FINM3405 Derivatives and risk management

Tutorial Sheet 6: Options - Delta hedging, implied volatility, trading strategies

August 30, 2024

In the following questions let $S = \$50$, $r = 0.05$, $\sigma = 0.25$, $T = \frac{1}{2}$ and $q = 0$ unless otherwise stated.

Delta hedging

Question 1. Suppose you wrote 10 ATM call options.

1. How many assets Q should you short or long to be delta hedged? Show the outcomes of your hedging strategy in 1 week if the asset price went both up and down by \$1.5. Using the above parameters I calculate that the delta of an ATM (at-the-money) call is

   $$\Delta_C = \mathcal{N}(d_1) = 0.5908802.$$

   To be delta neutral, noting that $h = -10$ since we wrote the calls, we should hold

   $$Q = -h\Delta_C = 10 \times 0.5908802 = 5.908802$$

   units of the underlying asset, meaning we should buy $Q = 5.908802$ units since $Q > 0$. So, we have a portfolio of $h = -10$ calls and $Q = 5.908802$ units of the asset, whose value is

   $$V = QS + hC = 254.14,$$

   with $C = 4.13$ using the Black-Scholes model.

   - Now suppose the asset price goes down by $\mathrm{d}S = 1.5$. The asset price is now $S = 48.5$ and noting that after 1 week we now have 25 weeks to expiry so $T = \frac{25}{52}$, using the Black-Scholes model I calculate the new call price to be $C = 3.206667$. Our new portfolio value is then

     $$V = QS + hC = 254.51,$$

     so we were fairly well hedged against the fall in the asset price.

- If the change in the asset price is $\mathrm{d}S = 1.5$, we have $T = \frac{25}{52}$, $S = 51.5$ and now $C = 4.9717941$. I calculate that

$$V = QS + hC = 254.59,$$

so again we're well hedged.

2. Consider an additional OTM put option with a strike of $K_1 = 47.5$. How many assets Q and puts k should you long and/or short to be delta and gamma hedged? Again, show the outcomes of your hedging strategy in 1 week if the asset price went both up and down by \$1.5. To be delta and gamma neutral we want to hold

$$Q = -h\frac{\Delta_C \Gamma_P - \Delta_P \Gamma_C}{\Gamma_P}$$

units of the asset and

$$k = Q\frac{\Gamma_C}{\Delta_C \Gamma_P - \Delta_P \Gamma_C}$$

units of the new OTM put. I first calculate that:

$$\Delta_C = 0.5908802, \quad \Gamma_C = 0.0439589, \quad \Delta_P = -0.3015428, \quad \Gamma_P = 0.0394281.$$

I then calculate that

$$Q = 9.270745 \qquad \text{and} \qquad k = 11.14914.$$

The original value of our portfolio of $h = -10$ ATM calls, $Q = 9.270745$ assets and $k = 11.14914$ puts is

$$V = QS + hC + kP = 443.04,$$

where I calculate that $P = 1.86598$. The new value when the asset price falls by $\mathrm{d}S = -1.5$ is

