Module 1.

Investment Analysis and Real Estate

This module illustrates how many of the principles and techniques outlined can be applied to real estate investment

The use of Investment analysis techniques is essential if property investors and their advisors are to test the viability of investment decisions objectively. Real estate investment markets are becoming increasingly complex and interrelated with other investment markets and it is no longer acceptable to base investment decisions on a 'hunch' alone.

Objectives

After studying this module you should be able:

- to describe the general context of real estate analysis
- to apply financial mathematics to problems relating to real estate investment analysis
- to apply the techniques of project evaluation (i.e. discounted cash flow) to real estate investment analysis.

The intent of this module is to offer readers a problem-based tutorial set around the project level in property mathematics. Taken from Bell, R.A. (1988) *Investment Property Income Analysis and Appraisal* and Worthington, J.E. (1990) *Financial Appraisal of Property and Equities*, it offers a mix of instruction and worked problems with Excel solutions and possibilities for extension or adaptation. You can solve these problems using the Excel spreadsheet functions in the file JC 3 Financial maths tutorial 1.xls on Blackboard (see Learning Resouces tab). Consider the illustrated examples and in addition, where possible, work out the financial factors manually with the use of a calculator

Property Investment Analysis in its Context

Remember that the analysis of investment value in property should always be placed in the wider context of the general investment market.

> By its nature, appraisal (of property investment) is a comparative or relative process and property investments must at some stage be appraised in comparison with alternative investment vehicles.
>
> (Baum and Crosby, 1988: 7)

It is argued by some that the investment value of real property can be differentiated from the concept of market value. It is proposed that the investment value of real estate to an individual need not be its market value. Market value can be defined simply as the estimated exchange price in the market place, whereas investment worth is often an analysis to determine whether the market value should be paid.

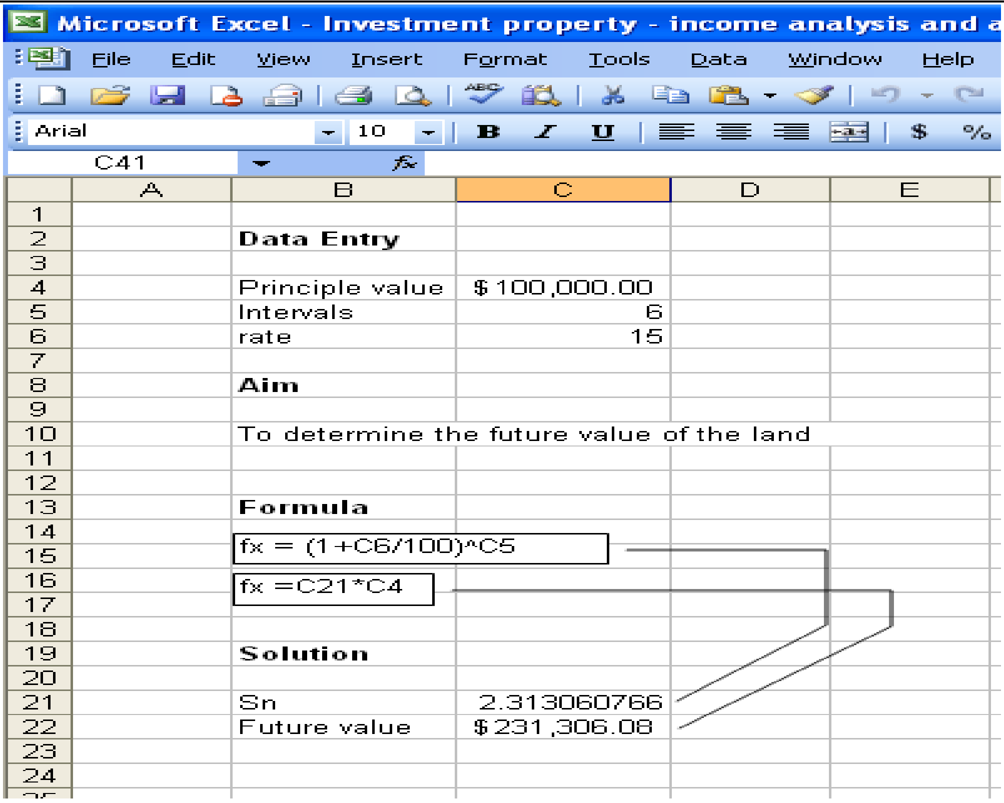
Future value

$FV = PV\ (1 + i)^n$ i.e. compound interest formula

Example

Land values are increasing at an annual compound rate of 15% p.a. A parcel of land has a current value of $100,000. What will the land be worth in 6 years time based on this information?

This problem can be solved with the use the Excel file JC 3 Financial maths tutorial 1.xls spreadsheet first tab ‘Future Value of $100,000’ as follows:



Using a calculator, the problem is solved as follows

$$
\begin{aligned}
FV &= PV\,(1 + i)^{n} \\
&= 100{,}000\,(1 + 0.15)^{6} \\
&= 100{,}000 \times 2.31306 \\
&= \$231{,}306.08
\end{aligned}
$$

Given any 3 of the 4 variables in the above type of problem (Present Value, Interest rate, Intervals/Period of time and Future value), you can solve for the unknown variable. For example

If an investor paid $100 000 for a parcel of land and sold it in 3 years for $200,000, what is the actual profit measured in terms of compound interest?

$$
\begin{aligned}
FV &= PV\,(1+i)^n \\
200,000 &= 100,000\,(1+i)^3 \\
(1+i)^3 &= \frac{200,000}{100,000} \\
1+i &= \sqrt[3]{\frac{200,000}{100,000}} \\
i &= \sqrt[3]{\frac{200,000}{100,000}} - 1 \\
i &= (2)^{1/3} - 1 \\
&= 1.25489 - 1 \\
&= .25989 \\
&= 25.989\%
\end{aligned}
$$

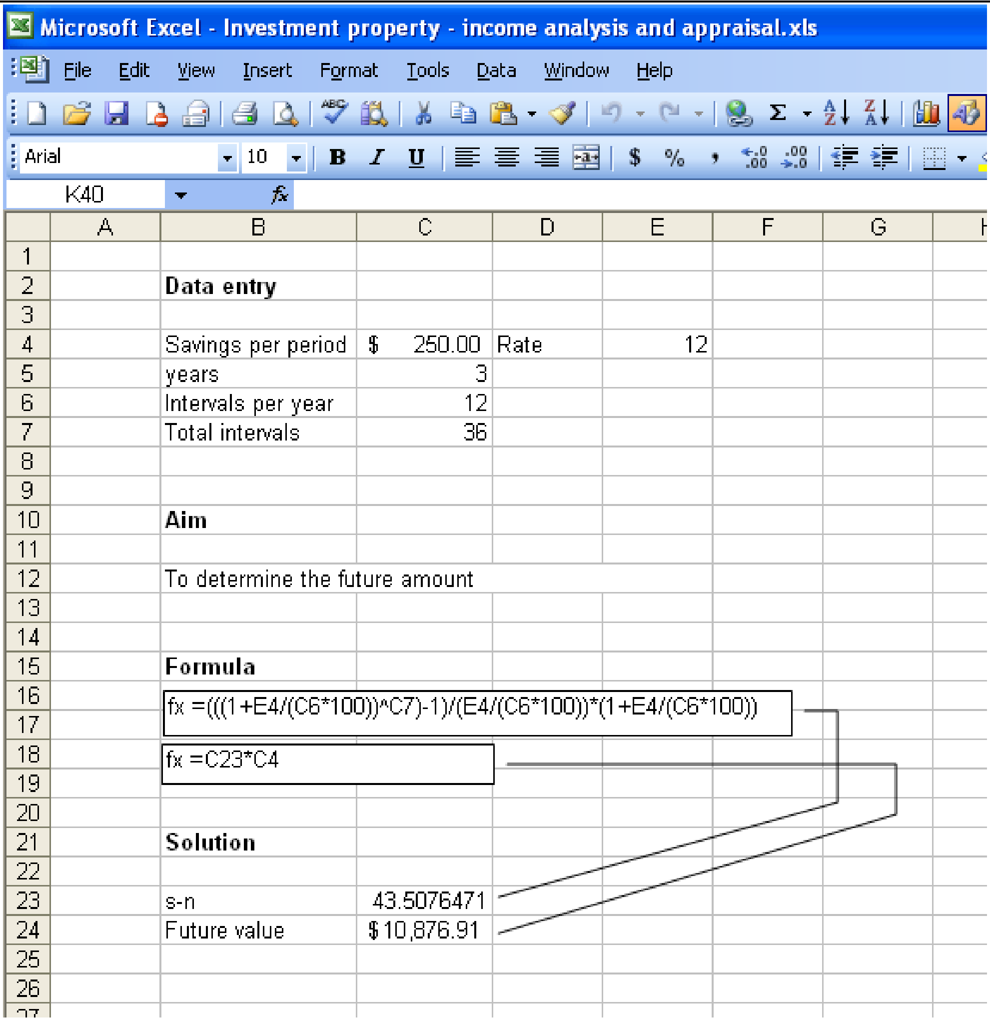
Future value of an annuity

$$FV_n = R\,\frac{(1+i)^n - 1}{i}$$

Example

You can save $250 per calendar month and have opened an investment account to save for the deposit on a property. If they make their first deposit today and intend to save regularly for 3 years, how much will they have saved if interest is to be credited to their account at 12% per year compounded monthly?

