FINM3405 Derivatives and risk management

Tutorial Sheet 12: VaR and ES
Suggested solutions

October 27, 2024

Question 1. Explain in words what VaR and ES are measuring, what are the potential shortfalls of VaR, and why ES may be advantageous over VaR.

Quite literally, VaR with tail risk $\alpha\%$ or level of confidence $p\% = 1 - \alpha$ is the outcome "not as bad as" the worst $\alpha\%$ of potential outcomes. Equivalently it is the outcome "worse than" the best $p\%$ of potential outcomes.

From a financial perspective VaR is often interpreted as the smallest loss with tail risk $\alpha\%$, or equivalently as the largest loss with confidence $p\%$.

ES with tail risk $\alpha\%$, or level of confidence $p\%$, is the mean or average or expected loss of the worst $\alpha\%$ of losses.

The BIS's Explanatory note on the minimum capital requirements for market risk explains this as follows:

The revised internal models approach replaces VaR and stressed VaR with a single ES metric that is calibrated to a period of significant market stress. Two features of this new metric address deficiencies in the Basel 2.5 framework:

- ES captures the tail risks that are not accounted for in the existing VaR measures. While VaR calculates the maximum potential loss at a certain confidence level (eg a 97.5% VaR measures a loss that is expected to be exceeded only 2.5% of the time), ES calculates the average loss above a certain confidence level (eg a 97.5% ES measures the average of the worst 2.5% of losses).[8]

In other words, whereas VaR calculates the losses at a single cut-off point in the distribution (eg 97.5%), ES looks at the average of any loss that exceeds the cut-off point in the distribution. Therefore, if the same cut-off point is used for VaR and for ES, the value of ES will be higher than the value of VaR. The difference between ES and VaR outcomes increases in cases of fat-tailed distributions. In the revised market risk framework, the 97.5th percentile ES is roughly equivalent to the 99th percentile VaR used in Basel 2.5.

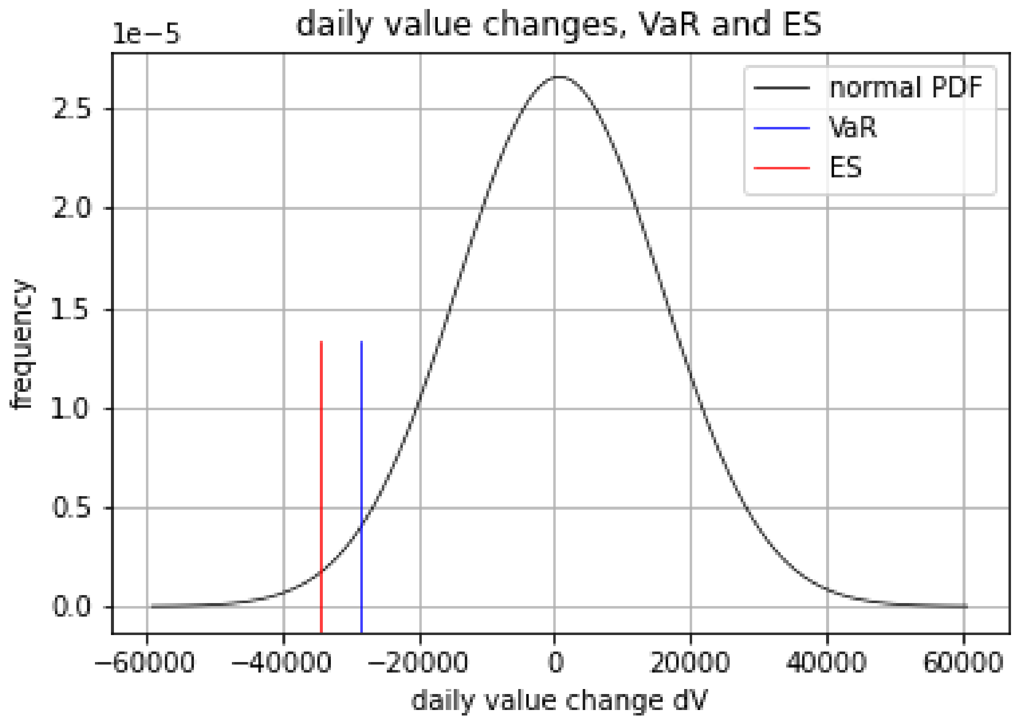
Question 2. How may VaR and ES be inaccurate when quantifying a share portfolio's market risk under the assumption of normally distributed returns? How might one get around this problem?

Financial market returns (and the distribution of profits/losses) have spiked means, narrow shoulders, fat tails, and extreme losses, relative to the normal distribution. Hence, VaR and ES under the assumption of normally distributed returns may underestimate the true VaR and ES. One way around this is to model returns (profits/losses) with a probability distribution that better models the distribution of returns (profits/losses), such as the Student's t-distribution. The other way is to use the nonparametric approach that does not assume a particular distribution, but instead directly uses historical data and ranking/sorting. Effectively, the nonparametric approach works directly with the distribution of returns (profits/losses) embedded in the historical data.

Question 3. Suppose returns are normally distributed. You hold a \$1m portfolio whose mean daily return is $\mu = 0.08\%$ and whose standard deviation (volatility) in daily returns is $\sigma = 1.5\%$. What is the daily $\alpha = 2.5\%$ (sometimes also referred to as the 97.5%) VaR and ES of your portfolio? Also calculate the daily VaR and ES for confidence levels of 95% and 99% (so $\alpha = 5\%$ and $\alpha = 1\%$). Plot the normal PDF modelling the distribution in the daily changes in your portfolio value (profits/losses) as well as the VaR and ES.

I'll do it for the case of $\alpha = 2.5\%$. I calculate that

$$\mu_{\mathrm{d}V} = \$800, \quad \sigma_{\mathrm{d}V} = \$15{,}000, \quad \mathrm{VaR}_{0.025} = \$28{,}599.46, \quad \mathrm{ES}_{0.025} = \$34{,}267.04.$$



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alpha = 0.025
z = norm.ppf(alpha)
V = 1000000 # $1m
mu = 0.0008
sigma = 0.015
mu_dV = mu*V
sigma_dV = sigma*V
VaR = -mu_dV - z*sigma_dV
ES = -mu_dV + (sigma_dV/alpha)*norm.pdf(z)
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Question 4. Continue to assume returns are normally distributed. You hold a portfolio of $Q_1 = 100$ units invested in share 1 whose current price is $P_1 = 50$, and $Q_2 = 150$ units invested in share 2 whose current price $P_2 = 35$. Suppose the daily expected returns are zero and the daily standard deviations in returns are $\sigma_1 = 1\%$ and $\sigma_2 = 1.3\%$. Calculate the individual 99% daily VaRs, the portfolio's worst case daily 99% VaR, and the gains from diversification if the correlation in returns is $\rho = 0.5$. Plot the normal PDF modelling the distribution in the daily changes in your portfolio value (profits/losses) as well as the VaR and ES, and the worst case VaR.

I calculate that:

$$\begin{aligned} \mathrm{VaR}_{0.01,1} &= \$116.32, \\ \mathrm{VaR}_{0.01,2} &= \$158.77, \\ \text{worst case } \mathrm{VaR}_{0.01} &= \$275.09, \\ \text{portfolio } \mathrm{VaR}_{0.01} &= \$239.18, \\ \text{diversification benefits} &= \$35.91. \end{aligned}$$