

# FINM3405 Derivatives and risk management

## Tutorial Sheet 4: Options - Introduction

August 15, 2024

**Question 1.** 1. What is the difference between call and put options?

- A call option gives the holder the right but not the obligation to buy the underlying asset on (European) or anytime up to and including (American) the expiry date.
- A put option gives the holder the right but not the obligation to sell the underlying asset on (European) or anytime up to and including (American) the expiry date.

Note that you can *buy* an option, making you the holder or taker of the option and having a long position, or *sell* the option, making you the writer and having a short position. Due to the risk of confusion in terminology between on the one hand calls (right to buy the asset) and puts (right to sell the asset) and on the other “buying” or “selling” an option, we typically say taking or writing an option instead of buying or selling it.

2. What is the difference between the taker/holder and writer of an option? I think I just answered that above.
3. What is the difference between European and American options? As well as this question.
4. Why do options have premiums and which party pays it to which? The payoff to the taker/holder of an option is nonnegative, so this party can never lose money (ignoring counterparty risk). Due to this, if there was no premium paid by the taker to the writer there would be an obvious arbitrage opportunity. Hence, the taker needs to compensate the writer in order to entice the writer to enter into an option contract.
5. What do the terms in-the-money, at-the-money and out-of-the-money mean? I'll explain these terms in a slightly different way to how they're presented in the lecture notes.
  - An option is in-the-money (ITM) if the spot price of the underlying

asset is currently “in the holder’s favour”, meaning that it’s greater than the strike price in the case of a call option, and less than the strike price in the case of puts.

- An option is at-the-money (ATM) if the spot price of the underlying asset currently equals the strike price.
  - An option is out-of-the-money (OTM) if the spot price of the underlying asset is currently “not in the holder’s favour”, meaning that it’s less than the strike price in the case of call options and greater than the strike price in the case of puts.
6. What is the difference between writing an option naked vs covered? Writing options naked does not mean writing them while wearing no clothes... It actually means writing them with “nothing backing them” and to understand what I mean by this, contrast it with writing covered options.
- A covered call position involves writing call options over an asset that you already hold, and to a total notional or face value  $F = hKm$  not exceeding the value  $V = QS$  of your asset holding. Hence, if you were exercised in the call option position and had to sell the underlying asset to the taker/holder, you have the holding in the underlying asset to cover this.
  - A covered put position involves writing a put options to a total notional or face value  $F = hKm$  not exceeding a total amount of cash (or capital) you have on hand available to cover having to buy the underlying asset if you get exercised.

*Remark:* Recall our notation that  $h$  is the number of contracts we enter into,  $K$  is the strike or exercise price,  $m$  is the contract multiplier (number of units in the underlying asset 1 contract is over),  $Q$  is the number of units of the asset we hold, and  $S$  is the spot price of the asset.

7. What is the intrinsic value and time value of an option? An option’s intrinsic value (IV) is its payoff if it was able to be exercised immediately, right now. So the intrinsic value of a call option is  $IV^{\text{call}} = \max\{0, S - K\}$  and of a put option is  $IV^{\text{put}} = \max\{0, K - S\}$ . The option premium is almost always greater than the option’s intrinsic value (just not in the case of deep in-the-money European puts), and we define the time value to be the difference in the two:

$$\text{time value} = \text{premium} - \text{intrinsic value}.$$

Note that deep in-the-money European puts can have negative time value because  $P^{\text{Eu}} \leq e^{-rT}K$  is an upper pricing bound for them: If  $S = 0$  then

their intrinsic value is  $K$  and in this case their time value is  $P^{\text{Eu}} - K < 0$  since  $e^{-rT}K < K$ . So as a European put option gets deeper in-the-money (as the price of the underlying asset gets closer to 0), there is a point in the price of the underlying asset at which the option transitions from having positive to negative time value (I wonder what that point is and if it can be quantified).

**Question 2.** What does it mean to exercise an option? Which party gets to choose whether they exercise an option? The taker/holder gets to choose whether to exercise the option; the writer is at their mercy. Exercising a call option means the holder exercising their right to buy the underlying asset off the writer for the strike price  $K$ . Exercising a put option means the holder exercising their right to sell the underlying asset to the writer for the strike price  $K$ .

