FINM3405 Derivatives and risk management

Tutorial Sheet 5: Options - Black-Scholes model and Greeks

August 22, 2024

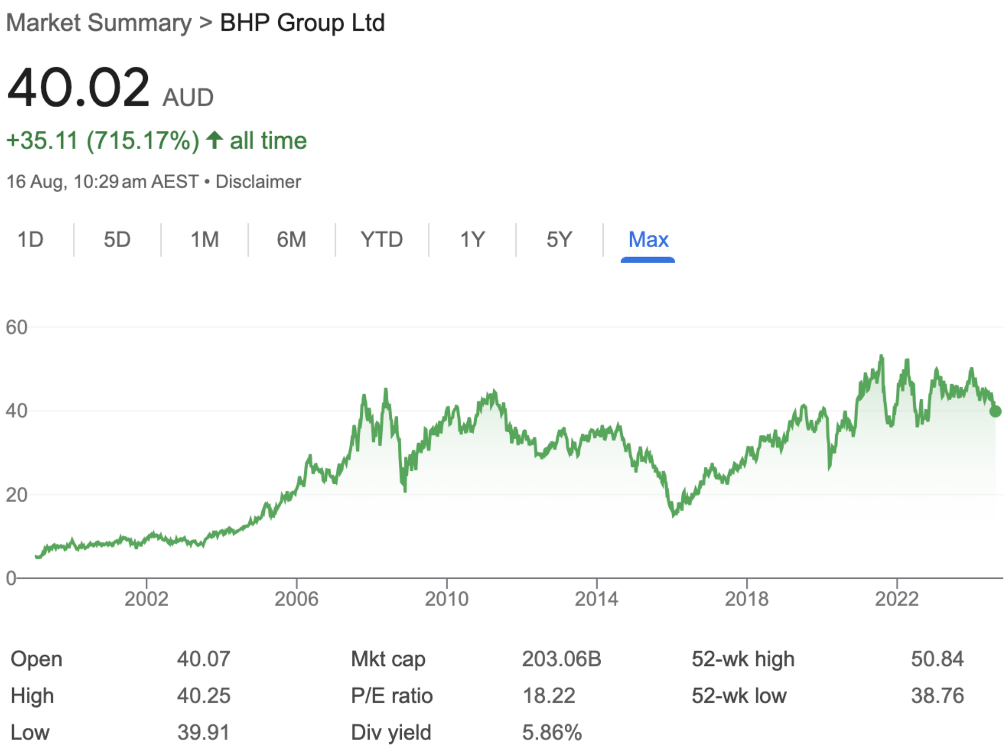
Question 1. 1. Plot on separate graphs the payoff diagrams for call and put options with a strike price of $K = 50$.

2. Let $r = 0.05$, $T = 1/2$, $\sigma = 0.25$ and $q = 0$. On the graphs above, plot a curve representing the value (or price or premium) of these options.

3. Visually, when do options have the most time value? When is it negative?

4. Now do the same for if $q = 0.05$.

Question 2. BHP is trading around $S = \$40.00$ and has a historical dividend yield of $q = 5.86\%$. yahoo!finance quotes a forward dividend yield of $q = 5.99\%$.



BHP Group Limited (BHP.AX) ☆ Follow · Compare

40.01 +0.80 (+2.04%)

As of 10:32 AM GMT+10. Market Open.

Previous Close	39.21	Day's Range	39.91 - 40.25	Market Cap (intraday)	203.064B	Earnings Date	Aug 26, 2024
Open	40.07	52 Week Range	38.76 - 50.84	Beta (5Y Monthly)	0.87	Forward Dividend & Yield	2.35 (5.99%)
Bid	40.03 x --	Volume	4,157,447	PE Ratio (TTM)	18.20	Ex-Dividend Date	Mar 7, 2024
Ask	40.04 x --	Avg. Volume	8,103,616	EPS (TTM)	2.20	1y Target Est	46.87

24 hour delayed BBSW rates

TENOR	BID	ASK	MID	METHOD	YIELD RANGE (BPS)
1 MONTH	4.3531	4.2531	4.3031	WLSR	0.9000
2 MONTH	4.3737	4.2737	4.3237	WLSR	0.9000
3 MONTH	4.4063	4.3063	4.3563	WLSR	2.0000
4 MONTH	4.4375	4.3375	4.3875	NBBO	--
5 MONTH	4.4725	4.3725	4.4225	NBBO	--
6 MONTH	4.5178	4.4178	4.4678	WLSR	1.5000

As of 14/08/2024 11am

1. Download BHP's historical daily share prices from yahoo!finance and calculate BHP's historical volatility (standard deviation) σ.
2. Calculate the Black-Scholes model prices of 6 and 3 month at-the-money European call and put options on BHP using the forward dividend yields, and the above BBSW rates.

