FINM3405 Derivatives and risk management

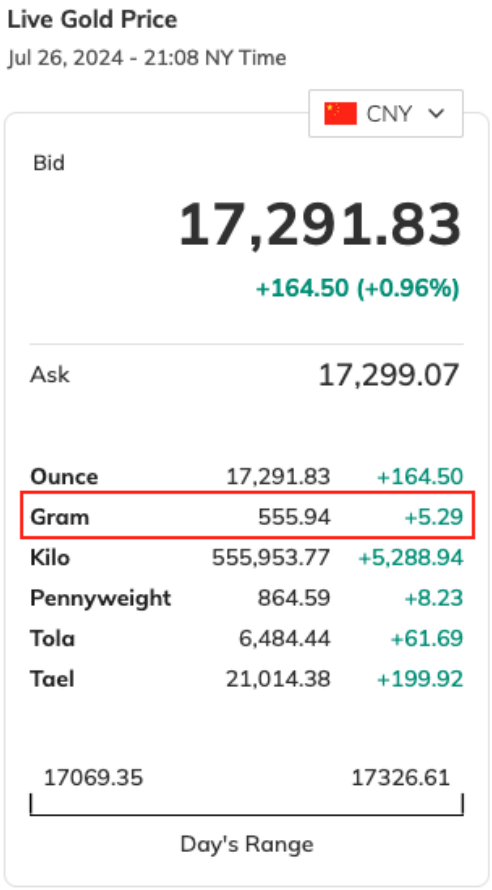
Tutorial Sheet 2: Futures and forwards examples and basic concepts

Suggested solutions

August 2, 2024

Commodities

Question 1. Here is the current gold price in CNY (the Chinese Yuan) and contract specifications for gold futures on the Shanghai Futures Exchange:



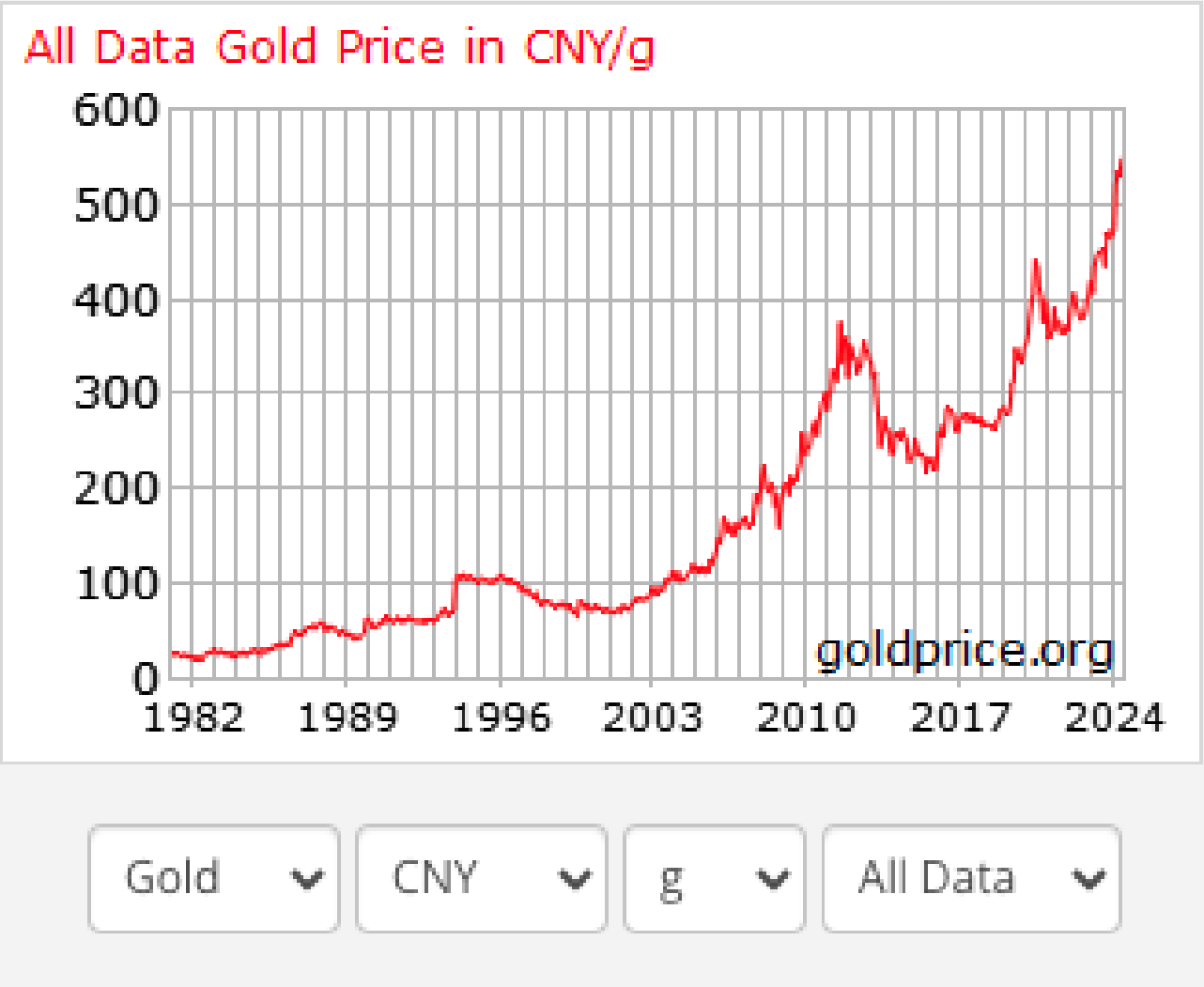
SHFE Gold Contract Specification

Updated on Aug 31,2020

Product	Gold
Contract Size	1,000 grams/lot
Price Quotation	Yuan (RMB)/gram
Minimum Price Fluctuation	0.02 yuan/gram
Range of Price Limit	Within 3% of the settlement price of the preceding trading day
Listed Contracts	Monthly contract for the most recent 3 months and bimonthly contract for the most recent 13 months
Trading Hours	9:00 a.m. to 11:30 a.m., 1:30 p.m. to 3:00 p.m., and other hours specified by the Exchange (Beijing Time)
Last Trading Day	15th day of the contract month (postponed accordingly if it is a legal holiday in China and subject to separate adjustment and announcement by the Exchange if it falls in the Spring Festival month or another month specially designated by the Exchange.)
Delivery Period	The first business day after the last trading day
Grade and Quality Specifications	Domestic gold ingots with gold content not less than 99.95% and standard gold ingots produced by a supplier or refiner certified by the London Bullion Market Association (LBMA) and recognized by the Exchange (refer to the Annex for detailed quality standards).
Delivery Venue	SHFE-designated gold delivery warehouses
Minimum Trade Margin	4% of contract value
Settlement Type	Physical delivery
Delivery Unit	3,000 grams
Contract Symbol	AU
Listing Exchange	Shanghai Futures Exchange

