FINM3405 Derivatives and risk management

Tutorial Sheet 2: Futures and forwards - Pricing and optimal hedging

Suggested solutions

August 4, 2024

Pricing

Question 1. The London Metal Exchange, formed in 1877 and owned since 2012 by Hong Kong Exchanges and Clearing Limited (HKEX Group), is one of the largest commodity futures exchanges in the world. It offers a wide range of futures contracts, as exemplified in the high grade aluminium contract:

Futures

Futures contracts are an agreement to buy or sell a fixed amount of metal for delivery on a fixed future date at a price agreed today.

Contract code	AH		
Underlying metal	High grade primary aluminium		
Lot size	25 tonnes		
Prompt dates	Daily: out to 3 months Weekly: 3 out to 6 months Monthly: 7 out to 123 months		
Price quotation	US dollars per tonne		
Clearable currencies	US dollar, Japanese yen, sterling, euro		
Minimum price fluctuation (tick size) per tonne		Outright	Carries
	Ring	\$0.50	\$0.01
	LMEselect	\$0.50	\$0.01
	Inter-office	\$0.01	\$0.01
Last trading day	Up until the close of the first Ring the day before the prompt date		
Settlement type	Physical		
Trading venues	Ring, LMEselect, inter-office telephone		
Margining	Contingent variation margin applied		

TRADING SUMMARY | Prices in US\$ | Dat

LME Aluminium Official Prices

CONTRACT	BID	OFFER
Cash	2228.00	2228.50
3-month	2286.00	2287.00
Dec-25	2445.00	2450.00
Dec-26	2538.00	2543.00
Dec-27	2622.00	2627.00

LME Aluminium Closing Prices

CONTRACT	PRICE
3-month	2296.00
Aug 24	2249.11
Sep 24	2266.37
Oct 24	2285.27
Nov 24	2301.34
Dec 24	2316.54
Jan 25	2331.89

DATE	CME TERM SOFR (%)			
	1 MONTH	3 MONTH	6 MONTH	12 MONTH
02 Aug 2024	5.35204	5.22773	5.00763	4.59608
01 Aug 2024	5.35025	5.24212	5.06657	4.7152

These futures are in USD and the quotes are as of 01-Aug-2024. The Cash price is the spot price. Assuming no storage costs s or convenience yield q, what do you calculate the 3 month forward price to be? What would the cost of carry rate $r + s - q$ need to be to realise the quoted 3 month forward price? Using the midpoint between the bid and offer prices, the spot-forward parity relation for commodities yields

$$K = S(1 + rT) = 2228.25\Big(1 + 0.0524212\frac{90}{360}\Big) = \$2,257.45.$$

The quoted 3 month forward price is $K = \$2,286.5$. Rearranging $K = S(1 + cT)$, we calculate the cost of carry $c = r + s - q$ to be

$$c = \Big(\frac{K}{S} - 1\Big)\frac{1}{T} = \Big(\frac{2286.5}{2228.25} - 1\Big)\frac{360}{90} = 10.457\%.$$

Assuming negligible convenience yield q, this equates to an annual simple storage rate of $s = 10.457 - 5.24212 = 5.215\%$.

Question 2. The current value of Australia's All Ordinaries Index, Australia's BBSW rates, and the All Ordinaries index's dividend yield are:

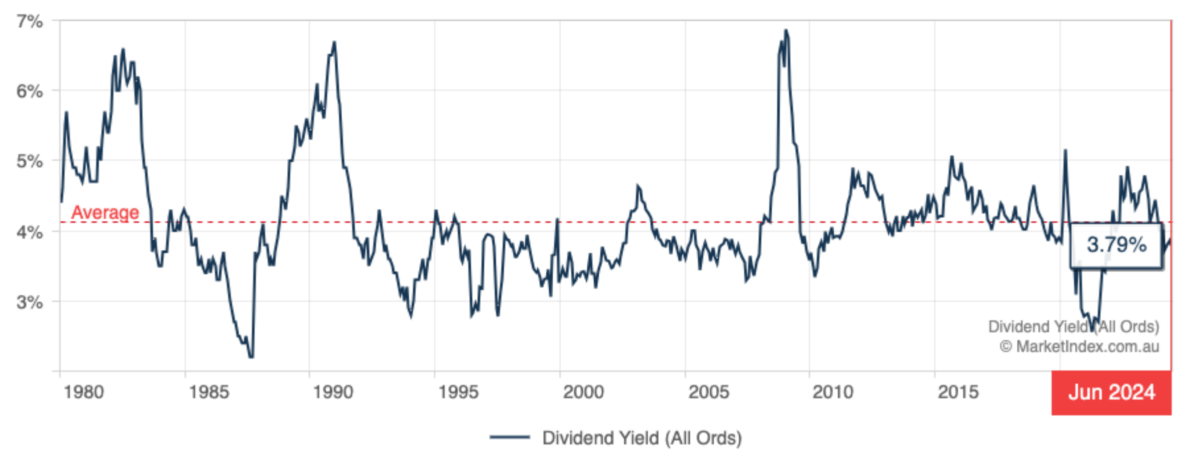
24 hour delayed BBSW rates

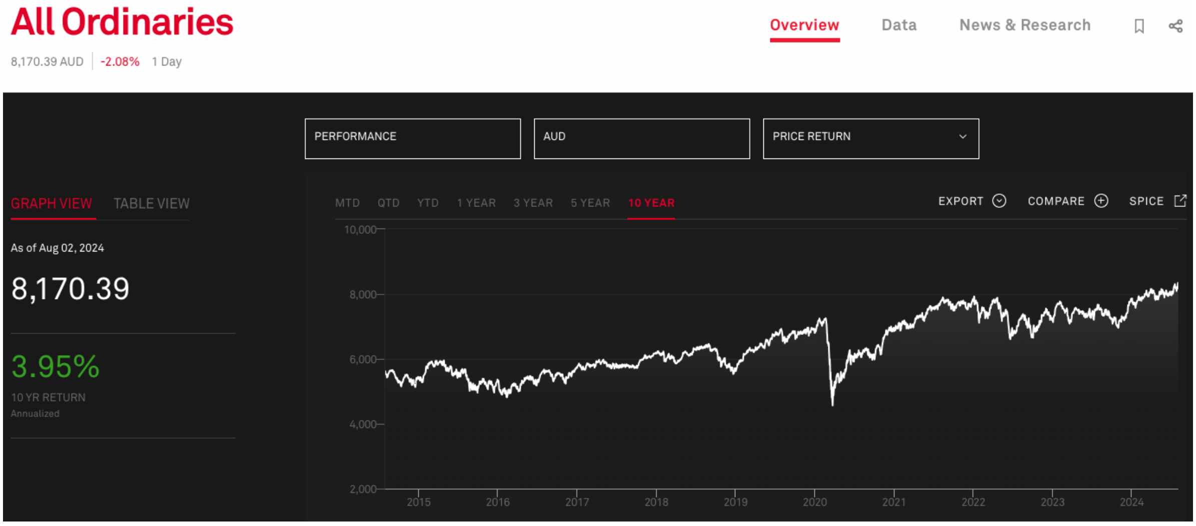
TENOR	BID	ASK	MID	METHOD	YIELD RANGE (BPS)
1 MONTH	4.3555	4.2555	4.3055	WLSR	3.5000
2 MONTH	4.4045	4.3045	4.3545	WLSR	4.0000
3 MONTH	4.4670	4.3670	4.4170	WLSR	7.0000
4 MONTH	4.5517	4.4517	4.5017	WLSR	2.5000
5 MONTH	4.6489	4.5489	4.5989	WLSR	3.8000
6 MONTH	4.7191	4.6191	4.6691	WLSR	2.8000

As of 01/08/2024 11am

Dividend Yield

Market-cap weighted Dividend Yield for the Australian stock market





What do you calculate the fair 1-6 month forward prices on the All Ordinaries index to be? If you were presented a 6 month forward quote of $K = 8,400$, how would you create an arbitrage profit? What if the quote was $K = 8,000$? I'll calculate the 6 month forward index value, and leave the others to you. Using the spot-forward parity relation for equities, we get

$$K = S\big[1 + (r - q)T\big] = 8170.39\Big[1 + (0.046691 - 0.0379)\frac{180}{365}\Big] = 8,205.81.$$

If you were presented with a forward quote of 8,400 then you'd perform a short forward trade. Suppose the index multiplier is $m = 1$:

- Borrow \$8,170.39 for 6 months at the BBSW rate of 4.6691% and "buy 1 unit of the index" (you could use an ETF for this?).
- Short $h = 1$ forward contract at a price of $K = \$8,400$.
- Your initial net cashflow is \$0. At maturity in 6 months time you:
  - Pay off the loan for $8,170.39\left(1 + 0.046691\frac{180}{365}\right) = \$8,358.52$.
  - Receive $8,170.39 \times 0.0379\frac{180}{365} = \$152.71$ in dividends.
  - Receive $K = \$8,400$ from your short forward position.

The net cashflow at maturity is

$$\text{cashflow} = 8400 + 152.71 - 8358.52 = 194.19 > 0.$$

If you were presented a forward contract price of $K = 8,000$ then you'd do the reverse of the above:

- "Short sell the index" (use an ETF?) for \$8170.39 and invest the proceeds at the BBSW rate of 4.6691%.

