2019

Taller Ecuaciones Diferenciales

TALLER VIRTUAL

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$y = -xe^{x} + c$

1) $y' + xy + x = \emptyset$

$\frac{dy}{dx} + xy + x = 0$

$\frac{dy}{dx} = -xy - x$

$\frac{dy}{dx} = -x(y+1)$

$\frac{dy}{y+1} = -x\,dx$

$\int \frac{dy}{y+1} = \int -x\,dx$

$\ln|y+1| = -\frac{x^2}{2}$

$y+1 = e^{-x^2/2}$

$y = e^{-x^2/2} - 1$

2) $y' + y = 2x$

$\frac{dy}{dx} + y = 2$

$y\mu(x) = \int Q($

$y(e^{x}) = \int (2xe^{-}$

$ye^{x} = \int 2xe^{0}$

$ye^{x} = \int 2x\,dx +$

$ye^{x} = \frac{2x^2}{2} + \int$

$ye^{x} = x^2 + \int e^{x}$

$ye^{x} = x^2 + x^2e^{x} -$

$ye^{0} = 0^2 + \cancel{0^2e^{0}} - 2$

$y = -2 + c$ $\quad c =$

$c - 2 = 5$ $\quad c$

2) $y' + y = 2xe^{-x} + x^2$ donde $y(0) = 5$ $\quad y = 5$, $x = 0$

$$\frac{dy}{dx} + y = \underbrace{2xe^{-x} + x^2}_{Q(x)}$$

$P(x) = 1$

$\int P(x) = \int dx$

$\int P(x) = x$

$\mu(x) = e^{x}$

$$y\mu(x) = \int Q(x)\mu(x)\,dx$$

$$y(e^{x}) = \int (2xe^{-x} + x^2)(e^{x})\,dx$$

$$ye^{x} = \int 2xe^{0} + e^{x}x^2\,dx$$

$$ye^{x} = \int 2x\,dx + \int e^{x}x^2\,dx$$

$$ye^{x} = \frac{2x^2}{2} + \int e^{x}x^2\,dx$$

$\int e^{x}x^2\,dx$

$u = x^2 \quad dv = e^{x}dx$

$du = 2x\,dx \quad v = e^{x}$

$x^2e^{x} - \int 2xe^{x}\,dx$

$x^2e^{x} - 2\int xe^{x}\,dx$

$u = x \quad dv = e^{x}dx$

$du = dx \quad v = e^{x}$

$x^2e^{x} - 2\left[xe^{x} - \int e^{x}dx\right]$

$x^2e^{x} - 2xe^{x} + 2e^{x}$

$$ye^{x} = x^2 + \int e^{x}x^2\,dx$$

$$ye^{x} = x^2 + x^2e^{x} - 2xe^{x} + 2e^{x} + C$$

$$ye^{0} = \cancel{0^2} + \cancel{0^2e^{0}} - \cancel{2(0)e^{0}} + 2e^{0} + C$$

$y = +2 + C \qquad C = 5 - 2$

$C + 2 = 5 \qquad C = 3$

Día Mes Año

3) $y'(e^y - x) = y$

Formulas

$-y + y'(e^y - x) = 0$

$M(x,y) + N(x,y)\,y' = 0$

Donde $M(x,y) = -y$

$N(x,y) = e^y - x$.

$\frac{\partial M(x,y)}{\partial y} = \frac{\partial N(x,y)}{\partial x}$

$-1 = -1$! Tautología

$\frac{\partial M(x,y)}{\partial y} = -1$

$\frac{\partial N(x,y)}{\partial x} = \cancel{e^y} - 1$ $\longrightarrow -xy + e^y = C \rightarrow y = -w\left(-\frac{1}{e^{c/x} \cdot x}\right) - \frac{c}{x}$

