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CST-201 Data Structures and Algorithms in C++

Wk 7 – Individual Assignment

Chapter 7 Exercise 8.14.32 below:

32. A tournament is a digraph in which there is exactly one edge between every two vertices.

**a. How many edges does a tournament have?**

A tournament that has *n* vertices will have *n(n – 1)/2* edges in total. So, a tournament of 3 vertices will have 3 edges, and a tournament of 50 vertices will have 1,225 vertices.

**b. How many different tournaments of n edges can be created?**

The size and connectivity of a tournament is bound to the amount of *n* edges it has. 1 edge can create only 2 tournament; 2 edges can create 4 tournaments (flipping orientation of edges), 3 edges can create 8 tournaments. Overall, a pattern seems to occur that *n* edges has the potential to create *2n* different tournaments. I am sure it is more complex than this, given special conditions. But, this seems like a good ballpark estimate, given the content I have read. <https://math.stackexchange.com/questions/2229270/how-many-tournaments-are-there> has a bit more info on the more complex scenarios, but much of it went over my head and the scope of what this class has taught me.

**c. Can each tournament be topologically sorted?**

A tournament can be topologically sorted, yes. There are a few ways to modify common sorting algorithms for trees/arrays for use with tournaments: <https://www.geeksforgeeks.org/topological-sorting/>. The more established way to topologically sort a tournament is to modify a Depth First Search to use an iterator on each “tier” of nodes in a graph, so as to not skip over vertices that ought to be in its path.

**d. How many minimal vertices can a tournament have?**

If by “minimal vertices”, a minimum vertex cover is meant, then there are *n* minimal vertices, since every vertex has an edge connecting it to every other vertex in the graph.

e. A transitive tournament is a tournament that has edge(vw) if it has edge(vu) and edge(uw). Can such a tournament have a cycle?

No, such a tournament is prevented from forming a cycle due to the rule imposed on the last edge (uw in the example). One of the edges will always have an opposite orientation of what is needed to form a complete cycle.