$$V = 443.44$$

and when $\mathrm{d}S = 1.5$ it is

$$V = 443.49,$$

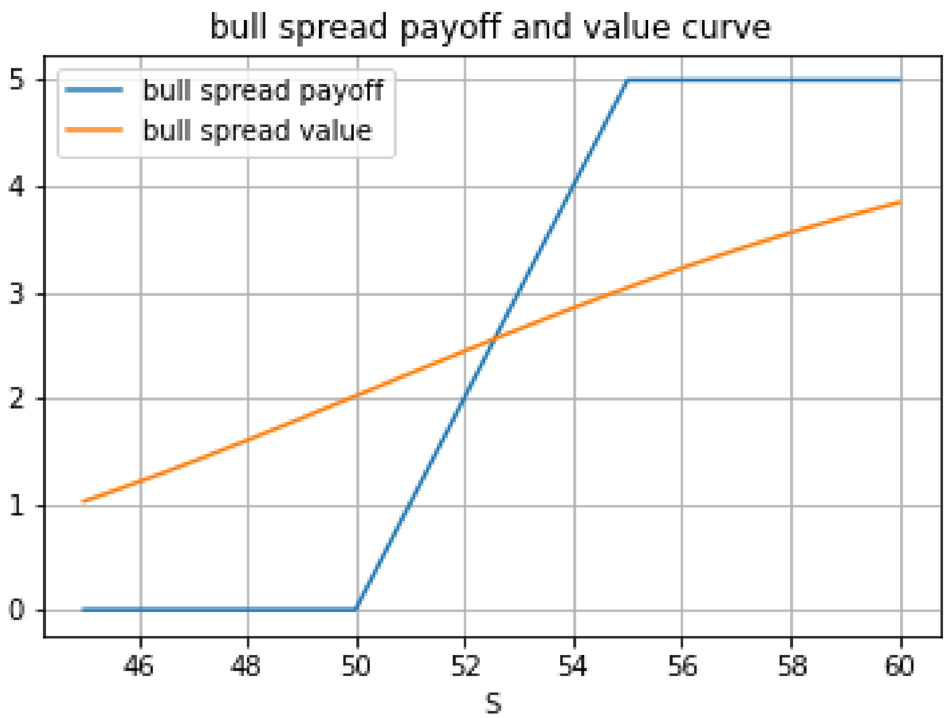
so in either case we're well hedged. *Remark*: Out of interest, when I wrote this question I had the asset price change by only 1.5 up and down, and form above we see that delta-gamma hedging doesn't really look better than delta hedging. But suppose I made the asset price change by say 3 up and down. Then

- Delta hedging: The portfolio value started as above at $V = 254.14$. If $\mathrm{d}S = -3$ then its value is $V = 252.91$. If $\mathrm{d}S = 3$ then its value is $V = 253.19$. In each case we're about a dollar or more out.
- Delta-gamma hedging: The portfolio value started as above at $V = 443.04$. If $\mathrm{d}S = -3$ then its value is $V = 443.47$. If $\mathrm{d}S = 3$ then its value is $V = 443.47$.

So under a larger change in the asset price, delta-gamma hedging shows its benefit.

Question 2. Suppose you wrote an ATM put but with 90 days to expiry. You can assume that there is 360 days in a year. Possibly using Excel, simulate 3 months of daily asset prices under geometric Brownian motion starting at $S = 50$ and calculate the mechanics of a daily dynamic delta hedging strategy. What was your final liquidation value? *Remark*: It should be as easy as writing formulas in only the top two rows of cells and then dragging them down. See the Excel spreadsheet provided.

Question 3. The delta of a portfolio of options is the sum of the delta of each option multiplied by the quantity of each option you hold in the portfolio. Suppose you implemented a bull spread with upper strike $K_2 = 55$, but are now unsure of your view of the market but don't want to close out the position. What is the delta of the position? How many assets Q should you short or long to be delta hedged? How many assets Q and OTM puts k with strike $K_1 = 47.5$ should you short and/or long in order to be delta-gamma hedged? What would be the outcome of each hedging position in 1 week if the asset fell by \$2? A bull spread means you went long 1 (let's assume ATM) call with a low strike $K = 50$ and short the OTM call with higher strike $K_2 = 55$. The basic idea here is you view the bull spread as position of $h = 1$ units in a "single derivative security" with a delta Δ_1 and gamma Γ_1, and proceed with delta and delta-gamma hedging as per usual. The delta of the long ATM call is calculated above as 0.5908802. The delta of the short OTM call is 0.378529. So the bull spread delta is $\Delta_1 = 0.212351$. The gamma of a portfolio is also the sum of the gamma of each option multiplied by the quantity of each option. The gamma of the long ATM call is 0.0439589 and of the short OTM call is 0.0430264, so the bull spread gamma is $\Gamma_1 = 0.0009325$. We see that the bull spread is already nearly gamma neutral (gamma is nearly 0), so we don't expect delta-gamma hedging to show much if any benefit over simple delta hedging. We can also see this gamma neutrality in the premium curve:



The value or premium curve is nearly a straight line (not much curvature).