**Question 3.** How do you use options to speculate on an increase in the price of the underlying asset? What about a decrease? The most basic strategy to speculate on an increase in the price of the underlying asset is to buy call options. To speculate on a decrease in the price of the underlying asset, one buys put options. We'll also cover more advanced strategies in a few weeks.

**Question 4.** What is the maximum profit an option writer can earn, and what is their potential loss? Why would you ever consider writing options? The most an option writer can earn is the premium received. Their loss is unlimited when writing a call option, and limited to the strike (minus the premium) when writing puts. I recommend the student look into the 2<sup>nd</sup> question. Some answers might be: It's common to write covered calls (roughly at-the-money) in order to generate additional income on a portfolio. It's also common to write covered puts (roughly at-the-money) in order to "leg in" to buying a stock for a cheaper price if you're exercised, or simply earn the premium if you're not exercised. There's also more advanced strategies involving combinations of short options that we'll cover a little later on. Note that time decay is basically always working against long option positions and for short option positions too, and we'll also touch on this (multiple times) as we go over the next few weeks.

**Question 5.** 1. You own 500,000 shares in the E Fund SSE STAR 50 ETF on the Shanghai Stock Exchange and are worried that the STAR 50 Index will fall in value. What options contracts are available for you to insure your holding and how would you do it? What would be the outcome if the CSI 300 Index fell 10% between now and next month? The SSE has various equity options including over the E Fund SSE STAR 50 ETF:

## E FUND STAR 50 ETF Options Contract Specification

Underlying	E FUND SSE Science and Technology Innovation Board 50 Index ETF (E FUND STAR 50 ETF;588080)
Contract Type	Call options and put options
Contract Size	10,000
Expiration Months	Current month, next month and the following two consecutive quarters
Strike Price	9 prices (1 at-the-money, 4 out-of-the-money and 4 in-the-money)
Strike Price Interval	RMB 0.05 for price below RMB 3 (inclusive), RMB 0.1 for price between RMB 3 and 5 (inclusive), RMB 0.25 for price between RMB 5 and 10 (inclusive), RMB 0.5 for price between RMB 10 and 20 (inclusive), RMB 1 for price between RMB 20 and 50 (inclusive), RMB 2.5 for price between RMB 50 and 100 (inclusive), RMB 5 for price above RMB 100
Exercise Style	Exercised at the expiration date (European style)
Contract Delivery	Physical delivery (unless otherwise specified in rules)
Expiration Date	The fourth Wednesday of each expiration month (if the expiration date falls on a holiday, it will be put off correspondingly)
Exercise Date	Same as the expiration date. Exercise orders are accepted between 9:15-9:25, 9:30-11:30, 13:00-15:30
Delivery Date	The day after the exercise date
Trading Hours	AM 9:15-9:25, 9:30-11:30 (the period between 9:15-9:25 is for opening auction) PM 13:00-15:00 (the period between 14:57-15:00 is for closing auction)
Order Types	Limit order, market order immediate to limit, market order immediate or cancel, kill or fill limit order, kill or fill market order and other types of orders as specified by the SSE
Long/Short Types	Long Open, Short Open, Long Close, Short Close, Open Covered Call, Close Covered Call and other types as specified by the SSE
Tick Size	RMB 0.0001
Order Size	One or its integral multiples
Price Limit	Price up limit for call option=max {previous close price of underlying×0.5%, min [(2×previous close price of underlying - strike price), previous close price of underlying]×20%} Price down limit for call option=previous close price of underlying×20% Price up limit for put option=max {strike price×0.5%, Min [(2×strike price - previous closing price of underlying), previous close price of underlying]×20%} Price down limit for put option=previous close price of underlying×20%
Circuit Breaker	During the continuous trading session period, if the percentage change of option price reaches or exceeds 50% of the latest reference price and the absolute value of price change reaches or exceeds 5 ticks, the option trading will enter a 3-minute call auction period
Minimum Initial Margin	Initial margin for short call=[previous settlement price + Max (12%×previous closing price of underlying - max (strike price - previous closing price of underlying, 0), 7%×previous close price of underlying)]×contract size Initial margin for short put=Min [previous settlement price +Max (12%×previous closing price of underlying - Max (previous close price of underlying-strike price, 0), 7%×strike price), strike price]×contract size
Minimum Maintenance Margin	Maintenance margin for short call=[settlement price +Max (12%×closing price of underlying - max(strike price- previous close price of underlying, 0), 7%×close price of underlying)]×contract size Maintenance margin for short put=Min [settlement price +Max (12%×close price of underlying - max (previous close price of underlying-strike price, 0), 7%×strike price), strike price]×contract size

