FINM3405 Derivatives and Risk Management

Week 6: Delta hedging, implied volatility, trading strategies

Dr Godfrey Smith

THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

August 25, 2024

Contents

Introduction

Last week we presented the basics of the Black-Scholes model for European options on non-income paying assets, income paying assets, and currencies. We also presented the Greeks, which quantify the relation between option premiums and the Black-Scholes model input parameters. This week we look at delta hedging, which is a common strategy employed by traders and market makers to manage risk and is the starting point for other "Greek neutral" strategies. We then cover the notion of implied volatility and finish with using the concepts of the Greeks, delta hedging and implied vols to present some basic trading strategies.

- Readings: Chapters 12, 15.11, 19.4 and 20 of Hull.

Introduction

There's also a number of additional useful resources that, while not compulsory, I recommend considering for this week's material:

- ASX: The Options Strategies booklet.
- CME Group: The 25 Proven Strategies booklet (also translated into Mandarin Chinese), and the Options Strategies online course.
- Alternative textbook: Chapter 27 Delta Hedging of Cuthbertson, Nitzsche and O'Sullivan, Derivatives: Theory and Practice.

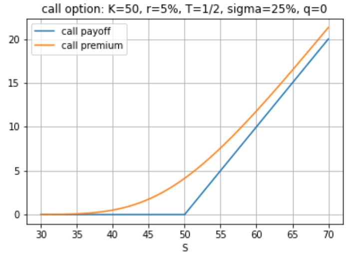
Introduction

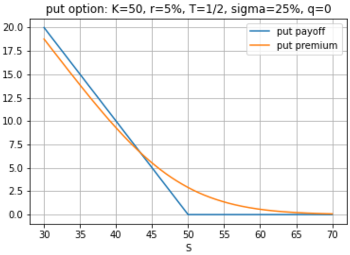
Disclaimer: These and all other lecture notes and materials in FINM3405 are for educational purposes only and do not in any way constitute investment advice, whether personal or general.

Static delta hedging

- We start off with the basic idea of static delta hedging.

Recall the basic long call and put option payoff and premium plots:





The orange lines are the premiums over the range of asset prices.

Static delta hedging

An option's delta is an approximation of the change in the premium due to a change $\mathrm{d}S$ in the price S of the underlying asset:

$$\mathrm{d}C \approx \Delta_C \mathrm{d}S \qquad \text{and} \qquad \mathrm{d}P \approx \Delta_P \mathrm{d}S.$$

We use an option's delta to hedge a position against changes $\mathrm{d}S$ in S:

> **Delta hedging** involves taking a position in the asset to hedge an option position against small movements in the asset's price.

So, given an option position, we calculate how many units Q in the asset we need in order to hedge against small changes $\mathrm{d}S$ in the asset price S.

- Negative Q means a short position in the asset.

Static delta hedging

The value of a portfolio of $h = 1$ call option and Q units in the asset is

$$V = QS + C.$$

The change $\mathrm{d}V$ in V due to a change $\mathrm{d}S$ in S is given by

$$\begin{aligned} \mathrm{d}V &= Q\mathrm{d}S + \mathrm{d}C \\ &\approx Q\mathrm{d}S + \Delta_C \mathrm{d}S = (Q + \Delta_C)\mathrm{d}S. \end{aligned}$$

Remark

The value of a portfolio of $h = 1$ put option and Q units in the asset is $V = QS + P$ and its change is $\mathrm{d}V \approx (Q + \Delta_P)\mathrm{d}S$.

Static delta hedging

Delta hedging means choosing Q so that $\mathrm{d}V = 0$.

- From the relations

$$\mathrm{d}V = (Q + \Delta_C)\mathrm{d}S \qquad \text{and} \qquad \mathrm{d}V = (Q + \Delta_P)\mathrm{d}S,$$

  to set $\mathrm{d}V = 0$ we easily see that Q should be chosen to equal

$$Q \approx -\Delta_C \qquad \text{or} \qquad Q \approx -\Delta_P.$$

Remark

Setting Q to the above so $\mathrm{d}V = 0$ means we're **delta neutral**.

Consider an example in which we are long $h = 1$ call:

Static delta hedging

Example

Let $K = S = 50$, $r = 5\%$, $T = \frac{1}{2}$, $\sigma = 25\%$ and $q = 0$. We set

$$Q = -\Delta_C = -\mathcal{N}(d_1) = -0.59088 \qquad \text{(short)}.$$

To delta hedge a long position of $h = 1$ call, we short 0.59088 units of the asset, receiving 29.544. Our initial call value is $C = 4.13$. Let $\mathrm{d}S = -2$, so $S_{\text{new}} = 48$. The call value falls to $C_{\text{new}} = 3.03855$, a loss of $\mathrm{d}C = -1.09146$. We close out the short stock position at a cost of 28.36, giving a profit of 1.182.

We can generalise delta hedging to a portfolio of $h > 1$ options:

Static delta hedging

The value of a portfolio of h calls and Q units in the underlying asset is

$$V = QS + hC$$

and its change in value is

$$\begin{aligned} \mathrm{d}V &= Q\mathrm{d}S + h\mathrm{d}C \\ &\approx Q\mathrm{d}S + h\Delta_C\mathrm{d}S = (Q + h\Delta_C)\mathrm{d}S. \end{aligned}$$

To be delta neutral, so $\mathrm{d}V = 0$, we set $Q = -h\Delta_C$.

Remark

The value of a portfolio of h puts and Q assets is $V = QS + hP$.

Its change is $\mathrm{d}V \approx (Q + h\Delta_P)\mathrm{d}S$ so we set $Q = -h\Delta_P$.

