FINM3405 Derivatives and Risk Management

Week 10: FRN and interest rate swaps

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Contents

Introduction

Last week we covered the first part of the swaps section of the course, namely credit default swaps. This week we finish the swaps section of the course by covering fixed-for-floating interest rate swaps. You may recall from the first week of the course that interest rate swaps basically dominate all other financial instruments in terms of market size. Note however that there is a wide variety of different kinds of interest rate swaps, and in this course we cover fixed-for-floating swaps, which are effectively the most basic "plain vanilla" type.

- Readings: Chapter 7 of Hull.

Also consider Chapter 33 Interest Rate Swaps, and Chapter 34 Pricing Interest Rate Swaps, of Cuthbertson, Nitzsche and O'Sullivan, Derivatives: Theory and Practice.

Floating rate notes (FRN)

- We start with FRN since they're needed for swap pricing.

A (forward looking) **floating rate note** (**FRN**) is a fixed interest security that promises to pay regular coupon payments which are calculated from a reference interest rate (such as Term SOFR, Euribor, etc) and also pay back the notional principal or face value at maturity.

- FRN are important in their own right but we mostly cover them since they're useful for pricing fixed-for-floating interest rate swaps.
- They're used to price the hypothetical "floating leg" of the swap.

But what do we mean by "forward looking"?

Floating rate notes (FRN)

Remark

By **forward looking** we mean that a given interest period's coupon (or interest) payment, which is payable at the end of the interest period, is calculated at the start of the interest period (using the spot reference rate whose maturity date is the end of the interest period). **Backward looking** means that the coupon is not only paid but also calculated at the end of the interest period based on "some kind of formula" involving the reference rate's values or behaviour over the interest period.

- They're not overly complicated, and they're quite common and interesting, but we don't cover backward looking FRN.

Floating rate notes (FRN)

Example

Consider a 6-month FRN with monthly coupons at a floating rate given by the BBSW rate (we'll just use the mid quotes):

24 hour delayed BBSW rates

TENOR	BID	ASK	MID
1 MONTH	4.3581	4.2581	4.3081
2 MONTH	4.4247	4.3247	4.3747
3 MONTH	4.4838	4.3838	4.4338
4 MONTH	4.5500	4.4500	4.5000
5 MONTH	4.6175	4.5175	4.5675
6 MONTH	4.6764	4.5764	4.6264

Floating rate notes (FRN)

Example

With a face value of $\$1,000,000$, the first interest or coupon payment C_1 due in 1 month time is

$$C_1 = F \times f_1 \times \frac{1}{12} = F \times 4.3081\% \times \frac{1}{12} = \$3,590.08,$$

where the floating rate $f_1 = 4.3081\%$ is the 1-month BBSW rate.

- We don't know that the next coupon C_2 due in 2 months will be until we get to the end of the 1$^{\text{st}}$ month, because only then will we know what the 1-month BBSW rate will be.

In general, we'll use the following notation, including for swaps:

Floating rate notes (FRN)

- F is the **face value** and T is the **maturity date** in years.
- t_i for $i = 1, \ldots, N$ are the **interest** or **coupon dates** in years.
- r_i for $i = 1, \ldots, N$ are the reference rate's **spot rates** for the time period $[0, t_i]$, thus the reference rate's yield curve.
- $r_{i-1,i}$ for $i = 1, \ldots, N$ are the **implied forward rates** for coupon period $[t_{i-1}, t_i]$ embedded in the yield curve.
- f_i is the **floating rate** for coupon period $[t_{i-1}, t_i]$, which is known only at time t_{i-1}, the start of the coupon period.
- C_i for $i = 1, \ldots, N$ are the **coupon payments** for period $[t_{i-1}, t_i]$, calculated from f_i at time t_{i-1}, paid at time t_i.

Floating rate notes (FRN)

So the time t_i coupon or interest payment C_i is calculate at time t_{i-1} as

$$C_i = F \times f_i \times d,$$

where d is the time in years between coupons (so $d = \frac{1}{4}$ for quarterly, $d = \frac{1}{2}$ for semiannual, etc) and the **floating rate** is usually given by

$$f_i = r + m,$$

where m is the **margin** (risk premium) over the spot reference rate r covering the period $[t_{i-1}, t_i]$ once it's known in the market at time t_{i-1}.

- But for simplicity, we assume $m = 0$.

FRN valuation

Question: How do we price a FRN, given that we don't know what the floating rates f_i and thus coupons C_i will be for $i = 2, \ldots, N$ until the start t_{i-1} of each coupon period $[t_{i-1}, t_i]$?

