FINM3405 Derivatives and risk management

Tutorial Sheet 7: Options - Numerical methods (binomial and Monte Carlo)

August 31, 2024

Question 1. Download the historical daily returns of your favourite stock or share market index from yahoo!finance or some other data provider. Then using Excel or some other means, on the same graph plot the (i) time series of daily returns and (i) a rolling annualised 180 day historical volatility. What does the plot tell you about stock or share market index volatilities over time?

In the following questions let $S = 50$, $K = 50$, $r = 5\%$, $T = \frac{1}{2}$, $\sigma = 25\%$ and $q = 0$ unless otherwise stated.

Binomial model

Question 2. Calculate by hand the 1-step binomial prices of ATM European call options and compare them to the Black-Scholes prices. Do the same for strikes of $K = 47.5$ and $K = 52.5$. Use the CRR or JR schemes.

Question 3. Use excel to create a 7 layer binomial model asset price tree and calculate the binomial model prices of ATM European call and put options, and compare your prices to the Black-Scholes model prices. Do the same for strikes of $K = 47.5$ and $K = 52.5$. Use the CRR or JR schemes.

Question 4. In the derivation of the binomial model in the lecture notes, we suggestively defined and found an equation for the parameter Δ, namely

$$\Delta = \frac{C_u - C_d}{S(u-d)} = \frac{C_u - C_d}{S_u - S_d}.$$

The notation was not a coincidence since it does in fact define the delta of a call option as calculated by the binomial model. Calculate by hand the 1-period binomial model deltas of ATM calls and puts and compare them to the Black-Scholes model deltas. You can use the CRR or JR schemes.

Question 5. Following on from questions 3 and 4, calculate the ATM put and call deltas at each node of your 7 layer asset price tree (not including the final expiry date nodes). Then compare the deltas at the time $t_0 = 0$ node to the Black-Scholes model deltas.

Question 6. A benefit of numerical option pricing methods is they can handle a very wide variety of payoffs at expiry, and hence price a wide variety of exotic options. Use the 7 layer binomial model to price the cash-or-nothing and asset-or-nothing binary options presented in a previous tutorial sheet and compare the results to their Black-Scholes prices. Use the CRR or JR schemes.

Monte Carlo method

Question 7. When pricing European options by the Monte Carlo method, we actually only need to calculate the N final asset price prices S_i at expiry for $i = 1, \ldots, N$. We can use the formula in the lecture notes to simulate them via

$$S_i = S e^{(r-\frac{1}{2}\sigma^2)T+\sigma\sqrt{T}Z_i} \qquad \text{for } i = 1, \ldots, N,$$

where each Z_i is an independent, identically distributed standard normal random variable. The option prices are then simply given by

$$C = e^{-rT}\frac{1}{N}\sum_{i=1}^{N}\max\{0, S_i - K\} \qquad \text{and} \qquad P = e^{-rT}\frac{1}{N}\sum_{i=1}^{N}\max\{0, K - S_i\}.$$

Use Excel, Python, R or Matlab, etc (whatever is your software of choice - I use these and Java and C^{++}), to price ATM call and put options via this "simplified" Monte Carlo method that simulates only the final asset prices at expiry. *Remark*: It's particularly easy in Excel by simulating 1 final asset price at expiry and then dragging this cell down for say another 499 cells to get 500 simulated final asset prices at expiry. Compare your prices to the Black-Scholes and binomial model prices. Do the same for strikes of $K = 47.5$ and $K = 52.5$.

Question 8. Now use this simplified Monte Carlo method to price the ATM cash-or-nothing and asset-or-nothing binary options and compare your prices to the Black-Scholes and binomial model prices.

Remark: Chapter 26 of Cuthbertson, Nitzsche and O'Sullivan, Derivatives: Theory and Practice provides some Matlab code for Monte Carlo pricing.