FINM3405 Derivatives and risk management

Some Final Exam practice questions
Suggested solutions

October 25, 2024

Remark: These solutions are for you to check your answers against, and are not necessarily how I'd recommend setting them out in an exam. Also, please alert me to any typos or outright mistakes that I have made in them.

Question 1. Let the current stock price be $S = \$100$, the risk-free rate be $r = 5.5\%$, the stock's volatility be $\sigma = 27.5\%$, the time to expiry be $T = \frac{1}{4}$, and the dividend yield be $y = 7\%$. Also let $N = 3$, so monthly time steps.

1. Starting at $S = \$100$, create the 3-step binomial model asset price tree using your choice of either the CRR or JR schemes. Using the JR scheme:

			125.23
		116.18	
	107.79		106.84
100.00		99.12	
	91.96		91.16
		84.57	
			77.77

2. Calculate the binomial model prices of ATM European and American call and put options. For the European options I get $C^{\text{Eu}} = 5.642$ and $P^{\text{Eu}} = 6.011$. The stock price tree (from the previous part) is:

	0	1	2	3
0	100	91.9627	84.5715	77.7742
1	nan	107.787	99.1237	91.1568
2	nan	nan	116.18	106.842
3	nan	nan	nan	125.226

The European call option price tree is:

	0	1	2	3
0	5.64152	1.69508	0	0
1	nan	9.63947	3.40558	0
2	nan	nan	15.9614	6.84217
3	nan	nan	nan	25.2265

The European put option price tree is:

	0	1	2	3
0	6.01071	9.88652	15.4632	22.2258
1	nan	2.19044	4.40118	8.84317
2	nan	nan	0	0
3	nan	nan	nan	0

The call intrinsic value tree is:

	0	1	2	3
0	0	0	0	0
1	nan	7.78676	0	0
2	nan	nan	16.1799	6.84217
3	nan	nan	nan	25.2265

The put intrinsic value tree is:

	0	1	2	3
0	0	8.03726	15.4285	22.2258
1	nan	0	0.876346	8.84317
2	nan	nan	0	0
3	nan	nan	nan	0

The American call option tree is:

	0	1	2	3
0	5.69564	1.69508	0	0
1	nan	9.7482	3.40558	0
2	nan	nan	16.1799	6.84217
3	nan	nan	nan	25.2265

The American put option tree is:

	0	1	2	3
0	6.01071	9.88652	15.4632	22.2258
1	nan	2.19044	4.40118	8.84317
2	nan	nan	0	0
3	nan	nan	nan	0

So the American call price is $C^{\text{Am}} = 5.69564$, which compares to the European price of $C^{\text{Eu}} = 5.64152$. In this example, the American and European put prices ended up equal. (But if we decreased the dividend yield, or increased the number of time steps, the put prices become different.)

3. Also calculate the time $t = 0$ binomial model deltas of ATM European and American calls and puts. For the European deltas I get $\Delta_{C^{\text{Eu}}} = 0.502$ and $\Delta_{P^{\text{Eu}}} = -0.4864$. For the American deltas I get $\Delta_{C^{\text{Am}}} = 0.50892$ and $\Delta_{P^{\text{Am}}} = -0.4864$.

4. Calculate the price of an ATM chooser option whose choice date is the end of month one. We can use the information we have above. At the end of the first month we have $C_u = 9.63947$ and $C_d = 1.69508$, as well as $P_u = 2.19044$ and $P_d = 9.88652$. Hence, in the up state we choose the call price of $C_u = 9.63947$, and in the down state we choose the put price of $P_d = 9.88652$. The chooser option value is then

$$V = e^{-rdt}\big[qC_u + (1-q)P_d\big] = 9.71835.$$

Question 2. Using the same parameters as above, consider the following five simulated paths of geometric Brownian motion again with monthly time steps:

	0	1	2	3
0	100	102.822	99.7769	94.5728
1	100	109.582	108.033	130.956
2	100	99.477	115.124	103.019
3	100	88.5383	87.0328	85.0011
4	100	90.586	86.2677	88.965

Calculate the Monte Carlo prices of:

1. European call and put options. I get:

Call payoffs	Put payoffs	Discounted call payoffs	Discounted put payoffs
0.0000	5.4272	0.0000	5.3531
30.9560	0.0000	30.5333	0.0000
3.0190	0.0000	2.9778	0.0000
0.0000	14.9989	0.0000	14.7941
0.0000	11.0350	0.0000	10.8843
	Premiums	**6.7022**	**6.2063**

