```
clear all;
close all;
clc;

syms x1 x2 x3 z

%Define the two sets
y = [1;0;2];

c1 = x1^2 + x2^2 + x3^2 == 4;
c2 = x1^2 - 4*x2 == 0;

%Solving the constrained Set
[solx1,solx2,solx3,parameters,conditions] = solve([c1 c2],[x1 x2 x3],"ReturnConditions",true)
```

solx1 =

$$\begin{pmatrix} -2\sqrt{-\sqrt{8-z^2}-2} \\ 2\sqrt{-\sqrt{8-z^2}-2} \\ -2\sqrt{\sqrt{8-z^2}-2} \\ 2\sqrt{\sqrt{8-z^2}-2} \end{pmatrix}$$

solx2 =

$$\begin{pmatrix} -\sqrt{8-z^2}-2 \\ -\sqrt{8-z^2}-2 \\ \sqrt{8-z^2}-2 \\ \sqrt{8-z^2}-2 \end{pmatrix}$$

solx3 =

$$\begin{pmatrix} z \\ z \\ z \\ z \end{pmatrix}$$

parameters = z

conditions =

$$\begin{pmatrix} \text{symtrue} \\ \text{symtrue} \\ \text{symtrue} \\ \text{symtrue} \end{pmatrix}$$

```
%Objective function (the distance formula)
obj = (x1-1)^2 +x2^2 + (x3-2)^2;

%Substitiuting all the solutions into the objective function, taking the first derivative and setting equal to zero
%and solving to get the extrema or crital points
for i = 1:4
    s(i) = subs(obj,[x1 x2 x3],[solx1(i) solx2(i) solx3(i)])
    ds(i) = diff(s(i))
    extrema(i) = solve(ds(i)==0,z)

end
```

s = $\left(2\sqrt{-\sqrt{8-z^2}-2}+1\right)^2+(z-2)^2+\left(\sqrt{8-z^2}+2\right)^2$

ds =

$$2z-\frac{2z\left(\sqrt{8-z^2}+2\right)}{\sqrt{8-z^2}}+\frac{2z\left(2\sqrt{-\sqrt{8-z^2}-2}+1\right)}{\sqrt{8-z^2}\sqrt{-\sqrt{8-z^2}-2}}-4$$

Warning: Unable to solve symbolically. Returning a numeric solution using vpasolve.

extrema = $2.9723768414845358471609969755157-0.066637733639532590130773966438042\,\mathrm{i}$

s =

$$\left(\left(2\sqrt{-\sqrt{8-z^2}-2}+1\right)^2+(z-2)^2+\left(\sqrt{8-z^2}+2\right)^2 \quad \left(2\sqrt{-\sqrt{8-z^2}-2}-1\right)^2+(z-2)^2+\left(\sqrt{8-z^2}+2\right)^2\right)$$

ds =

$$\left(2\,z-\sigma_1+\frac{2\,z\,\left(2\,\sqrt{-\sigma_3-2}+1\right)}{\sigma_2}-4\quad 2\,z-\sigma_1+\frac{2\,z\,\left(2\,\sqrt{-\sigma_3-2}-1\right)}{\sigma_2}-4\right)$$

where

$$\sigma_1=\frac{2\,z\,(\sigma_3+2)}{\sigma_3}$$

$$\sigma_2=\sigma_3\,\sqrt{-\sigma_3-2}$$

$$\sigma_3=\sqrt{8-z^2}$$

Warning: Unable to solve symbolically. Returning a numeric solution using vpasolve.

extrema =

(2.9723768414845358471609969755157 − 0.066637733639532590130773966438042 i   −2.9723768414845358471609969755157 − 0.06

s =

$$\left((\sigma_1+1)^2+(z-2)^2+(\sigma_2+2)^2\quad(\sigma_1-1)^2+(z-2)^2+(\sigma_2+2)^2\quad(z-2)^2+(\sigma_2-2)^2+\left(2\,\sqrt{\sigma_2-2}+1\right)^2\right)$$

where

$$\sigma_1=2\,\sqrt{-\sigma_2-2}$$

$$\sigma_2=\sqrt{8-z^2}$$

ds =

$$\left(2\,z-\sigma_1+\frac{2\,z\,(\sigma_3+1)}{\sigma_2}-4\quad 2\,z-\sigma_1+\frac{2\,z\,(\sigma_3-1)}{\sigma_2}-4\quad 2\,z-\frac{2\,z\,(\sigma_4-2)}{\sigma_4}-\frac{2\,z\,\left(2\,\sqrt{\sigma_4-2}+1\right)}{\sigma_4\,\sqrt{\sigma_4-2}}-4\right)$$

where

$$\sigma_1=\frac{2\,z\,(\sigma_4+2)}{\sigma_4}$$

$$\sigma_2=\sigma_4\,\sqrt{-\sigma_4-2}$$

$$\sigma_3=2\,\sqrt{-\sigma_4-2}$$

$$\sigma_4=\sqrt{8-z^2}$$

Warning: Unable to solve symbolically. Returning a numeric solution using vpasolve.