Static delta hedging

Example (Continued)

This time we're short 10 put options, so $h = -10$. Then we set

$$Q = -h\Delta_P = 10 \times -0.40912 = -4.0912.$$

To delta hedge we short 4.0912 units of the asset, receiving 204.56. Again let $\mathrm{d}S = -2$. The value of 10 puts increased from 28.955 to 38.04, a loss of 9.085. We close out the short stock position at a cost of 196.28, yielding a profit of 8.18.

Static delta hedging

Remark

We're assuming assets are "infinitely divisible". Options contracts are typically over m assets (the multiplier). Then the portfolio values are $V = QS + hCm$ or $V = QS + hPm$ with changes

$$\mathrm{d}V = (Q + h\Delta_C m)\mathrm{d}S \qquad \text{or} \qquad \mathrm{d}V = (Q + h\Delta_P m)\mathrm{d}S.$$

So for delta hedging we round $Q = -h\Delta_C m$ or $Q = -h\Delta_P m$ to whole numbers, which doesn't "badly damage" the hedge.

We can use gamma Γ to improve delta hedging:

Delta-gamma hedging

Recall from last week that we can make our approximations of the changes $\mathrm{d}C$ and $\mathrm{d}P$ in option premiums due to a change $\mathrm{d}S$ in the underlying asset price S more accurate by including Γ as follows:

$$\mathrm{d}C \approx \Delta_C \mathrm{d}S + \frac{1}{2}\Gamma \mathrm{d}S^2 \qquad \text{and} \qquad \mathrm{d}P \approx \Delta_P \mathrm{d}S + \frac{1}{2}\Gamma \mathrm{d}S^2.$$

We can use delta-gamma hedging to improve the delta hedging of an option position by taking a position in the asset <u>and a different option</u>.

Delta-gamma hedging

> **Delta-gamma hedging** involves taking a position in the asset and in another, different option to hedge an existing option position against small movements in the price of the underlying asset.

So, given a position of h options, for delta-gamma hedging we calculate how many units Q in the asset and k in another option we need in order to hedge against small changes $\mathrm{d}S$ in the price S of the underlying asset.

Delta-gamma hedging

The value of a portfolio of Q units in the asset, h units of one option, and k units of another, different option is given by

$$V = QS + hV_1 + kV_1,$$

where V_1 and V_2 are the respective option premiums.

Also let Δ_1 and Γ_1 be the delta and gamma of our existing option, and Δ_2 and Γ_2 be the delta and gamma of the new option.

Delta-gamma hedging

The change in portfolio value approximated by delta-gamma hedging is

$$\begin{aligned} \mathrm{d}V &= Q\mathrm{d}S + h\mathrm{d}V_1 + k\mathrm{d}V_1 \\ &\approx Q\mathrm{d}S + h\Big(\Delta_1\mathrm{d}S + \frac{1}{2}\Gamma_1\mathrm{d}S^2\Big) + k\Big(\Delta_2\mathrm{d}S + \frac{1}{2}\Gamma_2\mathrm{d}S^2\Big) \\ &= (Q + h\Delta_1 + k\Delta_2)\mathrm{d}S + \frac{1}{2}(h\Gamma_1 + k\Gamma_2)\mathrm{d}S^2. \end{aligned}$$

So when setting $\mathrm{d}V = 0$, for delta-gamma hedging we need to solve

$$Q + h\Delta_1 + k\Delta_2 = 0 \qquad \text{and} \qquad h\Gamma_1 + k\Gamma_2 = 0.$$

Delta-gamma hedging

Remark

Solving these equations for Q and k so that $\mathrm{d}V = 0$ means that we're both **delta** and **gamma neutral**.

We can show that the solution is

$$Q = -h\frac{\Delta_1\Gamma_2 - \Delta_2\Gamma_1}{\Gamma_2} \qquad \text{and} \qquad k = \frac{Q\Gamma_1}{\Delta_1\Gamma_2 - \Delta_2\Gamma_1}.$$

So to delta-gamma hedge an existing option position, we use the first equation to calculate Q and then the second equation to calculate k.

- Consider the previous example of long $h = 1$ call:

Delta-gamma hedging

Example

Let $S = 50$, $r = 5\%$, $T = \frac{1}{2}$, $\sigma = 25\%$ and $q = 0$. We hold $h = 1$ call with $K = 50$. We want to know how many units Q in the asset and k in another call with $K_2 = 55$ we need in order to delta-gamma hedge our exposure to changes $\mathrm{d}S = -2$ in S.

- Delta hedging was presented above. The loss on the call is $\mathrm{d}C = -1.09146$. The gain on a short stock position is 1.182.

For delta-gamma hedging we first calculate (noting $h = 1$) that

$$Q = -\frac{\Delta_1 \Gamma_2 - \Delta_2 \Gamma_1}{\Gamma_2} = -0.20415.$$

Delta-gamma hedging

Example (Continued)

We then calculate that

$$k = \frac{Q\Gamma_1}{\Delta_1\Gamma_2 - \Delta_2\Gamma_1} = -1.021673.$$

We short 0.20415 units of the asset for $0.20415 \times 50 = 10.2074$. We also short 1.021673 calls with a strike of $K_2 = 55$ to receive $1.021673 \times 2.113 = 2.1587$. We close out the short asset position at a cost of $0.20415 \times 48 = 9.7991$, yielding a profit of 0.4083. We also close out the short option position at a cost of $1.021673 \times 1.44 = 1.47$, giving a profit of 0.68. The combined hedging profit is 1.0951, which compares to $\mathrm{d}C = -1.09146$.