2. ATM fixed-strike lookback call and put options.
3. Floating-strike lookback call and put options. I get:

Max price	Min price	Fixed strike call payoff	Fixed strike put payoff	Floating strike call payoff	Floating strike put payoff	Discounted Fixed strike call payoff	Discounted Fixed strike put payoff	Discounted Floating strike call payoff	Discounted Floating strike put payoff
102.8220	94.5728	2.8220	5.4272	0.0000	8.2492	2.7835	5.3531	0.0000	8.1365
130.9560	100.0000	30.9560	0.0000	30.9560	0.0000	30.5333	0.0000	30.5333	0.0000
115.1240	99.4770	15.1240	0.5230	3.5420	12.1050	14.9175	0.5159	3.4936	11.9397
100.0000	85.0011	0.0000	14.9989	0.0000	14.9989	0.0000	14.7941	0.0000	14.7941
100.0000	86.2677	0.0000	13.7323	2.6973	11.0350	0.0000	13.5448	2.6605	10.8843
					Premiums	**9.6468**	**6.8416**	**7.3375**	**9.1509**

4. ATM fixed-strike arithmetic average Asian call and put options.
5. Arithmetic average-strike Asian call and put options. I get:

Average price	Fixed strike Asian call payoff	Fixed strike Asian put payoff	Average strike Asian call payoff	Average strike Asian put payoff	Discounted Fixed strike Asian call payoff	Discounted Fixed strike Asian put payoff	Discounted Average strike Asian call payoff	Discounted Average strike Asian put payoff
99.2929	0	0.7071	0	4.7201	0	0.6974	0	4.6557
112.1428	12.1428	0	18.8133	0	11.9769	0	18.5563	0
104.4050	4.4050	0	0	1.3860	4.3448	0	0	1.3671
90.1431	0	9.8570	0	5.1420	0	9.7223	0	5.0717
91.4547	0	8.5453	0	2.4897	0	8.4286	0	2.4557
				Premiums	**3.2644**	**3.7697**	**3.7113**	**2.7100**

Question 3. Consider these default and survival probabilities, and yield curve:

Quarter	Survival probability	Default probability	Risk-free rate
1	99.2528%	0.7472%	3.41%
2	98.5112%	0.7416%	3.30%
3	97.7751%	0.7361%	3.18%
4	97.0446%	0.7306%	3.03%
5	96.3194%	0.7251%	2.85%
6	95.5997%	0.7197%	2.64%

1. Calculate the breakeven CDS spread k on a 18-month CDS with quarterly coupon periods and recovery rate of $R = 40\%$. I calculate that:

Quarter	Survival probability	Default probability	Risk-free rate	Recovery rate	Payout	E[payout]	PV(E[payout])	F x d x survival prob	PV(F x d x survival prob)
1	99.2528%	0.7472%	3.41%	0.4	0.6	0.004483	0.004445	0.248132	0.246026
2	98.5112%	0.7416%	3.07%	0.4	0.6	0.004450	0.004382	0.246278	0.242527
3	97.7751%	0.7361%	2.90%	0.4	0.6	0.004416	0.004321	0.244438	0.239177
4	97.0446%	0.7306%	2.70%	0.4	0.6	0.004383	0.004267	0.242611	0.236145
5	96.3194%	0.7251%	2.47%	0.4	0.6	0.004351	0.004219	0.240799	0.233489
6	95.5997%	0.7197%	2.19%	0.4	0.6	0.004318	0.004179	0.238999	0.231283
						PV(E[payouts])	0.025812	Denominator	1.428648
								CDS spread	1.806767%

2. Calculate the upfront payment if the CDS spread was fixed at $k = 1\%$ over a notional principal of $F = \$10\text{m}$ I calculate that:

Quarter	Survival probability	Default probability	Risk-free rate	Recovery rate	Payout	E[payout]	PV(E[payout])	Premium at k = 1%	E[premium]	PV(E[premium])
1	99.2528%	0.7472%	3.41%	0.4	0.6	0.004483	0.004445	0.0025	0.0025	0.002460
2	98.5112%	0.7416%	3.07%	0.4	0.6	0.004450	0.004382	0.0025	0.0025	0.002425
3	97.7751%	0.7361%	2.90%	0.4	0.6	0.004416	0.004321	0.0025	0.0024	0.002392
4	97.0446%	0.7306%	2.70%	0.4	0.6	0.004383	0.004267	0.0025	0.0024	0.002361
5	96.3194%	0.7251%	2.47%	0.4	0.6	0.004351	0.004219	0.0025	0.0024	0.002335
6	95.5997%	0.7197%	2.19%	0.4	0.6	0.004318	0.004179	0.0025	0.0024	0.002313
						PV(E[payouts])	0.025812		PV(E[premiums])	0.014286
									Upfront cashflow	0.011526

3. Explain in words how you'd use CDS indices to speculate on market-wide credit risk perceptions. Enter into a CDS as the protection buyer (seller) if you think credit risk perceptions and hence default probabilities and CDS spreads will increase (decrease).