extrema =

(2.9723768414845358471609969755157 − 0.066637733639532590130773966438042 i   −2.9723768414845358471609969755157 − 0.06

s =

$$\left((\sigma_1+1)^2+(z-2)^2+(\sigma_3+2)^2\quad(\sigma_1-1)^2+(z-2)^2+(\sigma_3+2)^2\quad(z-2)^2+(\sigma_3-2)^2+(\sigma_2+1)^2\quad(z-2)^2+(\sigma_3-2)^2+(\sigma_2-1)^2\right)$$

where

$$\sigma_1=2\,\sqrt{-\sigma_3-2}$$

$$\sigma_2=2\,\sqrt{\sigma_3-2}$$

$$\sigma_3=\sqrt{8-z^2}$$

ds =

$$\left(2z-\sigma_2+\frac{2z\,(\sigma_5+1)}{\sigma_3}-4\quad 2z-\sigma_2+\frac{2z\,(\sigma_5-1)}{\sigma_3}-4\quad 2z-\sigma_1-\frac{2z\,(\sigma_6+1)}{\sigma_4}-4\quad 2z-\sigma_1-\frac{2z\,(\sigma_6-1)}{\sigma_4}-4\right)$$

where

$$\sigma_1=\frac{2z\,(\sigma_7-2)}{\sigma_7}$$

$$\sigma_2=\frac{2z\,(\sigma_7+2)}{\sigma_7}$$

$$\sigma_3=\sigma_7\,\sqrt{-\sigma_7-2}$$

$$\sigma_4=\sigma_7\,\sqrt{\sigma_7-2}$$

$$\sigma_5=2\,\sqrt{-\sigma_7-2}$$

$$\sigma_6=2\,\sqrt{\sigma_7-2}$$

$$\sigma_7=\sqrt{8-z^2}$$

```
Warning: Unable to solve symbolically. Returning a numeric solution using vpasolve.
extrema =
```

$(2.9723768414845358471609969755157-0.066637733639532590130773966438042\,i\quad -2.9723768414845358471609969755157-0.06$

```
x3crit = real(extrema)
```

```
x3crit =
```

$(2.9723768414845358471609969755157\quad -2.9723768414845358471609969755157\quad -1.8100643633589540125708462663186\quad 1.81006$

```
%Using the 4th value because it is the only real positive solution, setting
%it to x3
x3bar = x3crit(4)
```

x3bar = $1.8100643633589540125708462663186$

```
%Solving for x2 using our newfound x3
x2crit = real(subs(solx2,z,x3bar))
```

x2crit =

$$\begin{pmatrix}-4.1733998712841464826722001573208\\-4.1733998712841464826722001573208\\0.17339987128414648267220015732079\\0.17339987128414648267220015732079\end{pmatrix}$$

```
%Solving for x1 using our newfound x3
x1crit = real(subs(solx1,z,x3bar))
```

x1crit =

$$\begin{pmatrix}0\\0\\-0.83282620344018110792741305085171\\0.83282620344018110792741305085171\end{pmatrix}$$

```
x2bar = x2crit(4)
```

x2bar = $0.17339987128414648267220015732079$

```
x1bar = x1crit(4)
```

x1bar = $0.83282620344018110792741305085171$

```
x = [x1; x2; x3]
```

x =
$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

```
%Solutions
xbar = [x1bar;x2bar;x3bar]
```

xbar =
$$\begin{pmatrix} 0.83282620344018110792741305085171 \\ 0.17339987128414648267220015732079 \\ 1.8100643633589540125708462663186 \end{pmatrix}$$

```
%Plugging in the solution into the objective function and taking the sqrt
%to make it the actual distance value
minDist = sqrt(subs(obj,x,xbar))
```

minDist = $0.30674116072646940067324359552447$

```
%Finding the seperating hyperplane by plugging into the equation (y-xbar)^t(x-xbar)=0
hyperplane = simplify(transpose(y-xbar)*(x-xbar)==0)
```

hyperplane =
$0.16717379655981889207258694914829\, x_1 + 0.18993563664104598742915373368137\, x_3 = 0.17339987128414648267220015732079\, x_2 +$