Here is some gold futures quotes and a chart of the CNY gold price:

Contract	Last	Chg	Open Interest	Volume	Turnover	Bid-Ask	Pre-clear	Open	Low	High	Chart
au2408	558.76	5.66	16872	6591	3676234640	558.32/558.76	553.10	557.10	556.00	559.40	
au2409	558.72	4.62	211	57	31846460	559.66/559.94	554.10	557.72	557.22	560.02	
au2410	560.64	5.76	178248	132176	73980674220	560.60/560.62	554.88	559.58	557.84	561.26	
au2412	562.78	5.62	132121	27051	15197078600	562.74/562.78	557.16	560.92	559.82	563.36	
au2502	564.70	6.26	21013	3818	2151948920	563.52/565.00	558.44	563.48	561.78	565.26	
au2504	566.82	6.10	15754	1260	713010380	566.70/567.18	560.72	565.38	564.12	567.46	
au2506	569.14	6.36	9138	586	332974020	567.00/569.48	562.78	567.86	566.50	569.56	
au2508	570.46	6.20	862	100	56980700	570.18/572.80	564.26	569.08	568.32	570.98	



Right about now are you wishing you or your parents bought gold in 1980?

In the following questions you can use the last traded futures prices.

1. You run a gold mining company currently producing approximately 1 ton of gold per month, which you sell to Chinese businesses and government. What risk are you exposed to in your business and how would you use SHFE futures contracts over the next 3 months of August, September and October to manage this risk? You're exposed to spot gold prices falling over the next few months. To manage this risk you would short the August, September and October contracts. You produce approximately 1 ton of gold per month, and there is 1,000,000 grams in 1 ton. Each SHFE gold futures contract is over $m = 1,000$ grams of gold. Hence, you short $h = 1,000$ contracts for each month.

2. Following on from 1., alternatively, what risk is your Chinese buyers exposed to and how would they use futures contracts to manage this risk over the next few months? Your Chinese buyers are exposed to the risk of spot gold prices increasing. To manage this risk they would go long the same amount of contracts as you shorted.

3. Following on from 1., suppose the gold price was 575 CNY at the August contract maturity, 600 CNY at the September contract maturity, and 625 CNY at the October contract maturity. Are you happy with your hedging decision? How would you justify this to your board of directors? Are your Chinese buyers happy with their decision to hedge? The futures prices are $K_{\text{Aug}} = 558.76$ CNY, $K_{\text{Sep}} = 558.72$ CNY and $K_{\text{Oct}} = 560.64$ CNY. In general, the payoff from a short position is

$$\text{short payoff} = h(K - S_T)m,$$

where S_T is the spot price at expiry. Hence, your contract payoffs are

$$\begin{aligned}
\text{Aug payoff} &= 1000 \times (558.76 - 600) \times 1000 = -41,240,000 \,\text{CNY},\\
\text{Sep payoff} &= 1000 \times (558.72 - 625) \times 1000 = -66,280,000 \,\text{CNY},\\
\text{Oct payoff} &= 1000 \times (560.64 - 650) \times 1000 = -89,360,000 \,\text{CNY}.
\end{aligned}$$

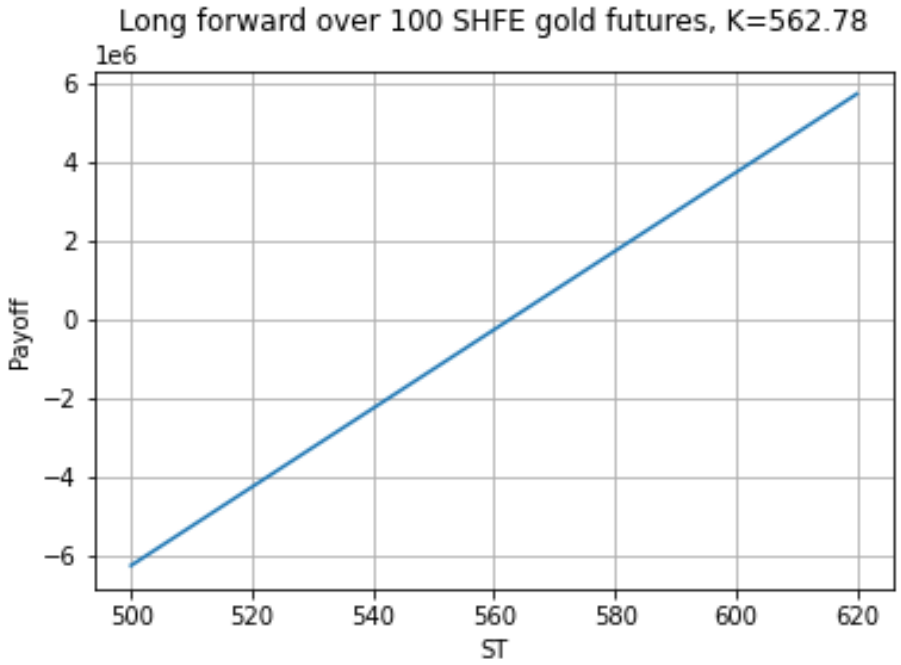
These amounts represent how much money you missed out on by locking in the gold price via futures contracts, but the gold price went up, which would have benefited you. You're not particularly happy about it, and about all you can say to your board is "what if gold went down?"

