Lecture 3: Parity Conditions

Reading: Eun & Resnick Ch. 6 (10th ed.)

Lecture Outline

- Introduction to Parity Conditions
- Purchasing Power Parity (PPP)
- Absolute and Relative PPP
- Real Exchange Rate
- Interest Rate Parity (IRP)
- International Fisher Effect (IFE)
- Forward Rate Unbiasedness Condition

Introduction

- Managers of multinational firms, international investors, importers and exporters, and government officials must deal with these fundamental issues:
  - Are changes in exchange rates predictable?
  - How are exchange rates related to interest rates?
  - What, at least theoretically, is the “proper” exchange rate?

- Our approach to answering these questions and more is to describe the economic fundamentals of international finance, known as parity conditions.

Parity Conditions

- Parity Conditions provide an intuitive explanation of the movement of prices and interest rates in different markets in relation to exchange rates.

- Parity conditions rely on **ARBITRAGE** to hold.

- The derivation of these conditions requires the assumption of Perfect Capital Markets (PCM).
  - no transaction costs
  - no taxes
  - complete certainty

Purchasing Power Parity (PPP)

- PPP is based on the notion of **arbitrage** across goods markets and the basic building block of PPP is given by the Law of One Price (LOP)
- LOP states that the price of an identical good should be the same in all markets (assuming no transactions costs).
  - Otherwise, one could make profits by buying the good in the cheap market and reselling it in the expensive market.

- Basic Idea: A can of coke should <u>cost the same</u> in Australia as it does in London when expressed in a <u>common currency</u> (either in AUD or GBP).

The Law of One Price

- States that a while product's price may be stated in different currency terms, but the price of the product in a common currency should remain the same.
  - Comparison of prices would only require conversion from one currency to the other:

London price in AUD terms

$$P_{\$} = \boxed{S_{\$/£} \times P_{£}}$$

  - Conversely, the exchange rate could be deduced from the relative local product prices:

$$S_{\$/£}^{\mathrm{PPP}} = \frac{P_{\$}}{P_{£}}$$

Law of One Price Example

- $P_{\text{wheat, Aust}}$ = $4/bushel ***and*** $P_{\text{wheat, UK}}$ = £2.5/bushel
- Spot rate (AUD/£) = 1.70

$$\boldsymbol{P_{Aust}^{wheat} = S_{\$/£} \times P_{UK}^{wheat}}$$

- A$ equivalent price of wheat <u>in the UK</u> is $^{AUD1.70}/_{£} \times £2.50$

  = AUD4.25/bushel

- **Implication:** The demand for Australian wheat will increase forcing up its price. The price of UK wheat will drop.

Absolute PPP

- A less extreme form of the Law of One Price is the **ABSOLUTE PPP** which says that the price of a basket of goods would be the same in each market.
- The PPP exchange rate between the two countries would then be:

Absolute PPP:

$$S_t^{A/B} = \frac{PI_{A,t}}{PI_{B,t}}$$

$PI_{A,t}$ ($PI_{B,t}$) are the price indices of the two countries (e.g., consumer price index) at time *t*

- The Big Mac Index is an example of this variant of PPP.

NB: $^{A}/_{B}$ $- A\ is\ the\ currency\ in\ numerator; B\ currency\ in\ denominator\ (i.e., base\ currency)$

The Big Mac Index

- The most famous test of the Law of One Price is *The Economist* magazine's Big Mac Hamburger standard.

Source: *The Economist*

- The Big Mac can be viewed as a standardized bundle of goods that are individually traded, so we would expect it will *only* hold approximately.

The Big Mac Index
Jan. 2024

IMAGE: THE ECONOMIST

The Big Mac index

Country		2000 —— 2024	Under/over valued, %
Switzerland	Franc		43.5
Norway	Krone		25.5
Uruguay	Peso		23.7
Euro area	Euro		3.1
Sweden	Krona		3.1
Costa Rica	Colón		0.4
Britain	Pound		0.4
Denmark	Krone		0.0
United States	US$		BASE CURRENCY
Sri Lanka	Rupee		0.0
Canada	C$		-2.4
Mexico	Peso		-8.7
Colombia	Peso		-10.6
Australia	A$		-10.8
Saudi Arabia	Riyal		-11.0
New Zealand	NZ$		-12.0
Poland	Zloty		-12.7
Singapore	S$		-12.9
Venezuela	Bolívar		-13.3
UAE	Dirham		-13.9

Sources: McDonald's; Refinitiv Datastream; IMF; Eurostat; LebaneseLira.org; Liban; *The Economist* Note: All prices include tax

How well does it work?

