

FINM3405 Derivatives and Risk Management

Week 4: Introduction to options

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August 12, 2024

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Time value

Introduction to options

Last week we finished futures and forwards, at least for now:

- ▶ We return to interest rate derivatives (including futures) again later.
- ▶ FRA can be viewed as single period interest rate swaps.

Today we start on options. We give a basic introduction to options, look at the main equity and currency options markets globally, and introduce put-call parity and some basic pricing bounds on the option premium.

- ▶ Readings: Chapters 10 and 11 of [Hull](#).

Due to their asymmetrical payoffs, options have an upfront cost to the taker or holder called the premium. Much of the options material we cover involves methods for valuing options: calculating the premium.

Introduction to options

Recall that there is two types of plain vanilla **European** options:

- ▶ **Call option:** Gives the holder the right but not the obligation to buy the underlying asset for the strike price K on the expiry date T .
- ▶ **Put option:** Gives the holder the right but not the obligation to sell the underlying asset for the strike price K on the expiry date T .

Also recall that an **American** option gives the holder these rights at any point in time up to and including the expiry date T .

The option writer is “at the mercy of” the buyer.

- ▶ We say that an option has **asymmetric rights**.

Introduction to options

Recall that we use the following notation:

- ▶ We work on a given time interval $0 \leq t \leq T$ where:
 - ▶ Time $t = 0$ is the date we enter into a contract.
 - ▶ Time T is the **expiry date**.
 - ▶ Time t is some intermediate date $0 \leq t \leq T$.
- ▶ S_t is the **underlying asset's price** at time t .
 - ▶ We write $S = S_0$ to reduce notation.
- ▶ r is the **risk free rate**.
- ▶ K is the **strike** or **exercise price** (some authors use X).
- ▶ C_t and P_t are the call and put **prices (premiums)** at time t .
 - ▶ Again, we write $C = C_0$ and $P = P_0$.

Option payoffs and profits

Asymmetric rights: The holder (long position) has **payoffs** at expiry of

$$\text{call holder payoff} = \max\{0, S_T - K\},$$

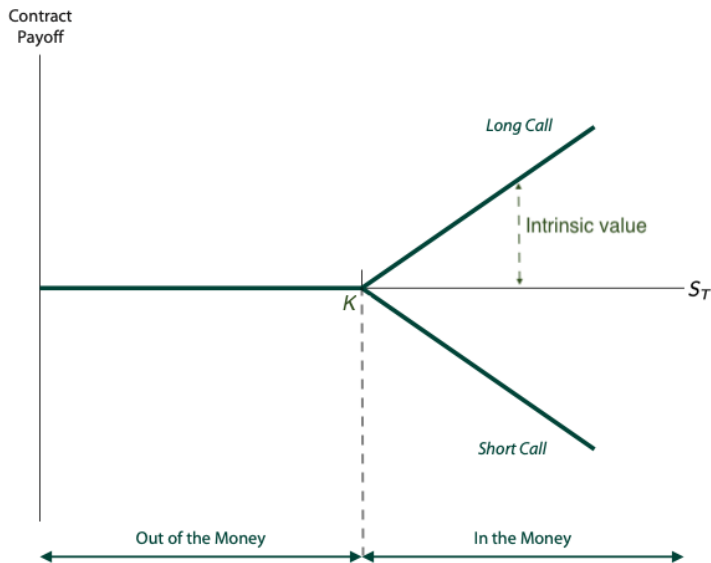
$$\text{put holder payoff} = \max\{0, K - S_T\}$$

and the writer's (short position) **payoffs** are the negative of these:

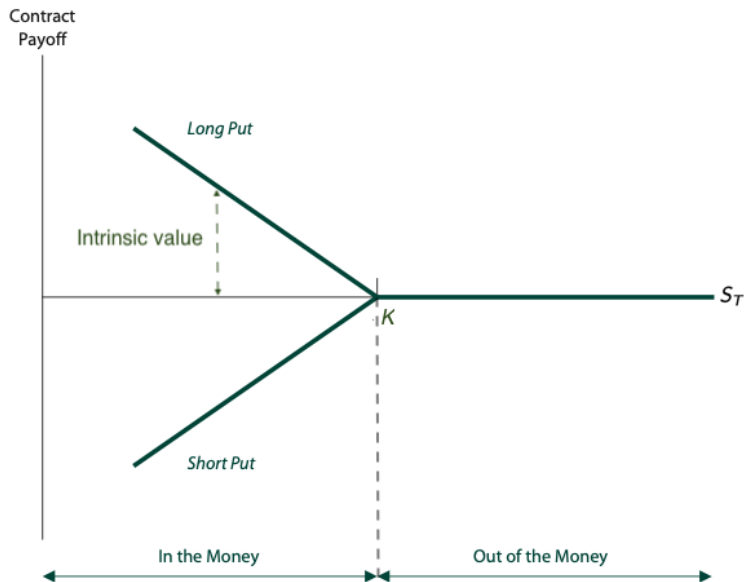
$$\text{call writer payoff} = -\max\{0, S_T - K\},$$

$$\text{put writer payoff} = -\max\{0, K - S_T\}.$$

Option payoffs and profits



Option payoffs and profits



Option payoffs and profits

At any time t , an option's **intrinsic** or **exercise value (IV)** is

$$\text{call IV}_t = \max\{0, S_t - K\} \quad \text{and} \quad \text{put IV}_t = \max\{0, K - S_t\}$$

(payoff if the option expired at time t). At time t an option is:

- ▶ **In the money** if it has positive intrinsic value:
 - ▶ $K < S_t$ for a call option.
 - ▶ $S_t < K$ for a put option.
- ▶ **At the money** if $S_t = K$ (intrinsic value is 0).
- ▶ **Out of the money** if its intrinsic value is 0 and $S_t \neq K$:
 - ▶ $S_t < K$ for a call option.
 - ▶ $K < S_t$ for a put option.

Option payoffs and profits

The above motivates using the idea of no arbitrage to justify the taker having to pay the option price or premium to the writer:

- ▶ The holder's payoff at expiry is nonnegative (positive or zero).
- ▶ If options required no upfront premium, then by definition this would be an arbitrage opportunity for the holder:
 - ▶ No upfront cashflow.
 - ▶ No risk of loss.
 - ▶ Chance of a positive payoff.

Remark

Developing complex mathematical models that calculate the fair value of an option's premium takes up a lot of space in the world of quant finance in both industry and academia.