4. Suppose you bought protection over a notional principal of $F = \$10\text{m}$ at the above data and with a fixed CDS spread of $k = 1\%$, and during the first quarter there was a macroeconomic shock resulting in the following new survival and default probabilities, and yield curve:

Quarter	Survival probability	Default probability	Risk-free rate
2	98.5112%	1.4888%	3.07%
3	97.0446%	1.4666%	2.90%
4	95.5997%	1.4448%	2.70%
5	94.1765%	1.4233%	2.47%
6	92.7743%	1.4021%	2.19%

What is your profit if you were to close out your CDS position at the end of the first quarter? This is just repeating part 2 with one less time period. I'm sure you can do it.

5. Explain in words how a bank might use CDS to hedge against default by a large lending exposure on their balance sheet. If a bank has a relatively large lending exposure to a particular reference asset of a reference entity, it could enter into a CDS over that reference asset as the protection buyer.

6. Provide an intuitive no-arbitrage argument for why the breakeven CDS spread should approximately equal the risk premium of the yield on the

reference entity's debt over the risk-free rate. A long position in a reference asset and entering into a CDS over that reference asset as the protection buyer effectively provides a risk-free net yield of the reference asset's yield minus the CDS spread. Hence, being risk free, this net yield should equal the risk-free rate.

Question 4. Consider the following Euribor yield curve:

Euribor 1 week	3.392 %
Euribor 1 month	3.272 %
Euribor 3 months	3.265 %
Euribor 6 months	3.058 %
Euribor 12 months	2.798 %

Let the 9-month Euribor rate be an average of the 6-month and 12-month rates.

1. What is the fixed rate k in a 1-year fixed-for-floating interest rate swap with quarterly interest periods and whose floating rate is Euribor? I get:

quarter	Euribor	discount factors
1	3.26500%	0.9919036
2	3.05800%	0.9849403
3	2.92800%	0.9785119
4	2.79800%	0.9727816
	sum disc facs	3.9281373
	k	**2.77164%**

2. Suppose you entered into the swap as the fixed rate receiver over a notional principal of $F = \$10\text{m}$, and the Euribor yield curve fell by 50 basis points during the first quarter. What would be your profit or loss if you closed out your position at the end of the first quarter? What would be your profit or loss if instead the Euribor yield curve went up by 50 basis points?

quarter	Euribor - 50bps	quarterly fwd rates	fixed coupon	floating coupon	pay fixed party NCF	discount factor	PV NCF
2	2.55800%	2.55800%	$ 69,290.93	$ 63,950.00	-$5,340.93	0.993645636	-$5,306.99
3	2.42800%	2.28340%	$ 69,290.93	$ 57,084.94	-$12,205.99	0.988005612	-$12,059.58
4	2.29800%	2.01356%	$ 69,290.93	$ 50,338.89	-$18,952.04	0.983057012	-$18,630.94
						pay fixed party value	**-$35,997.51**
						reveive fixed party value	**$35,997.51**

quarter	Euribor + 50bps	quarterly fwd rates	fixed coupon	floating coupon	pay fixed party NCF	discount factor	PV NCF
2	3.55800%	3.55800%	$ 69,290.93	$ 88,950.00	$ 19,659.07	0.991183423	$19,485.75
3	3.42800%	3.26892%	$ 69,290.93	$ 81,723.07	$ 12,432.14	0.983148829	$12,222.65
4	3.29800%	2.98681%	$ 69,290.93	$ 74,670.15	$ 5,379.22	0.975862052	$ 5,249.38
						pay fixed party value	**$36,957.77**
						reveive fixed party value	**-$36,957.77**

3. Explain in words how you would use interest rate swaps to speculate on shifts in the yield curve. If you expect the yield curve to shift up (down), enter into a fixed-for-floating interest rate swap as the pay fixed, receive floating (receive fixed, pay floating) party.

4. Explain in words how you would use interest rate swaps to hedge a floating interest rate borrowing exposure against an increase in interest rates. Here, you're exposed to increasing interest rates, so you want to lock in your cost of debt to a fixed interest rate. So you'd enter into an interest rate swap as the pay fixed, receive floating party. If interest rates went up, your floating payments on your existing debt in the market will increase, but they'll be offset by the swap's cashflows on its interest or coupon dates.