Answer: We use forward rate agreements (FRA) to calculate certain, risk-free cashflows $\bar{C}_i$ at each coupon date t_i called the **certainty equivalents** of the coupons C_i, and then use arbitrage arguments to show that the value at time $t = 0$ of an FRN is

$$\boxed{V = \sum_{i=1}^{N} \frac{\bar{C}_i}{1 + r_i t_i} + \frac{F}{1 + r_N T}.}$$

FRN valuation

After doing this we'll actually see that the value of an FRN is simply:

$$V = \begin{cases} F & \text{at time } t = 0 \text{ or an interest date } t_i, \\ \dfrac{C_i + F}{1 + \hat{r}(t_i - s)} & \text{at time } s \text{ in coupon period } [t_{i-1}, t_i], \end{cases}$$

where $\hat{r}$ is the spot reference rate covering the time period $[s, t_i]$.

Remark

- On the coupon dates t_i, when valuing a FRN we're assuming that the coupon C_i has just been paid "an instant ago".
- $V = \dfrac{C_i + F}{1 + \hat{r}(t_i - s)}$ is just the time s present value of $C_i + F$.

FRN valuation

We construct the certainty equivalent cashflows $\bar{C}_i$ at time t_i as follows:

- Arrange to borrow F at time t_{i-1} over the period $[t_{i-1}, t_i]$ at whatever the spot reference rate r is at time t_{i-1}.
  - Cashflow at time t_{i-1} is F and at time t_i is $-F(1 + rd)$.
- Enter a $t_{i-1} \times t_i$ FRA as the fixed rate $r_{i-1,i}$ receiver over F.
  - Cashflow at time t_{i-1} is $-F$ and at time t_i is $F(1 + r_{i-1,i}d)$.

Net cashflow is 0 and time t_{i-1} and $F(r_{i-1,i} - r)d$ at time t_i. If you also hold a FRN, you receive the coupon $C_i = Frd$ at time t_i.

- Then your net cashflow at time t_i is locked in as

$$\bar{C}_i = F(r_{i-1,i} - r)d + C_i = Fr_{i-1,i}d.$$

FRN valuation

The above $\bar{C}_i$ is the time t_i **certainty equivalent** cashflow, namely:

$$\bar{C}_i = F r_{i-1,i} d.$$

So if you held a FRN, from each coupon C_i you could construct its risk-free certainty equivalent $\bar{C}_i$ via the process above.

- The value of the FRN is sum of the present values of these risk-free cashflows $\bar{C}_i$ discounted at the risk-free reference rates r_i, namely

$$V = \sum_{i=1}^{N} \frac{\bar{C}_i}{1 + r_i t_i} + \frac{F}{1 + r_N T},$$

otherwise there's an obvious arbitrage opportunity:

FRN valuation

If someone offers you to an FRN for a price less than this theoretically correct market value, buy it and issue the same FRN in the market for its higher correct value. The coupons cancel out.

- You can do the case of the FRN priced greater than V.

What's important to us is to show that

$$V = \begin{cases} F & \text{at time } t = 0 \text{ or an interest date } t_i, \\ \dfrac{C_i + F}{1 + \hat{r}(t_i - s)} & \text{at time } s \text{ in coupon period } [t_{i-1}, t_i], \end{cases}$$

and we start with the case of time $t = 0$. We calculate that:

FRN valuation

$$
\begin{aligned}
V &= \sum_{i=1}^{N} \frac{F r_{i-1,i} d}{1 + r_i t_i} + \frac{F}{1 + r_N T} \\
&= \sum_{i=1}^{N} \frac{F r_{i-1,i} d \overbrace{+F - F}^{\text{standard trick}}}{1 + r_i t_i} + \frac{F}{1 + r_N T} \\
&= \sum_{i=1}^{N} \frac{F(1 + r_{i-1,i} d)}{1 + r_i t_i} - \sum_{i=1}^{N} \frac{F}{1 + r_i t_i} + \frac{F}{1 + r_N T} \\
&= \sum_{i=1}^{N} \frac{F}{1 + r_{i-1} t_{i-1}} - \sum_{i=1}^{N-1} \frac{F}{1 + r_i t_i} \\
&= F + \underbrace{\sum_{i=2}^{N} \frac{F}{1 + r_{i-1} t_{i-1}} - \sum_{i=1}^{N-1} \frac{F}{1 + r_i t_i}}_{=0}.
\end{aligned}
$$

FRN valuation

The above was for time $t = 0$, but for any other coupon date t_i, we can just view t_i as the new "current date", so we just have a "new" FRN with less time periods to maturity, and the above holds verbatim.