4. Now suppose you're a gold futures trader and you think that the gold price will keep going up at least until the end of the year. How would you use futures contracts to speculate on this view? Plot a payoff diagram of your speculative position in 100 December contracts. Alternatively, how would you speculate on the gold price falling? Plot a payoff diagram of your position speculating on a fall in price using 100 December futures. In this

case you'd go long the 100 December futures contracts, for a contract price of 562.78 CNY. Your payoff is

$$\text{long payoff} = h(S_T - K)m,$$

and using Python we get:



I'm sure you can do this for taking a short position to speculate on a fall in the gold price.

```
import numpy as np
import matplotlib.pyplot as plt
K=562.78
ST=np.linspace(500,620,1001)
h=100
m=1000
long_forward=h*(ST-K)*m
plt.figure()
plt.plot(ST,long_forward)
plt.grid()
plt.xlabel("ST")
plt.ylabel("Payoff")
plt.title("Long forward over 100 SHFE gold futures, K=562.78")
```

Equities

Question 2. The Tencent share price on the Hong Kong Exchange (HKEX):

Contract specifications for HKEX share futures, and Tencent's multiplier:

CONTRACT SUMMARY

Item	Contract Terms
Underlying Stocks & HKATS Codes	Please refer to list of stock futures
Contract Multiplier	Board lot size of the underlying shares *
Contracted Value	Contracted price multiplied by Contract Multiplier
Minimum Fluctuation	HK$0.01
Contract Months	Spot Month, the next two calendar months, & next two calendar quarter months
Trading Hours	9:30 am - 12:00 noon & 1:00 pm - 4:00 pm
Last Trading Day	The second last Trading Day of the Contract Month
Final Settlement Day	The first Trading Day after the Last Trading Day
Final Settlement Price **	The official closing price of the underlying stock as quoted by SEHK on the Last Trading Day.
Settlement Method	Cash settled contract of difference
Transaction Costs***	Exchange Fee Tier 1: HK$3.0 Tier 2: HK$1.0 Tier 3: HK$0.5 Commission Levy HK$0.10 Commission Rate Negotiable

*Relevant information regarding contract multiplier of individual stock futures can be found in the list of stock futures
**Starting form 3 July 2018, the Final Settlement Price (FSP) shall be revised to the official closing price of the underlying stock as quoted by SEHK on the Last Trading Day.
***Starting from 3 July 2018, a new 3-Tier trading fee structure will be introduced. Please refer to the circular MKD/EQD/16/18 for the tier level.

No.	SEHK Code	Underlying Stocks	HKATS Code	Contract Multiplier (shares)	Tier No.	Position Limit ** (Effective from 1 April 2024)	Market Makers	Approved by FSC Taiwan
18	700	Tencent Holdings Limited	TCH	100	1	25,000	Y	✓

Tencent share futures quotes:

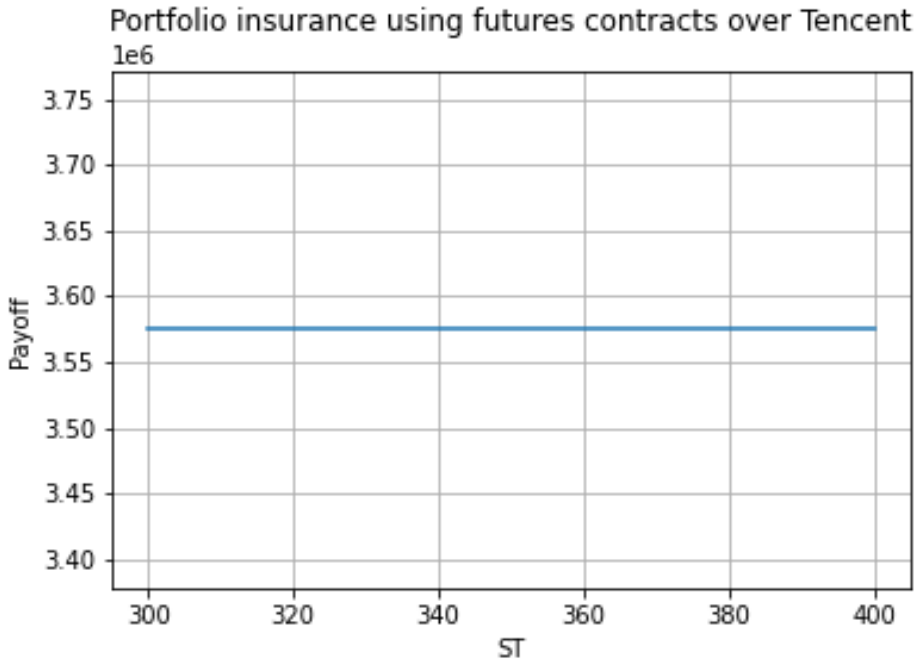
FUTURES Export to Excel

Contract	Last Traded	Net Change	Prev.Day Settlement Price	Bid Ask	Open	High Low	Volume	Prev.Day Open Interest
Total							749	2,203
Jul-24	354.80	+3.00	354.85	354.51 355.19	353.80	358.36 352.66	476	982
Aug-24	356.30	+3.12	356.37	356.00 356.73	355.15	359.66 355.00	243	1,129
Sep-24	357.50	+3.02	357.77	- 380.00	357.06	360.60 357.06	25	91
Dec-24	-	-	354.03	- -	-	- -	-	-
Mar-25	367.30	+14.20	360.00	360.00 -	360.00	368.00 360.00	5	1

1. You own 10,000 Tencent shares and are worried that the share price is heading back down again over the next 2 months. How would you use HKEX Tencent futures to insure your holding against this risk? Plot a payoff diagram of your position at maturity of your chosen contract. You own 10,000 Tencent shares and 1 futures contract is over $m = 100$ shares, so you would short $h = 100$ September futures contracts. The payoff of your combined Tencent portfolio and futures position is

$$\begin{aligned}\text{payoff} &= 10000S_T + h(K - S_T)m \\ &= 10000S_T + 100(K - S_T)100 = 10000K = 3,575,000\,\text{HKD},\end{aligned}$$

noting that $K = 357.50$ HKD. Also note that this is slightly more than the portfolio's current price of 3,544,000 due to a positive initial basis of $B = K - S = 357.50 - 354.4 = 3.1$. For a payoff diagram:

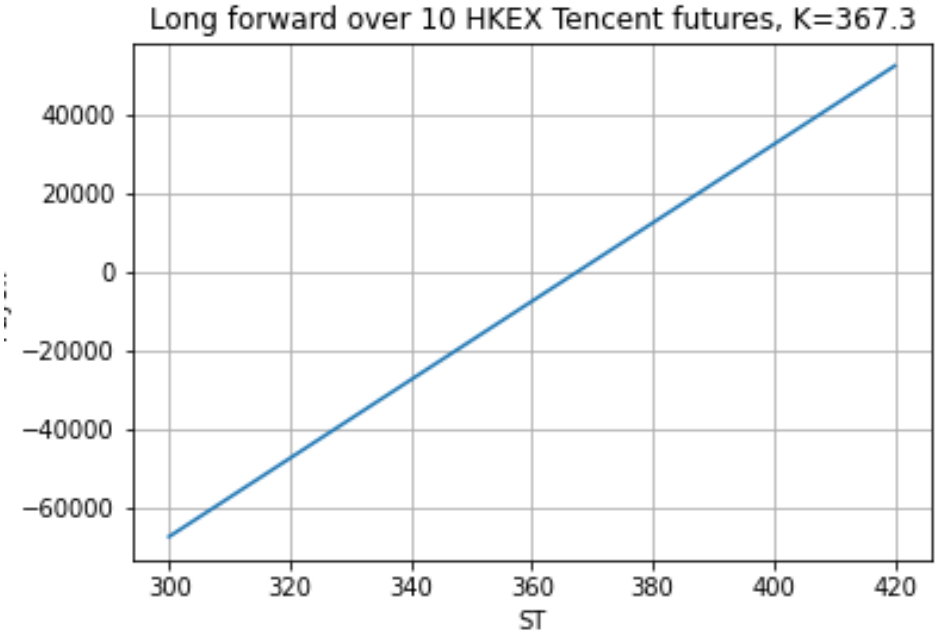


The Python code is below, after the question.

2. Now suppose you're a futures trader in Hong Kong and believe that Tencent will reach all-time highs over the next 9 or so months. How do you trade $h = 10$ March 2025 futures contracts to speculate on Tencent going up? Plot a payoff diagram of your trade. What would be your payoff if Tencent actually revisited its recent low of say 200 HKD? You go long the futures contracts. Your payoff is

$$\text{long payoff} = h(S_T - K)m = 10(S_T - 367.3)100.$$

For a payoff diagram:



Again, the Python code is at the end of the question.

3. Following on from 2., suppose Tencent hits 500 HKD after 2 months so you closed out your trade and took your profits. How much did you make? You profit from going long at $K = 367.3$ HKD and then closing out by going short at $K_t = 500$ HKD is

$$\text{profit} = h(K_t - K)m = 10(500 - 367.3)100 = 132,700\,\text{HKD}$$

(about $\$25,964$ AUD).

4. Following on from 2., suppose Tencent range traded between 200 HKD and 400 HKD over the life of your trade. The HKEX margin requirements on Tencent are below. How much capital in HKD would you need in order to maintain your position?

			Initial margin	Mantenance margin
Tencent Holdings Ltd.	TCH	Full Rate /lot	2,846	2,276

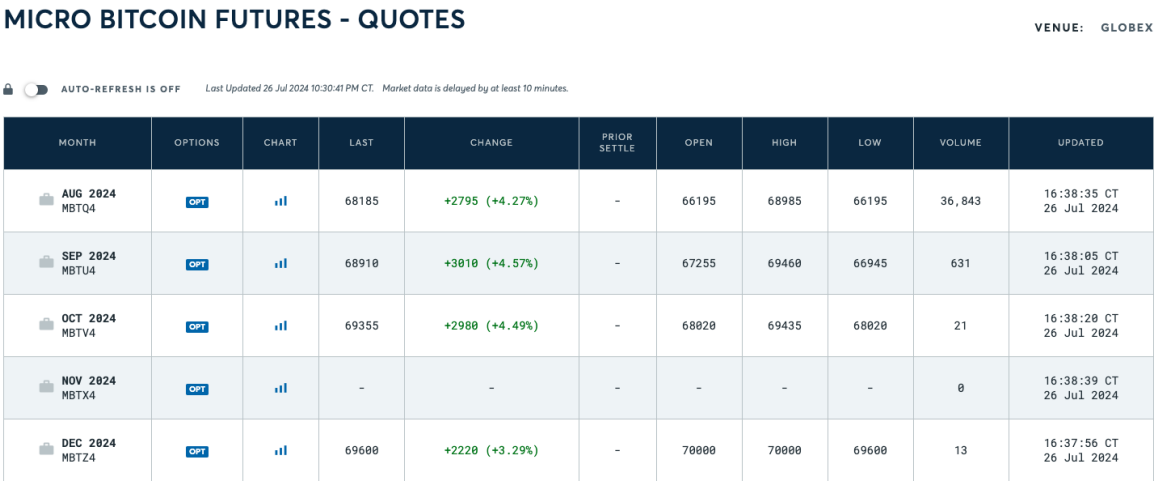
You'e long so you only need to consider your cumulative loss if the contract price hits at worst 200 HKD. In this case your payoff would be $h(K_t - K)m = 10(200 - 367.3)100 = -167,300$ HKD. On 10 contracts the initial margin is $28,460$. You would have needed to post the initial margin and then have made margin calls of 167,300 HKD to top up your account back to the initial margin, so rounding up you'd need 200,000 HKG in capital.

```
import numpy as np
import matplotlib.pyplot as plt

K=357.50
ST=np.linspace(300,400,1001)
h=100
m=100
short_forward=h*(K-ST)*m
payoff=10000*ST+short_forward
plt.figure()
plt.plot(ST,payoff)
plt.grid()
plt.xlabel("ST")
plt.ylabel("Payoff")
plt.title("Portfolio insurance using futures contracts over Tencent")

K=367.3
ST=np.linspace(300,420,1001)
h=10
m=100
long_forward=h*(ST-K)*m
plt.figure()
plt.plot(ST,long_forward)
plt.grid()
plt.xlabel("ST")
plt.ylabel("Payoff")
plt.title("Long forward over 10000 HKEX Tencent futures, K=367.3")
```

Currencies

Bitcoin's a currency, right?