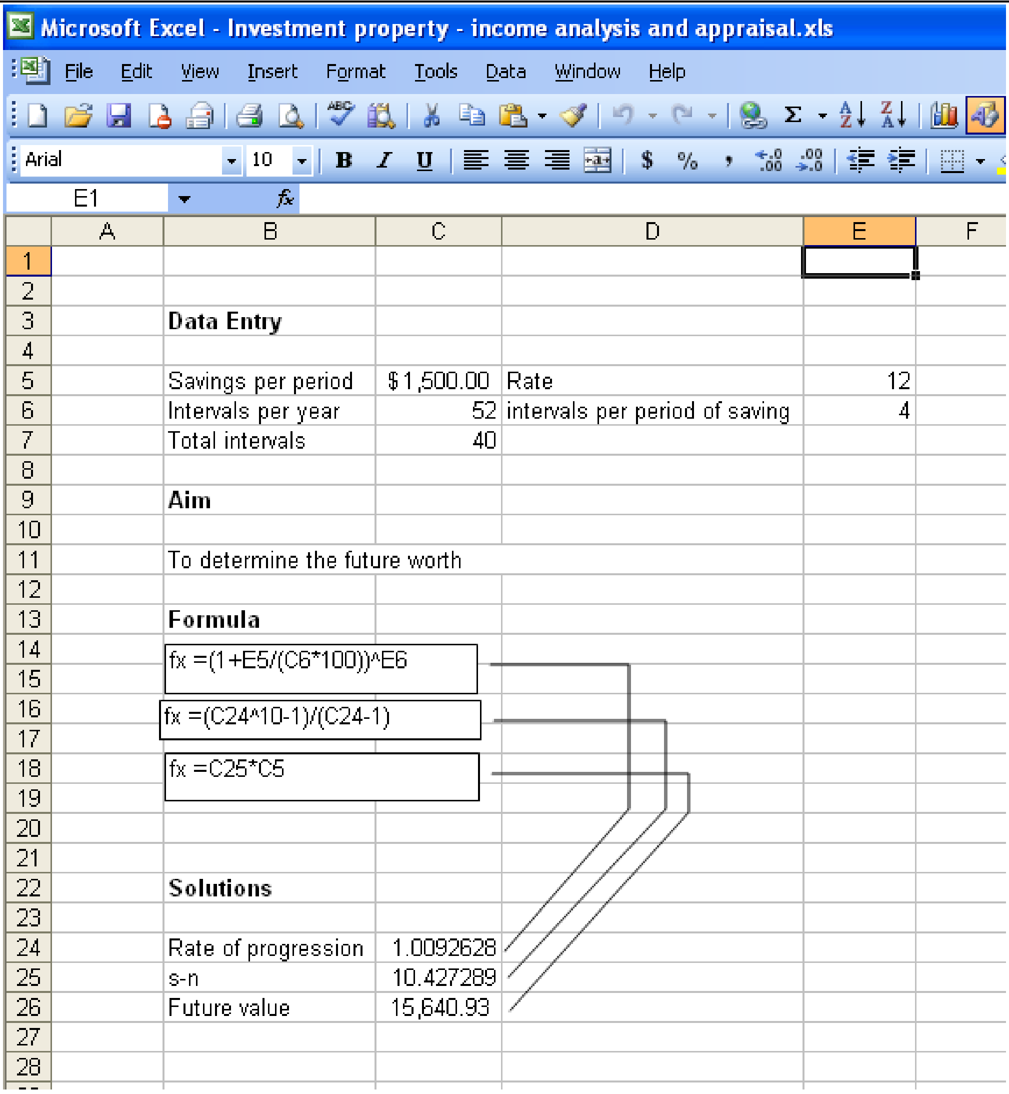
This problem can be solved with the use the Excel JC 3 Financial maths tutorial spreadsheet link on Blackboard under Learning Resources and ‘Future Value of an Annuity (A)’ as follows:



Example

A tenants' Association in a Shopping Centre has decided to levy its members for a promotion to be held in 40 weeks' time. A levy of $1500 will be collected every 4 weeks starting In 4 weeks time. The levy will be invested to earn Interest at 12 % per year compounded weekly. How much money will be In the promotion fund In 40 weeks' time?

This problem can be solved with the use of a spreadsheet (see the JC 3 spreadsheet for Financial maths tutorial link on Blackboard under learning resources and ‘Future Value of an Annuity (B))’ as follows:



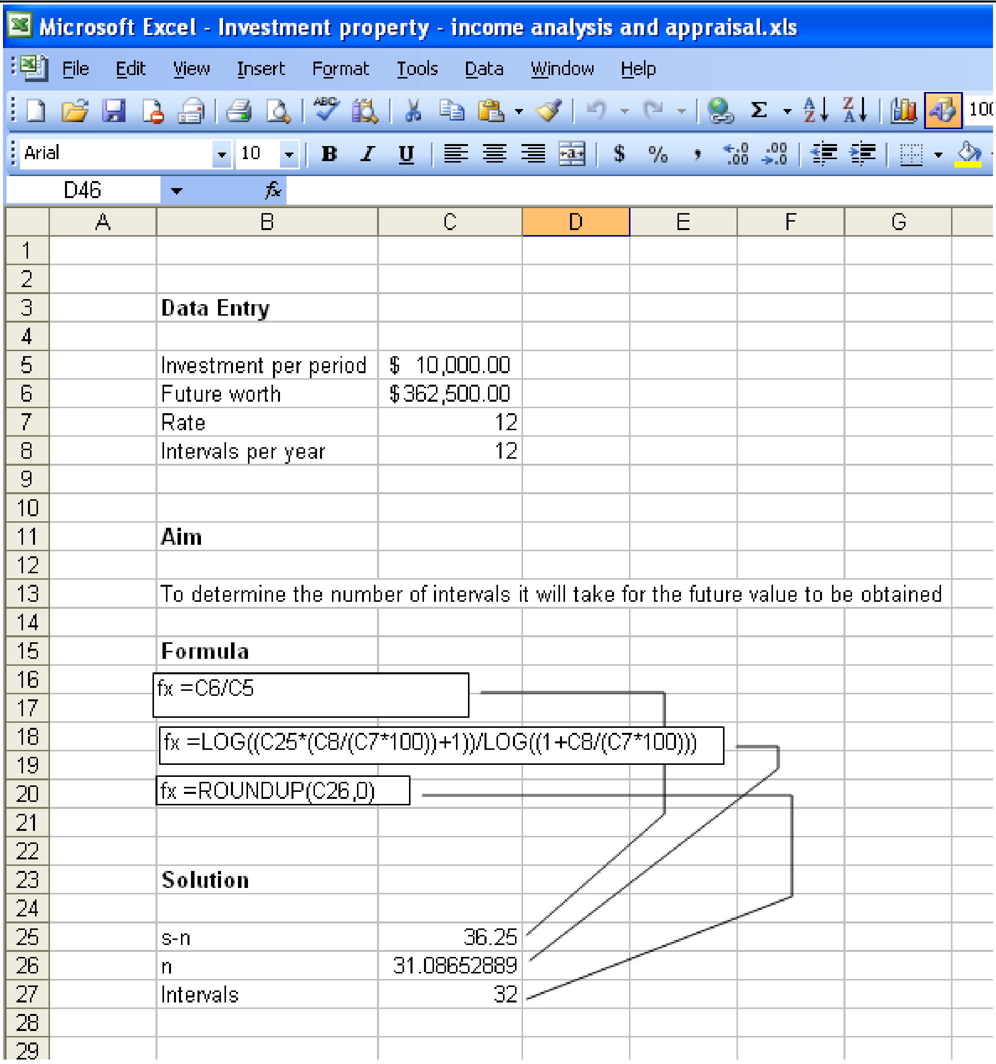
Microsoft Excel - Investment property - income analysis and appraisal.xls

