FINM3405 Derivatives and risk management

Tutorial Sheet 10: FRN and interest rate swaps

October 8, 2024

Question 1. Explain in words how you would use fixed-for-floating swaps to:

1. Speculate on shifts in the yield curve. If you thought interest rates would increase (decrease), then enter into an interest rate swap as the floating rate receiver (fixed rate receiver).
2. Hedge fixed and floating borrowing exposures to shifts in the yield curve. If you'd originally issued funds at a fixed (floating/variable), then to take advantage of a view that interest rates will fall (hedge against a view that interest rates would increase) you could enter into a swap as the pay floating (pay fixed) party.

Question 2. Consider the following Term SOFR rates:

CME TERM SOFR (%)			
1 MONTH	3 MONTH	6 MONTH	12 MONTH
4.84558	4.58398	4.28099	3.86582

Suppose the 9-month Term SOFR is an average of the 6-month and 12-month rates: $r_9 = 4.07341\%$.

1. What is the theoretically correct fixed rate k on a 1-year fixed-for-floating interest rate swap whose floating rate is Term SOFR plus 50 basis points over a notional principal of $F = \$10\text{m}$ and with quarterly coupon dates?

quarter	Term SOFR + 50bps	discount factors
1	5.08398%	0.9874496
2	4.78099%	0.9766532
3	4.57341%	0.9668369
4	4.36582%	0.9581681
	sum disc facs	3.8891078
	k	4.30247%

2. If the Term SOFR yield curve fell by 75 basis points during the 1st quarter, what is the value of the swap to both parties on the 1st coupon date? Interest rates went down so the swap value should be negative to the pay fixed party and positive to the receive fixed party:

quarter	Term SOFR - 25bps	quarterly fwd rates	fixed coupon	floating coupon	pay fixed party NCF	discount factor	PV NCF
2	4.03099%	4.03099%	$107,561.68	$ 100,774.75	-$6,786.93	0.990023067	-$6,719.21
3	3.82341%	3.57976%	$107,561.68	$ 89,493.88	-$18,067.80	0.981241556	-$17,728.87
4	3.61582%	3.14060%	$107,561.68	$ 78,515.02	-$29,046.65	0.973597354	-$28,279.74
						pay fixed party value	-$52,727.83

3. Following on from question 2, if you originally raised funds in capital markets at a fixed interest rate of $r = 5\%$ and entered into the swap as the floating rate payer, when standing at the end of the 1st quarter, what is your new calculated total interest payment due at the end of the 2nd quarter, and your remaining expected total interest payments? Here you would be paying a coupon of $F \times 5\% \times \frac{1}{4} = \$125,000$ in the market on your fixed interest financing. As the floating rate payer, interest rates moved in your direction so you receive the above swap net cashflows. Your next total interest payment is $125000 - 6844.49 = \$118,155.51$. Using the forward rates, your forecast future total interest payments are $125000 - 18125.36 = \$106,874.64$ and $125000 - 29104.21 = \$95,895.79$.

Question 3. 1. Show how a fixed-for-floating interest rate swap can be viewed as a sequence of FRA maturing on each coupon date.

2. Vice versa, show how a FRA can be viewed as an interest rate swap with a single coupon or interest payment date.

We can answer both together. Consider an arbitrary interest period $[t_{i-1}, t_i]$ and let r be the spot reference rate at time t_{i-1} and covering this period. Let k be the fixed rate of the swap and in a $t_{i-1} \times t_i$ FRA. Suppose both the FRA and swap are over the same notional principal F. Then we recall that:

- The net cashflow paid at time t_{i-1} to the fixed-rate receiver in the FRA is payoff $= \dfrac{F(k-r)d}{1+rd}$, where d is the interest period adjustment.
- The net cashflow paid at time t_i to the fixed-rate receiver in the swap is swap payment $= F(k-r)d$.

Note that the FRA payoff occurs *at the start of the interest period*, date t_{i-1}, but the swap cashflow occurs *at the end of the interest period*, date t_i. But these cashflows differ only by discounting the swap cashflow to the start of the period to get the FRA payoff, so from a financial perspective they're equivalent.