Option payoffs and profits

To calculate trading profits, the above payoffs need to be modified to incorporate the premium paid/received. The option taker's **profits** are:

$$\text{call holder profit} = \max\{0, S_T - K\} - C,$$

$$\text{put holder profit} = \max\{0, K - S_T\} - P$$

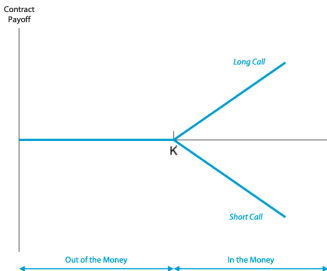
and the writer's (short position) **profits** are the negative of these:

$$\text{call writer profit} = C - \max\{0, S_T - K\},$$

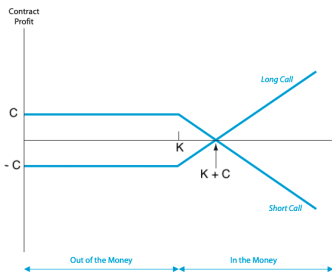
$$\text{put writer profit} = P - \max\{0, K - S_T\}.$$

Option payoffs and profits

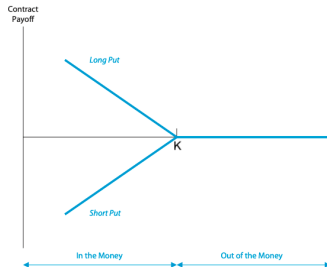
A. Contract Payoffs



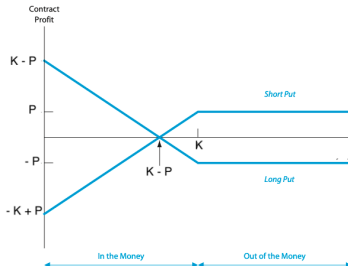
B. Contract Profits



A. Contract Payoffs



B. Contract Profits



Option payoffs and profits

So the premium gives the writer a chance of a profit, but exposes them to the risk of significant loss, unlimited in the case of call options:

- ▶ Calls and puts:
 - ▶ Holder: Loss is limited to the premium paid C or P .
 - ▶ Writer: Profit is limited to the premium received C or P .
- ▶ Call options:
 - ▶ Holder: Profit is unlimited and equal to $S_T - K - C$ for $S_T > K$.
 - ▶ Writer: Loss is unlimited and equal to $S_T - K - C$ for $S_T > K$.
- ▶ Put options:
 - ▶ Holder: Profit is potentially large and as much as $K - P$ for $S_T = 0$.
 - ▶ Writer: Loss is potentially large and as much as $K - P$ for $S_T = 0$.

Exchanges have margin mechanisms for short options positions.

Options vs futures/forwards

Fundamental differences between futures/forwards and options:

- ▶ Obligations:
 - ▶ Futures/forwards: Both parties must transact.
 - ▶ Options: The taker/holder gets to choose.
- ▶ Payoffs:
 - ▶ Futures/forwards: Symmetric.
 - ▶ Options: Asymmetric (favour the taker/holder).
- ▶ The “price” or value:
 - ▶ Futures/forwards: The actual contract price K_t .
 - ▶ Options: The premium C_t or P_t (the strike price K is fixed).
- ▶ Upfront cashflow:
 - ▶ Futures/forwards: K_t is set so they have 0 upfront value/cashflow.
 - ▶ Options: The taker pays premium C_t or P_t upfront to the writer.

Options markets

We look at the main exchange traded options markets and contracts for:

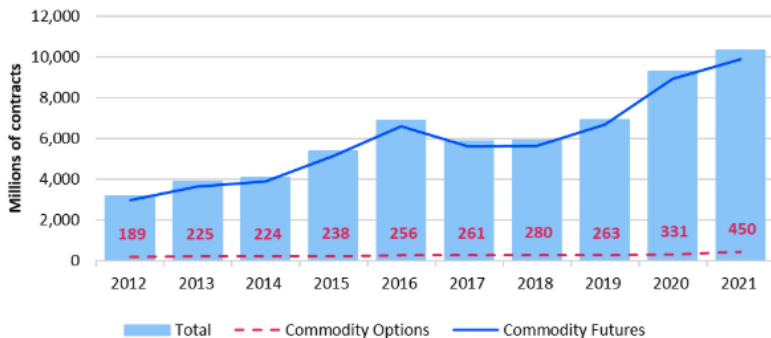
- ▶ Equities:
 - ▶ Individual share and ETFs.
 - ▶ Share index.
- ▶ Currencies.

While doing this we don't present basic speculation and hedging examples since these we devote a whole lecture to this in a few weeks time. We also cover interest rate derivatives later in the course.

Commodity options

Note also that commodity options are not as actively traded as commodity futures so we'll skip looking at commodity options:

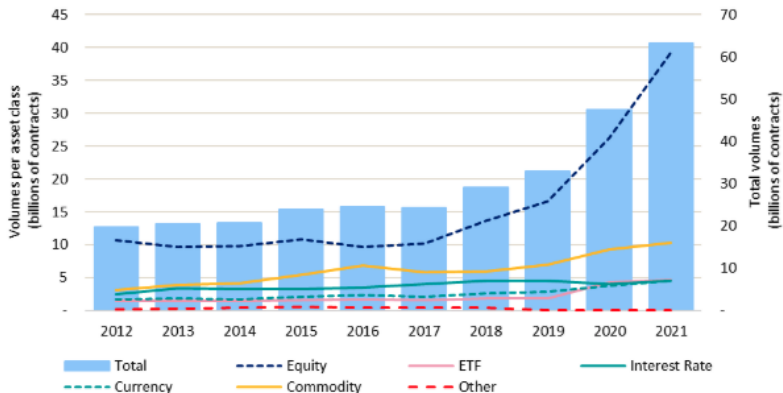
Figure 22: Volumes of commodity options and futures



Equity options

Equity options are very popular and heavily traded products. Below gives volumes for all exchange traded derivatives (options and futures).