- Violations of **Absolute PPP** occur in the short run, but it tends to hold in the long run (several years).
  - Describes a long-run phenomenon due to the fast speed adjustment in exchange rates relative to the “sticky” nature of goods prices.
    - Over **short** horizons, exchange rates are too volatile, and goods prices are too sticky, for this theory to work well in the short-term.
  - The presence of imperfections such as taxes, transaction costs, import tariffs and quotas.
  - Basket of commodities might be different for different countries. Also, it will **not** hold if the price indices do NOT have the same weights across countries.
  - The inclusion of non-tradeable goods and services in price indices.

Relative PPP

- **Relative PPP** claims that exchange rate movements should exactly offset any *inflation differential* between two countries:

Relative PPP:

$$\frac{S_{t+1}^{A/B}}{S_t^{A/B}} = \frac{1+\pi_A}{1+\pi_B}$$

Now, subtract 1 from both sides

$$\frac{S_{t+1}^{A/B} - S_t^{A/B}}{S_t^{A/B}} = \frac{\pi_A - \pi_B}{1+\pi_B}$$

- We can also write: $S_{t+1}^{PPP} = S_t^{A/B} \times \frac{1+\pi_A}{1+\pi_B}$

$\pi : inflation$

Relative PPP (2)

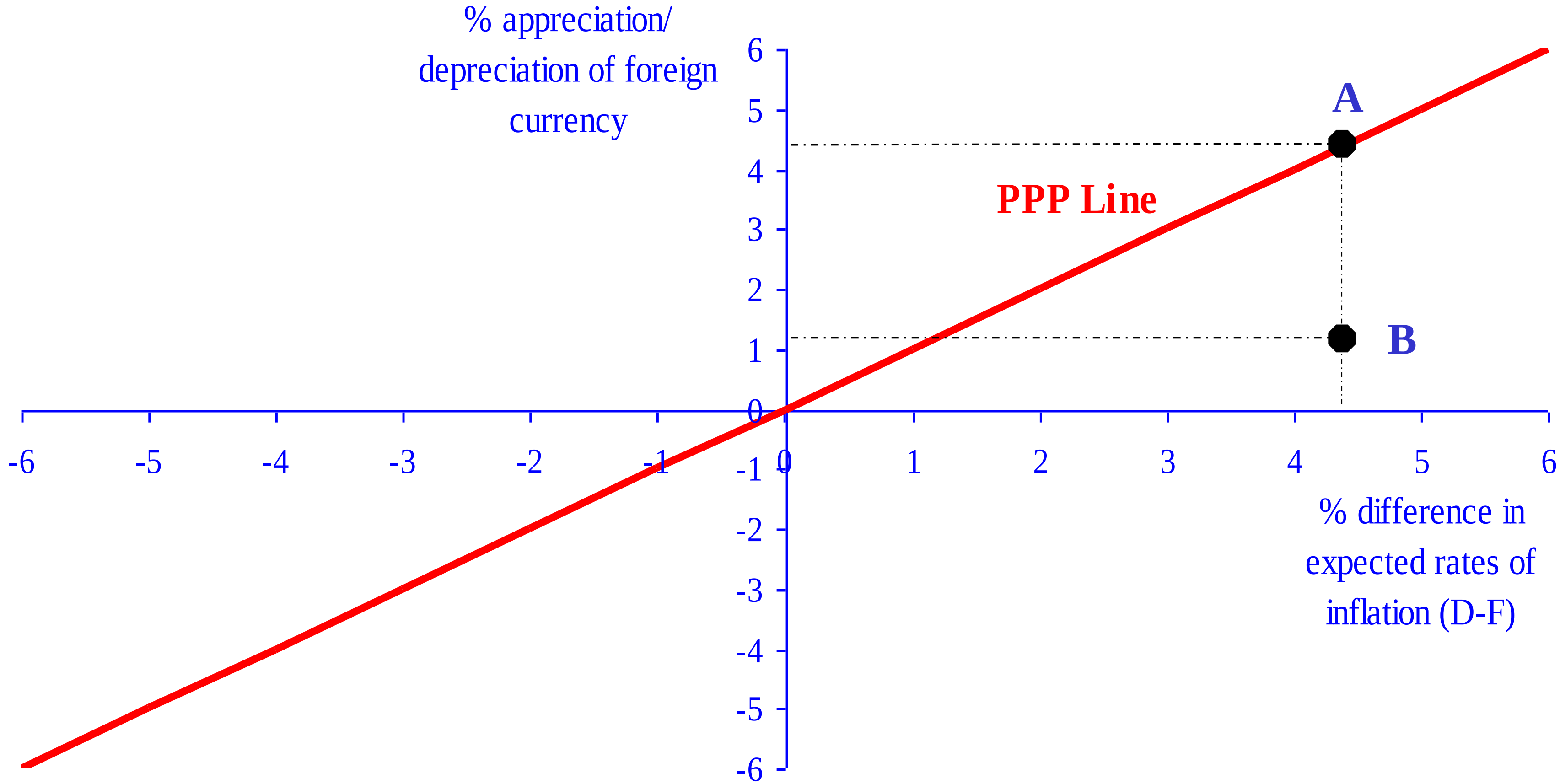
- Given inflation rates of 1.9% and 3% in Australia and the UK respectively, what is the prediction of PPP with regards to $A/GBP exchange rate?