	A	B	C	D	E	F
1						
2						
3		**Data Entry**				
4						
5		Savings per period	$1,500.00	Rate	12	
6		Intervals per year	52	intervals per period of saving	4	
7		Total intervals	40			
8						
9		**Aim**				
10						
11		To determine the future worth				
12						
13		**Formula**				
14		fx =(1+E5/(C6*100))^E6				
15						
16		fx =(C24^10-1)/(C24-1)				
17						
18		fx =C25*C5				
19						
20						
21						
22		**Solutions**				
23						
24		Rate of progression	1.0092628			
25		s-n	10.427289			
26		Future value	15,640.93			
27						
28						

Example

Following a rent review a property manager has $ 10,000 per month additional cah flow. The manager decides to invest the money in interest bearing securities in order to accumulate sufficient funds to carry out refurbishment of a property in the portfolio. By taking out a forward commitment to buy securities to the value of $10,000 each month, starting next month, interest will be paid at 12 % per year compounded monthly. The refurbishment is estimated to cost $362,500. When will the refurbishment be able to be undertaken?

This problem can be solved with the use of a spreadsheet (see the JC 3 spreadsheet for Financial maths tutorial link on Blackboard under learning resources and ‘Future Value of an Annuity (c))’ as follows:



The Sinking Fund Factor

The reciprocal of the future value of an annuity is sometimes termed the sinking fund factor, i.e.

$$SF = FV\frac{i}{(1+i)^n - 1}$$

Example

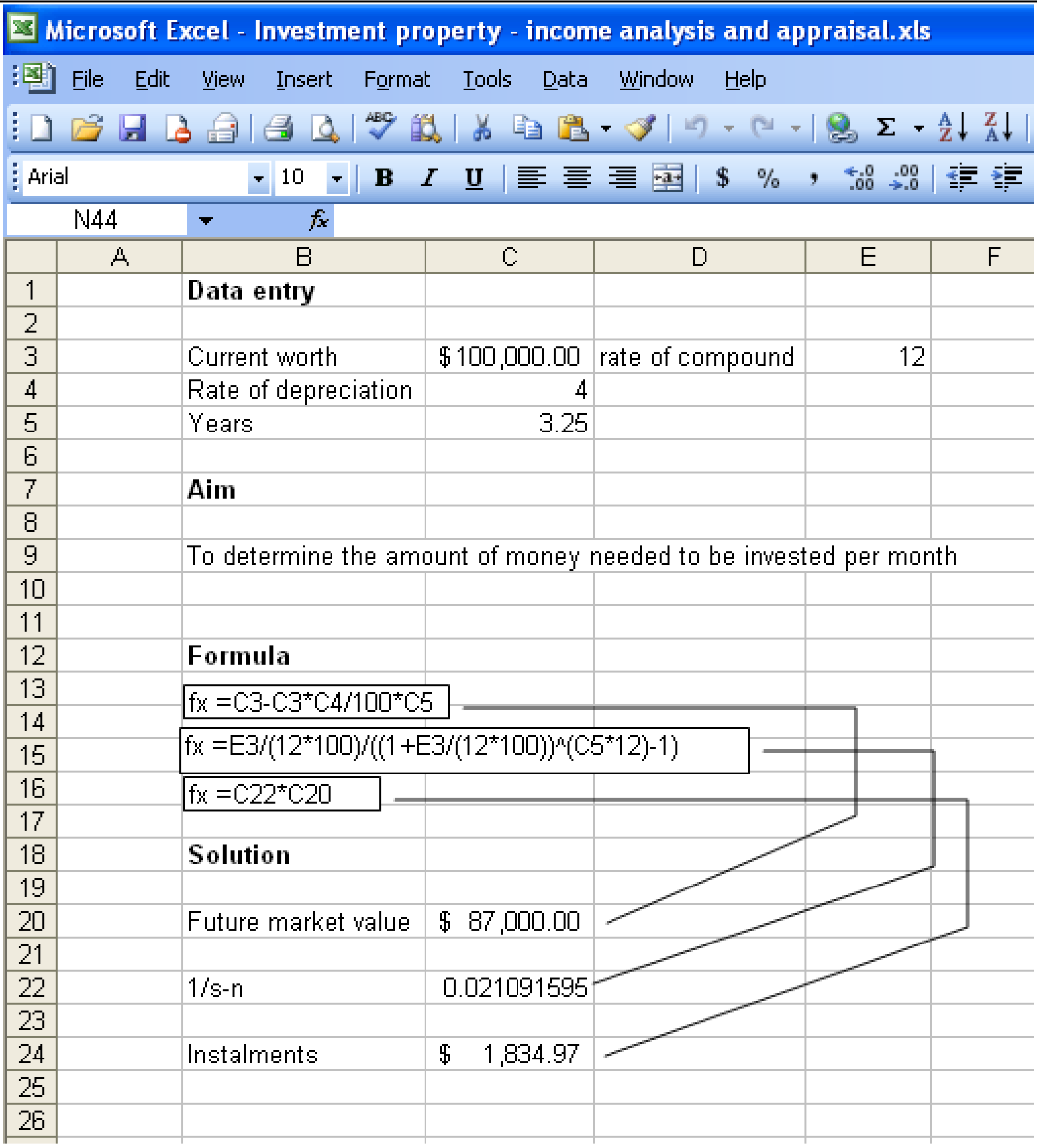
What amount would be necessary to deposit into a fixed interest savings account in which interest is credited at 12% p.a. compounded monthly to save a sum of $2,697.35 in 2 years time, the answer is:

$$
\begin{aligned}
\mathrm{SFF} &= \mathrm{FV}\frac{\mathrm{i}}{(1+\mathrm{i})^{\mathrm{n}}-1} \\
&= 2,697.35\frac{0.01}{(1+0.61)^{24}-1} \\
&= 2,697.35\frac{0.01}{.2697} \\
&= 2,697.35\times.037078 \\
&= 100.01 \\
\mathrm{say} &= \$100.
\end{aligned}
$$

The Sinking Fund Factor is particularly useful in property management. For example:

A ground lease expires in 3 years 3 months, the ground lessor (owner) having a reversionary value in the ground lessee's improvements (for which no compensation is payable to the ground lessee)of $100,000 at today's valuation. It is expected that a physical depreciation and market obsolescence will be incurred on the improvements at 4% per year on a straight line basis during the unexpired term of the lease. The ground lessor will agree to a new ground lease if the ground lessee purchases the ground lessor's reversionary interest on expiration of the current lease. If the ground lessee can obtain 12% per year compounded monthly on an investment account to be opened in one month with regular payments of equal installments thereafter, how much will the installments need to be to ensure the ground lessee secures a new lease.

This problem can be solved with the use the JC 3 Financial maths tutorial 1.xls Excel spreadsheet link on Blackboard under Learning Resources, ‘Sinking Fund Factor’ as follows:



	A	B	C	D	E	F
1		**Data entry**				
2						
3		Current worth	$100,000.00	rate of compound	12	
4		Rate of depreciation	4			
5		Years	3.25			
6						
7		**Aim**				
8						
9		To determine the amount of money needed to be invested per month				
10						
11						
12		**Formula**				
13		fx =C3-C3*C4/100*C5				
14						
15		fx =E3/(12*100)/((1+E3/(12*100))^(C5*12)-1)				
16		fx =C22*C20				
17						
18		**Solution**				
19						
20		Future market value	$ 87,000.00			
21						
22		1/s-n	0.021091595			
23						
24		Instalments	$ 1,834.97			
25						
26						

Present value

This formula is sometimes referred to as the present value of one to distinguish it from the present value of an annuity.

