

FINM3405 Derivatives and risk management

Tutorial Sheet 2: Futures and forwards - Pricing and optimal hedging

August 4, 2024

Pricing

Question 1. The **London Metal Exchange**, formed in 1877 and owned since 2012 by **Hong Kong Exchanges and Clearing Limited** (HKEX Group), is one of the largest commodity futures exchanges in the world. It offers a large range of futures contracts, as exemplified in the **high grade aluminium contract**:

Futures				TRADING SUMMARY	Prices in US\$	Date
Futures contracts are an agreement to buy or sell a fixed amount of metal for delivery on a fixed future date at a price agreed today.				LME Aluminium Official Prices		
Contract code	AH			CONTRACT	BID	OFFER
Underlying metal	High grade primary aluminium			Cash	2228.00	2228.50
Lot size	25 tonnes			3-month	2286.00	2287.00
Prompt dates	Daily: out to 3 months Weekly: 3 out to 6 months Monthly: 7 out to 123 months			Dec-25	2445.00	2450.00
Price quotation	US dollars per tonne			Dec-26	2538.00	2543.00
Clearable currencies	US dollar, Japanese yen, sterling, euro			Dec-27	2622.00	2627.00
Minimum price fluctuation (tick size) per tonne		Outright	Carries	LME Aluminium Closing Prices		
	Ring	\$0.50	\$0.01	CONTRACT	PRICE	
	LMeselect	\$0.50	\$0.01	3-month	2296.00	
	Inter-office	\$0.01	\$0.01	Aug 24	2249.11	
Last trading day	Up until the close of the first Ring the day before the prompt date			Sep 24	2266.37	
Settlement type	Physical			Oct 24	2285.27	
Trading venues	Ring, LMeselect, inter-office telephone			Nov 24	2301.34	
Margining	Contingent variation margin applied			Dec 24	2316.54	
				Jan 25	2331.89	
DATE	CME TERM SOFR (%)					
	1 MONTH	3 MONTH	6 MONTH	12 MONTH		
02 Aug 2024	5.35204	5.22773	5.00763	4.59608		
01 Aug 2024	5.35025	5.24212	5.06657	4.7152		

These futures are in USD and the quotes are as of 01-Aug-2024. The Cash price is the spot price. Assuming no storage costs s or convenience yield q , what do you calculate the 3 month forward price to be? What would the cost of carry rate $r + s - q$ need to be to realise the quoted 3 month forward price?

Question 2. The current value of Australia's **All Ordinaries Index**, Australia's **BBSW** rates, and the All Ordinaries index's **dividend yield** are:

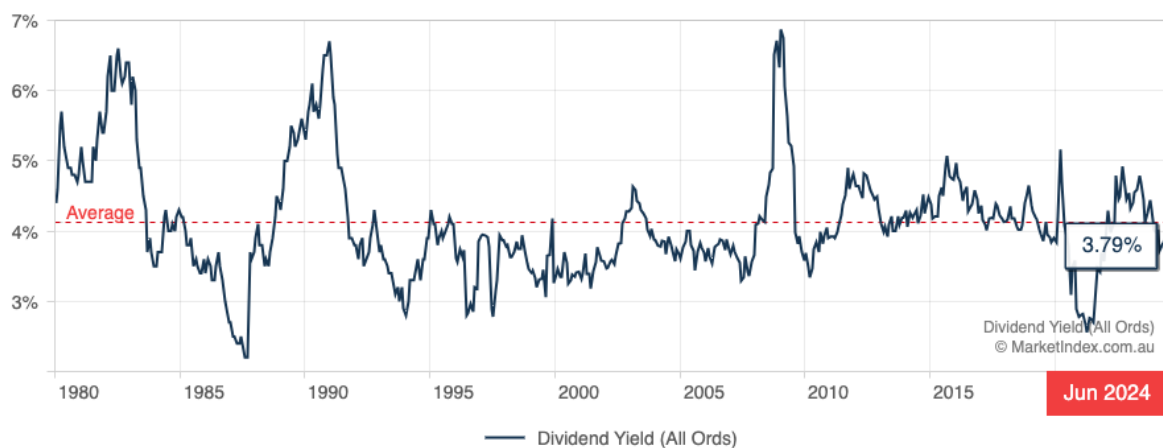
24 hour delayed BBSW rates

TENOR	BID	ASK	MID	METHOD	YIELD RANGE (BPS)
1 MONTH	4.3555	4.2555	4.3055	WLSR	3.5000
2 MONTH	4.4045	4.3045	4.3545	WLSR	4.0000
3 MONTH	4.4670	4.3670	4.4170	WLSR	7.0000
4 MONTH	4.5517	4.4517	4.5017	WLSR	2.5000
5 MONTH	4.6489	4.5489	4.5989	WLSR	3.8000
6 MONTH	4.7191	4.6191	4.6691	WLSR	2.8000

As of 01/08/2024 11am

Dividend Yield

Market-cap weighted Dividend Yield for the Australian stock market



All Ordinaries

8,170.39 AUD | -2.08% 1 Day

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What do you calculate the fair 1-6 month forward prices on the All Ordinaries index to be? If you were presented a 6 month forward quote of $K = 8,400$, how would you create an arbitrage profit? What if the quote was $K = 8,000$?