- Delta hedging: The bull spread's delta is $\Delta_1 = 0.212351$, so noting that $h = 1$, we want to short this many units of the underlying asset. When $S = 50$ the value of the bull spread is $C_{\text{ATM}} - C_{\text{OTM}} = 2.01712$. The value of a portfolio of a bull spread plus short $\Delta_1 = 0.212351$ units of the asset is

$$V = -\Delta_1 S + C_{\text{ATM}} - C_{\text{OTM}} = -8.6.$$

  If the asset price falls by 2 in 1 week then we calculate new call prices and the new value to be

$$V = -\Delta_1 48 + C_{\text{ATM}} - C_{\text{OTM}} = -8.61.$$

- Delta-gamma hedging: From above I calculate the delta of the OTM put to be $\Delta_2 = -0.3015428$ and its gamma to be $\Gamma_2 = 0.0394281$. I also calculate its premium to be $P = 1.86598$. To be delta-gamma neutral we need

$$Q = -h\frac{\Delta_1\Gamma_2 - \Delta_2\Gamma_1}{\Gamma_2} = -0.219483$$

  units in the underlying asset and

$$k = q\frac{\Gamma_1}{\Delta_1\Gamma_2 - \Delta_2\Gamma_1} = -0.023651$$

units in the OTM put. The initial value of the portfolio of $h = 1$ units in the bull spread, $Q = -0.219483$ units in the asset and $k = -0.023651$ units in the OTM put when $S = 50$ is

$$V = QS + C_{\text{ATM}} - C_{\text{OTM}} + kP = -19.6187.$$

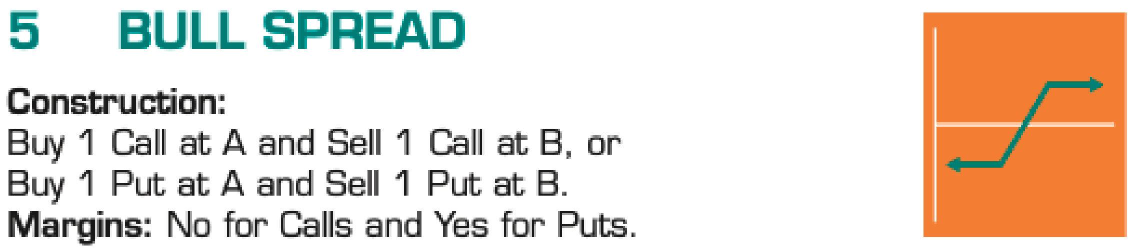
After the asset price falls to 48 and I recalculate the option values, I get

$$V = QS + C_{\text{ATM}} - C_{\text{OTM}} + kP = -19.20671.$$

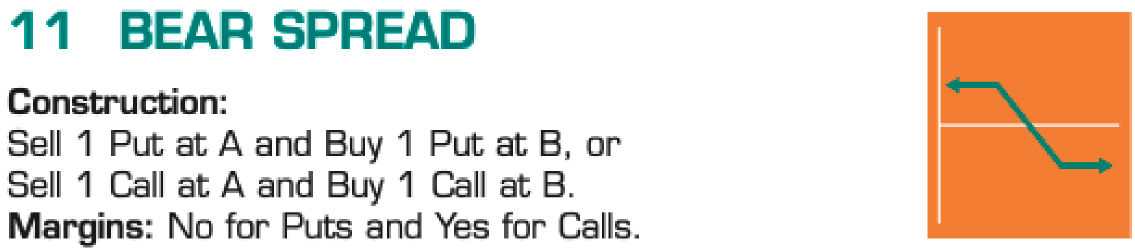
Question 4. Now suppose you're a market maker and implemented a bear spread with lower strike $K_1 = 45$ and at very favourable bid-ask spreads (which you can make up) relative to the Black-Scholes prices. Calculate a dynamic delta hedging strategy possibly again in Excel. What was your final liquidation value and hence profit? Would you rather be a market maker or retail trader? I'll leave this as a challenge question for the interested student who can modify the spreadsheet provided for the above dynamic delta hedging position.