The formula is merely the re-arrangement of the future value value formula, i.e.

$$PV = \frac{FV}{(1+i)^n} \quad (\text{or } FV(1+i)^{-n})$$

Example

A property sells for $1,000,000 on the basis of $100,000 deposit, the balance due on settlement in 2 years. What is the actual sale price in present value terms on the date of the sale, assuming an opportunity cost/discount rate of 10% per annum.

Present value of deposit = $100,000

$$\text{Present worth of balance} = \frac{FV}{(1+i)^n}$$

$$= \frac{900{,}000}{(1.10)^2}$$

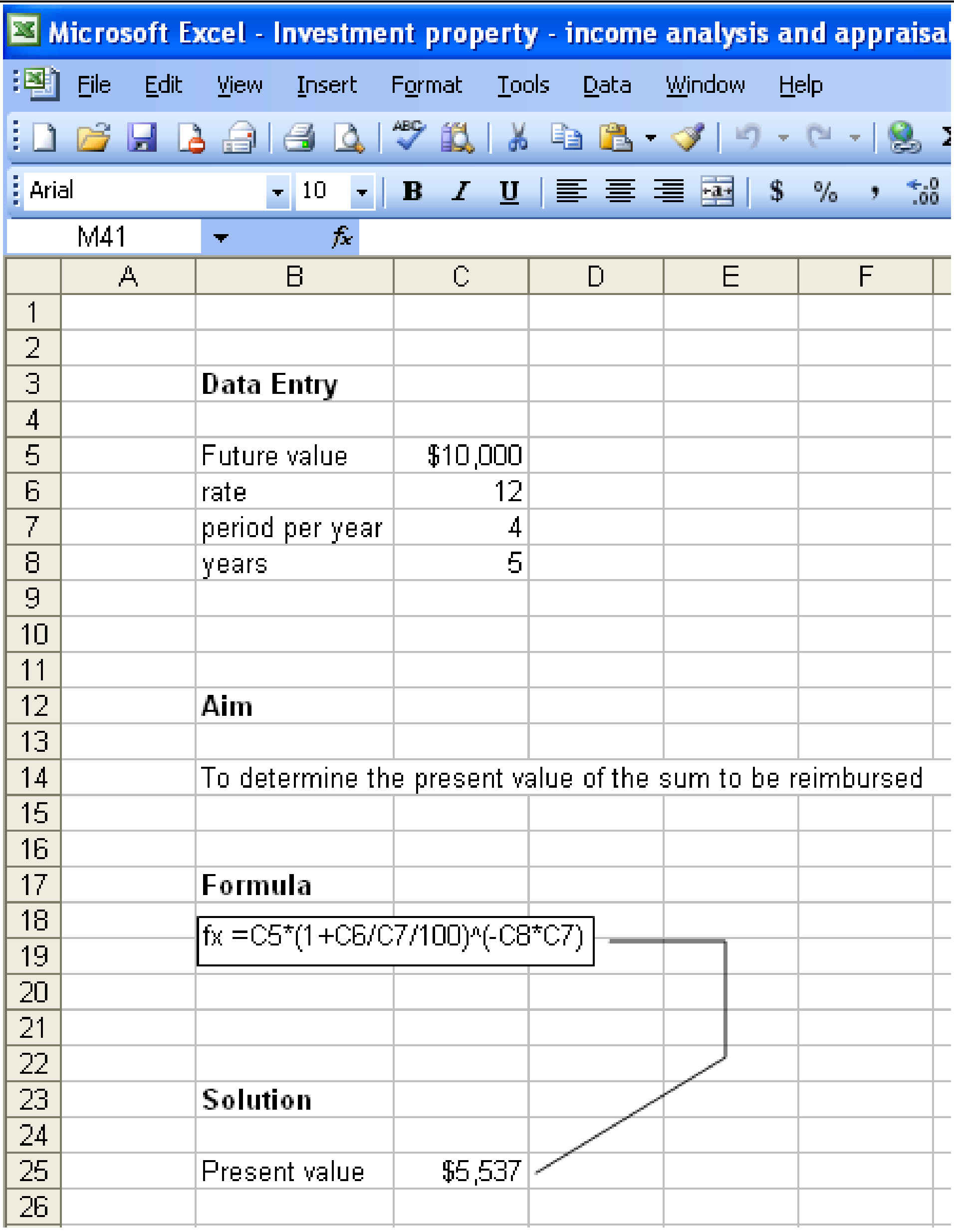
$$= \frac{900{,}000}{1.21} = 743{,}801$$

$$\text{Therefore present value of sale price} = \$843{,}801$$

Example

Under the terms of a lease just negotiated the lessee has agreed to install a shop front costing $10,000. On expiration of the lease the lessor will reimburse the lessee for this expenditure. If the lessee could have otherwise invested capital to return 12% per year compounded quarterly, what is the present value of the lessor's promise to reimburse $10,000 when the lease expires in 5 years time. This problem can be solved with the use of the file JC 3 Financial maths tutorial 1.xls spreadsheet Tab 6 'Present Value of $10,000' as follows:

This problem can be solved with the use of the file JC 3 Spreadsheet Financial maths tutorial link on Blackboard under Learning Resources as follows



Present value of an annuity

This formula is

$$\text{Present Worth} = R\frac{1-(1+i)^{-n}}{i} \qquad \text{(ordinary annuity)}$$

$$\text{or} = R\left[\frac{1-(1+i)^{-n}}{i}\times(1+i)\right] \qquad \text{(annuity due)}$$

Example

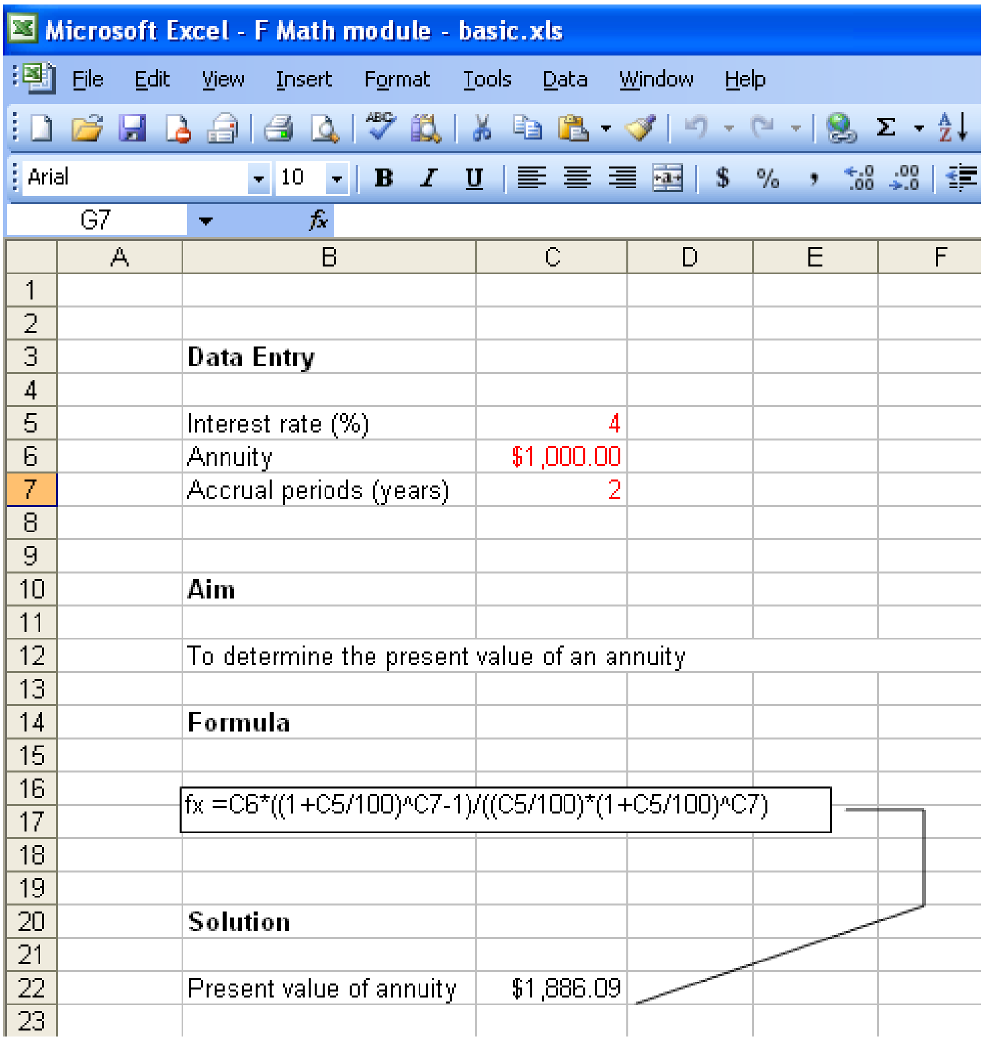
Calculate the present value of a tenant's interest where the tenant has two years remaining under a lease and pays $1,000 per annum less than market rent. The discount rate is 4% p.a.