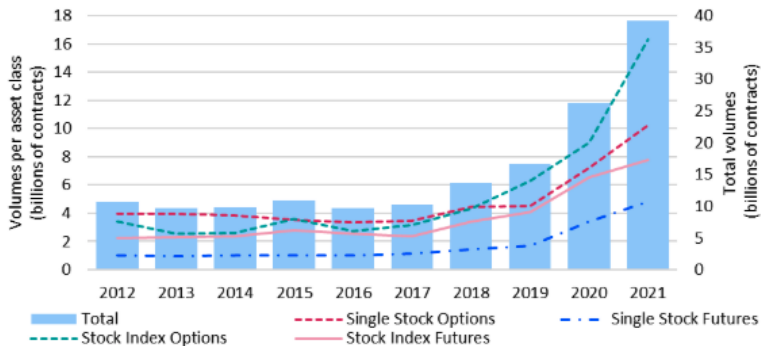
Figure 7: Volumes by asset class (billions of contracts)



Equity options

And out of the very large volume of equity derivatives trading worldwide from the previous slide, it's mostly equity options trading:

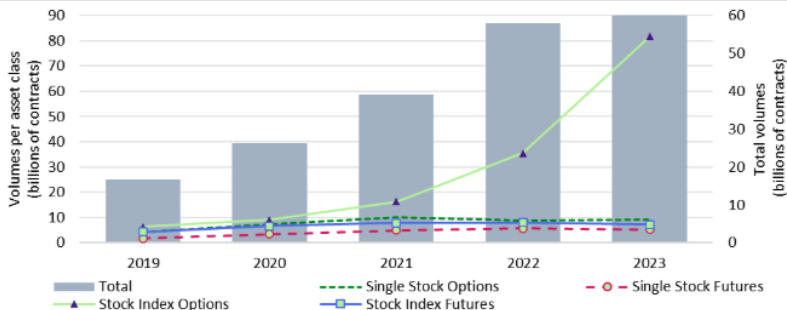
Figure 9: Volumes of equity derivatives (billions of contracts)



Equity options

This figure fills in the previous slide's figure with 2022 and 2023 data.

Chart 18: Volumes of equity derivatives over five years (billions of contracts traded)



It goes without saying that equity index options dwarf everything else.

Share and ETF options

Starting with individual share and ETF options, most trading by volume is in the USA ([NASDAQ](#), [CBOE](#), [NYSE](#), [MIAX](#), [ISX](#) combined).

Table 2: Top 10 exchanges by number of single stock options contracts traded in 2021

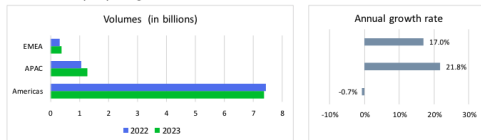
Exchange	Volume		Notional Value		Open interest	
	2021	YoY change	2021	YoY change	2021	YoY change
Nasdaq - US	2,337,412,860	37%	NA	NA	NA	NA
B3 - Brasil Bolsa Balcão	1,835,383,463	26%	393,631	-44%	48,206,676	4%
Cboe Global Markets	1,714,980,943	33%	NA	NA	326,576,982	23%
NYSE	1,231,950,440	57%	176,933	-51%	NA	NA
MIAX Exchange Group	963,410,916	90%	21,143,276	121%	NA	NA
International Securities Exchange	842,042,322	34%	NA	NA	NA	NA
National Stock Exchange of India	598,412,296	120%	7,003,360	158%	979,759	7%
Eurex	201,529,915	7%	929,148	0%	52,301,492	5%
Hong Kong Exchanges and Clearing	157,074,267	21%	885,042	60%	7,897,040	-11%
Euronext	84,868,421	22%	374,280	29%	19,336,315	54%
Others	237,433,905	11%	556,156	0%	21,759,699	-14%
Grand Total	10,204,499,748	40.9%	31,461,825	101%	477,057,963	17%

And the heaviest share options trading is in Apple and Tesla:

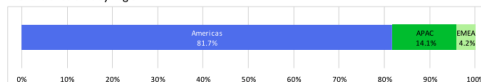
Share and ETF options

Chart 20: Single stock options

A. Volumes and yearly change



B. Share of volume by region



C. Share of volume by exchange

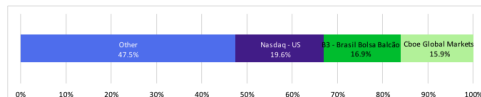


Table 1: Top 10 single stock options contracts traded in 2023¹

Contract Name	Contract size	Exchange name	Volume	Notional value	Open interest
			2023	2023	2023
Tesla Inc (TSLA)	100 shares	Cboe Global Markets	173,325,784	3,699,283	8,712,165
Tesla Inc (TSLA)	100 shares	Nasdaq - US	120,810,293	5,138,456	1,584,183,214
Apple Inc (AAPL)	100 shares	Nasdaq - US	107,794,532	1,504,293	1,964,399,392
Tesla Inc (TSLA)	100 shares	MIAX Exchange Group	105,437,899	NA	NA
Apple Inc (AAPL)	100 shares	Cboe Global Markets	73,472,047	1,259,029	7,008,029
NVIDIA Corp (NVDA)	100 shares	Cboe Global Markets	64,538,915	2,470,235	3,746,160

Share and ETF options

CBOE Equity Options Product Specifications

Exchange traded equity options are "physical delivery" options. This means that there is a physical delivery of the underlying stock to or from your brokerage account if the option is exercised. The owner of an equity option can exercise the contract at any time prior to the exercise deadline set by the investor's brokerage firm. Generally this deadline occurs on the option's last day of trading. The expiration date for equity options is the Saturday immediately following the third Friday of the expiration month until February 15, 2015. On and after February 15, 2015, the expiration date will be the third Friday of the expiration month. If this third Friday happens to be an exchange holiday, then the last day is the third Thursday of the month. Check with your brokerage firm about its procedures and deadlines for instruction to exercise any equity options. After the option's expiration date, the equity option will cease to exist.

For additional information on equity options, visit the [Equity Option Strategies](#) section of the web site.

Equity Options Product Specifications

Symbol

The option symbols are the same as for the underlying equity security. Visit the [Cboe Symbol Directory](#) for specific symbols.

Underlying

Generally, 100 shares of the underlying equity security.

Strike Price Intervals

Generally, 2 1/2 points when the strike price is between \$5 and \$25, 5 points when the strike price is between \$25 and \$200, and 10 points when the strike price is over \$200. Strikes are adjusted for splits, re-capitalizations, etc.

Strike (Exercise) Prices

In-, at- and out-of-the-money strike prices are initially listed. New series are generally added when the underlying trades through the highest or lowest strike price available.

Premium Quotation

Stated in decimals. One point equals \$100. Generally, minimum tick for options trading below \$3 is \$0.05 and for all other series, \$0.10. For classes participating in the *Penny Pilot Program*, the minimum tick for options trading below \$3 is \$0.01 and \$0.05 for options trading at \$3 or above.

Expiration Date

The third Friday of the expiration month.

Share and ETF options

Expiration Months

Two near-term months plus two additional months from the January, February or March quarterly cycles.

Exercise Style

American - Equity options generally may be exercised on any business day up to and including on the expiration date.

Settlement of Option Exercise

Exercise notices properly tendered on any business day will result in delivery of the underlying stock on the first business day following exercise.