Trading strategies

Question 5. 1. Show how to create a bull spread out of puts.

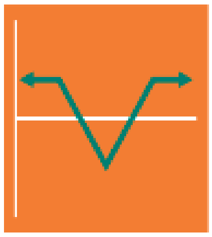


2. Show how to create a bear spread out of calls.



3. Show how to create a short butterfly payoff out of calls alone and out of puts alone. Plot their profit diagrams.

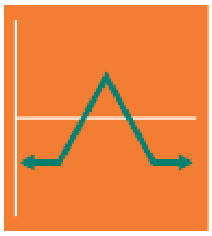
21 SHORT BUTTERFLY



Construction (any of the following):
Sell 1 Call at A and Buy 2 Calls at B and Sell 1 Call at C.
Sell 1 Put at A and Buy 2 Puts at B and Sell 1 Put at C.
Sell 1 Call at A and Buy 1 Call and 1 Put at B and Sell 1 Put at C.
Sell 1 Put at A and Buy 1 Put and 1 Call at B and Sell 1 Call at C.
Margins: Depends on how it is constructed.

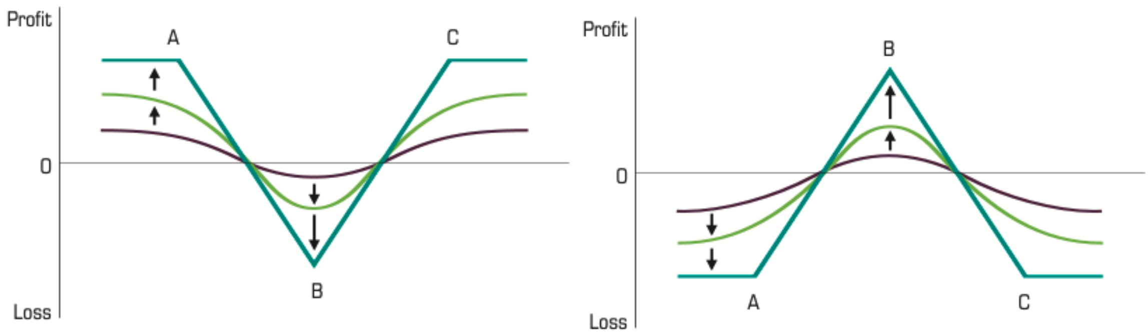
4. Show how to create a long butterfly payoff out of calls alone and out of puts alone. Plot their profit diagrams.

15 LONG BUTTERFLY



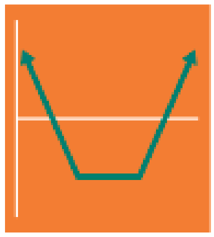
Construction (any of the following):
Buy 1 Call at A and Sell 2 Calls at B and Buy 1 Call at C.
Buy 1 Put at A and Sell 2 Puts at B and Buy 1 Put at C.
Buy 1 Call at A and Sell 1 Call and 1 Put at B and Buy 1 Put at C.
Buy 1 Put at A and Sell 1 Put and 1 Call at B and Buy 1 Call at C.
Margins: Depends on how it is constructed.

5. Plot the payoffs (not profits) diagrams and premium curves on the same graphs for long and short butterflies.



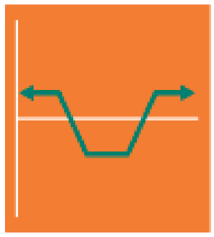
Question 6. What are long and short strangles and condors? Show how they can be created out of calls alone or puts alone. Plot their profit diagrams. Also plot their payoff diagrams and premium curves on the same graphs. Under what market expectations would you consider using these strategies? Why would you consider them instead of straddles and butterflies? How are these strategies also impacted by implied vols and time decay?

20 LONG STRANGLE



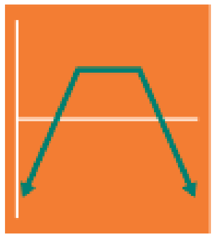
Construction:
Buy 1 Call at B and Buy 1 Put at A.
Margins: No.