Assume the calculation is on the basis of an annuity due, then:

$$\text{Present Worth} = 1000\left[\frac{1-(1.04)^{-2}}{0.04}\right].$$

$$= \$1{,}886.09.$$

This problem can be solved with the use of the file JC 3 Financial maths tutorial 1.xls spreadsheet link'Present Value of an Annuity (A)' on Blackboard as follows:



Microsoft Excel - F Math module - basic.xls

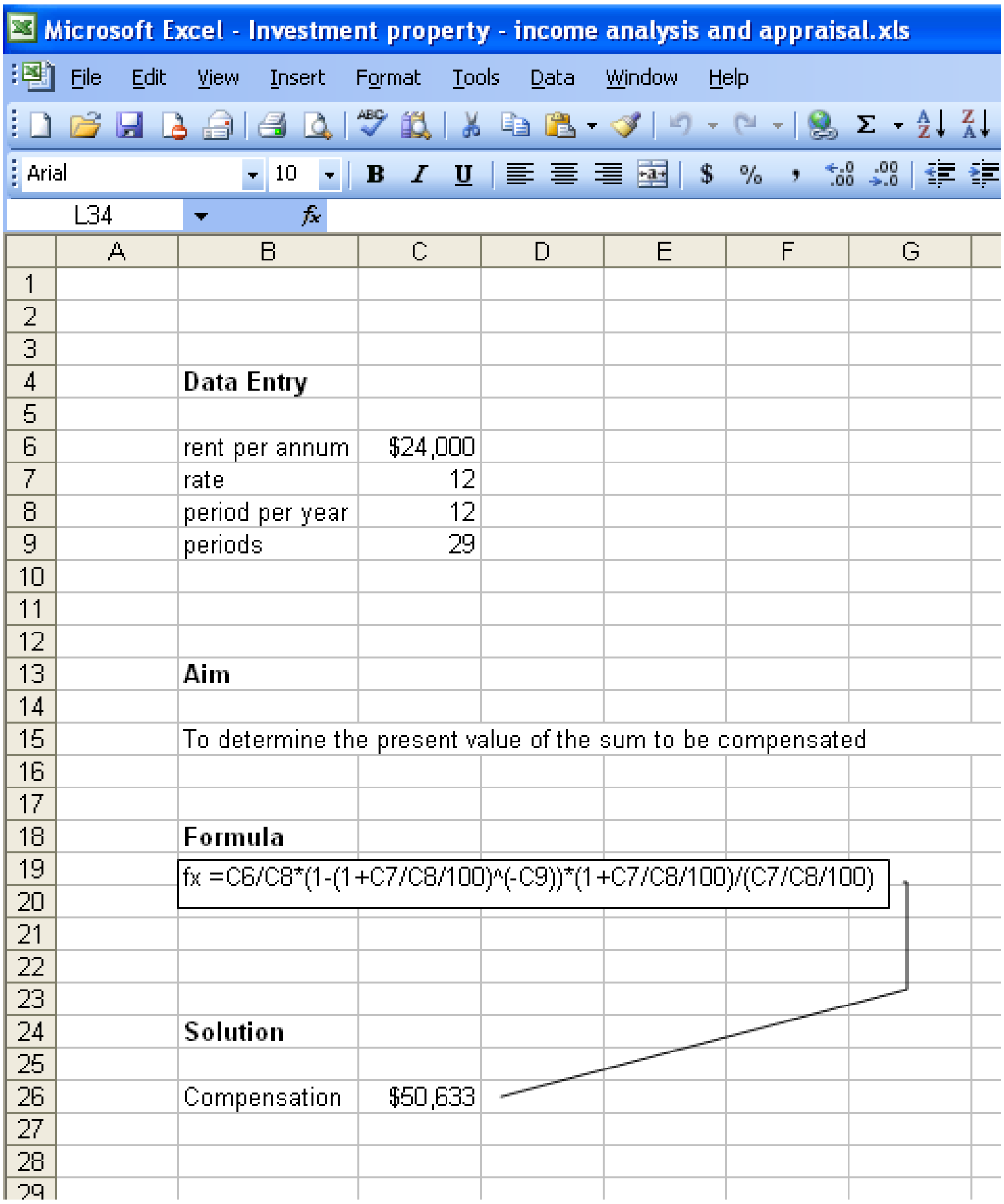
	A	B	C	D	E	F
1						
2						
3		**Data Entry**				
4						
5		Interest rate (%)	4			
6		Annuity	$1,000.00			
7		Accrual periods (years)	2			
8						
9						
10		**Aim**				
11						
12		To determine the present value of an annuity				
13						
14		**Formula**				
15						
16		fx =C6*((1+C5/100)^C7-1)/((C5/100)*(1+C5/100)^C7)				
17						
18						
19						
20		**Solution**				
21						
22		Present value of annuity	$1,886.09			
23						

Example

A factory lease has 2 years 5 months to run. The lessee wishes to surrender the lease now in order to relocate to other premises. What compensation should the lessor require if the rental

Is $24,000 per year paid monthly in advance and the lessor's net return on the property is 12% per year?

This problem can be solved with the use of the file JC 3 Financial maths tutorial spreadsheet link on Blackboard 'Present Value of an Annuity (B)') as follows:


Microsoft Excel - Investment property - income analysis and appraisal.xls

	A	B	C	D	E	F	G
1							
2							
3							
4		**Data Entry**					
5							
6		rent per annum	$24,000				
7		rate	12				
8		period per year	12				
9		periods	29				
10							
11							
12							
13		**Aim**					
14							
15		To determine the present value of the sum to be compensated					
16							
17							
18		**Formula**					
19		fx =C6/C8*(1-(1+C7/C8/100)^(-C9))*(1+C7/C8/100)/(C7/C8/100)					
20							
21							
22							
23							
24		**Solution**					
25							
26		Compensation	$50,633				
27							
28							
29							

Present Value of an Annuity in Perpetuity

The formula is $PV = \frac{R}{i}$ and is a corollary of the present value of an annuity, where the annuity continues for such a long period as a series to perpetuity. The factor is generally applied to the analysis/valuation of income streams (annuities) from real property held freehold in fee simple which are considered to be enjoyed in perpetuity.

This concept can be tested by calculating the present value of an annuity over say 50, 100 years, etc. using the present value of an annuity formula and comparing the answer with that produced by the present value of an annuity in perpetuity formula. As the period of the annuity increases, the present value will approach the present value of the annuity in perpetuity. After, say, 100 years the annuity can in effect be treated as one in perpetuity.

Example

A freehold property is let at a net income (which is estimated to be its net market rental value) of $21,000; the appropriate discount rate is 10%.