Position and Exercise Limits

Limits vary according to the number of outstanding shares and past six-month trading volume of the underlying stock. The largest in capitalization and most frequently traded stocks have an option position limit of 250,000 contracts (with adjustments for splits, re-capitalizations, etc.) on the same side of the market; smaller capitalization stocks have position limits of 200,000, 75,000, 50,000 or 25,000 contracts (with adjustments for splits, re-capitalizations, etc.) on the same side of the market. The number of contracts on the same side of the market that may be exercised within any five consecutive business days is equal to the position limit. Equity option positions must be aggregated with equity LEAPS positions on the same underlying for position and exercise limit purposes. Exemptions may be available for certain qualified hedging strategies.

Reporting Requirements

Please refer to Exchange Rule 4.13 for information pertaining to reporting requirements for positions of 200 or more contracts.

Margin

Purchases of puts or calls with 9 months or less until expiration must be paid for in full. Writers of uncovered puts or calls must deposit / maintain 100% of the option proceeds* plus 20% of the aggregate contract value (current equity price x \$100) minus the amount by which the option is out-of-the-money, if any, subject to a minimum for calls of option proceeds* plus 10% of the aggregate contract value and a minimum for puts of option proceeds* plus 10% of the aggregate exercise price amount. (*For calculating maintenance margin, use option current market value instead of option proceeds.) Additional margin may be required pursuant to Exchange Rule 12.10.

Last Trading Day

Trading in equity options will ordinarily cease at the close of business on their expiration date.

Trading Hours

8:30 a.m. - 3:00 p.m. Central Time (Chicago time).

Share and ETF options

Individual share and ETF options on the [ASX](#):

Equity options

Underlying security	Any share approved by ASX under guidelines for listing equity options
Security code	The first three characters are the ASX code eg. BHP, the fourth and fifth character are the clearing code which is randomly assigned by the ASX. Some codes will include a sixth numerical character which is a clearing code randomly assigned by ASX.
Contract size	Usually 100 shares per contract. This may be adjusted for rights, bonus issues and other capital adjustment events.
Tick size	\$0.001 per share = \$0.10 (contract size 100 shares) for premium below 1 cent. \$0.005 per share = \$0.50 (contract size 100 shares) for premium of 1 cent or more.
Exercise style	American (Exercisable on or before expiry date) or European (Exercisable on the expiry date only)
Exercise price	Generally set by ASX Clear subject to strike price intervals. Occasionally subject to an adjustment due to a corporate action by the underlying company.
Type	Call and put options
Contract months	As detailed in equity options expiry calendar
Expiry date	Monthly Contracts - 3rd Thursday of the calendar month. Weekly Contracts - Thursday Both may change due to public holidays
Trading hours	Normal trading 10.00am to 4.20pm (Sydney time). Late trading 4.20pm to 5.00pm and overseas trading in accordance with the ASX Market Rules
Settlement	Physical delivery of underlying security

Share and ETF options

ETP options

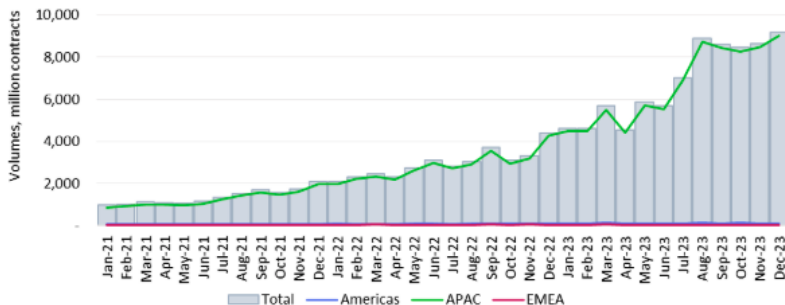
Underlying security	Any ETP approved by ASX under guidelines for listing equity options
Security code	The first three characters are the ASX code eg. STW, the fourth and fifth character are the clearing code which is randomly assigned by the ASX. Some codes will include a sixth numerical character which is a clearing code randomly assigned by ASX.
Contract size	Normally 100 securities per contract.
Tick size	\$0.001 per share = \$0.10 (contract size 100 shares) for premium below 1 cent. \$0.005 per share = \$0.50 (contract size 100 shares) for premium of 1 cent or more.
Exercise style	European (Exercisable on the expiry date only)
Exercise price	Generally set by ASX Clear subject to strike price intervals.
Type	Call and put options
Contract months	As detailed in equity options expiry calendar
Expiry date	Monthly Contracts - 3rd Thursday of the calendar month. Weekly Contracts - Thursday Both may change due to public holidays
Trading hours	Normal trading 10.00am to 4.20pm (Sydney time). Late trading 4.20pm to 5.00pm and overseas trading in accordance with the ASX Market Rules
Settlement	Physical delivery of underlying security

On the ASX, **exchange traded products** (ETPs) are mostly ETFs.

Share index options

Turning to index options, it has taken off worldwide:

Chart 23: Stock index options



Share index options

And most of it is on the [National Stock Exchange of India \(NSE\)](#):

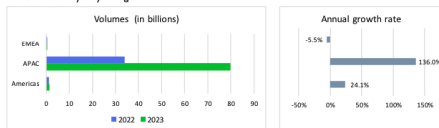
Table 6: Top 10 exchanges by number of stock index options contracts traded in 2021

Exchange	Volume		Notional Value		Open interest	
	2021	YoY change	2021	YoY change	2021	YoY change
National Stock Exchange of India	13,889,063,127	108%	172,588,703	183%	4,323,597	13%
Korea Exchange	758,533,660	3%	62,650,014	25%	2,654,019	19%
Cboe Global Markets	496,585,744	8%	NA	NA	22,193,282	12%
Eurex	365,199,107	-19%	14,503,112	-13%	44,343,990	-5%
Taiwan Futures Exchange	197,532,656	-2%	5,986,865	39%	368,100	-10%
CME Group	190,363,339	6%	42,863,133	37%	3,842,514	59%
India International Exchange	166,689,177	145%	NA	NA	565	743%
B3 - Brasil Bolsa Balcão	70,233,383	8%	113,311	-72%	447,610	-10%
China Financial Futures Exchange	30,241,547	81%	39,011	86%	195,960	37%
Tel-Aviv Stock Exchange	26,090,837	-7%	1,441,716	-32%	163,431	-18%
Others	124,880,801	-18%	4,368,761	-3%	7,383,189	-16%
Grand Total	16,315,413,378	80.7%	304,554,626	79%	85,916,257	1%