22 SHORT CONDOR



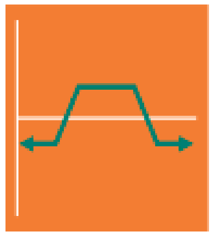
Construction (any of the following):
Sell 1 Call at A and Buy 1 Call at B and Buy 1 Call at C and Sell 1 Call at D.
Sell 1 Put at A and Buy 1 Put at B and Buy 1 Put at C and Sell 1 Put at D.
Sell 1 Call at A and Buy 1 Call at B and Buy 1 Put at C and Sell 1 Put at D. Sell 1 Put at A and Buy 1 Put at B and Buy 1 Call at C and Sell 1 Call at D.
Margins: Depends on how it is constructed.

14 SHORT STRANGLE



Construction:
Sell 1 Call at B and Sell 1 Put at A.
Margins: Yes.

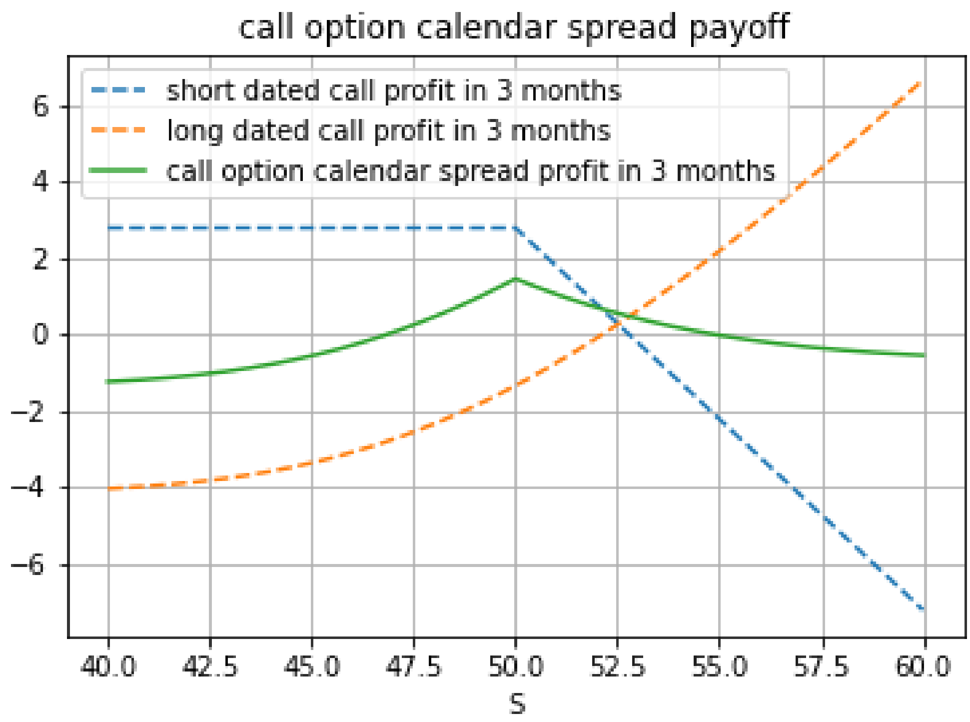
16 LONG CONDOR

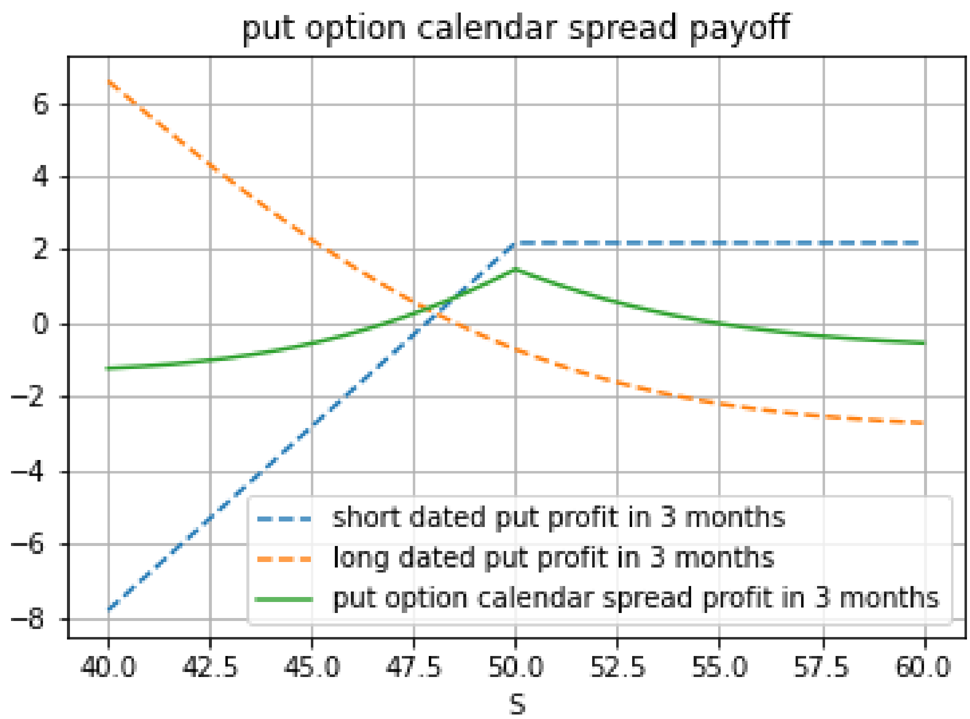


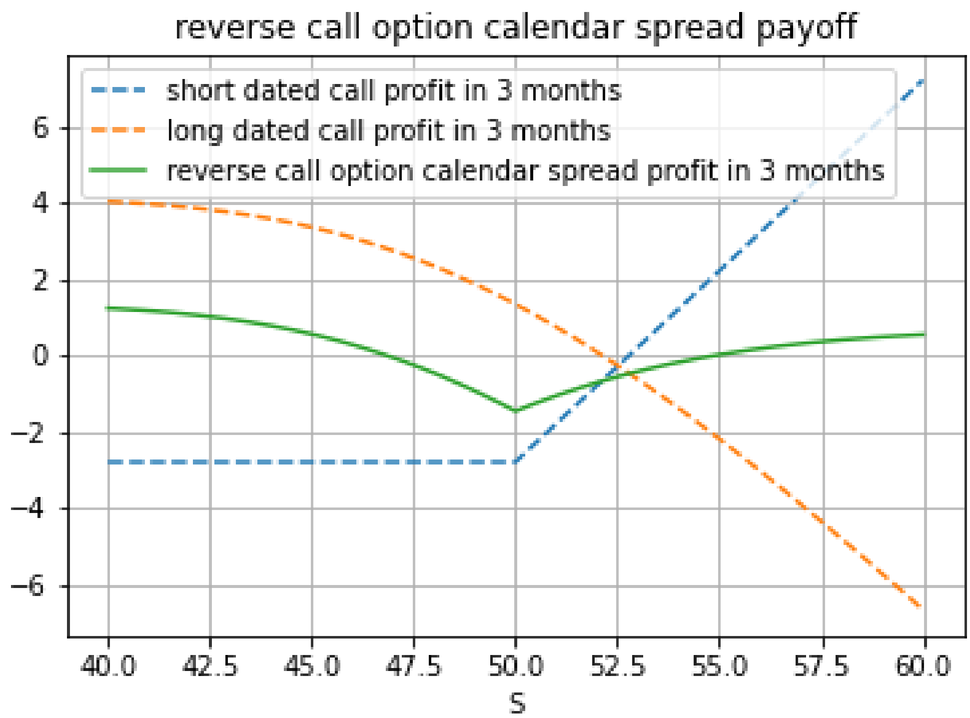
Construction (any of the following):
Buy 1 Call at A and Sell 1 Call at B and Sell 1 Call at C and Buy 1 Call at D.
Buy 1 Put at A and Sell 1 Put at B and Sell 1 Put at C and Buy 1 Put at D.
Buy 1 Call at A and Sell 1 Call at B and Sell 1 Put at C and Buy 1 Put at D.
Buy 1 Put at A and Sell 1 Put at B and Sell 1 Call at C and Buy 1 Call at D.
Margins: Depends on how it is constructed.