Dynamic delta hedging

So far we've been doing "1 period" delta hedging.

- But when the asset price S changes, so does the option's delta.
- Hence, to stay delta hedged over time, one needs to regularly rebalance the holding Q in the underlying asset.
- We also rebalance our bank account as we buy and/or sell the asset.
  - And pay and/or receive interest as appropriate.

Dynamic delta hedging involves maintaining a "rolling" delta hedge (staying delta neutral) over time via regular rebalancing.

An example best illustrates the idea, in which we use daily rebalancing.

Dynamic delta hedging

Example

We continue with the previous example of long $h = 1$ call with $K = 50$, $r = 5\%$, $T = \frac{1}{2}$, $\sigma = 25\%$ and initially $S = 50$.

- We simulate 6 months of daily asset prices via geometric Brownian motion, and perform a dynamic delta hedging strategy with daily rebalancing.

Consider the following table containing data for the first and last 5 rows (days) of our daily dynamic delta hedging strategy:

Dynamic delta hedging

Example (Continued)

Index	S	Delta	Q	Stock	C	Bank ac pre-rebalance	Bank ac post-rebalance	Liquidation
0	50	0.59088	−0.59088	nan	4.13001	25.414	25.414	nan
1	49.9006	0.586241	−0.586241	−29.4853	4.05845	25.4175	25.186	−0.00931082
2	48.5786	0.525704	−0.525704	−28.4788	3.31022	25.1895	22.2487	0.0209682
3	49.542	0.569615	−0.569615	−26.0445	3.82509	22.2518	24.4273	0.0324651
4	50.5061	0.612123	−0.612123	−28.769	4.3817	24.4306	26.5776	0.0433119
5	50.2623	0.601262	−0.601262	−30.7667	4.2205	26.5813	26.0353	0.0350393

Index	S	Delta	Q	Stock	C	Bank ac pre-rebalance	Bank ac post-rebalance	Liquidation
175	39.7393	4.33462e−15	−4.33462e−15	−1.05246e−11	6.31806e−16	−0.199251	−0.199251	−0.199251
176	40.053	2.57491e−17	−2.57491e−17	−1.73614e−13	3.14613e−18	−0.199279	−0.199279	−0.199279
177	39.5568	6.8228e−25	−6.8228e−25	−1.01855e−15	5.89466e−26	−0.199307	−0.199307	−0.199307
178	39.0363	1.98377e−40	−1.98377e−40	−2.66337e−23	1.07473e−41	−0.199334	−0.199334	−0.199334
179	38.9849	1.02212e−79	−1.02212e−79	−7.73371e−39	2.76516e−81	−0.199362	−0.199362	−0.199362
180	39.3679	nan	nan	−4.02387e−78	0	−0.19939	nan	−0.19939

The first few days are as follows:

t_0 Time t_0 delta hedge: Here $S = 50$ and we short 0.59088 stocks to receive 29.544. We initially bought $h = 1$ call for 4.13. So we invest 25.414 in the bank account at r.

Dynamic delta hedging

Example (Continued)

t_1 Time t_1 liquidation: At time t_1 we have $S_1 = 49.9$ and prior to rebalancing we could liquidate the whole position by:

- Buying back the 0.59088 shares for a cost of 29.4853.
- Sell the option for 4.05845.

The total cost of closing out is 25.4268. But after interest our bank account was 25.4175. So we're less than a cent out.

Suppose we don't liquidate, but delta hedge for another day:

Dynamic delta hedging

Example (Continued)

t_1 Time t_1 rebalancing: At time t_1 the new hedge quantity is $Q = -0.586241$. We buy $0.59088 - 0.586241 = 0.00464$ shares at a price of $S_1 = 49.9$, paying 0.2315. After this rebalancing, this brings our bank account to 25.186.

t_2 Time t_2 liquidation: At time t_2 we have $S_2 = 48.5786$ and prior to rebalancing we could liquidate our position by:

- Buying back the 0.58621 shares for a cost of 28.4788.
- Sell the option for 3.31022.

The total cost of closing out is 25.1686. But after interest our bank account was 25.1895, about 2 cents difference.

Dynamic delta hedging

Example (Continued)

If we continue daily delta hedging for the full 180 days of the option's life, we end up out of pocket by only about 20 cents.

Remark

We can make this delta hedging process more accurate by:

- Using a shorter time period: Say delta hedging every hour.
- Use another option and do dynamic delta-gamma hedging.

Also note that in reality dynamic hedging is not as "neat and clean" as this due to market imperfections such as bid-ask spreads, brokerage fees, different borrowing and lending rates, etc.

Dynamic delta hedging

Note also that in the above, we assumed that we initially bought the $h = 1$ call for the theoretically correct Black-Scholes value.

> *Question*: How do market makers make money?

One way market makers make money is by their bid-ask spread and hence trading options at a "more favourable" price than the theoretically correct price, and then dynamically delta-hedging to "lock in" their profit.

- Consider the following example:

Dynamic delta hedging

Example

Continuing on the above example, suppose that instead of paying the theoretically correct initial call price of $C = 4.13001$, we only paid $C = 3.5$. Then when I run the simulation 20 times I calculate the mean and standard deviation of the final liquidation values to be 0.5891 and 0.1261. The mean of 0.5891 is not too far off the difference $4.13 - 3.5 = 0.63$. In fact, if the dynamic delta hedging strategy was "perfect" (more accurate) then the outcome of each simulation would be $0.63e^{rT} = 0.64$.