Question 3. Consider the CME Group Micro Bitcoin Futures contract:

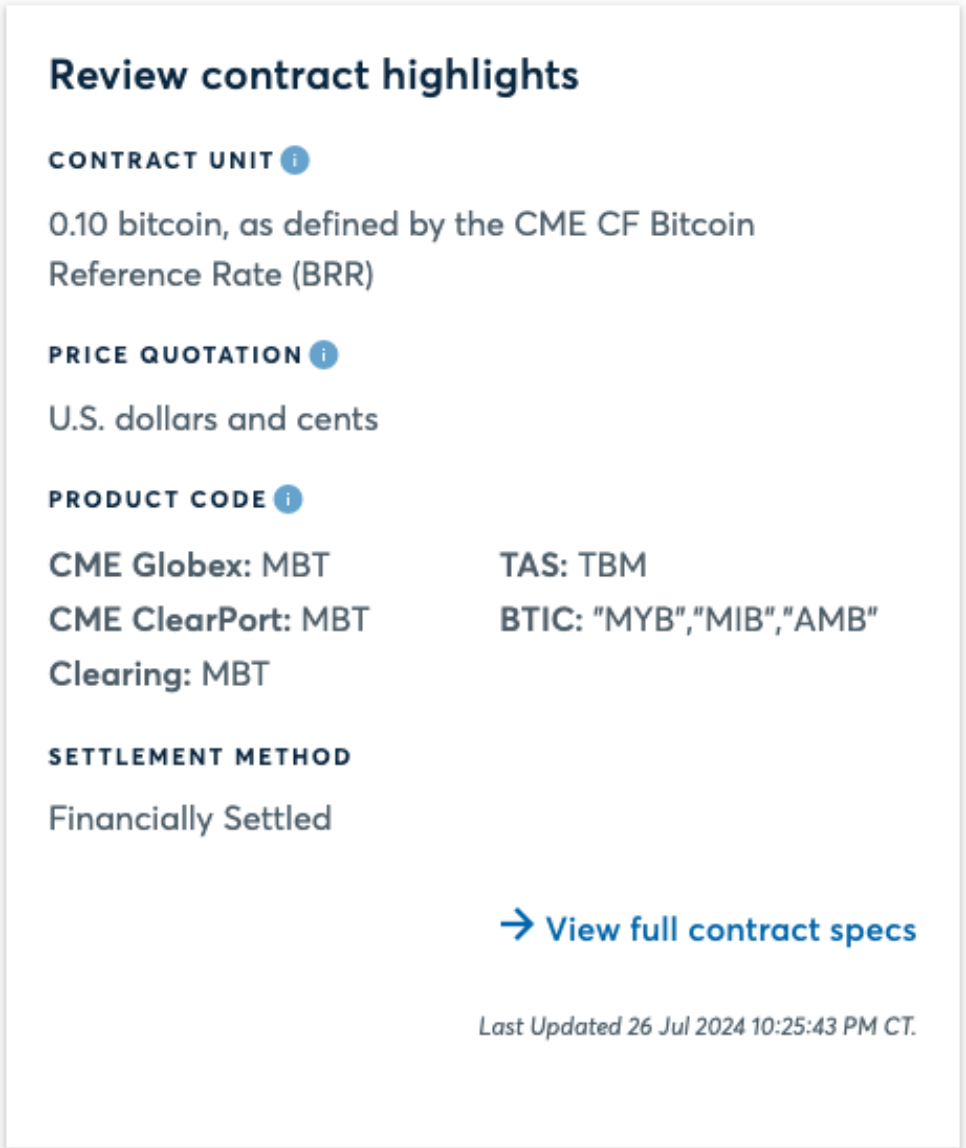


MICRO BITCOIN FUTURES - QUOTES

VENUE: GLOBEX

AUTO-REFRESH IS OFF *Last Updated 26 Jul 2024 10:30:41 PM CT. Market data is delayed by at least 10 minutes.*

MONTH	OPTIONS	CHART	LAST	CHANGE	PRIOR SETTLE	OPEN	HIGH	LOW	VOLUME	UPDATED
AUG 2024 MBTQ4	OPT		68185	+2795 (+4.27%)	-	66195	68985	66195	36,843	16:38:35 CT 26 Jul 2024
SEP 2024 MBTU4	OPT		68910	+3010 (+4.57%)	-	67255	69460	66945	631	16:38:05 CT 26 Jul 2024
OCT 2024 MBTV4	OPT		69355	+2980 (+4.49%)	-	68020	69435	68020	21	16:38:20 CT 26 Jul 2024
NOV 2024 MBTX4	OPT		-	-	-	-	-	-	0	16:38:39 CT 26 Jul 2024
DEC 2024 MBTZ4	OPT		69600	+2220 (+3.29%)	-	70000	70000	69600	13	16:37:56 CT 26 Jul 2024



Review contract highlights

CONTRACT UNIT

0.10 bitcoin, as defined by the CME CF Bitcoin Reference Rate (BRR)

PRICE QUOTATION

U.S. dollars and cents

PRODUCT CODE

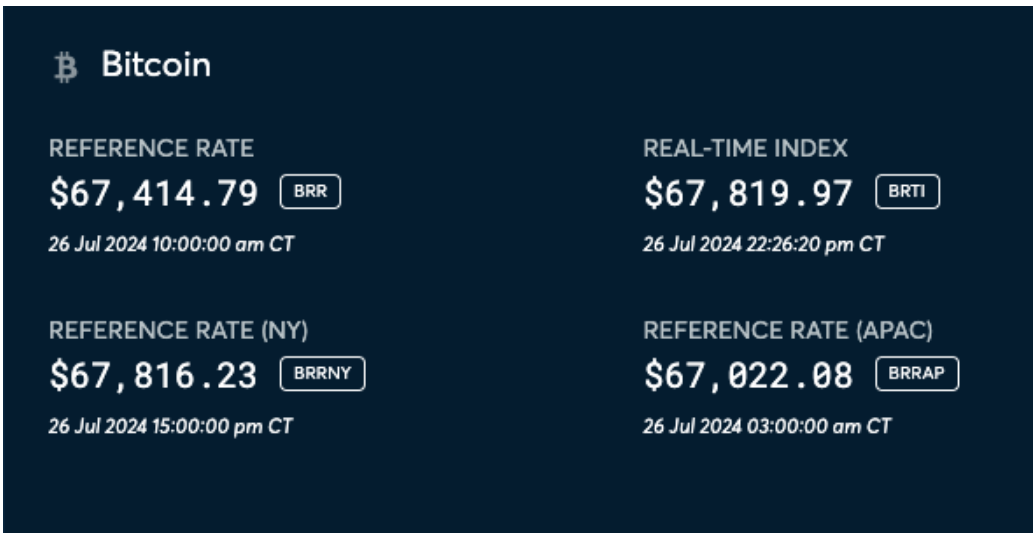
CME Globex: MBT
CME ClearPort: MBT
Clearing: MBT
TAS: TBM
BTIC: "MYB","MIB","AMB"

SETTLEMENT METHOD

Financially Settled

→ View full contract specs

Last Updated 26 Jul 2024 10:25:43 PM CT.