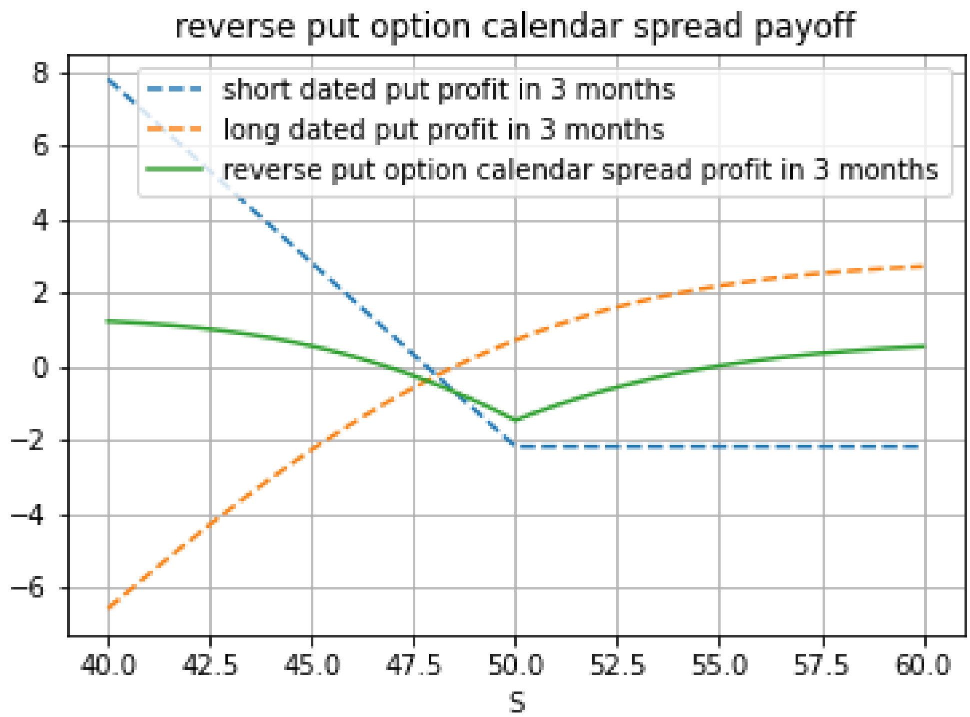
The interpretations of when one would consider these strategies and how they're impacted by implied vols and time decay are similar to the analogous straddle and butterfly strategies. I highly recommend reading the ASX booklet Options Strategies, which is where I cut and pasted the above diagrams from.

Question 7. In the lecture notes we gave an example of a calendar spread constructed out of calls. The market expectations behind it are a stable underlying asset. How is this strategy impacted by implied vols? Construct and plot the profit diagram for a calendar spread created out of puts which also hopes for a stable asset price. Also construct and plot the profit diagrams of calendar spreads created out of calls alone and out of puts alone which expect market volatility. How does time decay and implied vols impact these "reverse" calendar spreads?









The first graph (from the lecture notes) is a call calendar spread which writes a short dated call and takes a long dated call; this position benefits from time decay and steady to increasing (benefits the taken long dated call) implied vols, and it also wants a steady asset price. The second graph is a put calendar spread which writes a short dated put and takes a long dated put; it also benefits from time decay (since time decay is stronger for the written short dated put) and steady to increasing implied vols, and it also wants a steady asset price. The third graph is a reverse call calendar spread which takes a short dated call and writes a long dated call; it is damaged from time decay (time decay is stronger for the taken short dated call) and wants steady to decreasing (benefits the written long dated call) implied vols and a volatile asset price. The final graph is a reverse put calendar spread which takes a short dated put and writes a long dated put; it is damaged from time decay and wants steady to decreasing implied vols and a volatile asset price.