Market makers trade options at their quoted favourable bid-ask spreads and delta (and theta, vega, etc) hedge multiple times intraday.

Implied volatility

We now turn to the “mysterious” and “elusive” volatility parameter σ in the Black-Scholes model.

Implied volatility

In the Black-Scholes European option pricing model

$$C = S\mathcal{N}(d_1) - Ke^{-rT}\mathcal{N}(d_2) \quad \text{and} \quad P = Ke^{-rT}\mathcal{N}(-d_2) - S\mathcal{N}(-d_1),$$

where

$$d_1 = \frac{\log\frac{S}{K} + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \qquad \text{and} \qquad d_2 = d_1 - \sigma\sqrt{T},$$

the only unobservable parameter is the volatility σ.

- We've been estimating it using historical data but traders use market conventions, their knowledge of the market and even intuition when determining the volatility parameter σ to use for pricing options.

Here, we're interested in calculating σ from option prices:

Implied volatility

Given *observed* option prices C or P and observable market variables S, K, r and T, an option's **implied volatility** is the volatility parameter σ that yields Black-Scholes prices equal to C or P.

Example

From an above example, when $S = 50$, $K = 50$, $r = 0.05$, $T = \frac{1}{2}$, $\sigma = 0.25$ and $q = 0$, the call price is $C = 4.13$. But what σ yields a call price of $C = 4.5$? You can check that $\sigma = 0.277$ does.

- $\sigma = 0.277$ is the volatility *implied* by the call price $C = 4.5$.

The question is: How do we calculate implied vols?

Implied volatility

There's no "nice and neat" equation to calculate implied vols.

- We have to code numerical (iterative) techniques on computers.

I don't go into the mathematical details but I'll give you some Python code to calculate implied vols using Newton's method.

> **Remark**
>
> Newton's method is also used to calculate the YTM of bonds.

Python code to calculate implied vols:

Implied volatility

```python
import numpy as np; from scipy.stats import norm
# function to calculate Black-Scholes option prices
def black_scholes(S, K, r, T, sigma, q):
    d1 = (np.log(S/K) + (r - q + 0.5*sigma**2)*T)/(sigma*np.sqrt(T)); d2 = d1 - sigma*np.sqrt(T)
    C =  S*np.exp(-q*T)*norm.cdf(d1)  - K*np.exp(-r*T)*norm.cdf(d2) # call
    P = -S*np.exp(-q*T)*norm.cdf(-d1) + K*np.exp(-r*T)*norm.cdf(-d2) # put
    return [C, P]
# function to calculate option vega
def vega(S, K, r, T, sigma, q):
    d1 = (np.log(S/K) + (r - q + 0.5*sigma**2)*T)/(sigma*np.sqrt(T))
    return np.exp(-q*T)*S*norm.pdf(d1)*np.sqrt(T) # same for calls and puts
# observed call or put price
obs = 4.5 # call price, change for put price
# known/observed/given parameter values
S = 50; K = 50; r = 0.05; T = 1/2; q = 0
# Newton's method
sigma = np.sqrt(2*np.abs(np.log(S/(K*np.exp(-r*T))))/T) # initial guess of sigma
val = black_scholes(S, K, r, T, sigma, q)[0] # call price, change to [1] for puts
while (abs(val-obs)>10**-8):
    v = vega(S, K, r, T, sigma, q)
    sigma = sigma - (val - obs)/v # Newton step to update/improve estimate of sigma
    val = black_scholes(S, K, r, T, sigma, q)[0] # call price, change to [1] for puts
print("implied volatility =", sigma)
```

Smile and term structure

In markets, Black-Scholes implied vols are not constant but display:

- A **volatility smile** or **smirk** over the range of strike prices.
- A **volatility term structure** over the range of expiry dates.

Lets calculate some implied vols from S&P/ASX 200 Index option quotes.

Smile and term structure

Example

We'll use the bid-ask spread midpoints of the following quotes:

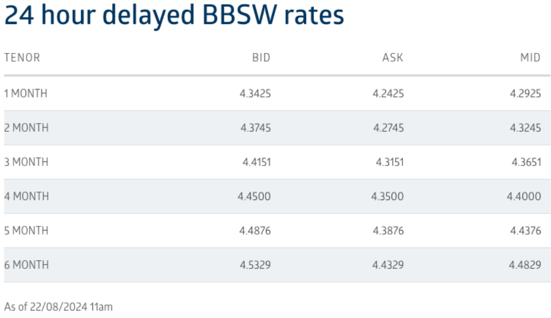
19 September 2024 | a month to expiry

Key: (E) = European (A) = American * = Theoretical Price

CALLS CODE	STYLE	BID	OFFER	OPEN INTEREST	VOLUME	LAST TRADE	STRIKE	PUTS CODE	STYLE	BID	OFFER	OPEN INTEREST	VOLUME	LAST TRADE
XJOBO7	(E)	176.00	192.00	2,206	--	197.40 *	7,850.00	XJOBP7	(E)	55.00	61.00	3,991	--	56.00 *
XJOQT7	(E)	158.00	173.00	616	--	178.90 *	7,875.00	XJOQU7	(E)	61.00	68.00	524	--	62.40 *
XJOHG7	(E)	141.00	156.00	3,094	--	161.20 *	7,900.00	XJOHH7	(E)	68.00	76.00	2,938	46	72.00
XJOQW8	(E)	126.00	138.00	427	--	144.40 *	7,925.00	XJOQX8	(E)	77.00	84.00	414	25	317.00
XJO6C7	(E)	112.00	121.00	714	--	128.50 *	7,950.00	XJO6D7	(E)	86.00	94.00	641	--	86.80 *
XJO0C7	(E)	98.00	106.00	2,189	26	95.00	7,975.00	XJO0D7	(E)	97.00	106.00	201	30	112.00
XJOMB7	(E)	85.00	91.00	4,139	7	79.00	8,000.00	XJOMC7	(E)	108.00	118.00	2,104	16	123.00
							LAST: 8,014.90							
XJO0E7	(E)	73.00	81.00	1,091	1	70.00	8,025.00	XJO0L7	(E)	121.00	130.00	1,009	--	119.90 *
XJOU39	(E)	62.00	70.00	394	--	75.30 *	8,050.00	XJOU49	(E)	132.00	147.00	1,223	--	133.20 *
XJO897	(E)	53.00	60.00	277	--	64.70 *	8,075.00	XJO8A7	(E)	147.00	162.00	163	--	147.60 *
XJOMD7	(E)	45.00	51.00	4,102	--	55.20 *	8,100.00	XJOME7	(E)	162.00	178.00	528	--	163.20 *
XJO9W7	(E)	37.00	43.00	1,170	--	46.80 *	8,125.00	XJO9Y7	(E)	179.00	195.00	33	--	179.80 *
XJOU59	(E)	31.00	37.00	2,459	130	30.00	8,150.00	XJOU69	(E)	196.00	216.00	4	--	197.60 *
XJORA9	(E)	26.00	31.00	1,339	160	25.00	8,175.00	XJORB9	(E)	216.00	236.00	57	--	216.40 *

Smile and term structure

Example



24 hour delayed BBSW rates

TENOR	BID	ASK	MID
1 MONTH	4.3425	4.2425	4.2925
2 MONTH	4.3745	4.2745	4.3245
3 MONTH	4.4151	4.3151	4.3651
4 MONTH	4.4500	4.3500	4.4000
5 MONTH	4.4876	4.3876	4.4376
6 MONTH	4.5329	4.4329	4.4829

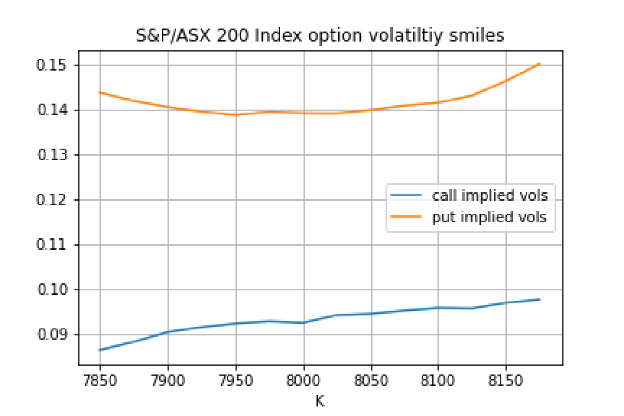
As of 22/08/2024 11am

There's 27 days to expiry so I'll use the 1 month BBSW rate of $r = 4.2925\%$. We also have that $S = 8,014.90$ and $q = 4.2\%$.

- We get the following volatility smiles:

Smile and term structure

Example



Why are puts priced at higher implied vols than calls?

VIX index

- The Cboe VIX Index measures "market wide" implied vols:

It "*is a calculation designed to produce a measure of constant, 30-day expected volatility of the U.S. stock market, derived from real-time, mid-quote prices of S&P 500 Index (SPX) call and put options*."

- See here for a detailed explanation, plus wiki and Investopedia.

The Cboe VIX index is effectively an average of the implied volatilities of a large range of 30 day Cboe S&P 500 Index options.

- One can even trade futures and options on the VIX (also see here)!

VIX index

A value of 17.55 means that average implied vols of 30 day S&P 500 Index options, thus the market's 30 day volatility expectations, is 17.55%.

VIX index

Australia also has its own S&P/ASX 200 VIX Index (also see here):

The implied vol of the S&P/ASX 200 Index is quite a bit lower at 11.69 than that of the S&P 500 Index of 17.55, and is roughly "in the middle" of our above S&P/ASX 200 Index option volatility smile and smirk.

VIX index

As an observation, the VIX index tends to spike when the market falls:

Trading strategies

We now present some "standard" options trading strategies designed to take advantage of movements in the asset price, changes in market sentiment and implied vols, and the passage of time:

- Directional strategies: Speculate on the direction of the price of the underlying asset, similar to taking calls and puts.
- Volatility strategies: Speculate on high or low asset volatility, or changes in implied vols, often incorporating delta neutrality.
- Time: Strategies that seek to take advantage of time decay, typically assuming low asset volatility and relatively constant implied vols.

Trading strategies

Remark

We've already present some basic options strategies, including:

- Taking calls (puts) to speculate on an increase (decrease) in the price of the underlying asset.
- Covered calls or a "buy-write" involving writing calls on a portfolio of the underlying asset.
- Portfolio insurance, involving taking puts over a portfolio of the asset in order to protect it against market falls.

This week we're interested in additional, more complex strategies.

Trading strategies

It's common to divide trading strategies into two different classes:

- **Spreads**: Positions in the same type (call alone or puts alone) of options on an asset, but with different strikes and possibly expiries.
  - Bull and bear spreads.
  - Backspreads and ratio spreads.
  - Butterfly and condor spreads.
  - Calendar spreads.
- **Combinations**: Positions in both puts and calls on an asset.
  - Straddles and strangles.
  - Butterfly and condor combinations.
  - Strips and straps.

The payoffs of many strategies can be created in different ways, and we can create strategies containing both spread and combination positions.