$$\frac{S_t - S_{t-1}}{S_{t-1}} = \frac{\pi_A - \pi_B}{1 + \pi_B}$$

$$= \frac{0.019 - 0.03}{1 + 0.03} = -0.010678 = -1.07\%$$

- *The general implication of relative PPP is that countries with **high** rates of inflation will see their currencies **depreciate** against those with **low** rates of inflation*

Relative PPP



Relative PPP

- Applications of Relative PPP:

    1. Forecasting future spot exchange rates.

    2. Calculating appreciation in “**real**” exchange rates. This will provide a measure of how expensive a country’s goods have become (relative to another country’s).

Forecasting Future Spot Rates

- Suppose the ¥/$ spot exchange rate and expected inflation for Japan and Australia are:

$$S_{¥/\$,t_0} = ¥87.86/\$; \pi_{Aus} = 1.9; \pi_{Japan} = 1\%$$

- What is the expected ¥/$ exchange rate if relative PPP holds?

$$S^{PPP}_{¥/\$,t_1} = S_{¥/\$,t_0} \cdot \left(\frac{1+\pi_{¥}}{1+\pi_{\$}}\right)$$

$$= (87.86) \cdot \left(\frac{1.01}{1.019}\right) = 87.08\ ¥/\$$$

The Real Exchange Rate

- The real exchange rate measures deviations from PPP.
  - That is, changes in the spot exchange rate that do not reflect differences in inflation rates between the two currencies in question.

Real Exchange Rate:

$$E = \frac{S_{t+1}^{Actual}}{S_{t+1}^{PPP}}$$

E = 1 + % over- (under) valuation
of denominator currency

Note: We will re-visit this issue in Lecture 6.

Real Exchange Rate

- Appreciation in the real exchange rate measures deviations from PPP.
  - When $E = 1$, the denominator currency is valued correctly. *The competitiveness of this country is unaltered.*
  - When $E < 1$, the denominator currency is undervalued. Products from the other country seem expensive relative to the base year. That is, *the competitiveness of the denominator country improves.*
  - When $E > 1$, the denominator currency is overvalued. Products from the other country seem cheap relative to the base year. That is, *the competitiveness of the denominator country deteriorates.*

Interest Rate Parity

- Interest rate parity (IRP) is an arbitrage condition that provides the linkage between the foreign exchange markets and the international money markets.

$$\boxed{\frac{F_{t,t+1}^{A/B}}{S_t^{A/B}} = \frac{1+i_A}{1+i_B}}$$

$$\boxed{\frac{F_{t,t+1}^{A/B}}{S_t^{A/B}} - 1 = \frac{1+i_A}{1+i_B} - 1} \Rightarrow \boxed{\frac{F_{t,t+1}^{A/B} - S_t^{A/B}}{S_t^{A/B}} = \frac{i_A - i_B}{1+i_B}}$$

- In general, the currency trading at a forward premium (discount) is the one from the country with the lower (higher) interest rate.

