MAT 5800 – Mathematical Programming – Fall 2021
California State Polytechnic University, Pomona

Programming Project

The project will involve a computer programming part and a written part. The computer programming part of the project can be done using any computer language of your choice (for example, basic, Pascal, Fortran, c, c++, Matlab, Maple, Mathematica, etc.). You must submit a **formal** written report that includes the following:

1) **Abstract.** A summary of your results and conclusions
2) **Introduction**. A description of the project.
3) **Discussion of the results and project.** This is a very important part of the project. Here you should discuss your results in the context of the project and the course material. You may include your observations and what you have learned from the project. You may include difficulties you encountered and interesting things you observed while working on the project.
4) **Source code.** Include a copy of the source code of all the program(s) you used.

Due date: Tuesday November 30, 2021. Submit your report to Canvas as a single file in pdf format.

The Minimum Distance Between two Orbits

A tilted elliptical orbit having (0, 0) as one focal point is given by

$$\begin{bmatrix} x_1(t) \\ y_1(t) \end{bmatrix} = \begin{bmatrix} \cos(\phi_1) & \sin(\phi_1) \\ -\sin(\phi_1) & \cos(\phi_1) \end{bmatrix} \begin{bmatrix} \frac{P_1 - A_1}{2} + \frac{P_1 + A_1}{2}\cos(t) \\ \sqrt{P_1 A_1}\,\sin(t) \end{bmatrix}.$$

Think of the Sun as being the focus (0, 0). The tilt angle is given by ϕ_1, A_1 and P_1 are fixed parameters.

A second orbit is given by

$$\begin{bmatrix} x_2(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} \cos(\phi_2) & \sin(\phi_2) \\ -\sin(\phi_2) & \cos(\phi_2) \end{bmatrix} \begin{bmatrix} \frac{P_2 - A_2}{2} + \frac{P_2 + A_2}{2}\cos(t) \\ \sqrt{P_2 A_2}\,\sin(t) \end{bmatrix}.$$

The objective is to find the minimum distance from a point (A_1, P_1, ϕ_1) on the first orbit to a point (A_2, P_2, ϕ_2) on the second orbit. For a measure of distance we can use the function

$$dis(t_1, t_2) = \frac{1}{2}\left[(x_1(t_1) - x_2(t_2))^2 + (y_1(t_1) - y_2(t_2))^2\right].$$

This is a function of two variables.

The objective is to minimize $dis(t_1, t_2)$. Use the parameters $(A_1, P_1, \phi_1) = (10, 2, \pi/8)$ and $(A_2, P_2, \phi_2) = (4, 1, -\pi/7)$.

The following steps may help you obtain the minimum for $dis(t_1, t_2)$.

1) Plot the two orbits on the same graph.
2) Plot contour lines of $dis(t_1, t_2)$ to get an idea of the solution.
3) Determine the gradient of $dis(t_1, t_2)$ and use it in the steepest decent method to find the minimum.