Note that the Bitcoin price defined in the contract is the CME CF Bitcoin Reference rate, as described here. But just think of it as the USD Bitcoin price.

1. You hold 10 Bitcoin and don't think it will break through its all-time highs, and will most likely fall over the next month or two. How might you use CME Bitcoin futures to insure your holding against this risk? It's no different to the above: 1 contract is over $m = 0.1$ Bitcoin, so you'd short 100 September contracts for $K = \$68,910$ USD.

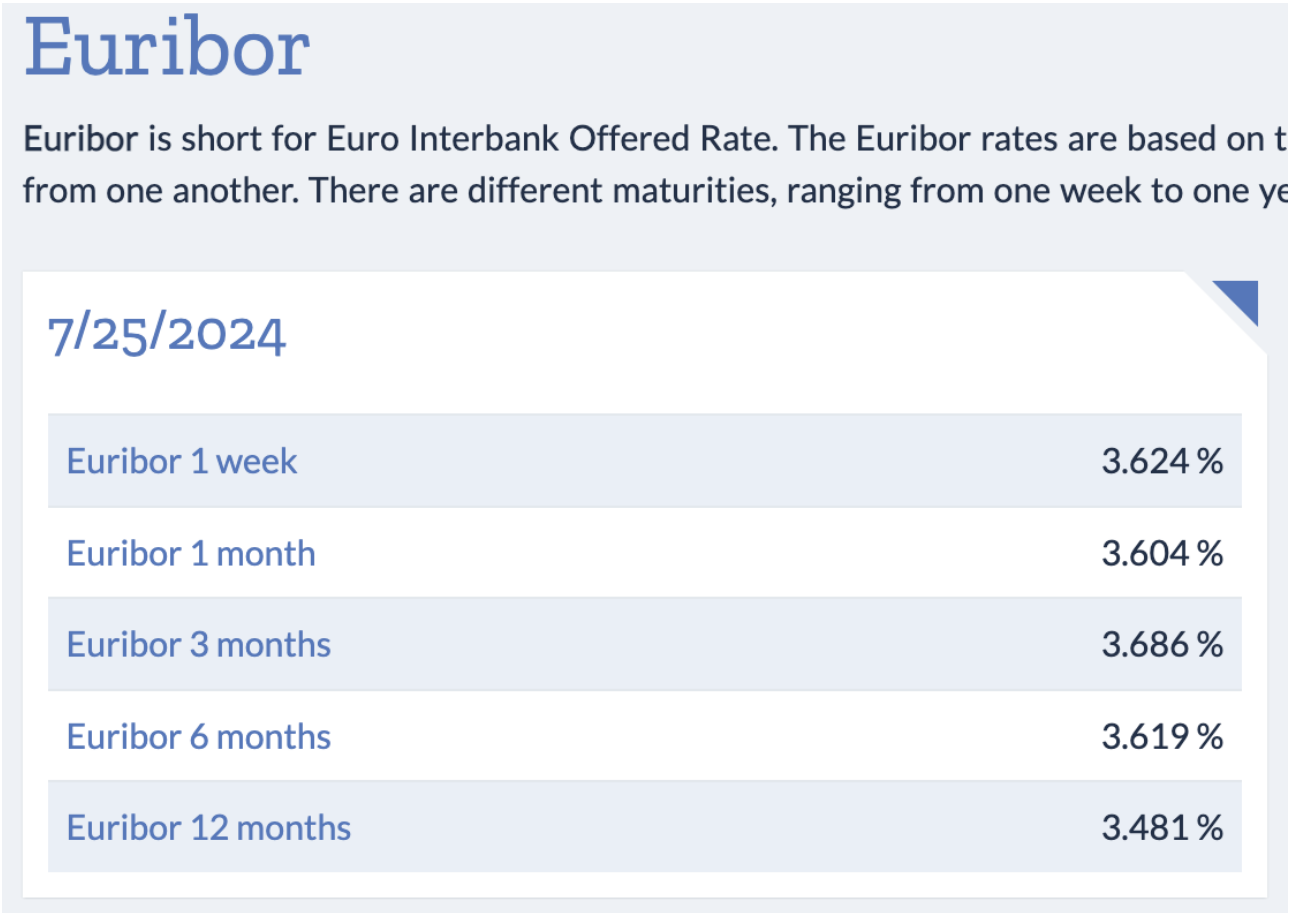
2. How do you use futures to speculate on Bitcoin breaking through its all-time highs by December, by getting an exposure to 10 Bitcoin? Plot a payoff diagram of your trade. In this case you go long 100 December contracts for $K = \$69,600$ USD. Here $h = 10$ and $m = 0.1$, so with a payoff of $h(S_T - K)m$, I'm sure you can plot a payoff diagram.

3. You think Elon Musk will come out and say something negative about cryptocurrencies in the next month, or maybe just post some cryptic meme on $\mathbb{X}$, causing the Bitcoin price to fall significantly. How do you use September Bitcoin contracts to speculate on this, via an exposure to 10 Bitcoin? Plot a payoff diagram of your position. Musk is always the bad guy... To hedge against his reckless meme posting you'd short 100 September contracts.

Interest rates

FRA

Question 4. Consider the following EURIBOR yield curve:



Note that we show in the week 3 lecture that the correct fixed rate k in a FRA is the implied forward rate embedded in the yield curve.

1. You need to borrow €1,000,000 in 3 months time for a period of 3 months, paying back the loan in one payment. What risk are you exposed to? How do you use a FRA to manage this risk? You're exposed to the risk of interest rates increasing in 3 months time, thus increasing your cost of funds. To manage this risk you'd enter into a 3×6 FRA as the fixed rate payer over the notional amount of $F = €1,000,000$.