Therefore investment value of an annuity in perpetuity:

$$PV = \frac{R}{i}$$

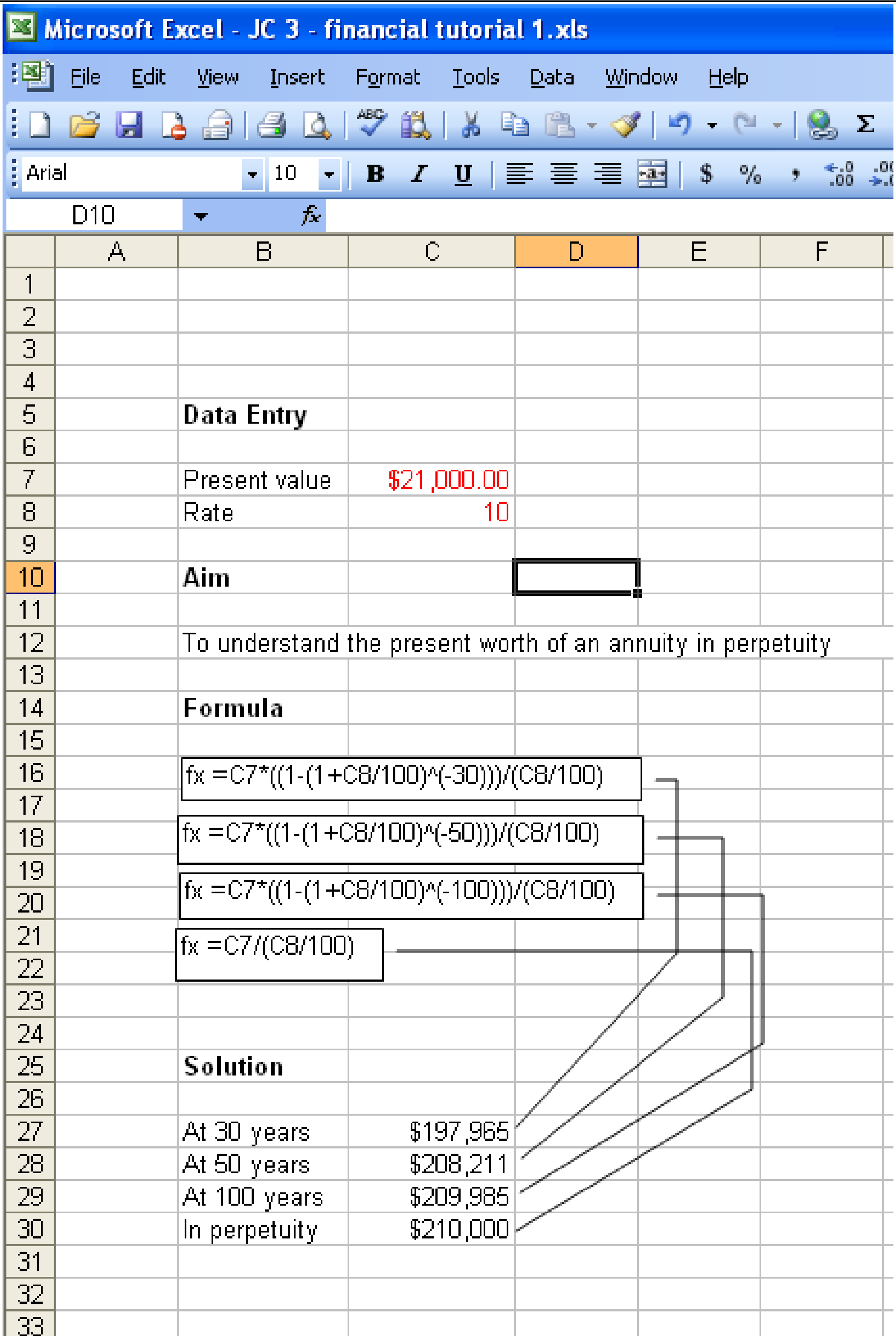
$$= \frac{21{,}000}{.1}$$

$$= \$210{,}000$$

Note the present value of $21,000 p.a. discounted at 10% per annum over

30 years	=	$197,965
40 years	=	$205,360
50 years	=	$208,211
100 years	=	$209,984
In perpetuity	=	$210,000

This problem can be solved with the use of the file JC 3 Financial maths tutorial spreadsheet link ‘An Annuity in Perpetuity’ as follows:


Microsoft Excel - JC 3 - financial tutorial 1.xls
File Edit View Insert Format Tools Data Window Help
Arial 10
D10
Data Entry
Present value $21,000.00
Rate 10
Aim
To understand the present worth of an annuity in perpetuity
Formula
fx =C7*((1-(1+C8/100)^(-30)))/(C8/100)
fx =C7*((1-(1+C8/100)^(-50)))/(C8/100)
fx =C7*((1-(1+C8/100)^(-100)))/(C8/100)
fx =C7/(C8/100)
Solution
At 30 years $197,965
At 50 years $208,211
At 100 years $209,985
In perpetuity $210,000

Calculating the Investment Value of Real Estate

The capitalisation process

In assessing the investment value of real estate the discount rate used is traditionally obtained by analysing market sales to derive a discount rate which, in effect, represents the net market return expected on the investment in the first year. This market return is then expressed as a decimal point and substituted in the PV of an annuity formula (in perpetuity).

Example

Freehold Property A has just sold for $100,000 with a net market rent of $10,000. Therefore the net market (%) return is 10%. This percentage return is expressed as a decimal and used to discount the net market rent in perpetuity from Property B (which is perceived to have very similar investment characteristics) in order to estimate its investment value.

For example:

$$PV = \frac{R}{i}$$

$$= \frac{\$12,000}{0.1}$$

$$= \$120,000$$

This process is known as the capitalisation process, and the discount rate is usually referred to as the capitalisation rate or ‘all risk yield’ or ‘all risk rate’.

> In simple terms the capitalisation rate is a mathematical relationship which exists between the net income derived from the property and the value or price which a probable purchaser would pay for the privilege of receiving that net income stream.
>
> D.J. Wilson, ‘Elements of the capitalisation rate’, *Canadian Appraiser*, Fall 1991: 30

The capitalisation rates or ‘all risk yields’ applicable to the following hypothetical transactions taken from the market place are illustrated below:

Transaction A	Net Rent	$10,000	Capitalisation Rate = .10 or 10%
	Sale Price	$100,000)	
Transaction B	Net Rent	$12,500)	Capitalisation Rate = .125 or12.5%
	Sale Price	$100,000)	
Transaction C	Net Rent	$20 000)	Capitalisation Rate = 111 or 11.1%
	Sale Price	$180,000)	

The capitalisation rate as a discount rate and a measure of risk

It will be noticed that in capitalising income from a freehold property (i.e. capitalising in perpetuity) in order to determine investment value, there is a relationship between the capitalisation or discount rate and investment value. As the capitalisation rate rises, investment value falls. Consider the following example.

Example

Calculate the investment value of a net income of $100,000 expected from a freehold property at the following capitalisation rates: 8%, 10%, 12.5%.

Capitalisation or All Risk Rates	**Formula** ($PV = \frac{R}{i}$)	**Value**
(a) 8%	$\frac{\$100,000}{.08}$	$1,250,000
(b) 10%	$\frac{\$100,000}{.1}$	$1,000,000
(c) 12.5%	$\frac{\$100,000}{.125}$	$800,000

Clearly the greater the capitalisation rate the lower the investment value, and it is logical to assume that the investment value assumes a greater risk attached to some net incomes which result in greater capitalisation (discount rates) and lower investment values to reflect this situation. The rate of return is directly related to the perceived risk inherent in the investment and the capitalisation (discount rates) derived from the analysis of sales in the market place is a reflection of the market's assessment of risk.

It is important to distinguish this market derived concept of a discount (risk) rate and the discount rate based on 'the cost of capital' as employed in **Finance**. The market derived capitalisation (discount) rate is also different from the discount (risk) rate usually applied in discounted cash flow techniques relating to property investment. The Discounted Cash Flow approach to property investment uses the discount rate as an opportunity cost concept, i.e. the discount rate implies that a rate of return must be paid to an investor sufficient to meet the competition of alternative investment outlets for the investor's funds.

Years Purchase

The above term is to be found in most text books relating to real estate valuation and property investment text books. It is an alternative way of explaining the relationship between net income and purchase price and is the number of years of net income needed to buy the property (Rost and Collins (1984) *Land Valuation and Compensation in Australia* pp. 97 and 207).

Using the figures above to illustrate the relationship between capitalisation rate and price, the calculation of the Years Purchase (in perpetuity) is as follows:

(i) $\dfrac{\text{Worth}}{\text{Net income}}$ = 1,250,00 / 100,000 = 12.5 Years Purchase in perpetuity

(ii) $\dfrac{\text{Worth}}{\text{Net income}} = \dfrac{1{,}000{,}000}{100{,}000}$ = 10 Years Purchase in perpetuity

(iii) $\dfrac{\text{Worth}}{\text{Net income}} = \dfrac{800{,}000}{100{,}000}$ = 8 Years Purchase in perpetuity

From these figures one can now deduce that there is a relationship between the Years Purchase and the Capitalisation Rate, i.e.

$$\frac{1}{\text{Capitalisation Rate}} = \text{Years Purchase}$$

For example, using the same figures as previously:

(i) $\dfrac{1}{.08}$ = 12.5 Years Purchase

(ii) $\dfrac{1}{.1}$ = 10 Years Purchase

(iii) $\dfrac{1}{.125}$ = 8 Years Purchase

It can be concluded that the Years Purchase is the reciprocal of the capitalisation rate.