The contract multiplier (size) is  $m = 10,000$  so to insure your  $Q = 500,000$  unit ETF portfolio you'd take (buy)  $h = Q/m = 50$  put option contracts. Your position at expiry is

$$\text{portfolio insurance position} = QS_T + h \max\{0, K - S_T\}m - hPm.$$

At the time of writing, the CNY price of this ETF was  $S = 0.7220$ , so the value of your portfolio is  $V = QS = 500000 \times 0.722 = 361,000$  CNY.

Shanghai - Delayed Quote - CNY

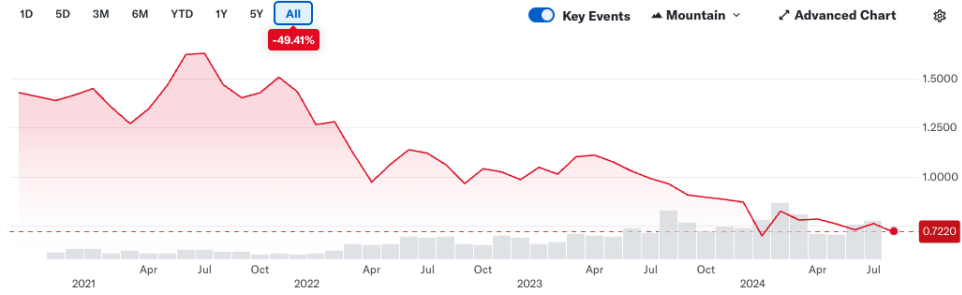
## E Fund SSE STAR 50 ETF (588080.SS)

☆ Follow

↔ Compare

**0.7220** +0.0020 (+0.2778%)

As of 10:05 AM GMT+8. Market Open.



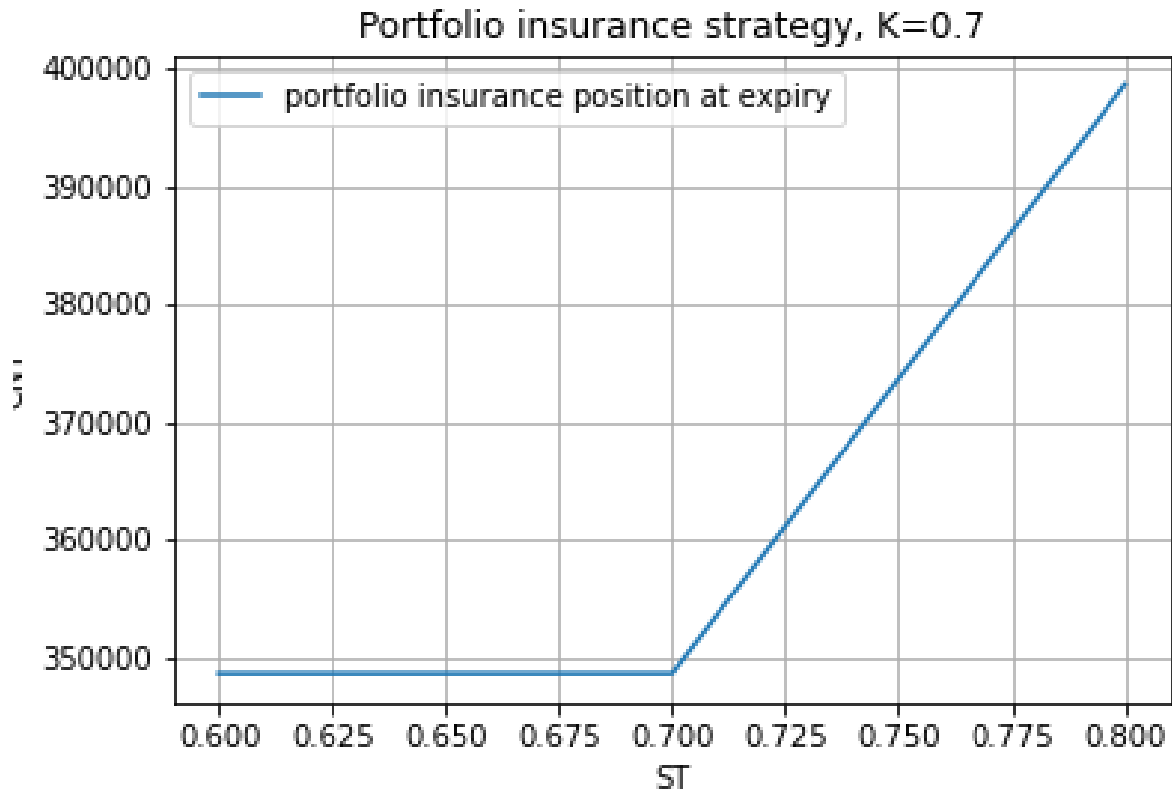
Here  $Q = hm$ . If  $S_T \geq K$  then from above your final position is  $QS_T - QP = Q(S_T - P)$ . If  $S_T < K$  your final position is  $QS_T + Q(K - S_T) - QP = Q(K - P)$ . Here's some option quotes:

Call Option				Aug 2024	Put Option				中文
Trading Code	Current Price	Change (%)	Last Settlement Price	Strike Price	Trading Code	Current Price	Change (%)	Last Settlement Price	
588080C 2408M00950	0.0001	0.00%	0.0001	0.950	588080P 2408M00950	--	0.00%	0.2307	
588080C 2408M00900	0.0002	100.00%	0.0001	0.900	588080P 2408M00900	--	0.00%	0.1809	
588080C 2408M00850	0.0001	-50.00%	0.0002	0.850	588080P 2408M00850	0.1161	-11.17%	0.1307	
588080C 2408M00800	0.0013	160.00%	0.0005	0.800	588080P 2408M00800	0.0680	-16.15%	0.0811	
588080C 2408M00750	0.0075	78.57%	0.0042	0.750	588080P 2408M00750	0.0238	-32.00%	0.0350	
588080C 2408M00700	0.0367	47.98%	0.0248	0.700	588080P 2408M00700	0.0027	-54.24%	0.0059	
588080C 2408M00650	0.0840	20.00%	0.0700	0.650	588080P 2408M00650	0.0003	-57.14%	0.0007	
588080C 2408M00600	0.1231	2.58%	0.1200	0.600	588080P 2408M00600	0.0001	0.00%	0.0001	
588080C 2408M00550	--	0.00%	0.1700	0.550	588080P 2408M00550	0.0001	0.00%	0.0001	
588080C 2408M00500	--	0.00%	0.2200	0.500	588080P 2408M00500	0.0001	0.00%	0.0001	

Last Update: 2024-08-15

10:32:44

Let's take the just out-of-the-money put with a strike price of  $K = 0.7000$  and last traded price of 0.0027. You pay  $hPm = 50 \times 0.0027 \times 10000 = 2,950$  CNY in premium upfront. Your position at expiry is:



If  $S_T < K$  you're locking in  $QK$  minus the premium you paid, yielding 348,650 CNY. This is slightly less than your current portfolio value of 361,000 CNY because you took out-of-the-money puts and also had to pay the premium. The upshot is you still get to participate in an increase in the price of the underlying asset: If  $S_T > K$  then your ETF position is  $QS_T$  minus the premium you paid (and the option expires out-of-the-money and hence worthless). Some Python code is below.

2. You also own 500,000 shares in the **Harvest CSI 300 ETF** on the **Shenzhen Stock Exchange**. You seek additional income on your holding and expect the **CSI 300 Index** to go sideways. What options contracts are available for you to do this and how would you do it? What would be the outcome if the CSI 300 Index went sideways between now and next month?