Interest Rate Parity

- Suppose Gattinara Corp. has funds that it can place in the money market for 3 months. The options are
  - Invest in Australia
  - Invest in foreign currency denominated securities

- The returns on an Australian investment is given by

$$\$A\left(1+\frac{i_{\$A}}{4}\right)$$

Interest Rate Parity

- The return on a foreign investment is given by

$$\frac{1}{S_{\$/CHF}}\left(1+\frac{i_{CHF}}{4}\right)\times F_{3(\$/CHF)}$$

- Gattinara Corp. will be **indifferent** between the two investment opportunities if

$$\frac{1}{S_{\$/CHF}}\left(1+\frac{i_{CHF}}{4}\right)\times F_{3(\$/CHF)}=\$A\left(1+\frac{i_{\$A}}{4}\right)$$

Interest Rate Parity

- **Basic idea:** Two alternative ways to transform from currency A at time 0 to currency B at time 1 should earn the same return.

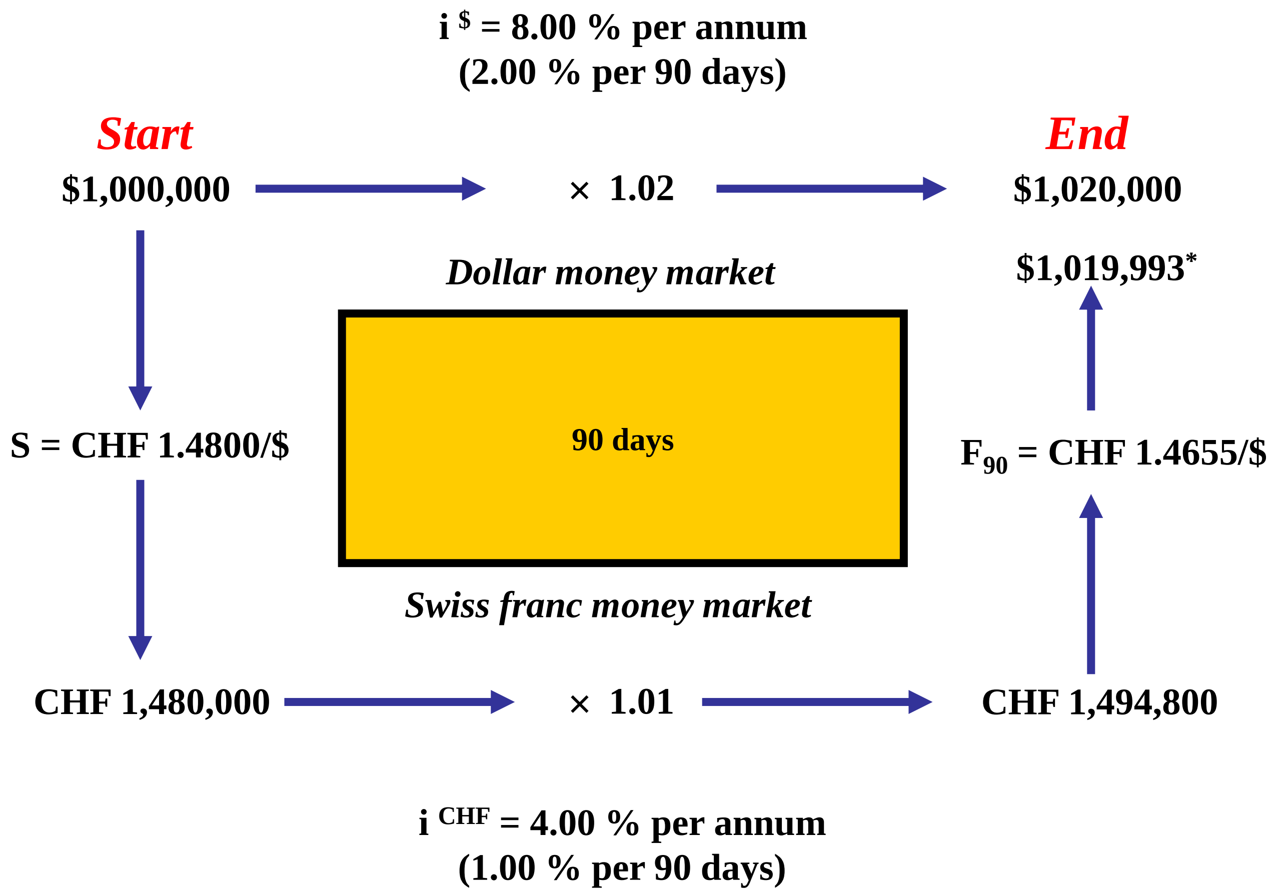
- Thus, an AUS investor with $1 million to invest should be indifferent between holding dollar-denominated securities for 90 days earning 8.00% per annum ***and*** holding Swiss franc-denominated securities of similar risk and maturity earning 4.00% per annum, when "cover" against currency risk is obtained with a forward contract.