- When we're at an intermediate date s in $[t_{i-1}, t_i]$, there is a known cashflow of C_i at time t_i, and the FRN has value $V = F$ at time t_i, and thus could also be sold for $V = F$ at time t_i. So

$$V = \frac{C_i + F}{1 + \hat{r}(t_i - s)},$$

  at time s in coupon period $[t_{i-1}, t_i]$, where $\hat{r}$ is the spot reference rate known at time s for the period $[s, t_i]$, so maturing at time t_i.

Interest rate swaps

- We mostly covered FRN valuation since it's used in swap pricing.

We now turn to fixed-for-floating interest rate swaps, in particular their pricing and using them for hedging and speculating.

- Recall that a **fixed-for-floating** interest rate swap is a financial instrument that enables parties to "swap" or exchange their interest rate exposure or obligations, or their investment or lending income:

Interest rate swaps

Interest payments, exposure or obligations:

- A fixed interest borrower, or issuer of a fixed coupon bond, can turn their fixed interest payments into variable interest payments, or their fixed coupon bond into a FRN.
- A variable interest borrower, or issuer of a FRN, can turn their variable interest payments into fixed interest payments, or their FRN into a fixed coupon bond.

Interest rate swaps

Interest or lending income:

- A fixed interest investor, or holder of a fixed coupon bond, can turn their fixed interest rate into a variable interest rate, or the fixed coupon bond into a FRN.
- A variable interest investor, or holder of a FRN, can fix their interest rate, or turn the FRN into a fixed coupon bond.

We will see below how this enables borrowers and lenders/investors to hedge their interest rate risk, by which we basically mean their exposure to roughly parallel shifts in the yield curve.

Interest rate swaps

But, like CDS, interest rate swaps can also be traded as "standalone" instruments purely for speculative (or other, such as arbitrage) purposes, and that's how we'll treat them from now on:

- We just view interest rate swaps as "just another derivative security" that can be traded, and ignore any "underlying" exposure, cashflow obligations or investment income stream of either counterparty.

And we'll use the following additional notation:

- k is the **fixed interest rate** in the swap.
- $C = Fkd$ is the **fixed coupon** in the swap.

Interest rate swaps

In this light there is two parties to a fixed-for-floating interest rate swap:

1. **Receive fixed, pay floating**: This party agrees to receive a fixed investment interest rate k and pay a floating borrowing interest interest. (Also called the pay floating, received fixed party.)
2. **Pay fixed, receive floating**: This party agrees to pay the fixed borrowing interest rate k and receive a floating investment interest rate. (Also called the receive floating, pay fixed party.)

An example probably best illustrates the basic idea:

Interest rate swaps

Example

Consider the following BBSW rate yield curve and a 6-month fixed-for-floating interest rate swap with monthly interest dates:

24 hour delayed BBSW rates

TENOR	BID	ASK	MID
1 MONTH	4.3581	4.2581	4.3081
2 MONTH	4.4247	4.3247	4.3747
3 MONTH	4.4838	4.3838	4.4338
4 MONTH	4.5500	4.4500	4.5000
5 MONTH	4.6175	4.5175	4.5675
6 MONTH	4.6764	4.5764	4.6264

Interest rate swaps

Example

Let $F = \$1,000,000$ and suppose the fixed rate in the swap is $k = 5\%$. Then the fixed coupon C in the swap is always

$$C = F \times k \times \frac{1}{12} = \$4,166.67.$$

The first floating coupon C_1 is also calculated to be

$$C_1 = F \times f_1 \times \frac{1}{12} = \$3,590.08,$$

where $f_1 = 4.3081$ is the 1-month BBSW rate. Since $C > C_1$, the fixed rate payer pays $C - C_1 = \$576.58$ at time t_1.

Interest rate swaps

Remark

Note that we don't actually know what the other, later net cashflows or payments in the swap will be because we don't know what the reference rate will be at the start of each coupon period, until we arrive at that date. At time $t = 0$ we only know what the first net cashflow will be (we're covering forward looking swaps).

Pricing

Pricing a fixed-for-floating swap involves determining the theoretically correct or "fair" fixed rate k in the swap.

Remark

The floating rates f_i for $i = 1, \ldots, N$ are already specified in the swap as being the reference rate (usually plus a risk premium or margin m), but each period's floating rate f_i is known only at the start t_{i-1} of the coupon period $[t_{i-1}, t_i]$, and not at time $t = 0$.

Question: How do we price a swap, given that we don't know what the floating rates f_i and hence coupons C_i and net cashflows will be for dates t_i for $i = 2, \ldots, N$, until the start t_{i-1} of the coupon periods $[t_{i-1}, t_i]$?