SZSE offers **CSI 300 ETF options** (see specifications below). The strategy in this question is covered call, also sometimes called a buy-write (buy the stock and write the calls). The contract multiplier is also  $m = 10,000$  so you'd write 50 say just out-of-the-money calls. If the market went sideways you'd keep the premium and your portfolio, so you achieved your goal of generating additional income on the portfolio. If the market went down, then you'd have the same outcome except of course the value of your portfolio is less. If the market goes up, you'll get exercised and have to sell your holding; here you lock in the price of your overall position as the  $QK$  plus the premium you received.

### CSI 300 ETF Options Product Specification

Underlying	Harvest CSI 300 Exchange Traded Fund (CSI 300 ETF, 159919)																	
Contract Types	Call options and put options																	
Exercise Style	European style																	
Contract Multiplier	10,000																	
Expiration Months	4 contract expiration months (spot month, next calendar month, and next two calendar quarter months)																	
Contract Delivery	Physical delivery (unless otherwise specified)																	
Strike Prices	At least 9 prices (1 at-the-money, 4 out-of-the-money, 4 in-the-money)																	
Strike Price Intervals	<table><thead><tr><th>Price</th><th>Strike Price Interval</th></tr></thead><tbody><tr><td>≤ RMB 3</td><td>RMB 0.05</td></tr><tr><td>&gt; RMB 3 and ≤ RMB 5</td><td>RMB 0.1</td></tr><tr><td>&gt; RMB 5 and ≤ RMB 10</td><td>RMB 0.25</td></tr><tr><td>&gt; RMB 10 and ≤ RMB 20</td><td>RMB 0.5</td></tr><tr><td>&gt; RMB 20 and ≤ RMB 50</td><td>RMB 1</td></tr><tr><td>&gt; RMB 50 and ≤ RMB 100</td><td>RMB 2.5</td></tr><tr><td>&gt; RMB 100</td><td>RMB 5</td></tr></tbody></table>	Price	Strike Price Interval	≤ RMB 3	RMB 0.05	> RMB 3 and ≤ RMB 5	RMB 0.1	> RMB 5 and ≤ RMB 10	RMB 0.25	> RMB 10 and ≤ RMB 20	RMB 0.5	> RMB 20 and ≤ RMB 50	RMB 1	> RMB 50 and ≤ RMB 100	RMB 2.5	> RMB 100	RMB 5	
Price	Strike Price Interval																	
≤ RMB 3	RMB 0.05																	
> RMB 3 and ≤ RMB 5	RMB 0.1																	
> RMB 5 and ≤ RMB 10	RMB 0.25																	
> RMB 10 and ≤ RMB 20	RMB 0.5																	
> RMB 20 and ≤ RMB 50	RMB 1																	
> RMB 50 and ≤ RMB 100	RMB 2.5																	
> RMB 100	RMB 5																	
Expiration Date	The fourth Wednesday of the contract expiration month (if the expiration date is a holiday, it will be put off to the next business day)																	
Trading Hours	Morning: 9:15–9:25 (for opening auction), 9:30-11:30 Afternoon: 13:00–15:00 (14:57-15:00 for closing auction)																	
Exercise Date	Same as the expiration date. Exercise orders should be submitted between 9:15-11:30, 13:00-15:30 on the exercise date and will not be accepted otherwise.																	
Delivery Date	The day after the exercise date																	
Tick Size	RMB 0.0001																	
Order Size	1 contract																	
Order Types	Ordinary limit order, Fill-or-Kill limit order, Opposite-side Best Price market order, Same-side Best Price market order, Five Best market orders Immediate or Cancel, Immediate or Cancel market order, Fill-or-Kill market order and other types of orders as specified by the SZSE																	
Trading Types	Buy to open, buy to close, sell to open, sell to close, open covered call, close covered call and other types as specified by the SZSE																	
Price Limit	<b>Call</b> Price up limit = max {previous closing price of underlying×0.5%, min [(2×previous closing price of underlying – strike price), previous closing price of underlying]×10%} Price down limit = previous closing price of underlying×10% <b>Put</b> Price up limit = max{strike price×0.5%, min [(2×strike price –previous closing price of underlying), previous closing price of underlying]×10%} Price down limit = previous closing price of underlying×10%																	
Circuit Breaker	During continuous auction, if the intraday trading price of a contract moves by 50% or more above or below the latest reference price and the absolute value of such movement reaches or exceeds 10 times the tick size, the contract will enter a 3-minute call auction session																	
Minimum Initial Margin	<b>Short Call</b> Initial margin = {previous settlement price + max [12%×previous closing price of underlying – max (strike price – previous closing price of underlying, 0), 7%×previous closing price of underlying]}×contract size <b>Short Put</b> Initial margin = min {previous settlement price +max [12%×previous closing price of underlying – max (previous closing price of underlying–strike price, 0), 7%×strike price], strike price}×contract size																	
Minimum Maintenance Margin	<b>Short Call</b> Maintenance margin ={settlement price +max [12%×closing price of underlying – max(strike price–closing price of underlying, 0), 7%×closing price of underlying]}×contract size <b>Short Put</b> Maintenance margin=min {settlement price +max [12%×closing price of underlying – max (closing price of underlying–strike price, 0), 7%×strike price], strike price}×contract size																	
Maximum Contracts Per Order	50 contracts maximum for each limit order. 10 contracts maximum for each market order																	

```

1 import numpy as np
2 import matplotlib.pyplot as plt
3 # portfolio insurance strategy
4 Q = 500000
5 m = 10000
6 h = Q/m
7 K = 0.7
8 P = 0.0027
9 premium = h*P*m
10 ST = np.linspace(0.6, 0.8, 1001)
11 put_payoff = h*np.maximum(0, K-ST)*m
12 payoff = Q*ST + put_payoff - premium
13 plt.figure()
14 plt.plot(ST, payoff, label='portfolio insurance position at expiry')
15 plt.grid()
16 plt.legend()
17 plt.xlabel("ST")
18 plt.ylabel("CNY")
19 plt.title("Portfolio insurance strategy, K=0.7")