An Example

- Suppose the 3-month money market rate is 8% p.a. (2% for 3-months) in the U.S. and 4%p.a. (1% for 3-months) in Switzerland, and the spot exchange rate is CHF1.48/$.

- The 3-month forward rate must be CHF1.4655/$ to prevent arbitrage opportunities (i.e., interest rate parity must hold).

An Example (2)



* rounding error

Why Parity Holds?

- This must hold by **arbitrage**. Otherwise, riskless profits could be made. This is known as covered interest arbitrage (CIA) and occurs whenever IRP does **NOT** hold.

  CIA **can** involve the following steps:

  - Borrow the domestic currency;
  - Exchange the domestic currency for the foreign currency in the spot market;
  - Invest the foreign currency in an interest-bearing instrument; and then
  - Sign a forward contract to “lock in” a future exchange rate at which to convert the foreign currency proceeds back to the domestic currency.

Example #2

- The annual interest rate in the AUS and UK are 5% and 8% respectively. The current spot rate is \$1.50/£ and the 1 year forward rate is \$1.48/£. Can arbitrage profits be made?

$$\frac{F_t^{A/B} - S_t^{A/B}}{S_t^{A/B}} = \frac{i_A - i_B}{1 + i_B} \longrightarrow \frac{1.48 - 1.50}{1.50} = \frac{0.05 - 0.08}{1 + 0.08}$$