2. Following on from 1., suppose that the spot 3 month EURIBOR rate in 3 months time unexpectedly increases to $r = 5\%$. Show the mechanics of how you locked in your borrowing rate at k. We first calculate the 3 month fixed rate k. It is the simple interest rate satisfying

$$1 + r_6 \frac{180}{360} = \Big(1 + r_3 \frac{90}{360}\Big)\Big(1 + k \frac{90}{360}\Big),$$

where r_3 and r_6 are the above 3 and 6 month EURIBOR rates. We get:

$$k = \left(\frac{1 + r_6 \frac{180}{360}}{1 + r_3 \frac{90}{360}} - 1\right) \frac{360}{90} = 3.52\%.$$

Noting that $T = \dfrac{90}{360}$, the payoff to you as the FRA fixed rate payer is

$$\text{fixed rate payer payoff} = \frac{F(r-k)T}{1+rT} = €3,655.39.$$

The hypothetical loan payment you agreed to make in the FRA is

$$F(1 + kT) = €1,008,798.92.$$

You got €3,655.39 from the FRA, so to raise F, you only need to borrow $F - 3655.39$ in the market at the spot 3 month EURIBOR rate of $r = 5\%$. The loan payment on borrowing this amount is

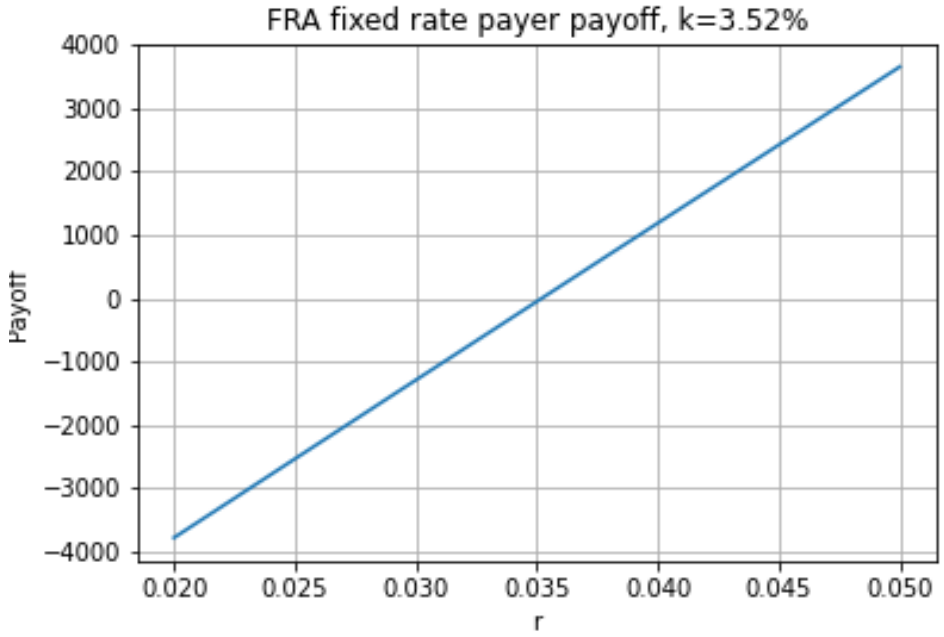
$$(F - 3655.39)(1 + rT) = €1,008,798.92.$$

Hence, you locked in the FRA fixed rate k on raising F in capital.

3. Following on from 1., plot a payoff diagram of your FRA position over a range of different spot 3 month EURIBOR rates in 3 months time. Here, we have all the variables in

$$\text{fixed rate payer payoff} = \frac{F(r-k)T}{1+rT}$$

except r, so we plot this payoff on the y-axis against fixed rates r on the x-axis. Hence, we view r as the "underlying asset" in the FRA. We get:



The Python code that produced this is below.

4. Suppose you wanted to speculate on 3 month EURIBOR rates being lower than market expectations in 3 months time. How do you do this via a FRA? Plot a payoff diagram of your position again on a notional or face value of €1,000,000.

You enter into a 3×6 FRA as the fixed rate receiver. Your payoff diagram is the negative of the above for the fixed rate payer (downsloping).

```
import numpy as np
import matplotlib.pyplot as plt
k=0.03519567188359307
T=90/360
F=1000000
r=np.linspace(0.02,0.05,1001)
payoff=F*(r-k)*T/(1+r*T)
plt.figure()
plt.plot(r,payoff)
plt.grid()
plt.xlabel("r")
plt.ylabel("Payoff")
plt.title("FRA fixed rate payer payoff, k=3.52%")
```

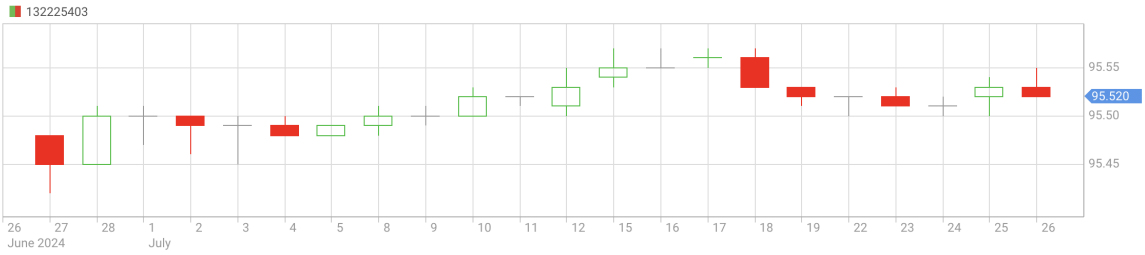
BAB Futures

Question 5. Use the last traded price of the following BAB futures quotes:

90 day bank bill IR

FUTURES OPTIONS

1 MONTH



EXPIRY DATE	BID	OFFER	LAST TRADE	CHANGE	VOLUME	PREVIOUS SETTLEMENT
Sep 24	95.5200	95.5300	95.5300 As of 26/07/24	--	6073	95.5300 As of 26/07/24
Dec 24	95.5900	95.6000	95.6000 As of 26/07/24	△ 0.0100	7379	95.5900 As of 26/07/24
Mar 25	95.7300	95.7400	95.7300 As of 26/07/24	△ 0.0100	5238	95.7200 As of 26/07/24
Jun 25	95.8700	95.8800	95.8700 As of 26/07/24	△ 0.0200	4178	95.8500 As of 26/07/24
Sep 25	95.9900	96.0100	96.0100 As of 26/07/24	△ 0.0400	6175	95.9700 As of 26/07/24
Dec 25	96.0900	96.1100	96.1000 As of 26/07/24	△ 0.0300	6576	96.0700 As of 26/07/24
Mar 26	96.1700	96.1800	96.1800 As of 26/07/24	△ 0.0400	4965	96.1400 As of 26/07/24
Jun 26	96.2200	96.2400	96.2300 As of 26/07/24	△ 0.0300	4207	96.2000 As of 26/07/24
Sep 26	96.2600	96.2800	96.2600 As of 26/07/24	△ 0.0300	247	96.2300 As of 26/07/24

1. You work in the liquidity management function within the treasury department of a large company and forecast that you will have approximately $\$5,000,000$ AUD in surplus funds to invest in December for 90 days. What risk are you exposed to? How do you use BAB futures to manage this risk? You're exposed to the risk of interest rates decreasing, resulting in a lower investment yield on your money. To manage this risk you enter into $h = 5$ BAB futures as the fixed rate receiver, effectively locking in the interest rate on investing in BABs with a face value of $\$5,000,000$ in December.