   Remark: Consider these dividend yields and the BBSW rates to be simple interest rates and convert them to continuously compounded rates.
3. Calculate all of the Greeks for these 6 and 3 month at-the-money options.
4. Recalculate the prices and Greeks for these 6 and 3 month at-the-money options if BHP's share price decreases and increases by 15% and 30%.
5. Calculate the time values of these 6 and 3 month options when at-the-money and after BHP decreases and increases by 15% and 30%

Question 3. Following on from Question 2, price the BHP 19-Sep-2024 European options displayed below and compare your prices to the quoted bid-ask spreads. For these options we know the time to expiry, strike prices and current price of BHP. We also have a good estimate of BHP's dividend yield, and the BBSW rate is a decent proxy for the risk-free rate. Hence, the only input parameter in the Black-Scholes model under question is the volatility σ (we're currently using historical data to estimate it). Have a few guesses for different values of σ to see if you can get your Black-Scholes option prices within the ranges of the quoted bid-ask spreads.

BHP GROUP LIMITED BHP

LAST PRICE / TODAY'S CHANGE: $40.005 +$0.795 (2.027%)
VOLUME: 4,482,739
BID / OFFER RANGE: $40.000 - $40.010
MARKET CAPITALISATION: $198.85B

Industry Group: Materials
Listed on 13 August 1885

Prices delayed by at least 20 minutes | Currently trading

BHP Overview | Options | Warrants

REFRESH DATA

Filters

OPEN INTEREST ONLY THEORETICAL PRICING ONLY

ALL EXPIRIES | ALL EXERCISE PRICES | CALLS & PUTS | ALL STYLES

19 September 2024 | a month to expiry

Key: (E) = European (A) = American * = Theoretical Price

CALLS								PUTS						
CODE	STYLE	BID ($)	OFFER ($)	OPEN INTEREST	VOLUME	LAST TRADE ($)	STRIKE ($)	CODE	STYLE	BID ($)	OFFER ($)	OPEN INTEREST	VOLUME	LAST TRADE ($)
BHPC79	(E)	1.59	1.98	20	--	1.31 *	38.01	BHPC89	(E)	0.61	0.81	407	--	1.02 *
BHPBY9	(A)	1.79	2.18	30	--	1.43 *	38.50	BHPC69	(A)	0.74	1.00	1,987	--	1.25 *
BHP6V7	(E)	1.26	1.66	150	--	1.05 *	38.51	BHP6X7	(E)	0.74	1.03	12	--	1.24 *
BHPVD9	(A)	1.42	1.82	50	--	1.13 *	39.00	BHPVE9	(A)	0.95	1.21	1,839	--	1.52 *
BHP4V7	(E)	1.03	1.32	89	--	0.83 *	39.01	BHP4X7	(E)	0.95	1.24	301	--	1.51 *
BHPWY8	(A)	1.15	1.44	298	--	0.87 *	39.50	BHPWZ8	(A)	1.20	1.46	1,075	--	1.83 *
BHP4Z7	(E)	0.78	1.06	2,934	--	0.64 *	39.51	BHP5F7	(E)	1.20	1.48	814	--	1.81 *
BHPVF9	(A)	0.87	1.15	125	39	1.01	40.00	BHPVY9	(A)	1.44	1.80	1,390	--	2.18 *
							LAST: 40.00							
BHPG68	(E)	0.62	0.82	9,656	--	0.48 *	40.01	BHPGM8	(E)	1.44	1.82	1,886	--	2.16 *
BHPSN9	(A)	0.63	0.91	687	--	0.48 *	40.50	BHPSO9	(A)	1.77	2.12	1,116	--	2.57 *
BHPIR8	(E)	0.44	0.64	250	130	0.53	40.51	BHPIS8	(E)	1.77	2.14	960	--	2.55 *
BHPVZ9	(A)	0.48	0.67	461	--	0.34 *	41.00	BHPW19	(A)	2.08	2.52	2,351	--	3.00 *
BHPRO8	(E)	0.32	0.47	102	100	0.40	41.01	BHPRP8	(E)	2.05	2.55	2,388	--	2.96 *
BHPSP9	(A)	0.35	0.49	744	86	0.40	41.50	BHPSQ9	(A)	2.45	2.91	380	--	3.45 *
BHPQE9	(E)	0.22	0.37	5,540	200	0.32	41.51	BHPQL9	(E)	2.45	2.95	782	--	3.41 *
BHPW29	(A)	0.23	0.37	470	1	0.30	42.00	BHPW39	(A)	2.84	3.42	1,536	--	3.91 *

Question 4. Still following on from Question 2, starting with the 3 month at-the-money options, use Δ and Γ to calculate the predicted change in call and put option prices from a 1% increase and decrease in BHP's share price. Then compare your predictions to the new true Black-Scholes prices.