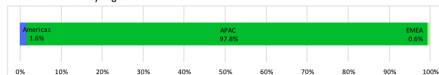
Share index options

Chart 24: Stock index options

A. Volumes and yearly change



B. Share of volume by region



C. Share of volume by exchange

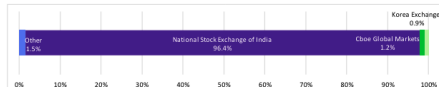


Table 3: Top 10 stock index options contracts traded in 2023

Contract Name	Contract size	Exchange name	Volume 2023	Notional value 2023	Open interest 2023
BANKNIFTY	15 units	National Stock Exchange of India	39,031,865,039	392,652,524	4,918,343
NIFTY	50 units	National Stock Exchange of India	22,726,670,033	261,044,459	5,049,083
FINNIFTY	40 units	National Stock Exchange of India	14,837,992,075	139,920,881	953,785
MIDCPNIFTY	75 units	National Stock Exchange of India	2,219,689,894	18,476,403	762,906
S&P 500 Index options (SPX)	100 shares of the underlying index	Cboe Global Markets	729,346,246	312,956,780	20,821,531
DRV option KOSPI 200 Weekly	KRW 250,000	Korea Exchange	448,080,320	86,117	1,859,995

Share index options

BANK NIFTY INDEX

Options Contract Specification - Weekly

[Futures Contract Specifications](#) | [Monthly Options Contract Specifications](#)

BANK NIFTY OPTIONS CONTRACT SPECIFICATIONS		
TICKER SYMBOL	BANKNIFTY	
CONTRACT SIZE	40 units.	
TICK SIZE	0.05	
TRADING HOURS	As in equity derivative segment	
NO. OF STRIKES/STRIKE INTERVALS	INDEX LEVEL	STRIKE INTERVAL
	≤ 2000	50
	> 2000 upto ≤ 3000	100
	> 3000 upto ≤ 4000	100
	> 4000 upto ≤ 6000	100
	> 6000	100
	NO OF STRIKES	
	8-1-8	
	6-1-6	
	8-1-8	
	12-1-12	
	16-1-16	

Share index options

EXPIRY DATE	Every Thursday of the week. In case the Thursday is a trading holiday, the previous trading day shall be the expiry/last trading day. All contracts shall expire at the normal market closing time on the expiry day or such other time as decided by Exchange.
CONTRACT MONTHS	7 weekly expiry contracts excluding the expiry week of monthly contract New serial weekly options contract shall be introduced after expiry of the respective week's contract.
OPTION TYPE	The options contracts shall be European styled which can be exercised only on the expiration date
DAILY SETTLEMENT PRICE	Daily premium settlement
FINAL SETTLEMENT PRICE	Final settlement price for an option contract shall be the closing price of the underlying index in the Normal Market of the Capital Market segment of National Stock Exchange on the last trading day of such futures contract.
FINAL SETTLEMENT PROCEDURE	Final settlement will be Cash settled in INR based on final settlement price. long positions of in-the money contracts shall be assigned to open short positions in option contracts.
FINAL SETTLEMENT DAY	All open positions on expiry date shall be settled on the next working day of the expiry date (T+1)
POSITION LIMITS	The trading member/MF/FII position limits in BANKNIFTY options contracts shall be higher of Rs.500 crores or 15% of the total open interest in the market . This limit would be applicable on open positions in all options contracts on underlying index

Note that the **Nifty Bank Index** *“is comprised of the most liquid and large capitalised Indian Banking stocks. It provides investors and market intermediaries with a benchmark that captures the capital market performance of Indian Banks.”*

Share index options

There's various sized **CBOE S&P500 Index** options:

CBOE INDEX OPTIONS	SPX®	XSP® (MINI-SPX®)	NANOS SM
Contract Size	1	1/10th of SPX	1/100th of XSP
Contract Multiplier	\$100	\$10	\$1
Approx. Notional Size (if S&P 500 is \$4,500)	\$450,000	\$45,000	\$450
Settlement Type	Trading account credited or debited in cash		
Exercise Style	European style, exercised at expiration, no risk of early exercise or assignment		
Tax Treatment	Capital gains may benefit from 60% / 40% tax treatment*		
Extended Trading Hours	Global trading hours available**		Standard market trading hours
Certainty of Settlement	Settlement and exercise style eliminate potential economic and tax risk for writers		
	More SPX details	More XSP details	More Nanos details

Share index options

CBOE SPX Options Product Specification

S&P 500® Index Options

Symbol

SPX/SPXW

Underlying

The Standard & Poor's 500 Index is a capitalization-weighted index of 500 stocks from a broad range of industries. The component stocks are weighted according to the total market value of their outstanding shares. The impact of a component's price change is proportional to the issue's total market value, which is the share price times the number of shares outstanding. These are summed for all 500 stocks and divided by a predetermined base value. The base value for the S&P 500 Index is adjusted to reflect changes in capitalization resulting from mergers, acquisitions, stock rights, substitutions, etc.

Multiplier

\$100.

Strike Price Intervals

Generally, \$5, \$10, \$25, \$50, \$100, and \$200 strike prices are available. Newly added and longer-term expirations have less granularity. Granularity is added over the life of the expiration with nearer term options having the most granularity. Standard 3rd Friday AM and PM options always have the most granularity and widest ranges.

Strike (Exercise) Prices

In-, at- and out-of-the-money strike prices are initially listed. New strikes can be added as the index moves up or down.

Premium Quote

Stated in decimals. One-point equals \$100. Minimum tick for options trading below 3.00 is 0.05 (\$5.00) and for all other series, 0.10 (\$10.00). For complex orders legs may trade in .01 (\$1.00) increments, but net package price must be in .05 (\$5.00) increments except for Boxes and Box Swaps which are eligible to trade in .01 (\$1.00) increments.

Exercise Style

European - SPX/SPXW options generally may be exercised only on the expiration date.

Last Trading Day

Trading in SPX options will ordinarily cease on the business day (usually a Thursday) preceding the day on which the exercise-settlement value (i.e., the expiration date) is calculated, 3:15pm CT.

Trading in SPXW options will ordinarily cease on the day of expiration, 3:00 pm CT.

Share index options

Expiration Date for Standard, Weekly and EOM Option

SPX AM options expire on the third Friday of the expiration month or the immediately preceding business day if the Exchange is not open on that Friday.

SPXW PM expiring options including Monday thru Friday, EOM and EOQ settle on their expiration date. If the Exchange is not open on that Friday, SPXW options expire on a Monday, Tuesday, Wednesday, Thursday, or a Friday. If the Exchange is not open on a Tuesday, Wednesday, Thursday, or Friday, the normally expiring Tuesday, Wednesday, Thursday, or Friday Weekly will expire on the immediately preceding business day. If the Exchange is not open on a Monday, the normally expiring Monday XSP Weekly will expire on the first business day immediately following that Monday. EOM XSP options expire on the last business day of the expiration month.

Expiration Months

Cboe may list up to twelve standard monthly expirations. Cboe may list up to ten (10) SPX LEAPS® monthly expirations at one time that expire from 12 to 180 months from the date of issuance.