4) $(2x^2 - ye^x)\,dx - e^x dy = 0$ $y(0) = 1$

$(2x^2 - ye^x)\,dx = e^x\,dy$ $y = 1$, $x = 0$

$\frac{dx}{dy}(e^x) = 2x^2 - ye^x$

$P(x) = 1 \rightarrow \int P(x) = x$

$y'e^x = 2x^2 - ye^x$

$Q(x) = 2x^2 e^{-x}$

$y'e^x + ye^x = 2x^2$

$\mu(x) = e^x$

$e^x(y' + y) = 2x^2$

$y' + y = 2x^2 e^{-x}$

$y\mu(x) = \int Q(x)\mu(x)\,dx$

$ye^x = \int (2x^2 e^{-x})(e^x)\,dx$

$ye^x = 2\int x^2 e^{-x+x}\,dx$

$ye^x = 2\int x^2 e^0\,dx$

$ye^x = 2\int x^2\,dx$

$ye^x = \frac{2x^3}{3}\,dx$

$\boxed{y = \frac{2}{3}x^3 e^{-x} + C}$

$1 = \frac{2}{\cancel{3}}(0)^3 e^{-(0)} + C \longrightarrow C = 1$

5) $(x+2)^2 y' = 5 - 8y - 4xy$. $(x+2)^2 = x^2 + 2x + 4$.

$\frac{(x+2)^2}{(x+2)^2}y' = \frac{5}{(x+2)^2} - \frac{8y}{(x+2)^2} - \frac{4xy}{(x+2)^2}$.

$\left(\frac{1}{x^2+2x+4}x^2 + \frac{4}{x^2+4x+4}x + \frac{4}{x^2+4x+4}\right)y' + \frac{4}{2+x}y = \frac{5}{x^2+4x+4}$

$P(x) = \frac{4}{2+x} \rightarrow \int P(x) = \int \frac{4}{2+x} \rightarrow 4\ln|2+x| \rightarrow \ln|(2+x)^4|$

$Q(x) = \frac{5}{x^2+4x+4}$ $\mu(x) = e^{\ln|(2+x)^4|} \rightarrow (2+x)^4$

Design

Mes Año

$$y\mu(x) = \int Q(x)\mu(x)\,dx$$

$$y(x+2)^4 = \int \left(\frac{5}{x^2+4x+4}\right)(x+2)^4\,dx.$$

$$y(x+2)^4 = \int \frac{5}{\cancel{(x+2)^2}}(x+2)^{\cancel{4}}\,dx$$

$$y(x+2)^4 = \int 5(x+2)^2\,dx$$

$$y(x+2)^4 = 5\int (x+2)^2\,dx$$

$$y(x+2)^4 = 5\left(\frac{(x+2)^3}{3}\right)$$

$$\int (x+2)^2\,dx \rightarrow \int u^2\,du.$$

$u = x+2$ $\quad \frac{u^3}{3}$

$du = dx$

$$\frac{(x+2)^3}{3}$$

$$y = \frac{5}{3}\frac{\cancel{(x+2)^3}}{(x+2)^{\cancel{4}}}$$

$$y = \frac{5}{3(x+2)}$$

6) $y - x\frac{dy}{dx} = y^2 e^y \frac{dy}{dx}.$

$$y - xy' = y^2 e^y y'$$

Reescribiendo como E.D. Exacta

$$\frac{1}{y} + \left(\frac{-x-e^y y^2}{y^2}\right)y' = 0$$

$$M(x,y) + N(x,y)y' = 0$$

$\downarrow \frac{1}{y}$ $\qquad$ $\downarrow \frac{-x-e^y y^2}{y^2}$

$$\left.\begin{aligned}\frac{\partial M(x,y)}{\partial y} &= \frac{-1}{y^2}\\ \frac{\partial N(x,y)}{\partial x} &= \frac{-1}{y^2}\end{aligned}\right\} \text{Tautología}$$

$$\downarrow$$

$$\frac{x}{y} - e^y = C$$

7) $\frac{dy}{dx} = \frac{y}{y-x}$ $\qquad y(5) = 2$ $\quad \begin{cases} y = 2 \\ x = 5 \end{cases}$

$$y' = \frac{y}{y-x} \longrightarrow y'(y-x) = y. \longrightarrow y'(y-x) - y = 0$$