Directional strategies

Directional strategies seek to speculate on a directional change in the price of the asset, but at a lower upfront cost to the basic strategies of taking calls and puts, while still maintaining limited downside risk.

- However, in lowering the upfront cost relative to taking calls and puts, these strategies sacrifice some upside potential, as we will see.

Remark

So we don't repeat ourselves, we state upfront here that in the following options trading examples we will always let $S = 50$, $r = 0.05$, $T = \frac{1}{2}$, $\sigma = 25\%$ and $q = 0$ unless otherwise stated.

Directional strategies

Suppose we want to speculate on an increase in the asset price:

Example

We could take an ATM call with strike $K = 50$, costing $C_1 = 4.13$. But also write an OTM call with strike $K_2 = 55$.

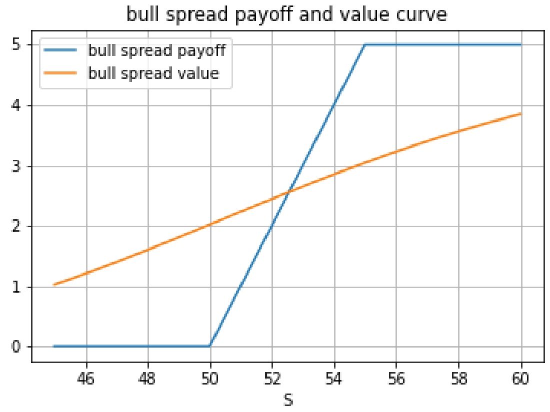
- The payoff of the combined position, called a **bull spread** is:

$$\text{payoff} = \max\{0, S_T - 50\} - \max\{0, S_T - 55\}.$$

We receive $C_2 = 2.11$ from writing the OTM call, so the net cost of the bull spread is 2.02. Some payoff and profit diagrams:

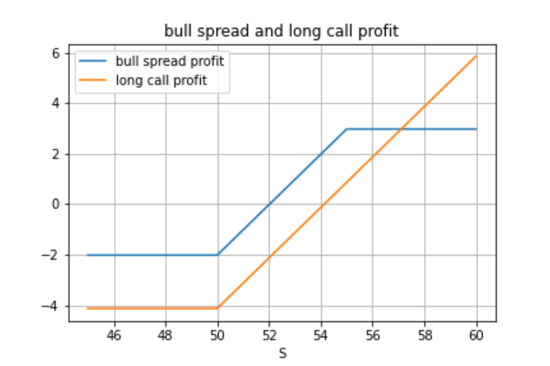
Directional strategies

Example



Directional strategies

Example



A bull spread has lower downside risk and "profits sooner" compared to the basic long call, but its upside profits are capped.

Directional strategies

Remark

Bull spreads can also be constructed from puts (tutorial question).

Now suppose we want to speculate on a decrease in the asset price:

Example

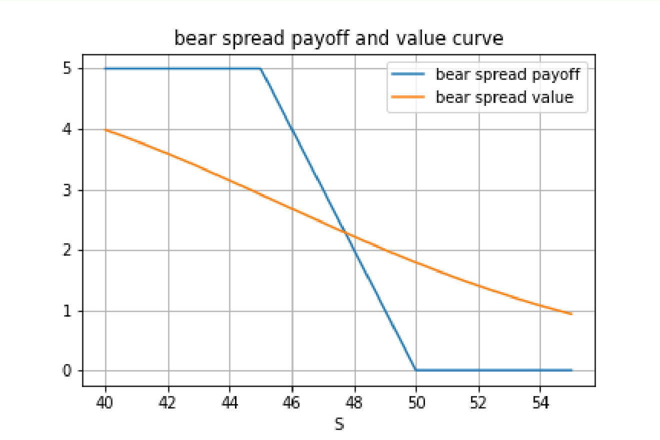
We can lower the cost of taking a put with strike $K = 50$ by also writing a put with strike $K_2 = 45$. This **bear spread** payoff is

$$\text{payoff} = \max\{0, 50 - S\} - \max\{0, 45 - S\}.$$

The premiums are $P_1 = 2.9$ and $P_2 = 1.11$, for a net cost of 1.79.

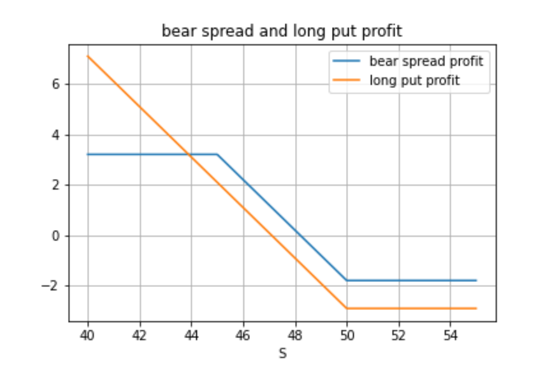
Directional strategies

Example



Directional strategies

Example



A bear spread has lower downside risk and "profits sooner" compared to the basic long put, but its upside profits are capped.

Directional strategies

Remark

Bear spreads can also be created from calls (tutorial question).

The basic strategies of taking calls, as well as bull and bear spreads, seek to speculate on the asset price moving in a given direction.

- But what if we think the asset price will be very volatile but we're not sure about which direction it will go?
- Also, what if we think the asset price will be stable?

How might we trade options given these views or beliefs?