Pricing

Answer: The cashflows of the say receive fixed, pay floating party to an interest rate swap can be replicated via the following portfolio:

- Buying a fixed coupon bond with face value F, maturity date T, and coupon rate k and thus fixed coupons $C = F \times k \times d$.
- Issuing a FRN with face value F, maturity date T, and floating coupons given by $C_i = F \times f_i \times d$ at time t_i for $i = 1, \ldots, N$.

So the value of a swap to the receive fixed, pay floating party must be the value of a fixed coupon bond minus the value of a FRN, both with face value F and maturity date T, or else there's an arbitrage opportunity.

Pricing

Example (Continued)

The above example of the cashflows at time t_1 illustrates this:

- The fixed coupon was $C = F \times k \times d = \$4,166.67$.
- The 1st floating coupon was $C_1 = F \times f_1 \times d = \$3,590.08$.
- The receive fixed, pay floating party received the net difference of \$576.58 as if they were literally long the fixed coupon bond and short the FRN. The same scenario happens at each future coupon payment date t_i for $i = 2, \ldots, N$.

Pricing

Hence, the time $t = 0$ **value** V of a fixed-for-floating interest rate swap to the receive fixed, pay floating party (becoming a tongue-twister) is

$$\begin{aligned} V &= \text{value of fixed coupon bond} - \text{value of FRN} \\ &= \sum_{i=1}^{N} \frac{C}{1 + r_i t_i} + \frac{F}{1 + r_N T} - \sum_{i=1}^{N} \frac{\bar{C}_i}{1 + r_i t_i} - \frac{F}{1 + r_N T} \\ &= \sum_{i=1}^{N} \frac{C}{1 + r_i t_i} - \sum_{i=1}^{N} \frac{\bar{C}_i}{1 + r_i t_i}, \end{aligned}$$

where $C = Fkd$, and $\bar{C}_i = Fr_{i-1,i}d$ are the certainty equivalent cashflows.

Pricing

But we previously showed that the value of a FRN at time $t = 0$ is simply F. So the value of an interest rate swap to the received fixed party is

$$V = \sum_{i=1}^{N} \frac{C}{1 + r_i t_i} + \frac{F}{1 + r_N T} - F.$$

An interest rate swap is priced via the usual principal:

The fixed rate k in a fixed-for-floating interest rate swap is set so that the swap has 0 value to either party at initiation, time $t = 0$.

Pricing

As a result of the above usual principal, the fixed rate k is set so that

$$F = \sum_{i=1}^{N} \frac{C}{1 + r_i t_i} + \frac{F}{1 + r_N T},$$

which, after recalling $C = Fkd$ and $\bar{C}_i = Fr_{i-1,i}d$, rearranges to give

$$k = \frac{1 - \frac{1}{1+r_N T}}{d \sum_{i=1}^{N} \frac{1}{1+r_i t_i}}.$$

Remark

This ugly formula is provided in the Final Exam formula sheet.

Pricing

It's time for an example...!

Example

Continue on with the above BBSW rate yield curve, with the floating rate in the swap being set to the BBSW rate (so no risk premium m). We have $d = \frac{1}{12}$, $T = \frac{1}{2}$, $N = 6$ and r_i for $i = 1, \ldots, 6$ given in the table. We simply calculate that

$$k = \frac{1 - \frac{1}{1+r_N T}}{d \sum_{i=1}^{N} \frac{1}{1+r_i t_i}} = 4.5908\%.$$

But it might be easier to calculate this in a table, as follows:

Pricing

Example (Continued)

First, to simplify notation, let

$$D_i = \frac{1}{1 + r_i t_i}$$

be the **discount factors**. Also note that in this example, $d = \frac{1}{12}$, so that $\frac{1}{d} = 12$. Then the above formula becomes

$$k = 12 \frac{1 - D_N}{\sum_{i=1}^{N} D_i}.$$

We could then use the following table:

Pricing

Example (Continued)

time	bbsw	discount factor
1	0.04308	0.9964
2	0.04374	0.9928
3	0.04338	0.9893
4	0.04500	0.9852
5	0.04568	0.9813
6	0.04636	0.9773
	sum of discount factors	**5.9223**

From it we calculate that

$$k = 12\frac{1 - D_N}{\sum_{i=1}^{N} D_i} = 12\frac{1 - 0.9773}{5.9223} = 4.5908\%$$

as before.

Speculation

We give an example that illustrates speculating with interest rate swaps.

Example (Continued)

Continuing with the above example, suppose that at the end of the first period, the BBSW rate yield curve shifted up by 100 basis points, so 1%. The value of the swap to the receive fixed, pay floating party is still

$$V = \text{value of fixed coupon bond} - \text{value of FRN}.$$

Here, the new "current" date is t_1, we're assuming the first swap payment was just made, and there is now 5 months to maturity.