$$\mathbf{-0.0133 \neq -0.0278}$$

An Example #2

1 **Borrow** $1,000,000 for 1 year at $i_\$ = 5\%$ → <$1,050,000>

2 **Convert Spot** at $1.50/£ → £666,667

3 **Invest** for 1 year at $i_£ = 8\%$ → £720,000

4 **Cover Forward** at $1.48/£ → $1,065,600

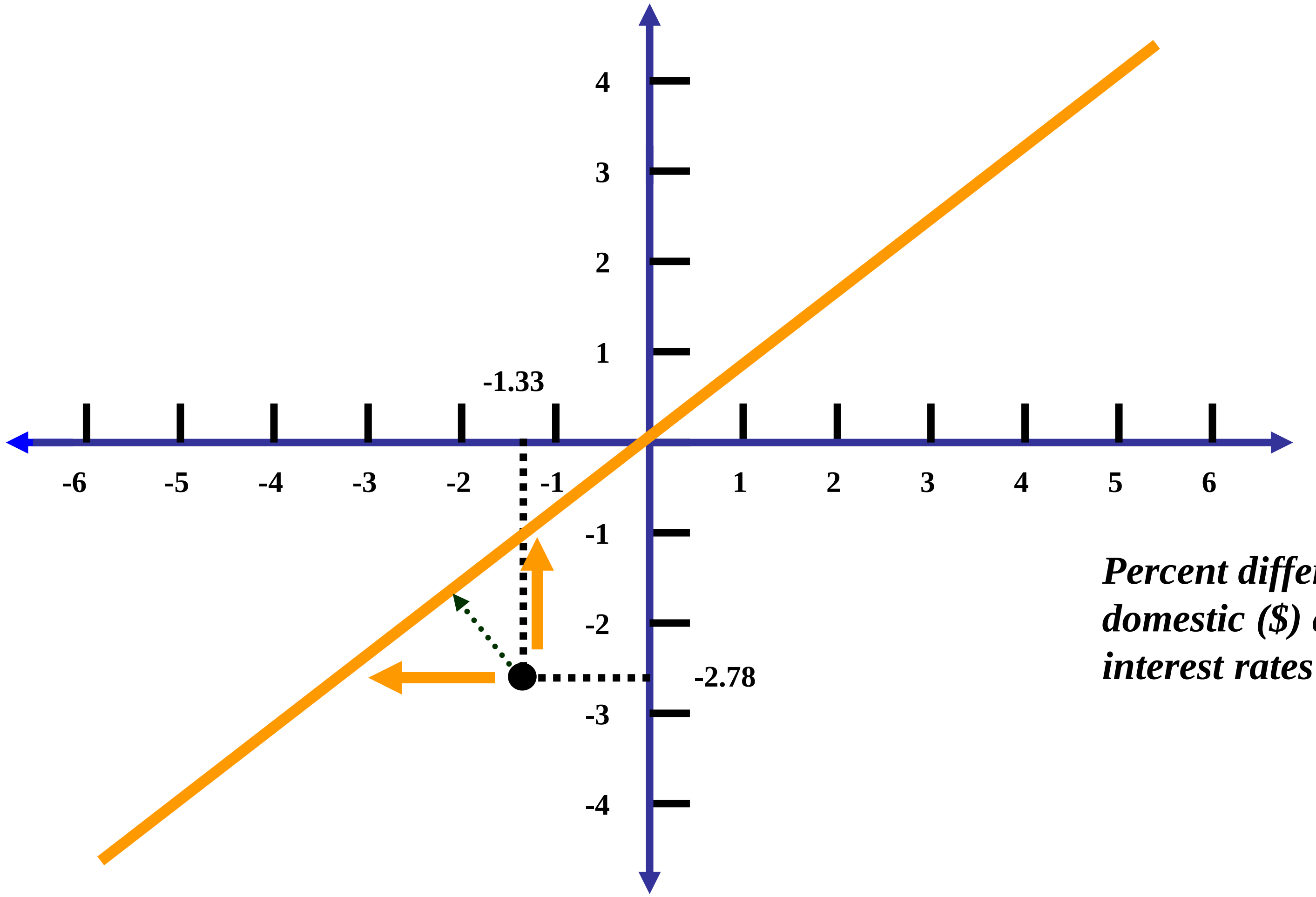
Profit = $15,600

Covered Interest Arbitrage

- Covered interest arbitrage (CIA) should continue until interest rate parity is re-established, because the arbitrageurs are able to earn risk-free profits by repeating the cycle.
- But their actions nudge the foreign exchange and money markets back toward equilibrium:

1. *Purchase of Pounds in the spot market and sale of ~~$s~~ pounds in the forward market narrow the premium on forward pounds.*
2. *The demand for pound-denominated securities causes the pound interest rates to fall, while the higher level of borrowing in Australia causes dollar interest rates to rise.*

Interest Rate Parity and Equilibrium



Percent difference between domestic ($) and foreign (£) interest rates

The Fisher Effect

- The Fisher effect (also called Fisher-closed) postulated by Irving Fisher (1930) states:

$$(1+i) = (1+r) \times (1+\pi)$$

$$i = r + \pi + r\pi$$

- This relation is often presented as a linear approximation stating that the nominal interest rate (i) is equal to a *real* interest rate (r) plus ***expected inflation*** (π):

$$i \approx r + \pi$$

- Applied to two different countries, like the Australia and Japan, the Fisher Effect would be stated as:

$$i^{\$} \approx r^{\$} + \pi^{\$} \qquad i^{¥} \approx r^{¥} + \pi^{¥}$$

The International Fisher Effect

- The International Fisher Effect (also called Fisher-open or *Uncovered Interest rate parity condition*) states that *the spot exchange rate should change to adjust for differences in interest rates between two countries:*

$$\boxed{\frac{S_{t+1}^{A/B}}{S_t^{A/B}} = \frac{1+i_A}{1+i_B}}$$

The International Fisher Effect

- The Fisher (closed) effect applied to two different countries say Australia and Japan would be:

(1) $$(1+i_\$) = (1+r_\$)(1+\pi_\$)$$

(2) $$(1+i_¥) = (1+r_¥)(1+\pi_¥)$$

- Dividing (1) by (2), we get:

(3) $$\frac{1+i_\$}{1+i_¥} = \underbrace{\left(\frac{1+r_\$}{1+r_¥}\right)}_{=1}\left(\frac{1+\pi_\$}{1+\pi_¥}\right)$$

The International Fisher Effect

- If ***real*** interest rates are equalized across countries, then for equation (3) we get $r_\$ = r_¥$:

(4) $$\frac{1+i_\$}{1+i_¥} = \frac{1+\pi_\$}{1+\pi_¥}$$

Subtract 1 from both sides

(5) $$\frac{i_\$ - i_¥}{1+i_¥} = \boxed{\frac{\pi_\$ - \pi_¥}{1+\pi_¥}}$$

Remember this?

International Fisher:

$$\frac{S_{t+1} - S_t}{S_t} = \frac{i_A - i_B}{1+i_B}$$

E(S$_{t+1}$)

Uncovered Interest Rate Parity

- A deviation from CIA is uncovered interest arbitrage, **UIA**, wherein investors borrow in currencies exhibiting relatively low interest rates and convert the proceeds into currencies which offer higher interest rates

- The transaction is "uncovered" because the investor does **not** sell the currency forward, thus remaining uncovered (or unprotected) to any risk of the currency deviating

UIA – An example

- Suppose you observe the following quotes:

$$i_{¥} = 0.4\%\ \text{p.a.},\quad i_{\$} = 5\%\ \text{p.a.}$$

$$S_0 = 120\ ¥/\$,\quad F_{0,360} = 114.7429\ ¥/\$$$

- You check IRP first, but find no CIA opportunity:

$$\frac{F_{0,360}}{S_0} \overset{?}{=} \frac{1+i_{¥}}{1+i_{\$}} \Rightarrow \frac{114.7429}{120} \overset{?}{=} \frac{1.004}{1.05}$$

0.9562 0.9562

UIA – An example

- Suppose you forecast the spot rate next year to remain unchanged at 120 ¥/$
- Even though you *cannot* make a riskless profit through CIA, given your estimation of the future spot rate, you can enter into an uncovered interest "arbitrage"
- In an uncovered position, rather than locking in a forward rate today, you take your chances and hope you're right about your assessment of the future spot rate

$$\frac{S_{360}}{S_0} \stackrel{?}{=} \frac{1+i_{¥}}{1+i_{\$}} \Rightarrow \frac{120}{120} > \frac{1.004}{1.05} \quad \Rightarrow \textbf{Borrow Yen!}$$

UIA – An example

Investors borrow yen at 0.40% per annum

Start | ***End***

¥ 10,000,000 → **× 1.004** → **(¥ 10,040,000) Repay**

¥ 10,500,000 Earn

¥ 460,000 Profit

Then exchanges the yen proceeds for US dollars,

S =¥ 120.00/$

investing in US dollar money markets for one year

Japanese yen money market

360 days

US dollar money market

S_{360} = ¥ 120.00/$

$ 83,333 → **× 1.05** → **$ 87,500**

Invest dollars at 5.00% per annum

Forward Exchange Expectations

- The Forward Rate as an *Unbiased Predictor* of the Future Spot Rate
  - Some forecasters believe that for the major floating currencies, foreign exchange markets are "efficient" and forward exchange rates are unbiased predictors of future exchange rates.
  - The **forward exchange hypothesis** states that *the forward exchange rate, quoted at time* ***t*** *for delivery at time* ***t+1****, is equal to the expected value of the spot exchange rate at time* ***t+1****.*