2. Following on from 1., show the mechanics of your trading and investing activities at maturity of the December contract if the RBA reduced interest rates and the 90 day BBSW rate dropped to 4% by December. The face value of 1 BAB futures is $F = \$1,000,000$ AUD, so you enter into $h = 5$ contracts as the fixed rate receiver. The last traded December BAB futures price was 95.6, so a fixed rate of $k = 4.4\%$. The payoff to you as the fixed

rate receiver in the $h = 5$ BAB futures contracts is

$$\begin{aligned}\text{fixed rate receiver payoff} &= h \times F\left(\frac{1}{1 + r\frac{90}{365}} - \frac{1}{1 + k\frac{90}{365}}\right) \\ &= \$4,830.93\,\text{AUD}.\end{aligned}$$

In December you invest the following in BABs with a face value of F:

$$\frac{F}{1 + r\frac{90}{365}} = \$4,951,166.58.$$

You received $\$4,830.93$ AUD in the BAB futures so you effectively only invested $4951166.58 - 4830.93 = \$4,946,335.65$ AUD. We calculate that

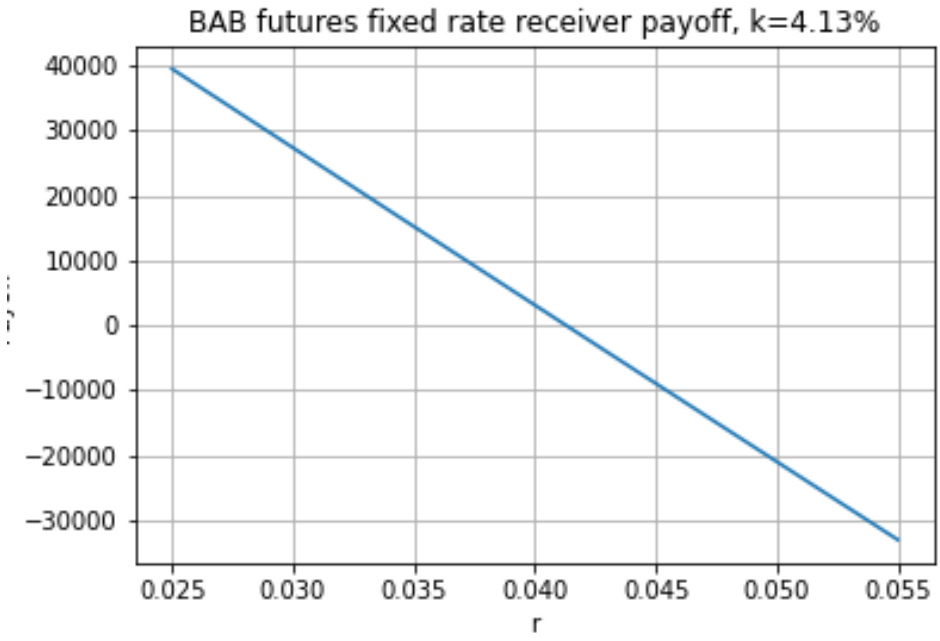
$$\frac{F}{1 + k\frac{90}{365}} = \$4,946,335.65\,\text{AUD},$$

so you locked in the fixed rate $k = 4.4\%$ on your investment.

3. How do you use BAB futures to speculate on 90 day BAB yields being lower than market expectations by June next year, with a notional or face value of $10,000,000? Plot a payoff diagram of your position for a range of 90 day BBSW rates at maturity. From 2., when market spot yields at maturity were lower than the BAB futures fixed rate, the BAB futures contract had a positive payoff to the fixed rate receiver. If we expect this to happen, we enter into BAB futures as the fixed rate receiver. The June contract's last traded price is 95.87, equating to a fixed rate of $k = 4.13\%$. Viewing the June spot 90 day BBSW rate r as the "underlying asset" in the BAB futures, we plot the payoff to the fixed rate receiver:

$$\text{fixed rate receiver payoff} = h \times F\left(\frac{1}{1 + r\frac{90}{365}} - \frac{1}{1 + k\frac{90}{365}}\right)$$

as a function of r, noting that here $h = 10$. We get:



4. Suppose the RBA decreases interest rates more than expected, and the BAB futures fixed rate falls to 3.5% in March, so you close out your position. What was your payoff? Your payoff is the difference between the prices of 90 day BABs at the fixed rates at which you opened and closed your BAB futures trade. You opened as the fixed rate receiver in $h = 10$ June contracts at $k = 4.13\%$, effectively meaning you agreed to purchase 90 day BABs with a face value of $F = \$10,000,000$ AUD in June for

$$P_k = \frac{F}{1 + k\frac{90}{365}} = \$9,899,190.98 \text{ AUD}.$$

You closed out your trade in March as the fixed rate payer of $h = 10$ June contracts for $k_t = 3.5\%$, meaning that you effectively agreed to issue $F = \$10,000,000$ AUD worth of 90 day BABs in June for

$$P_{k_t} = \frac{F}{1 + k_t\frac{90}{365}} = \$9,914,437.05 \text{ AUD}.$$

Hence, your profit was

$$P_{k_t} - P_k = \$15,246.07 \text{ AUD}.$$

In general, the payoff of trade opened as the fixed rate receiver of h BAB futures at fixed rate k and then closed as the fixed rate payer of h BAB futures at a fixed rate of k_t is

$$\text{fixed rate receiver trade payoff} = h \times F\left(\frac{1}{1 + k_t\frac{90}{365}} - \frac{1}{1 + k\frac{90}{365}}\right).$$

The payoff to the reverse trade is $h \times F\left(\frac{1}{1 + k\frac{90}{365}} - \frac{1}{1 + k_t\frac{90}{365}}\right)$.