Volatility strategies

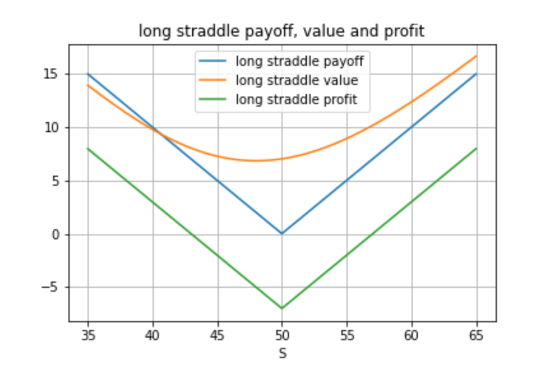
Volatility strategies seek to take advantage of the asset either staying stable or displaying high volatility. Another class of volatility strategies also seek to take advantage of changing market sentiment and hence implied vols, while typically staying delta (and possibly gamma and theta) hedged. They're best described with examples.

- Suppose we think that the asset price will be volatile:

Volatility strategies

Example

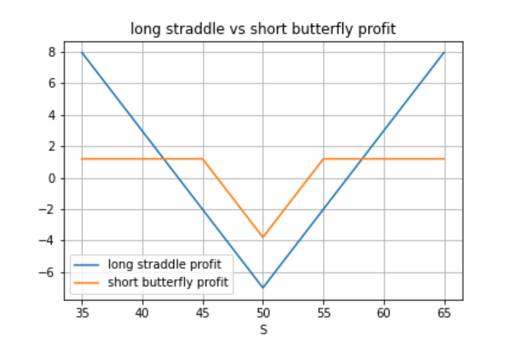
We could take ATM calls and puts both with strike $K = 50$, a strategy called a **long straddle** with payoff and profit:



Volatility strategies

Example (Continued)

Long straddles cost a lot! We can reduce it by writing an OTM put with strike $K_1 = 45$ and a OTM call with strike $K_2 = 55$:



Volatility strategies

Remark

This strategy has the same payoff as a **short butterfly**, which is a spread involving only calls (or puts). If created from calls you:

- Write 1 ITM call with strike $K_1 = 45$.
- Take 2 ATM calls with strike $K = 50$.
- Write 1 OTM call with strike $K_2 = 55$.

A butterfly spread can also be created from puts (tutorial question).

Volatility strategies

Now suppose that we expect a period of low volatility?

Example

Short straddles and long butterflies hope the asset stays stable:

- **Short straddle**: Write a call and put with strike $K = 50$.
- Floor the short straddle's downside by also taking a put with strike $K_1 = 45$ and call with strike $K_2 = 55$.

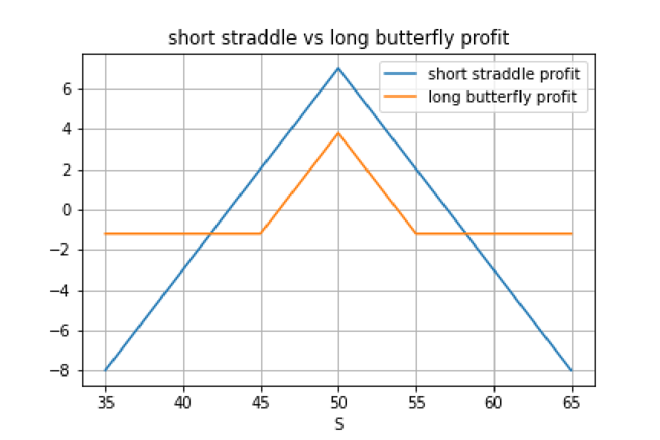
The diagrams are below.

Remark

The second strategy has the same payoff as a long butterfly spread, which can be created out of calls alone, or puts alone.

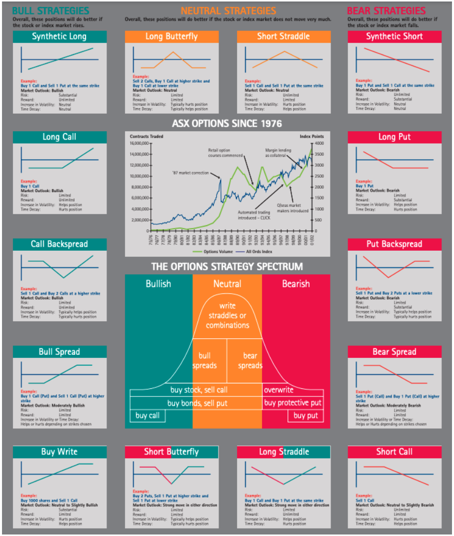
Volatility strategies

Example (Continued)



Volatility strategies

The above strategies are all just speculating on the asset price, as are:



Volatility strategies

But can we speculate on a change in volatility σ and hence implied vols?

- Suppose we think that implied vols will increase in 1 month.

Example

Option premiums increase when implied vols increase, so we want to take options. Suppose we use a long straddle, which involves taking a call and put with strike $K = 50$. The cost is 7.026 and the options have deltas of $\Delta_C = 0.59088$ and $\Delta_P = -0.40912$. The straddle's delta is simply the sum of these, so $\Delta_{\text{str}} = 0.182$.

- So our option position (long straddle) is slightly exposed to a fall in the asset price, which we can delta hedge against.

To be delta neutral we short 0.182 assets. Scenarios in 1 month:

Volatility strategies

Example (Continued)

1. Stable asset and implied vols: Close out the straddle for 6.42, a *loss* of $7.026 - 6.42 = 0.61$ due to time decay.
2. Stable asset but implied vols increase to 30%: Close out the straddle for 7.68, a *profit* of $7.68 - 7.026 = 0.65$.
3. $\mathrm{d}S = -2$ and stable implied vols: Close out the strangle for 6.28, a *loss* of $7.026 - 6.28 = 0.74$.
4. $\mathrm{d}S = -2$ and implied vols increase to 30%: Close out the strangle for 7.51, a *profit* of $7.51 - 7.026 = 0.494$.