$$M(x,y) + N(x,y)y' = 0$$

$\downarrow -y$ $\qquad$ $\downarrow y-x$

$$\left.\begin{aligned}\frac{\partial M(x,y)}{\partial y} &= -1.\\ \frac{\partial N(x,y)}{\partial x} &= -1\end{aligned}\right\} \text{Iguales}$$

$$\hookrightarrow -xy + \frac{y^2}{2} = C.$$

Año

8) $y' - \left(\frac{2}{x+1}\right) y = (x+1)^3$

$P(x)$ $\quad$ $Q(x)$

$P(x) = \frac{-2}{x+1} \rightarrow \int P(x) = -2 \ln|x+1|$

$\hookrightarrow \ln|(x+1)^{-2}|$

$\mu(x) = e^{\ln|(x+1)^{-2}|}$

$\mu(x) = (x+1)^{-2} \rightarrow \frac{1}{(x+1)^2}$

$y\left(\frac{1}{(x+1)^2}\right) = \int (x+1)^{\cancel{3}} \left(\frac{1}{\cancel{(x+1)^2}}\right) dx$

$\frac{y}{(x+1)^2} = \int (x+1)\, dx$

$\frac{y}{(x+1)^2} = \int x\, dx + \int dx$

$\frac{y}{(x+1)^2} = \frac{x^2}{2} + x + C$

$y = \frac{x^2(x+1)^2}{2} + x(x+1)^2 + C(x+1)^2$

9) $(6-2xy)y' + y^2 = \emptyset$ $\quad$ $y(0)=1$ $\begin{cases} y=1 \\ x=0 \end{cases}$

$(6-2xy)y' = -y^2$

$y' = \frac{-y^2}{6-2xy}$ $\quad$ Sustituir $y = -\left(\frac{v-6}{2x}\right)$

$\left(-\frac{v-6}{2x}\right)' = \frac{-\left(-\frac{v-6}{2x}\right)}{6-2x\left(-\frac{v-6}{2x}\right)} \longrightarrow \left(-\frac{v-6}{2x}\right)' = -\frac{xv' - v + 6}{2x^2}$

$-\left(\frac{xv' - v + 6}{2x^2}\right) = -\frac{(v-6)^2}{4x^2 v} \longrightarrow \underbrace{-\frac{1}{2}\ln|-v^2+8v-12| + \ln\left(\frac{\left|\frac{v-2}{2}\right|}{\left|\frac{v-6}{2}\right|}\right)}_{\frac{-3}{2}\ln x}$

Sustituir $v = 6-2y$

$-\frac{1}{2}\ln|-(6-2y)^2 + 8(6-2y) - 12| + \ln\left(\frac{\left|\frac{6-2y-2}{2}\right|}{\left|\frac{6-2y-6}{2}\right|}\right) = \frac{-3}{2}\ln x$

Design

$$-\frac{\ln(-4x^2y^2+8xy)}{2}+\ln\left(\frac{|-xy+2|}{|xy|}\right)=-\frac{3}{2}\ln x+C$$

10) $y\,dx+(xy+2x-ye^y)\,dy=0$

$$\frac{y\,\cancel{dx}}{\cancel{dx}}+(xy+2x-ye^y)\frac{dy}{dx}=0$$

$$y+(xy+2x-ye^y)\frac{dy}{dx}=0$$

$$y+(xy+2x-ye^y)y'=0$$

$$e^y y^2+e^y y(xy+2x-e^y y)y'=0$$

$$M(x,y)=e^y y^2$$

$$N(x,y)=e^y y(xy+2x-e^y y)$$

$$\frac{\partial M(x,y)}{\partial y}=e^y y(y+2)$$

$$\frac{\partial N(x,y)}{\partial x}=e^y y(y+2)$$

$$xe^y y^2-\frac{1}{2}e^{2y}y^2+\frac{1}{2}e^{2y}y-\frac{1}{4}e^{2y}+C$$

$$xe^y y^2-\frac{1}{2}e^{2y}y^2+\frac{1}{2}e^{2y}y-\frac{1}{4}e^{2y}=C$$