For SPXW, the exchange lists 5 weeks of daily expirations for each Monday-Thursday expirations, if an EOM or EOQ falls on a Monday-Thursday it is counted towards that total. The Exchange lists 5 weeks of EOW expiring options, if a 3rd Friday, EOM or EOQ expiration falls on the EOW it is not counted towards the total. The Exchange may list series that expire at the end of the next consecutive four calendar quarters (EOQ), as well as the fourth quarter of the next calendar year. The Exchange may list 12 expirations for Monthly Options Series (EOM). Monthly Option Series expirations need not be for consecutive months; however, the expiration date of a nonconsecutive expiration may not be beyond what would be considered the last expiration date if the maximum number of expirations were listed consecutively.

Settlement of Options Exercise

Exercise will result in delivery of cash on the business day following expiration. The exercise-settlement value, SET, is calculated using the opening sales price in the primary market of each component security on the expiration date. The exercise-settlement amount is equal to the difference between the exercise-settlement value and the exercise price of the option, multiplied by \$100.

SPXW exercise will result in delivery of cash on the business day following expiration. The exercise-settlement value is calculated using the closing sales price in the primary market of each component security on the expiration date. The exercise-settlement amount is equal to the difference between the exercise-settlement value and the exercise price of the option, multiplied by \$100.

Position and Exercise Limits

No position and exercise limits are in effect.

Margin

Purchases of puts or calls with 9 months or less until expiration must be paid for in full. Writers of uncovered puts or calls must deposit / maintain 100% of the option proceeds* plus 15% of the aggregate contract value (current index level x \$100) minus the amount by which the option is out-of-the-money, if any, subject to a minimum for calls of option proceeds* plus 10% of the aggregate contract value and a minimum for puts of option proceeds* plus 10% of the aggregate exercise price amount. (*For calculating maintenance margin, use option current market value instead of option proceeds.) Additional margin may be required pursuant to Exchange Rule 12.10.

Share index options

Finally, on the ASX we have **S&P/ASX200 Index Options**:

S&P/ASX 200 index options XJO

Underlying index	S&P/ASX 200 index
Security codes	The first three characters are the ASX code e.g. XJO, the fourth and fifth characters are the clearing code randomly assigned by ASX. Certain codes include a sixth character which is always numeric.
Index multiplier	\$10. Each index point is equal to AUD \$10.
Tick size	Quoted as the number of points of the index. (Tick size 1.0 point)
Exercise style	European, ie. exercisable only on expiry day.
Exercise intervals	25 Index points
Type	Call and put options
Contract months	March, June, September, December up to six quarter months ahead and serial months up to four non-financial quarter months ahead
Expiry day	Monthly Contracts - The third Thursday of the contract month, unless otherwise specified by ASX. Weekly Contracts - Thursday, unless otherwise specified by ASX.
Last trading day	Trading will cease at 12 noon on expiry Thursday.
Trading hours	9.50am to 5.00pm and 5.30pm to 7.00pm (Sydney time)
Settlement	Index options are cash settled using the opening price index calculation on expiry morning. This means trading will continue after the settlement price has been determined.
Settlement day	The first business day following the Last Trading Day

Currency options

Turning to currency options, they previously didn't but now do have roughly the same trading volume as currency futures:

Figure 20: Volumes of currency derivatives

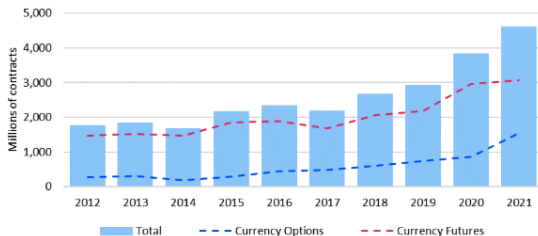
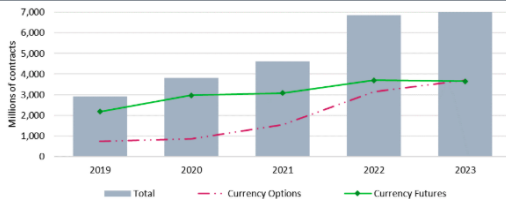


Chart 27: Currency derivatives



Currency options

And again, most of the trading is on the [NSE of India](#):

Table 15: Top 10 exchanges by number of currency options and futures contracts traded in 2021

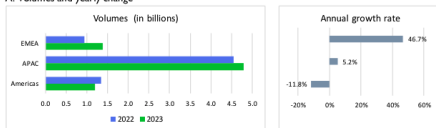
Exchange	Volume		Notional Value		Open interest	
	2021	YoY change	2021	YoY change	2021	YoY change
National Stock Exchange of India	2,267,659,129	50%	2,303,140	47%	9,843,109	29%
Moscow Exchange	906,732,901	4%	920,686	5%	5,005,194	23%
B3 - Brasil Bolsa Balcão	771,574,015	-1%	10,333,376	-6%	2,837,511	-63%
CME Group	201,255,084	-8%	19,339,802	-6%	2,661,130	17%
Matba Rofex	110,771,755	-2%	109,902	4%	4,316,024	11%
Korea Exchange	100,221,006	-7%	965,747	-25%	837,885	-7%
Borsa Istanbul	96,995,469	52%	73,056	15%	2,398,488	-22%
Johannesburg Stock Exchange	51,993,895	-17%	50,029	-31%	6,219,424	-5%
Singapore Exchange	26,569,612	4%	NA	NA	152,628	14%
Bolsa y Mercados Argentinos	11,720,720	-22%	11	-100%	14,120	-15%
Others	51,362,968	1%	5,759,241	259%	2,879,117	71%
Grand Total	4,596,856,554	20.3%	39,854,989	7%	37,164,630	-2%

Note that these statistics include both options and futures, as do:

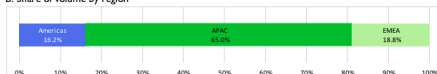
Currency options

Chart 30: Currency derivatives

A. Volumes and yearly change



B. Share of volume by region



C. Share of volume by exchange

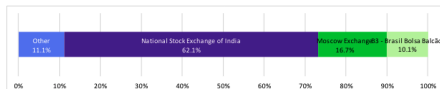


Table 5: Top 10 currency derivatives contracts traded in 2023

Contract Name	Contract size	Exchange name	Volume 2023	Notional value 2023	Open interest 2023
USD/INR options	US\$ 1,000	National Stock Exchange of India	3,620,372,162	3,596,697	11,574,294
USD/INR futures	US\$ 1,000	National Stock Exchange of India	788,813,312	783,472	6,019,974
DLR futures	US\$ 1,000	Matba Rofex	189,413,867	80,426	919,663
DRV futures USD	US\$ 10,000	Korea Exchange	108,034,964	1,087,524	891,938
GBP/INR futures	GBP 1,000	National Stock Exchange of India	77,131,286	95,417	333,974
EURO FX futures	EUR 125,000	CME Group	58,393,916	7,901,544	716,725
USD/TRY futures	US\$ 1,000	Borsa Istanbul	56,359,634	46,661	1,847,364
EUR/INR futures	EUR 1,000	National Stock Exchange of India	55,931,315	60,198	454,663
USD/INR futures	US\$ 1,000	Metropolitan Stock Exchange of India	55,139,166	27,393	115,998
JAPANESE YEN futures	JPY 12,500,000	CME Group	44,265,092	3,965,732	186,824

