5320HW1

Daniel Silva

2/17/22

Problem 1

```
set.seed(5320)
n <- 10
n2 <- 100000
x1 <- rnorm(n,8,4)
x2 <- runif(n,0,12)
X <- rbind(x1,x2)
ex1 <- rnorm(n2,8,4)
ex2 <- runif(n2,0,12)
eX <- rbind(x1,x2)
xtest <- cbind(x1,x2)
cov(xtest)
```

```
##           x1        x2
## x1 10.499272 -2.736052
## x2 -2.736052  8.634985
```

Part a

Find the expectation E[X] and the variance matrix Var[X] of the random vector X

$E[X] = [8, 6]'$

$Var[X] = [16, 0; 0, 12]$

```
mu_x = matrix(c(8,6),2,1,byrow=TRUE)
var_x = matrix(c(16,0,0,12),2,2,byrow=TRUE)
```

Part b

Find a matrix A and a column vector b such that Y = [Y1,Y2,Y3]' = AX+b

$[1, 1, 0; 1, -1, 0; 5, 0, 0] * [x1, x2, x3]' = A * [x1, x2, x3]' + b$

where $A = [1, 1, 0; 1, -1, 0; 5, 0, 0]$ and $b = [0, 0, 10]'$

```
y1 = x1+x2
y2 = x1-x2
y3 = 5*x1+10
Y = rbind(y1,y2,y3)

ey1 = ex1+ex2
ey2 = ex1-ex2
```

```
ey3 = 5*ex1+10
eY = rbind(ey1,ey2,ey3)


A = matrix(c(1,1,0,1,-1,0,5,0,0),3,3,byrow=TRUE)
b = matrix(c(0,0,10),3,1,byrow=TRUE)
```

Part c

Find the expectation E[Y] and Var[Y] of the random variable Y.

$E[Y] = E[[Y1, Y2, Y3]'] = [E[Y1], E[Y2], E[Y3]]' = [E[X1] + E[X2], E[X1] - E[X2], 5 * E[X1] + 10]'$
$= [8 + 6, 8 - 6, 5 * 8 + 10]' = [14, 2, 50]'$

$Var(Y) = Var([Y1, Y2, Y3]')$

$$Var(Y) = \begin{bmatrix} Var(Y1) & Cov[Y1, Y2] & Cov[Y1, Y3] \\ Cov[Y2, Y1] & Var(Y2) & Cov[Y2, Y3] \\ Cov[Y3, Y1] & Cov[Y3, Y2] & Var(Y3) \end{bmatrix}$$

$$Var(Y) = \begin{bmatrix} Var(X1 + X2) & Cov[X1 + X2, X1 - X2] & Cov[X1 + X2, 5 * X1 + 10] \\ Cov[X1 - X2, X1 + X2] & Var(X1 - X2) & Cov[X1 - X2, 5 * X1 + 10] \\ Cov[5 * X1 + 10, X1 + X2] & Cov[5 * X1 + 10, X1 - X2] & Var(5 * X1 + 10) \end{bmatrix}$$

$$Var(Y) = \begin{bmatrix} var(x1) + var(x2) & var(x1) - var(x2) & 5 * var(x1) \\ var(x1) - var(x2) & Var(x1) - var(x2) & 5 * var(x1) \\ 5 * var(x1) & 5 * var(x1) & 25 * var(x1) + 10 \end{bmatrix}$$

$$Var(Y) = \begin{bmatrix} 28 & 4 & 80 \\ 4 & 4 & 80 \\ 80 & 80 & 310 \end{bmatrix}$$

Part d

Simulations n = 10

```
m = cbind(y1,y2,y3)
colMeans(m)
```

```
##        y1        y2        y3
## 13.322609  5.866606 57.973038
```

```
cov(m)
```

```
##           y1        y2        y3
## y1 13.662153  1.864288  38.81610
## y2  1.864288 24.606362  66.17662
## y3 38.816101 66.176624 262.48181
```

Part e

Simulations n = 100,000

```
em = cbind(ey1,ey2,ey3)
colMeans(em)
```

```
##       ey1       ey2       ey3
## 14.004214  1.972506 49.941799
```

```
cov(em)
```

```
##          ey1      ey2       ey3
## ey1 28.03076  3.85481  79.71393
## ey2  3.85481 27.74793  79.00686
## ey3 79.71393 79.00686 396.80199
```

Problem 2

Seber Lee 1a.1

Prove that if a is a vector of constants with the same dimension as the random vector X ,., then

$E[(X-a)(X-a)'] = Var[X] + (E[X]-a)(E[X]-a)'$

Proof:

Let

$E[X] = \mu$

$E[(X-a)(X-a)'] = E[(X-\mu+\mu-a)(X-\mu+\mu-a)']$
$= E[(X-\mu)(X = `-\mu)' + (X-\mu)(\mu-a)' + (\mu-a)(X-\mu)' + (\mu-a)(\mu-a)']$
$= E[(X-\mu)(X = `-\mu)'] + E[(X-\mu)(\mu-a)'] + E[(\mu-a)(X-\mu)'] + E[(\mu-a)(\mu-a)']$
$= Var(X) + (E[X]-a)(E[X]-a)'$

Problem 3

```
sigma_X = matrix(c(1,1,3/4,1,2,3,3/4,3,3),3,3,byrow=TRUE)
B = matrix(c(2,-1,1),1,3,byrow=TRUE)
B%*%sigma_X%*%t(B)
```

```
##      [,1]
## [1,]    2
```

```
B%*%matrix(c(1,1,-1),3,1,byrow=TRUE)
```

```
##      [,1]
## [1,]    0
```

```
B%*%sigma_X%*%t(B)
```

```
##      [,1]
## [1,]    2
```