Question 3. Consider the following 01-Aug-2024 **BBSW** and CME Group **Term SOFR** rates, and AUD:USD exchange rate:

24 hour delayed BBSW rates

TENOR	BID	ASK	MID	METHOD	YIELD RANGE (BPS)
1 MONTH	4.3555	4.2555	4.3055	WLSR	3.5000
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As of 01/08/2024 11am

DATE	CME TERM SOFR (%)			
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Australian Dollar to United States Dollar

0.6511 ↓ 0.64% -0.0042 5 D

2 Aug, 21:25:00 UTC · Disclaimer



What are the fair 1, 3 and 6 month forward exchange rates? If you were presented a 6 month forward exchange rate of $K_{\text{AUD:USD}} = 0.7$, how would you construct an arbitrage profit? What if it was $K_{\text{AUD:USD}} = 0.6$? (You can assume that you're in the USA so the USD is the “domestic” currency.)

Question 4. Consider the following 02-Aug-2024 **Term SOFR** rates:

DATE	CME TERM SOFR (%)			
	1 MONTH	3 MONTH	6 MONTH	12 MONTH
02 Aug 2024	5.35204	5.22773	5.00763	4.59608

1. Calculate the correct fixed rates k for 3×6 , 3×12 and 6×12 FRA.
2. You're presented a fixed rate of $k = 5\%$ in the 6×12 FRA, which you can take as the receiver or payer. How would you construct an arbitrage profit on a notional principal of say $F = \$1,000,000$? What about if $k = 3\%$?

Question 5. 1. Use similar reasoning as for FRA to derive a formula for the fixed rate k in an Australian 90 day bank accepted bill futures contract.

2. Using the mid BBSW rates above, calculate the theoretically correct fixed rate k for BAB futures contracts maturing in 1, 2 and 3 months time.

Optimal hedging

Question 6. You're a corn farmer whose grade of corn does not match the CME Group **corn futures specifications**. You plan on selling 500,000 bushels of corn in December and want to use the CME Group corn futures to hedge your risk of corn prices falling.

1. Download the daily historical CME Group corn futures data from **yahoo!finance** and use them to calculate the standard deviation $\bar{\sigma}_K$ of the CME Group corn futures contract daily returns.
2. Suppose your corn prices have a daily standard deviation of $\bar{\sigma}_A = 1.25\%$ and correlation with the CME Group corn futures price of $\bar{\rho} = 0.8$. Also let the current price of your corn be $A = 380$ cents per bushel. How many contracts should you short to hedge your corn price risk?

Question 7. In the lecture notes we presented optimal hedging with the objective of reducing the beta β of a share portfolio to 0. We can generalise this to modifying the portfolio's beta to some **target beta** of $\hat{\beta}$. The following formula tells us how many futures contracts \hat{h} to short in order to modify the portfolio's beta to the target beta:

$$\hat{h} = (\beta - \hat{\beta}) \frac{V}{F},$$

where β is the beta of our share portfolio, V is the current value of our share portfolio (is given by $V = QA$ if we hold Q shares of one company with share price A) and $F = Km$ is the notional or face value of 1 index futures contract.

1. Using the Tesla optimal hedging example in the lecture notes, how many NYSE FANG+ index futures contracts should we short if we want to reduce Tesla's beta relative to the NYSE FANG+ index to $\hat{\beta} = 0.5$?
2. Using the NYSE FANG+ index portfolio example in the lecture notes, how many NYSE FANG+ index futures contracts should we short if we want to reduce our portfolio beta to $\hat{\beta} = 0.7$?
3. How many futures would you need to "short" in each case here in order to increase the beta of your Tesla holding and your index holding to $\hat{\beta} = 2$?
Remark: Hedge funds often use futures like this as a **market timing** strategy: They use futures to reduce their portfolio betas (deleverage or risk off) when they believe the market will fall and increase their portfolio beta (leverage up or risk on) if they believe the market will rise.