Question 5. Using the below Euribor and Term SOFR rates, and the quoted EURO:USD spot rate, calculate the prices and greeks of at-the-money 1 month, 3 month, 6 month and 12 month European call and put FX options for both cases of (i) EURO being the "underlying asset" and (ii) USD being the "underlying asset". Note that EURIBOR and Term SOFR are simple interest rates. Also, use the daily EURO:USD and USD:EURO exchange rates from yahoo!finance to calculate the volatility parameter, possibly in Excel :-(.

HOME > EUR / USD • CURRENCY

Euro to United States Dollar

1.0980 ↓5.99% -0.0700 MAX

16 Aug, 01:44:00 UTC · Disclaimer

1 D 5 D 1 M 6 M YTD 1 Y 5 Y MAX

DATE	CME TERM SOFR (%)			
	1 MONTH	3 MONTH	6 MONTH	12 MONTH
15 Aug 2024	5.34157	5.10155	4.78062	4.28661

8/14/2024

Euribor 1 week	3.609 %
Euribor 1 month	3.606 %
Euribor 3 months	3.542 %
Euribor 6 months	3.398 %
Euribor 12 months	3.148 %

Question 6. 1. Suppose you take 3 month at-the-money European call and put options over the same underlying asset whose current price is $S = 50$. Also let $r = 0.05$, $\sigma = 0.30$ and $q = 0$. Plot the payoff diagram of your combined position. Then on the same graph plot the value of your combined position as well as your profit diagram considering the total premium you paid. If you held this position in the market, what are you hoping the underlying asset does here? Then on the same graph plot the value of your combined position in 2 months time. If the price of the underlying asset remained unchanged in 2 months time and you closed out your position, what would be your profit or loss?

2. Now do the same as the above but for writing the options.

Question 7. A **binary option** pays out a fixed amount if an event happens, or nothing if the event does not happen. For example, a European **cash-or-nothing** option pays out \$1 if the option finishes in-the-money:

$$\text{cash-or-nothing call payoff} = \begin{cases} 0 & \text{if } S_T \leq K \\ 1 & \text{if } S_T > K, \end{cases}$$

$$\text{cash-or-nothing put payoff} = \begin{cases} 0 & \text{if } S_T \geq K \\ 1 & \text{if } S_T < K. \end{cases}$$

They turn out to have very simple pricing equations:

$$C = e^{-rT}\mathcal{N}(d_2) \qquad \text{and} \qquad P = e^{-rT}\mathcal{N}(-d_2),$$

where d_2 is as defined in the lecture notes:

$$d_2 = \frac{\log \frac{S}{K} + (r - q - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}.$$

Calculate the prices of 3 month at-the-money cash-or-nothing options on BHP.

Question 8. Another example of a binary option is a European **asset-or-nothing**, which pays out the current value S of the underlying asset if the option finishes in-the-money:

$$\text{asset-or-nothing call payoff} = \begin{cases} 0 & \text{if } S_T \leq K \\ S & \text{if } S_T > K, \end{cases}$$

$$\text{asset-or-nothing put payoff} = \begin{cases} 0 & \text{if } S_T \geq K \\ S & \text{if } S_T < K. \end{cases}$$

They also turn out to have very simple pricing equations:

$$C = Se^{-qT}\mathcal{N}(d_1) \qquad \text{and} \qquad P = Se^{-qT}\mathcal{N}(-d_1),$$

where d_1 is as above:

$$d_1 = \frac{\log \frac{S}{K} + (r - q + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}.$$

Calculate the prices of 3 month at-the-money asset-or-nothing options on BHP.

Question 9. A final example of a binary option is a European **supershare**, which has a payoff of \$1 if the underlying asset's price S_T at expiry falls within a certain range of values K and X, with $K < X$:

$$\text{supershare payoff} = \begin{cases} 1 & \text{if } K < S_T < X \\ 0 & \text{otherwise.} \end{cases}$$

Its pricing equation is also surprisingly simple and given by

$$V = e^{-rT}\left[\mathcal{N}(d_2^K) - \mathcal{N}(d_2^X)\right],$$

where

$$d_2^K = \frac{\log\frac{S}{K} + (r - q - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \qquad \text{and} \qquad d_2^X = \frac{\log\frac{S}{X} + (r - q - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}.$$

Calculate the price of 3 month supershare on BHP with $K = \$38$ and $X = \$42$.