Speculation

Example (Continued)

We know that the value of the FRN is F at the coupon dates. So we need only calculate the value of the fixed coupon bond:

$$P = \frac{C}{1 + 0.05374\frac{1}{12}} + \frac{C}{1 + 0.05338\frac{2}{12}} + \frac{C}{1 + 0.055\frac{3}{12}} + \frac{C}{1 + 0.05568\frac{4}{12}} + \frac{C + F}{1 + 05636\frac{5}{12}} = \$995,921.93.$$

So the value to the receive fixed, pay floating party is thus $P - F = -\$4,078.07$, which makes sense because the yield curve shifted up and you can calculate that the new fixed rate would be $k = 5.583\%$, but they're only receiving 4.5908% in the swap.

Speculation

Remark

The other pay fixed, receive floating party could close out their profitable swap position with a swap dealer, thus realising their \$4,078.07 profit (net of fees and the swap dealer's spread etc).

Consequently, to speculate with interest rate swaps:

- If you believe that the yield curve will shift up, then enter into an interest rate swap as the pay fixed, receive floating party.
- If you believe that the yield curve will shift down, then enter into an interest rate swap as the receive fixed, pay floating party.

Hedging

We now give an example illustrating hedging with interest rate swaps.

Example

Suppose you raised capital by issuing a 6-month FRN with face value $F = \$1,000,000$ and monthly floating coupons at the BBSW rate, but you're now worried that the yield curve will shift up, thus increasing your (variable interest) cost of funds.

- To hedge your exposure, you decide to enter into an interest rate swap as the pay fixed, receive floating party.

We first calculate your monthly swap cashflows and thus total loan payments using the above BBSW rate yield curve.

Hedging

Example (Continued)

From the previous example, the fixed rate in the swap is $k = 4.5908\%$ and the theoretical monthly fixed coupon is thus $C = Fkd = \$3,825.64$. If the yield curve stays unchanged, we calculate the floating rates and coupons from the 1-month forward rates embedded in the BBSW rate yield curve. We get:

time	BBSW	1-month forward rates	fixed coupon	floating coupon	pay fixed, receive floating cashflow
1	0.043081	0.043081	$3,825.64	$3,590.08	-$235.56
2	0.043740	0.044240	$3,825.64	$3,686.68	-$138.96
3	0.043380	0.042351	$3,825.64	$3,529.27	-$296.37
4	0.045000	0.049325	$3,825.64	$4,110.42	$284.78
5	0.045675	0.047660	$3,825.64	$3,971.67	$146.03
6	0.046364	0.048879	$3,825.64	$4,073.23	$247.59

Hedging

Example (Continued)

- You issued a FRN at the BBSW rate. The swap converts your FRN coupons into the fixed coupon of $C = \$3,825.64$.

Suppose the yield curve shifts up by 100 basis points in the first month. You are still paying the same fixed rate $k = 4.5908\%$ you agreed to in the swap, but you are now the receiver of the higher BBSW rates. Your future swap net cashflows now become:

time	BBSW	1-month forward rates	fixed coupon	floating coupon	pay fixed, receive floating cashflow
2	0.053740	0.053740	$3,825.64	$4,478.33	$652.69
3	0.053380	0.052784	$3,825.64	$4,398.63	$572.99
4	0.055000	0.057726	$3,825.64	$4,810.54	$984.89
5	0.055675	0.056917	$3,825.64	$4,743.12	$917.47
6	0.056364	0.058043	$3,825.64	$4,836.90	$1,011.26

Hedging

Example (Continued)

- Your FRN coupons go up due to the higher interest rates.
- But the swap's net cashflows also move in your favour
- The final overall result is you still keep paying the fixed coupon of $C = \$3,825.64$, even after interest rates increased.

So you're hedged against increasing interest rates.

Hedging

Remark

You could imagine the following hedging scenarios:

- If you borrowed via FRN, we have the above example.
- If you borrowed via fixed coupon bonds, to hedge your exposure to falling interest rates you enter into a swap as the pay floating party.
- If you invested in FRN, to hedge your exposure to falling interest rates you enter into a swap as the receive fixed party.
- If you invested in fixed coupon bonds, to hedge your exposure to increasing interest rates you enter into a swap as the receive floating party.

Hedging

Question: Is there some seemingly very simple and obvious steps via fixed-for-floating interest rate swaps that Silicon Valley Bank (SVB) could have taken to prevent its collapse...? Did they possibly think that interest rates would never increase again? So many questions...

Summary