$$F_{t\ t+1} = E_t(S_{t+1})$$

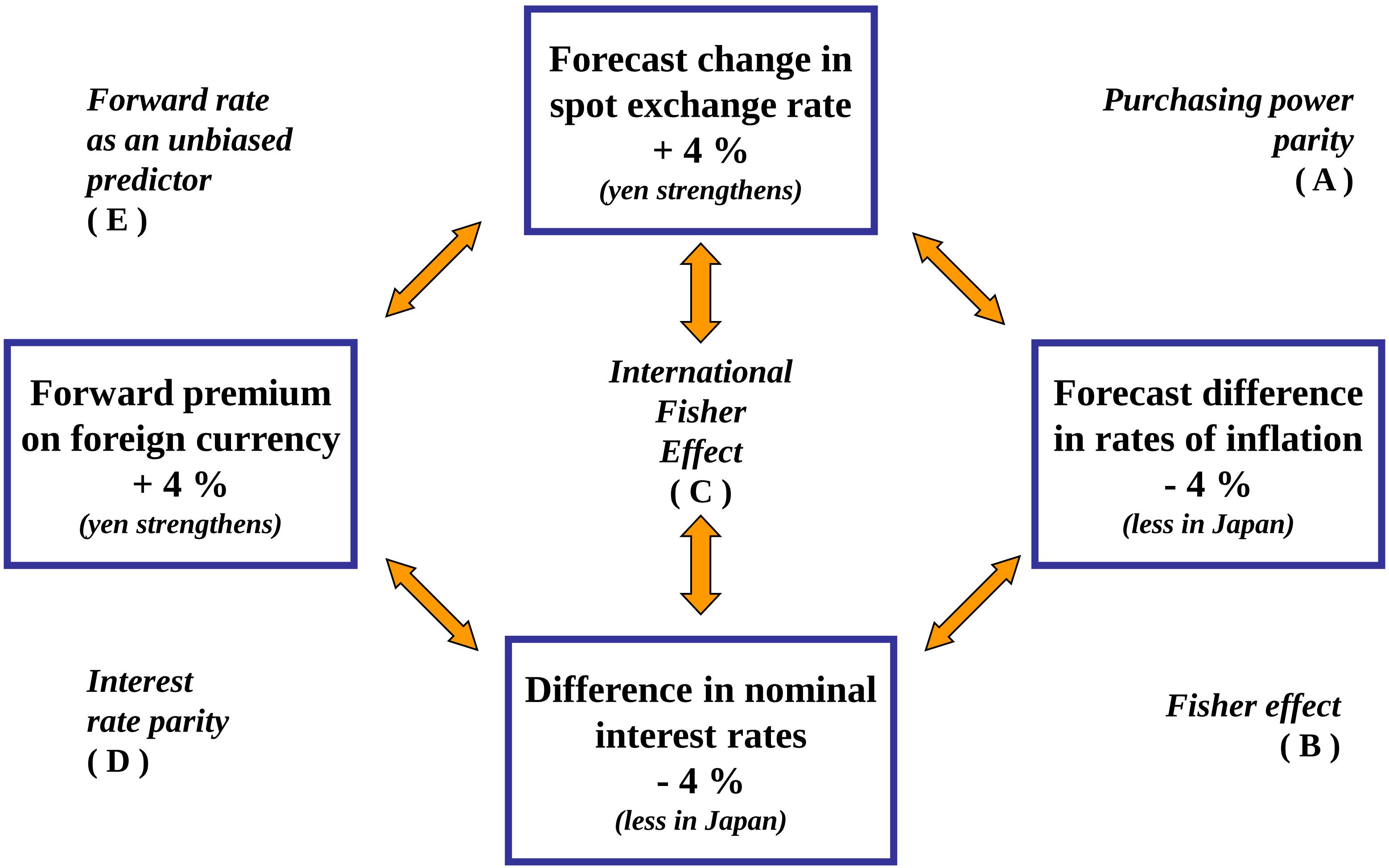
  - The Forward rate is said to be an *unbiased* predictor. Unbiased prediction means that the forward rate will, on average, overestimate and underestimate the actual future spot rate in equal frequency and degree. It implies that the expected forecast error is zero.

Prices, Interest Rates & Exchange Rates in Equilibrium

- Suppose expected inflation in Japan is 1% and expected inflation in the U.S. is 5%. The current spot exchange rate is ¥104.00/$ and the one-year forward rate is ¥100.00/$.

- See that all of our parity conditions predict that the ¥ will appreciate 4% relative to the $ over the next year.

Prices, Interest Rates & Exchange Rates in Equilibrium



Glossary of Terms

- Law of One Price – Identical goods should trade for the same price in different markets in the absence of “frictions”.
- Relative PPP – The change in the exchange rate should reflect the change in the price levels in two markets.
- Interest Rate Parity – The interest rate differential between two currencies should equal the forward premium (or discount).
- International Fisher Effect – The interest rate differential between two currencies should equal the expected change in the exchange rate.