Hence an alternative approach to calculating investment value is to use Years Purchase. Take the example used above to illustrate the present value of an annuity in perpetuity.

$$\text{Years purchase in perpetuity} = \frac{1}{\textit{Capitalisation rate}}$$

$$= \frac{1}{.1}$$

$$= 10$$

$$\text{Net market rental} = \$21{,}000$$

$$\text{Therefore Investment worth} = \$210{,}000 \text{ (ie } 10 \text{ YP} \times \$21{,}000)$$

That is, it would take ten years of net income to justify the investment value of $210,000.

Project Evaluation (DCF) Techniques

The project evaluation techniques used in the analysis of property investment are generally referred to as the Discounted Cash Flow techniques and the two most widely used techniques are Internal Rate of Return and Net Present Value.

The techniques of Internal Rate of Return and Net Present Value can be used in:

- cash flows expressing a return on the total investment capital
- cash flows expressing a return on the equity investment capital
- cash flows expressing a return on the 'after tax' equity cash flow.

The cash flows classified in (2) and (3) above are not considered in this module.

For the purpose of this subject, we restrict ourselves to the application of Net Present Value and Internal Rate of Return as measures of a return on total capital. This will facilitate an understanding of their use as a contemporary method of valuation (i.e. as a means of predicting the most likely selling price subject to all other necessary conditions). Their use as such is addressed in the next topic.

The selection of an appropriate discount rate in the Discounted Cash Flow is of course crucial to the technique. The rate is influenced by a host of factors and not least the degree of likely risk/rates of return available from alternative investment opportunities.

The following simple example relating to the investment decision involved in the acquisition of property A or B is provided to demonstrate the application of the IRR and NPV techniques.
Example

A property investment analyst has been appointed adviser to a tax exempt investor who has to make a choice between two alternative property investments which have been offered. Each is for sale at $100,000 inclusive of acquisition costs and the following information is available.

Investment A

Based on current lease terms the anticipated **net** cash flow is $10,000 p.a. for the first two years and $12,000 p.a. for the next three years, selling at the end of the fifth year at an anticipated price of $125,000 net of sale fees.

Investment B

Based on current lease terms the anticipated **net** cash flow is $11,000 p.a. for the first three years and $11,500 for the next two years, selling at the end of the fifth year at an anticipated price of $115,000 net of sale fees.

Assume rents are received annually in arrears and allowance has been made for inflation/real growth in projected rental and sale prices.

Solution

Investment A

	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
Costs						
1. Initial outlay	$(-100,000)					
Benefits						
1. Projected Net Rental Cash Flow		10,000	10,000	12,000	12,000	12,000
2. Estimated Sale Price						125,000
Cash flow	(100,000)	10,000	10,000	12,000	12,000	137,000
Discount Factor	–	0.8929	0.7972	0.7118	0.6355	0.567427
DCF	(100,000)	8,929	7,972	8,541	7,626	77,738

IRR 14.76%
NPV $10,805(based on a required rate of return, i.e. a discount rate of 12%)

Investment B

	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
Costs						
1. Initial outlay	$(-100,000)					
Benefits						
1. Projected Net Rental Cash Flow		11,000	11,000	11,000	11,500	11,500
2. Estimated Sale Price						115,500
Cash flow	(100,000)	11,000	11,000	11,000	11,500	127,000
Discount Factor		0.8929	0.7972	0.7118	0.6355	0.567427
DCF	(100,000)	9,821	8,769	7,830	7,308	72,063

IRR 13.53%
NPV $5,792 (based on a required rate of return i.e. a discount rate of 12%)

The NPV is obtained by adding the inflow and outflows of the bottom line, i.e. the Discounted Cash Flow. The IRR is best obtained by using a financial calculator or Excel spreadsheet.

The results, using the data and assumptions above, show Investment A to be preferable to Investment B by both NPV and IRR criteria (which are measures of return). However, the IRR results do not take into account the risks of the investment, and it should be remembered that the investment decision process should take place in the context of the ‘return risk trade off’, i.e. the uncertainty of expected rates of return need to be borne in mind when measuring the expected rates of return from investment alternatives in order to decide on which investment to select.

The above example is sometimes described as a monetarist approach (as opposed to a real cost approach) in that the cash flow is projected at ‘nominal rates’ taking into account the ‘inflationary element’ of rental growth and capital gain. (See discussion on ‘Inflation and Project Evaluation’ Peirson, Bird and Brown, p. 149) – you should have this text from last year).

The internal rate of return or net present value produced from the example’s cash flow is the measure of financial performance of a property (i.e. return on total capital) from its acquisition cost date until the assumed or actual sale including all the cash inflows and outflows of the investment during the actual or presumed term of ownership.

These techniques can be used as a measure of past or future performance of a property investment.

The operations involved in establishing the NPVs and the IRR are found in the file JC 4 Financial maths tutorial spreadsheet link ‘NPVs’ and Tab 2 ‘IRR’on Blackboard as follows:

Microsoft Excel - JC 4 - Financial maths tutorial 2.xls

File Edit View Insert Format Tools Data Window Help

Arial 10 100%

F51 =1/(1+C46/100)^B51

Investment A NPV (8 per cent)

Rate (%) 8

At end of Year	Cash outflow	Net cash Flow	Discount Factor	Discounted Cash Flow	Discounted Cash Flow
0	$3,800	$0	-$3,800	1.0000	-$3,800
1	$0	$1,000	$1,000	0.9259	$926
2	$0	$2,000	$2,000	0.8573	$1,715
3	$0	$2,000	$2,000	0.7938	$1,588
					$4,228
					-$3,800
				NPV =	$428

Investment B NPV (8 percent)

Rate (%) 8

At end of Year	Cash outflow	Cash inflow	Net cash flow	Discount Factor	Discounted Cash Flow
0	$3,000	$0	-$3,000	1.0000	-$3,000
1	$0	$500	$500	0.9259	$463
2	$0	$2,500	$2,500	0.8573	$2,143
3	$0	$1,000	$1,000	0.7938	$794
					$3,400
					-$3,000
				NPV =	$400

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Investment A NPV (15 per cent)

rate 15

At the end of year	Cash Outflow	Cash inflow	Net cash flow	Discount Factor	Discounted Cash Flow
0	$3,800	$0	-$3,800	1.0000	-$3,800
1	$0	$1,000	$1,000	0.8696	$870
2	$0	$2,000	$2,000	0.7561	$1,512
3	$0	$2,000	$2,000	0.6575	$1,315
					$3,697
					-$3,800
				NPV =	-$103

Investment B NPV (15 per cent)

Rate (%) 15

At the end of Year	Cash Outflow	Cash inflow	Net cash Flow	Discount factor	Discounted Cash flow
0	$3,000	$0	-$3,000	1.0000	-$3,000
1	$0	$500	$500	0.8696	$435
2	$0	$2,500	$2,500	0.7561	$1,890
3	$0	$1,000	$1,000	0.6575	$658
					$2,983
					-$3,000
				NPV =	-$17

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Feasibility study for proposed land sub division estimated internal rate of return

Period	Date	Cash distribution	Cash Receipt	Net cash flow	Discount 38 %		Factor 40 %	
0	30/04/1990	$1,810,000	$0	-$1,810,000	1.000	-$1,810,000	1.000	-$1,810,000
1	31/10/1990	$6,700,000	$0	-$6,700,000	0.851	-$5,703,419	0.845	-$5,662,534
2	30/04/1991	$16,290,000	$0	-$16,290,000	0.725	-$11,804,348	0.714	-$11,635,714
3	31/10/1991	$13,400,000	$0	-$13,400,000	0.617	-$8,265,824	0.604	-$8,089,334
4	30/04/1992	$16,700,000	$0	-$16,700,000	0.525	-$8,769,166	0.510	-$8,520,408
5	31/10/1992	$20,100,000	$0	-$20,100,000	0.447	-$8,984,592	0.431	-$8,667,143
6	30/04/1993	$10,050,000	$0	-$10,050,000	0.381	-$3,824,096	0.364	-$3,662,536
7	31/10/1993	$0	$153,000,000	$153,000,000	0.324	$49,558,098	0.308	$47,124,126
						-$49,161,445		-$48,047,669
					Total	$396,653	Total	-$923,543

IRR= 38.60 %

Conclusion

This module has outlined the basic mathematical techniques in relation to property investment. A valuer or advisor on property investment matters must have a knowledge of the mathematics of finance and the theory of compounding and discounting. This enables advice to be given on the present investment value of the future benefits that are derived from the ownership of freehold and leasehold interests in land and property. The benefits are the net income returns derived from the capital investment in real estate interests.

