graphs as inspiration.

4 min

# Protocols for vaccination in temporal graphs

Brainstorm protocols that could be used to vaccinate individuals in a temporal graph.  
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4 min

# Protocols for vaccination in temporal graphs

## Recent protocol

iteratively ask a random individual  $i$  to name the most recent contact and vaccinated this person

## Weight protocol

iteratively asked a random individual  $i$  to name its most frequent contact since some time  $t$  in the past

# Protocols for vaccination in temporal graphs

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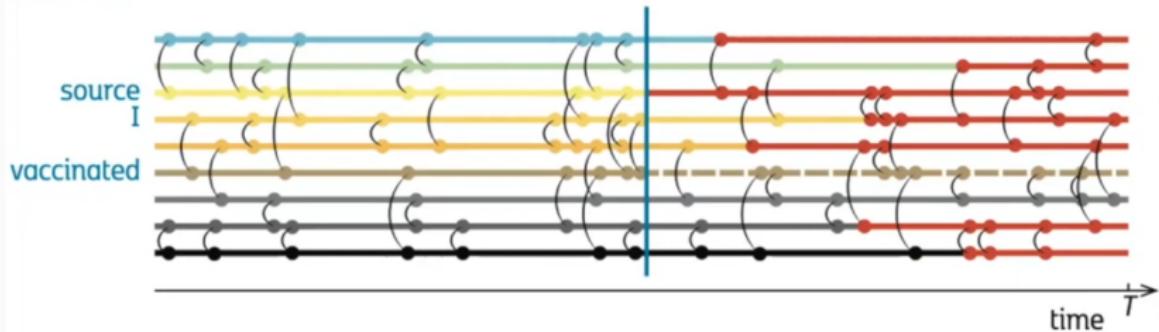
iteratively asked a random individual  $i$  to name its most frequent contact since some time  $t$  in the past

Apply those two protocols to the given temporal graphs on your handout. Then, discuss how the performance of the protocols might scale in real-world data (e.g. contacts in hospital, on a dating website, ...).

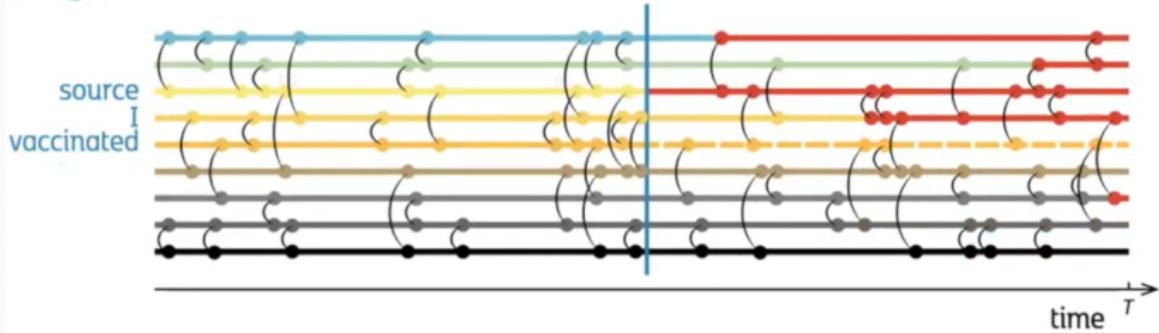
6min

# Solution

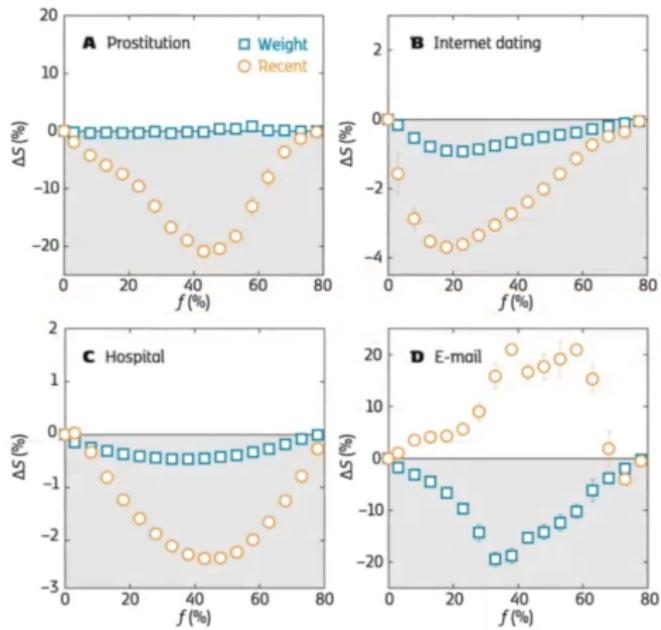
## Recent



## Weight



# Performance on bigger datasets



[2]

## Teasers

---

## Further interesting problems on temporal graphs

- Centrality of nodes in a temporal graph
- temporal graph metrics
- temporal connectivity
- Temporal Graph Neural Networks

## Sources i

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**The journey of graph kernels through two decades.**  
*Computer Science Review*, 27:88–111, 2018.
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*PLoS ONE*, 7(5):e36439, May 2012.

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**Path problems in temporal graphs.**  
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