```

**Question 6.** CBA options quotes either side of the money on the ASX website:

COMMONWEALTH BANK OF AUSTRALIA. CBA

LAST PRICE / TODAY'S CHANGE ⓘ

VOLUME ⓘ

BID / OFFER RANGE ⓘ

MARKET CAPITALISATION ⓘ

\$128.145 ▲+\$0.805 (0.632%)

731,003

\$128.140 - \$128.150

\$213.11B

Industry Group: Banks

Listed on 12 September 1991

Prices delayed by at least 20 minutes | Currently trading

CBA Overview

Options

Warrants

Hybrids

REFRESH DATA

Filters

☐ OPEN INTEREST ONLY

☐ THEORETICAL PRICING ONLY

ALL EXPIRIES

ALL EXERCISE PRICES

CALLS & PUTS

ALL STYLES

15 August 2024 | 7 days to expiry

Key: (E) = European (A) = American \* = Theoretical Price

CALLS

PUTS

CODE	STYLE	BID (\$)	OFFER (\$)	OPEN INTEREST	VOLUME	LAST TRADE (\$)	STRIKE (\$)	CODE	STYLE	BID (\$)	OFFER (\$)	OPEN INTEREST	VOLUME	LAST TRADE (\$)
CBA4Q8	(E)	8.06	8.91	1,808	--	7.84 *	120.01	CBA4U8	(E)	0.20	0.38	272	10	0.26
CBA7Z7	(A)	6.32	6.99	233	--	6.08 *	122.00	CBA8F7	(A)	0.37	0.50	1,061	200	0.50
CBA1Q9	(E)	6.30	6.99	1,895	--	6.07 *	122.01	CBA1U9	(E)	0.37	0.50	369	30	0.64
CBA9G7	(A)	4.65	5.23	306	--	4.45 *	124.00	CBA9H7	(A)	0.61	0.78	1,177	241	0.70
CBA2J9	(E)	4.64	5.23	1,867	--	4.44 *	124.01	CBA2K9	(E)	0.61	0.81	929	--	1.08 *
CBA9J7	(A)	3.10	3.70	504	--	3.03 *	126.00	CBA9K7	(A)	1.05	1.32	2,391	30	1.30
CBAG67	(E)	3.09	3.70	3,111	133	7.60	126.01	CBAG77	(E)	1.03	1.34	427	30	1.70
CBA3J8	(A)	1.98	2.41	1,302	538	2.20	128.00	CBA3K8	(A)	1.79	1.95	2,547	20	2.50
CBAKX7	(E)	1.97	2.38	1,317	4	1.74	128.01	CBAKY7	(E)	1.77	2.13	664	10	1.89
							LAST: 128.11							
CBA1M9	(A)	1.09	1.35	687	217	1.10	130.00	CBA1N9	(A)	2.66	3.23	1,537	--	3.69 *
CBAXP9	(E)	1.05	1.35	3,926	21	1.22	130.01	CBAXQ9	(E)	2.66	3.28	599	--	3.68 *
CBA2G9	(A)	0.53	0.70	1,863	20	0.66	132.00	CBA2H9	(A)	4.07	4.69	188	22	4.89
CBA2G7	(E)	0.50	0.70	3,125	18	0.69	132.01	CBA2H7	(E)	4.06	4.69	1,389	--	5.12 *
CBAG47	(A)	0.23	0.31	3,513	300	0.34	134.00	CBAG57	(A)	5.70	6.38	227	20	6.32
CBA6Z8	(E)	0.20	0.30	2,965	--	0.27 *	134.01	CBA7F8	(E)	5.64	6.39	2,399	--	6.81 *
CBAKV7	(A)	0.10	0.14	4,397	--	0.12 *	136.00	CBAKW7	(A)	7.50	8.35	242	--	8.70 *
CBAKH7	(E)	0.07	0.14	1,750	--	0.13 *	136.01	CBAKI7	(E)	7.48	8.23	165	--	8.65 *
CBAXN9	(A)	0.04	0.10	4,273	--	0.06 *	138.00	CBAXO9	(A)	9.46	10.31	1	--	10.66 *