Currency options

Futures Contracts Specifications		Options Contracts Specifications		
SYMBOL	USDINR	EURINR	GBPINR	JPYINR
MARKET TYPE	N			
INSTRUMENT TYPE	OPTCUR	OPTCUR	OPTCUR	OPTCUR
OPTION TYPE	Premium style European Call & Put Options			
PREMIUM	Premium quoted in INR			
UNIT OF TRADING	1 contract unit denotes USD 1000	1 contract unit denotes EUR 1000	1 contract unit denotes GBP 1000	1 contract unit denotes JPY 100000
UNDERLYING / ORDER QUOTATION	The exchange rate in Indian Rupees for US Dollars. Exchange rate published by FBIL.	The exchange rate in Indian Rupees for Euro. Exchange rate published by FBIL	The exchange rate in Indian Rupees for Pound Sterling. Exchange rate published by FBIL	The exchange rate in Indian Rupees for 100 Japanese Yen. Exchange rate published by FBIL
TICK SIZE	0.25 paise i.e. INR 0.0025			
TRADING HOURS	Monday to Friday 9:00 a.m. to 5:00 p.m. IST			

Currency options

CONTRACT TRADING CYCLE	11 serial weekly contracts (excludes monthly contract expiring on Friday), 3 serial monthly contracts followed by 3 quarterly contracts of the cycle March/June/September/December		
NO. OF STRIKES AND STRIKE PRICE INTERVALS	Minimum 12 In-the-money, Minimum 12 Out-of-the-money and 1 Near-the-money. (25 CE and 25 PE)		Minimum 12 In-the-money, Minimum 12 Out-of-the-money and 1 Near-the-money. (25 CE and 25 PE) Strike price intervals - INR 0.25
	Strike price intervals - INR 0.25		
	*Effective from September 04, 2023, below revised strike scheme will be applicable:		
	Particulars	Strike Scheme and Strike Interval	
	All weekly and near monthly expiry	Strike Interval of INR 0.125 with 3-1-3 number of strikes (ITM-ATM-OTM) AND Strike Interval of INR 0.25 with 12-1-12 number of strikes (ITM-ATM-OTM) (includes strikes generated due to 0.125 strike interval)	
Middle-month, Far-month and Quarterly expiries	Strike Interval of INR 0.25 with 12-1- 12 number of strikes (ITM-ATM-OTM)		
PRICE OPERATING RANGE	A contract specific price range based on its delta value is computed and updated on a daily basis		
QUANTITY FREEZE	10,001 or greater		
BASE PRICE	Theoretical price on the 1st day of the contract. On all other days, DSP of the contract.		

Currency options

INITIAL MARGIN	SPAN Based Margin
EXTREME LOSS MARGIN	1.5% of Notional Value of open short position
SETTLEMENT OF PREMIUM	Premium to be paid by the buyer in cash on T+1 day
SETTLEMENT	Daily settlement: T + 1 Final settlement: T + 2
EXPIRY/LAST TRADING DAY	Friday of the expiring week in case of weekly expiration contracts. Two working days prior to the last business day of the expiry month at 12:30 pm and trade modification end time will be till 01:00 PM. If expiry month date falls on holiday date, the expiry date revision may done of the existing available expiry contract and same will be intimated in a separate circular.
FINAL SETTLEMENT DAY	Last working day (excluding Saturdays) of the expiry month for monthly contracts. The last working day will be the same as that for Interbank Settlements in Mumbai.
POSITION LIMITS	Position Limits for CDS Segment
MODE OF SETTLEMENT	Cash settled in Indian Rupees
FINAL SETTLEMENT PRICE (FSP)	RBI reference rate on the date of the expiry of the contact

Pricing relationships and bounds

That's enough for looking at the major exchange traded options contracts worldwide. Note that, as we also know, OTC markets are huge and options contracts traded on them tend to be less standardised, with more complex and exotic options being traded. We cover some of these exotic options later in the course.

- ▶ We now turn to option pricing bounds and put-call parity.

Put-call parity

Developing mathematical models for pricing options (calculating the fair value of the option premium), such as the Black-Scholes option pricing model, is a major part of the theory and practice of options.

- ▶ We cover the basics in later lecture notes, starting next week with the celebrated Black-Scholes option pricing model.

But before covering formal option pricing models, we can use no arbitrage arguments to show that there are pricing relations and bounds that at least plain vanilla options prices must adhere to and satisfy.

- ▶ We start with the (possibly already familiar) put-call parity relation.

Put-call parity

Put-call parity gives a strict relation that must hold, due to no arbitrage arguments, between the prices P and C of European put and call options over the same underlying asset and with the same strike K and expiry T :

Form a portfolio long 1 European call and short 1 European put, both with the same underlying, strike and expiry.

This portfolio's current price is $C - P$ and its payoff at expiry is

$$\begin{aligned}\text{portfolio payoff} &= \underbrace{\max\{0, S_T - K\}}_{\text{long call}} - \underbrace{\max\{0, K - S_T\}}_{\text{short put}} \\ &= S_T - K.\end{aligned}$$

Put-call parity

This is easy to show:

- ▶ If $S_T > K$ then the payoff is $S_T - K - 0 = S_T - K$.
- ▶ If $S_T < K$ then the payoff is $0 - (K - S_T) = S_T - K$.
- ▶ Finally, if $S_T = K$ then the payoff is $0 - 0 = 0 = S_T - K$.

But $S_T - K$ is precisely the payoff of a long futures contract position over the underlying with contract price K and maturity date T .

So the value $C - P$ of this portfolio long 1 call and short 1 put must equal the value of a *long* futures position with price K .

- ▶ Since their payoffs are equal, or else there is an arbitrage opportunity.