These basic mathematical techniques form the foundation to what is generally classified as the income (investment) approach to property valuation which is the subject matter of the next topic. The valuation method adopts the investment technique of discounting future net income flows expected from investment in real estate, and is utilised by valuers because it most closely reflects the behaviour of the various parties operating in particular property submarkets.

Summary of Formulae

Future Value $= PV(1+i)^n$
(Compounding of single amount)

Future Value of an ordinary annuity $= R\dfrac{(1+i)^n - 1}{i}$
(Compounding of regular flow of payments)

Present Value $= \dfrac{FV}{(1+i)^n}$ or $FV(1+i)^{-n}$
(Discounting of a single amount)

Present Value of an annuity (ordinary) $= R\dfrac{1-(1+i)^{-n}}{i}$
Discounting of a regular flow of payments in arrears)

Present Worth of an annuity (annuity due) $= R\left[\dfrac{1-(1+i)^{-n}}{i} \times (1+i)\right]$
(in advance)

(Present Value of an annuity in perpetuity $= \dfrac{R}{i}$

Capitalisation Rate $= \dfrac{\text{Net Income}}{\text{Sale Price}}$ expressed as a decimal

or $= \dfrac{1}{\text{Years Purchase}}$

Years Purchase $= \dfrac{1}{\text{Capitalisation Rate}}$ expressed as a decimal

Net Present Value $= Cf_0 + \dfrac{Cf_1}{(1+i)^1} + \dfrac{Cf_2}{(1+i)^2} \cdots \dfrac{Cf_n}{(1+i)^n}$

where
R = net income
i = discount rate
n = number of periods
Cf = cash flow

Alternative Valuation Mathematic Formulae

The above formulae can be written in alternative ways and every text book will use different expressions for such formulae. An alternative set of formulae commonly used is also listed below for your information:-

Note: 'i' is the rate of interest expressed as a decimal.
'n' is the number of years or periods.

The Amount of $1

'The amount to which $1 invested now will accumulate in a given number of years at a selected rate or rates of interest, compounded over the period at appropriate intervals. This is the reciprocal of the present value of $1.' †

$$A = (1 + i)^n$$

Note:
a. interest is paid at end of each period;
b. amount must always be greater than 1;
c. forms the basis of all other tables;
d. based on compound interest theory.

The Amount of $1 per annum (also known as Future Value)

'The amount to which $1 invested at the end of each year for a given period of years will accumulate at a selected rate or rates of interest, compounded over the period at yearly or other intervals. The reciprocal of this is the annual sinking fund.' †

$$A\ \$1\ pa = \frac{(1 + i)^n - 1}{i}$$

Annual Sinking Fund (Compounding of a regular flow of payments to build up to a future amount)

'(A.S.F.) A sinking fund where payments and interest accumulated are calculated yearly.' †

Reciprocal of Amount of $1 p.a. table

Amount of $1 pa. $= \dfrac{(1 + i)^n - 1}{i} = \dfrac{A - 1}{i}$

THUS: $ASF = \dfrac{i}{(1 + i)^n - 1} = \dfrac{i}{A - 1}$

The Present Value of $1

'At a particular date, the value of $1 receivable in a given number of years discounted at a selected rate of interest. It is the reciprocal of the Amount of $1.'†

If $1 were invested now at 'i' for 'n' years, then at the end of the period it would be value $(1 + i)^n$ i.e. = 'A'.

If $ 'x' were to be invested now at 'i' for 'n' years and assuming it will accumulate to $1

Then $x\,(1 + i)^n = \$1$

And $'x' = \dfrac{1}{(1 + i)^n} = \dfrac{1}{A} = \text{Present value of } \$1 = PV$

This is the reciprocal of the Amount of $1.

N.B. a. Based on compound interest calculations
b. PV $1 is always less than 1.00.

The Present Value of $1 per annum

'Today's worth, as a capital sum, of the right to receive $1 per annum for a given number of years, when each yearly receipt has been discounted at a selected rate of interest and their individual present values aggregated. It is synonymous with years purchase.'†

$$\text{PV \$1 pa.} = \frac{1 - \left\{\dfrac{1}{(1 + i)^n}\right\}}{i}$$

or more simply, $\text{PV \$1 pa.} = \dfrac{1 - PV}{i}$

As 'n' approaches perpetuity, PV closes on zero (the PV of $1 receivable in infinite time is infinitely small)

so, $\dfrac{1 - PV}{i}$ tends towards $\dfrac{1 - 0}{i} = \dfrac{1}{i} = \text{YP in perpetuity}$

YP rev. perp. = YP perp. * PV $1

$$= \frac{1}{i} * \frac{1}{(1 + i)^n}$$

$$= \frac{1}{iA}$$

Nominal and Effective Rates of Interest (i and r)

Effective Rate of interest 'The annual rate of interest equivalent to the terms applying to a specific financial arrangement, where interest is payable at intervals other than yearly, and there may be other special provisions, eg rolled up interest. In considering the use of money it is essential to have a common denominator for the purposes of direct comparison and this is provided by the effective rate of interest.’ †

$$\text{r (effective yield)} \quad = \quad \left\{1 + \frac{i}{p}\right\}^{p} - 1$$

Where, r is the effective yield expressed as a decimal, i is the nominal yield expressed as a decimal, and p is the number of times that interest is received and compounded during the year.

Conversely to convert an effective yield to a nominal yield:

$$\text{i (nominal yield)} \quad = \quad p\left((1 + r)^{1/p} - 1\right)$$

Where, i is the nominal yield expressed as a decimal,
r is the effective yield expressed as a decimal, and
p is the number of times that interest is received and compounded during the year

Incomes in advance (annuities due) and non-annual incomes

$$\text{PV \$1 (in advance)} \quad = \quad \frac{1}{(1 + i)^{n-1}}$$

$$\text{A \$1 p.a. (in advance)} \quad = \quad \left\{\frac{(1 + i)^{n+1} - 1}{i}\right\} - 1$$

$$\text{YP (in advance)} \quad = \quad \frac{1 - \dfrac{1}{(1 + i)^{n-1}}}{i} + 1$$

$$\text{YP (›ly in advance)} \quad = \quad \frac{1 - \dfrac{1}{(1 + r)^{n}}}{4 \left\{\dfrac{1}{1 - \sqrt[4]{(1 + r)}}\right\}}$$

Where r is the effective yield expressed as a decimal.

References

Baum A. and Mackmin D. 1989, *The income approach to property valuation*, Chapter 1, Routledge, London.

Baum, A. and Mackmin, D. 1989, *The Income Approach to Property Valuation*, Chapter 1.

Bell, R.A. 1988, *Investment Property Income Analysis and Appraisal*, Chapters 2–9.

Britton, W. et al. 1989, *Modern methods of valuation*, Chapter 8, Estates Gazette, London.

Fibbens, M.J.W. 1987, *Investment Analysis*, Chapter 2 and 4.

Jones Lang Wootton et al. 1989, *The glossary of property terms*, Estate Gazette, London.

Millington, A.F. 1988, *An introduction to property valuation*, Chapter 14, The Estates Gazette, London.

Richmond D. 1989, *Introduction to valuation*, Chapter 5, Macmillan, London.

Robinson, J. 1989, *Property Valuation and Investment Analysis*. Chapter 1 and 2.

Rost and Collins 1984, *Land Valuation and Compensation in Australia*. Chapter 10-12.