CBAKJ7	(E)	0.01	0.10	405	--	0.06 *	138.01	CBAKK7	(E)	9.38	10.23	330	--	10.58 *



<b>CBAKX7</b> 128.01 CALL OPTION EXPIRING 15-AUG-2024 Monthly   7 days to expiry	<b>CBAKY7</b> 128.01 PUT OPTION EXPIRING 15-AUG-2024 Monthly   7 days to expiry
<b>CBA1M9</b> 130.00 CALL OPTION EXPIRING 15-AUG-2024 Monthly   7 days to expiry	<b>CBA1N9</b> 130.00 PUT OPTION EXPIRING 15-AUG-2024 Monthly   7 days to expiry

1. What do I mean when I say “either side of the money”? Basically just close to at-the-money.
2. Which options are in-the-money? Which are out-of-the-money? In the screengrab, the last traded price of CBA was  $S = \$128.11$ , so calls with a strike less than this are in-the-money and with a strike greater than this are out-of-the-money. Puts with a strike greater than this are in-the-money and with a strike less than this are out-of-the-money.
3. What is your profit if you took 10 just out-of-the-money calls and CBA closed at \$135 in 7 days? What about at \$125? What if you wrote the calls? What about for puts? Use the last traded prices. Plot profit diagrams. The just out-of-the-money calls have a strike of  $K = 130$  and last traded premium of  $C = \$1.10$  AUD (I’m referring to the American option series here). The contract multiplier (size) is  $m = 100$  so 1 option contract represents a total exposure of  $F = 100K = \$13,000$  AUD. If you took  $h = 10$  options it will cost you  $hCm = 10 \times 1.10 \times 100 = \$1,100$  AUD. If CBA closed at  $S_T = \$135$  on expiry, you’re in-the-money so your profit is  $h(S_T - K)m - hCm = \$3,900$  AUD. If CBA closes at  $S_T = \$125$  AUD then you’re out-of-the-money and it only cost you the premium. I’m confident that you can work it out for puts and plot payoff/profit diagrams by now!
4. What is the intrinsic and time values of the options close to at-the-money? Let’s just consider the American options whose strikes are 130 and 128. The last traded premiums of the calls with these strikes are 1.10 and 2.20. The call with a strike of 130 is out-of-the-money so its intrinsic value is 0 and hence its premium is all time value. The call with a strike of 128 has intrinsic value of 0.11 and hence time value of  $2.2 - 0.11 = 2.09$  (note that it’s pretty well at-the-money). I’m sure you can work it out for the puts.