In both 3 and 4 we also take a *profit* on the short asset of 0.364.

Volatility strategies

So, the point of this example is:

- We were expecting implied vols to increase.
- So we held a long straddle for 1 month that was exposed to:
  - A small fall in the asset price.
  - Time decay.
- We delta hedged the asset price exposure by shorting the asset.

In doing this, we concentrated (most of) our exposure to implied vols.

- But note that we were still exposed to time decay, and traders will often hedge against this as well. But we can also "trade" time decay:

Time decay

We can also use delta hedging to take advantage of time decay, which options close to ATM are exposed to. The following strategy also benefits when implied vols fall and requires the asset to stay relatively stable.

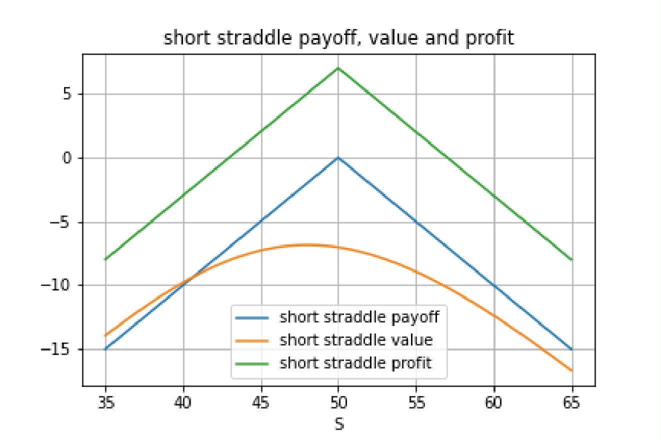
- Suppose we think that implied vols will stay stable or possibly fall, and that the asset price will remain relatively stable.

Example

Time decay (and falling implied vols) reduce option premiums, thus benefiting short positions. Suppose we use a short straddle, which involves writing a call and put with strike $K = 50$. We receive 7.026 upfront in premium. The short straddle's delta is $\Delta_{\text{str}} = -0.182$, so it's exposed to an increase in the asset price.

Time decay

Example (Continued)



Time decay

Example (Continued)

To be delta neutral we buy 0.18176 assets. Scenarios in 1 month:

1. Stable asset and implied vols: Close out the straddle for 6.42, a *profit* of 0.61 due to time decay.
2. Stable asset but implied vols fall to 20%: Close out straddle for 5.16, a *profit* of 1.87 from time decay and implied vols.
3. $\mathrm{d}S = 2$ and stable implied vols: Close out straddle for 6.9, a small profit of 0.09 from time decay. Asset profit is 0.364.
4. $\mathrm{d}S = 2$ and implied vols fall to 20%: Close out straddle for 5.75, a profit of 1.28. Asset position profit is also 0.364.

Calendar spreads

A final class of option trading strategies we mention are calendar spreads. They involve positions in the same type of option (calls alone, or puts alone) with different expiry dates instead of different strike prices.

- They're typically held until the expiry of the shorter dated option.
- They have similar profits and payoffs to straddles and are thus used to trade similar market expectations.
- They're also another way to take advantage of time decay.

Again, and example best illustrates the idea.

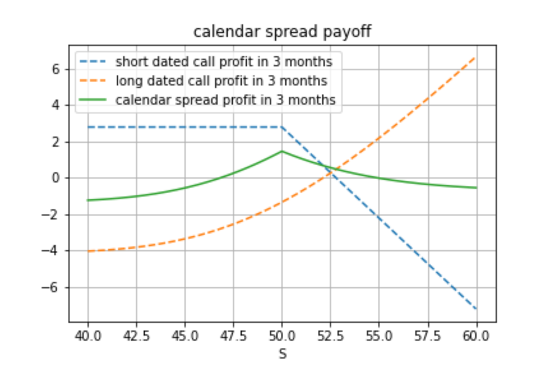
Calendar spreads

Example

Suppose we write a short dated call option with strike $K = 50$ and 3 month expiry, so $T_1 = \frac{1}{4}$, and we also take a long dated call also with a strike $K = 50$ but 6 month expiry, so $T = \frac{1}{2}$. The net cost is 1.33. The payoff diagram in 3 months, hence the expiry date of the short dated call we wrote, is:

Calendar spreads

Example (Continued)



The “profit” in 3 months of the longer dated call that we took is its premium curve of this call less the initial premium paid.

Calendar spreads

Example (Continued)

This kind of calendar spread is also benefits from time decay if implied vols and the asset price are stable: We wrote the shorter dated 3 month call, whose time decay is stronger than the longer dated 6 month call that we took. In fact, when we entered into the position, the theta of the short dated 3 month call is $\theta_3 = -6.193$ and of the long dated 6 month call is $\theta_6 = -4.705$. If implied vols and the asset price are unchanged, we exit the position for the value of the longer dated 6 month call of 2.78, for a profit of $2.78 - 1.33 = 1.47$, as per the above profit diagram.

Disclaimer

Remark

As a final remark, and somewhat of a disclaimer, options strategies like those above - spreads and combinations - involve positions in multiple options and possibly even trades in the asset, so they are never this "neat and clean" in practice: These strategies are *very badly* damaged by bid-ask spreads and brokerage costs and fees, as well as different borrowing and savings account interest rates. It's very difficult to overcome these market realities to consistently make money out of options trading strategies as presented above. Market makers benefit from the bid-ask spread and pay relatively low brokerage.

Summary