Put-call parity

- ▶ Let $X = Se^{rT}$ be the theoretically correct futures contract price.
- ▶ If we're long at K we could go short to close out the position at X .
- ▶ The value of this *long* futures contract position is

$$V^{\text{long}} = e^{-rT}(X - K).$$

So **put-call parity** is $C - P = e^{-rT}(X - K)$, which we rearrange to

$$C - P = S - e^{-rT}K,$$

noting that $e^{-rT}X = e^{-rT}Se^{rT} = S$.

Put-call parity

Remark

Put-call parity is also useful for equity options, in which we assume a continuously compounded annual dividend yield q .

- ▶ Here, the theoretically correct futures price is $X = Se^{(r-q)T}$ so put-call parity $C - P = e^{-rT}(X - K)$ becomes

$$C - P = e^{-qT}S - e^{-rT}K.$$

Note also that we tend to use continuous compounding for options.

Option pricing bounds

We now present pricing bounds that option prices must adhere to.

- ▶ If they don't satisfy these bounds then arbitrage opportunities exist.

Option pricing bounds

First note that American options are worth at least as much as European options over the same underlying and with the same strike and expiry:

$$0 \leq C^{\text{Eu}} \leq C^{\text{Am}} \quad \text{and} \quad 0 \leq P^{\text{Eu}} \leq P^{\text{Am}}$$

because American options can be exercised any time up to and including expiry, but European options can only be exercised at expiry.

- ▶ American options provide more flexibility and opportunity to profit.

Option pricing bounds

And American options are worth at least their intrinsic (exercise) value:

$$\max\{0, S - K\} \leq C^{\text{Am}} \quad \text{and} \quad \max\{0, K - S\} \leq P^{\text{Am}}$$

(lower pricing bounds) because they can be exercised immediately.

Option pricing bounds

Also, American calls can never be worth more than the underlying, and American puts can never be worth more than the strike:

$$C^{\text{Am}} \leq S \quad \text{and} \quad P^{\text{Am}} \leq K$$

(upper pricing bounds). Proving this is left as a tutorial exercise.

Option pricing bounds

Combining the lower and upper bounds from the previous two slides leads to the important pricing bounds for American options:

$$\max \{0, S - K\} \leq C^{\text{Am}} \leq S \quad \text{and} \quad \max \{0, K - S\} \leq P^{\text{Am}} \leq K.$$

Option pricing bounds

Turning to European options, we can tighten an upper bound for puts to

$$0 \leq P^{\text{Eu}} \leq Ke^{-rT}$$

because a European put can only be exercised at expiry, where it is known with certainty that their maximum payoff at expiry is K .

Remark

From this, deep in-the-money European puts can have negative time value (their premium can be less than their intrinsic value).

- Their maximum payoff is K . So if they're already deep in the money, not much more payoff can be realised at expiry, but the underlying asset could still move unfavourably.

Option pricing bounds

Furthermore, we can use put-call parity

$$C^{\text{Eu}} - P^{\text{Eu}} = e^{-qT} S - e^{-rT} K$$

to derive further lower bounds on European options.

- ▶ Since option prices are nonnegative, for European calls we get

$$\max \{0, e^{-qT} S - e^{-rT} K\} \leq C^{\text{Eu}} \leq S$$

and for European puts we get

$$\max \{0, e^{-rT} K - e^{-qT} S\} \leq P^{\text{Eu}} \leq K e^{-rT}.$$

No dividends: No early exercise of American calls

We actually have everything needed to show that early exercise is never optimal for an American call option on a non-dividend-paying stock:

- ▶ Under no dividends, so $q = 0$, from above we start with

$$\max \{0, S - e^{-rT} K\} \leq C^{\text{Eu}} \leq C^{\text{Am}}.$$

- ▶ If $r > 0$ then $e^{-rT} K < K$ and we get a strict inequality

$$\max\{0, S - K\} < \max \{0, S - e^{-rT} K\} \leq C^{\text{Eu}} \leq C^{\text{Am}}.$$

But $\max\{0, S - K\}$ is just the call's intrinsic value. So if there's no dividends, call options will always have strictly positive *time value*.

No dividends: No early exercise of American calls

- ▶ Under no dividends you never exercise an American call early.
 - ▶ The early exercise feature of American calls is worthless.
- ▶ So the American and European call prices are equal: $C^{\text{Eu}} = C^{\text{Am}}$.
- ▶ And the pricing bounds for European and American calls combine:

$$\max \{0, S - e^{-rT} K\} \leq C^{\text{Eu}} = C^{\text{Am}} \leq S.$$

Remark

If there *is* dividends, it may be optimal to exercise an American call on the day before the ex-dividend date. In any case early exercise may be optimal for deep in the money American puts.

Summary of the pricing bounds

$$\max \{0, S - K\} \leq C^{\text{Am}} \leq S,$$

$$\max \{0, K - S\} \leq P^{\text{Am}} \leq K.$$

$$\max \{0, e^{-qT} S - e^{-rT} K\} \leq C^{\text{Eu}} \leq S,$$

$$\max \{0, e^{-rT} K - e^{-qT} S\} \leq P^{\text{Eu}} \leq Ke^{-rT}.$$

Call options on non-dividend paying underlying:

$$\max \{0, S - e^{-rT} K\} \leq C^{\text{Eu}} = C^{\text{Am}} \leq S.$$

Time value

- ▶ And what exactly is *time value*?

From above, we noticed that call options on non-dividend-paying stocks have a premium that is strictly larger than the option's intrinsic value.

- ▶ We define an option's **time value** as the difference between the option premium and its intrinsic value:

$$\text{time value} = \text{premium} - \text{intrinsic value}.$$

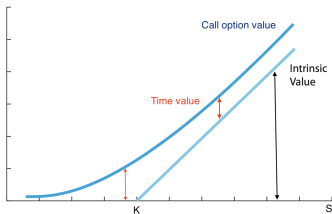
A better way to think of it is that the option's premium is made up of the option's intrinsic value plus the option's time value:

$$\text{premium} = \text{intrinsic value} + \text{time value}.$$

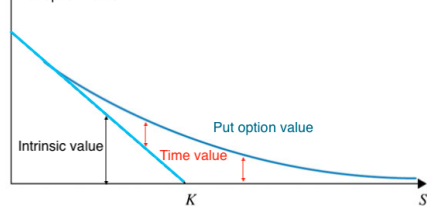
Time value

- **Intrinsic value:** Represents the option's payoff if the option expired at that moment in time - the exercise value.
- **Time value:** Represents the possibility that the price of an option's underlying will move favourably for the option holder before expiry.

Call Option Value



Put option value



Summary

Introduction to options

- Option payoffs and profits

- Options vs futures/forwards

Options markets

- Equity options

 - Share and ETF options

 - Share index options

- Currency options

Pricing relationships and bounds

- Put-call parity

- Option pricing bounds

- Early exercise of American calls

Time value