**Question 7.** Using the data and market outcomes from Question 5, what would be your final position if you held 500 CBA shares and insured it with just out-of-the-money options? What about if you wrote just-in-the-money call options? This is just repeating the above questions on the SSE and SZSE options.

**Question 8.** What is the relation between the prices  $C$  and  $P$  of European call and put options over the same underlying asset and with expiry  $T$  if their strike price  $K$  equals the fair price  $X$  of a futures/forward contract also with maturity  $T$  and over the same underlying asset? Ignoring dividends, the fair forward price is given by  $X = e^{rT}S$ . The question assumes that  $K = X = e^{rT}S$ . Now use put-call parity to see that

$$\begin{aligned} C - P &= S - e^{-rT}K \\ &= S - S = 0. \end{aligned}$$

Hence, if  $K$  is equal to the theoretically fair forward price, the call and put premiums are equal.

**Question 9.** 1. What is the payoff of a portfolio consisting of 1 long European put with strike  $K$  and expiry  $T$ , and 1 stock with price  $S$ ? The payoff is  $S_T + \max\{0, K - S_T\}$ . If  $S_T < K$  then the payoff is  $K$ . If  $S_T > K$  then the payoff is  $S_T$ . If  $S_T = K$  then the payoff is  $S_T = K$ .

2. What is the payoff of a portfolio consisting of 1 long European call also with strike  $K$  and expiry  $T$ , and  $e^{-rT}K$  invested at the risk-free rate  $r$ ? The payoff is  $K + \max\{0, S_T - K\}$ . If  $S_T < K$  then the payoff is  $K$ . If  $S_T > K$  then the payoff is  $S_T$ . If  $S_T = K$  then the payoff is  $S_T = K$ .

3. What can you say about the values of these portfolios? What do you get? Both portfolios have the same payoff so by the law of one price they must have the same value:

$$P + S = C + e^{-rT}K,$$

which is just put-call parity. So this is another way of deriving put-call parity.

**Question 10.** 1. Why are American options worth at least as much as their European counterparts over the same underlying asset with the same strike price and time to expiry? Because American options can be exercised to realise their intrinsic value anytime but for European options this can only occur at expiry.

2. Why are American options worth at least their intrinsic value? Because they can be exercised to realise their intrinsic value at anytime, so this amount is always there to be realised for an American option.
3. We claimed that American calls cannot be worth more than the underlying asset and American puts cannot be worth more than the strike price:

$$C^{\text{Am}} \leq S \quad \text{and} \quad P^{\text{Am}} \leq K.$$

Use no-arbitrage arguments to show these two inequalities.

- Suppose  $C^{\text{Am}} > S$ . Then you can buy the asset for  $S$  and short the call for  $C^{\text{Am}}$ . But at expiry if  $S_T < K$  then the option expires out-of-the-money and you can sell the asset for  $S_T$ . If  $S_T > K$  then the option expires in-the-money and you get exercised, meaning you sell the asset for  $K$ . Either way, you get positive cashflows upfront and at expiry.
- Suppose  $P^{\text{Am}} > K$ . In this case you short the option and invest  $Ke^{-rT}$  at the risk-free rate  $r$ . At expiry if  $S_T < K$  the option expires in-the-money and you get exercised, meaning you buy the asset for  $K$  using the proceeds of your investment, and you can sell the asset for  $S_T$ . If  $S_T > K$  then the option expires out-of-the-money and you get  $K$  from your investment. Again, either way, you get positive cashflows upfront and at expiry.

**Question 11.** In the following scenarios, state which European option pricing bounds (or put-call parity) are violated and outline how you can construct an arbitrage trade to take advantage of the mispricing. In each scenario, the price of the underlying is  $S = \$53$  and the risk-free rate is  $r = 2\%$ .

1. A 6 month European call with strike  $K = \$50$  is quoted at  $C = \$3.10$ . You can check that  $\max\{0, S - e^{-rT}K\} \leq C$  fails.
2. A 6 month European put with strike  $K = \$55$  is quoted at  $P = \$1.20$ . You can check that  $\max\{0, e^{-rT}K - S\} \leq P$  fails.
3. A 3 month European call is quoted at  $C = \$1.20$  while the put is  $P = \$2.50$ . The strike price for both is  $K = \$55$ . Here put-call parity fails.