- Go long $h = 1$ forward contract at a price of $K = \$8,000$.
- Your initial cashflow is \$0. At maturity you:
  - Receive $8,170.39\left(1 + 0.046691\frac{180}{365}\right) = \$8,358.52$ from investing at the BBSW rate.
  - Pay $8,170.39 \times 0.0379\frac{180}{365} = \$152.71$ in dividends.
  - Pay $K = \$8,000$ to buy the asset in your long forward position.

The net cashflow at maturity is

$$\text{cashflow} = 8170.39 - 152.71 - 8000 = \$17.68 > 0.$$

Remark: Note that these strategies are often referred to as **index arbitrage**. More generally, at any moment in time there is a very large number of computer programs scanning all financial markets looking for and trading arbitrage opportunities. They may be purely profit driven but they perform important, beneficial functions in markets including market integration, liquidity provision, volatility reduction and price discovery. Regarding price discovery, derivative markets are often much larger than the spot market for the underlying asset itself. Consequently, even though derivatives are defined as financial securities whose payoffs and value are derived from the underlying asset, in fact it is often the case that pricing in derivative markets leads pricing in the underlying asset.

Question 3. Consider the following 01-Aug-2024 BBSW and CME Group Term SOFR rates, and AUD:USD exchange rate:

24 hour delayed BBSW rates

TENOR	BID	ASK	MID	METHOD	YIELD RANGE (BPS)
1 MONTH	4.3555	4.2555	4.3055	WLSR	3.5000
2 MONTH	4.4045	4.3045	4.3545	WLSR	4.0000
3 MONTH	4.4670	4.3670	4.4170	WLSR	7.0000
4 MONTH	4.5517	4.4517	4.5017	WLSR	2.5000
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6 MONTH	4.7191	4.6191	4.6691	WLSR	2.8000

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DATE	CME TERM SOFR (%)			
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What are the fair 1, 3 and 6 month forward exchange rates? If you were presented a 6 month forward exchange rate of $K_{\text{AUD:USD}} = 0.7$, how would you construct an arbitrage profit? What if it was $K_{\text{AUD:USD}} = 0.6$? (You can assume that you're in the USA so the USD is the "domestic" currency.) Again, I'll calculate the 6 month forward exchange rate and leave the rest to you. The covered interest parity formula yields

$$K_{\text{AUD:USD}} = S_{\text{AUD:USD}} \frac{1 + r_{\text{USD}} T}{1 + r_{\text{AUD}} T} = 0.6498 \frac{1 + 0.0506657 \frac{180}{360}}{1 + 0.046691 \frac{180}{365}} = 0.6513.$$

The forward exchange rate implies that AUD buys more USD than the theoretically correct forward exchange rate. To take advantage of this:

- Borrow \$1 USD at 5.06657% for 6 months. The amount paid back is $1 + 0.0506657 \dfrac{180}{360} = \$1.0253$ USD.
- Invest $S_{\text{USD:AUD}} = 1/S_{\text{AUD:USD}} = 1/0.6498 = \$1.5389$ AUD in Australia at the 6 month BBSW rate of 4.6691%.

- Arrange to exchange the proceeds back into USD at the forward rate of $K_{\text{AUD:USD}} = 0.7$.

The final proceeds of your roundtrip investment in Australia is

$$\text{proceeds} = 1.5389 \times \left(1 + 0.046691 \frac{180}{365}\right) \times 0.7 = \$1.102\,\text{USD}.$$

Hence, your net profit is $1.102 - 1.0253 = \$0.0767$ USD. If you were presented with a forward rate of $K_{\text{AUD:USD}} = 0.6$ then you'd do the "reverse" transaction: Borrow AUD, exchange it for USD spot, invest at Term SOFR, and arrange to exchange the USD amount into AUD forward to pay off the AUD loan.

Question 4. Consider the following 02-Aug-2024 Term SOFR rates:

DATE	CME TERM SOFR (%)			
	1 MONTH	3 MONTH	6 MONTH	12 MONTH
02 Aug 2024	5.35204	5.22773	5.00763	4.59608

1. Calculate the correct fixed rates k for 3×6, 3×12 and 6×12 FRA. I'll calculate the fixed rate for a 6×12 FRA, and leave the others to you. We showed in the lecture notes that the fixed rate k is the 6 month rate covering the period $[T_6, T_{12}]$ satisfying

$$1 + r_{12} = (1 + r_6 T_6)(1 + kT),$$

where here $T = T_6 = \dfrac{180}{360}$. Rearranging, we get

$$k = \left(\frac{1 + r_{12}}{1 + r_6 T_6} - 1\right)\frac{1}{T} = 4.08232\%.$$

2. You're presented a fixed rate of $k = 5\%$ in the 6×12 FRA, which you can take as the receiver or payer. How would you construct an arbitrage profit on a notional principal of say $F = \$1,000,000$? What about if $k = 3\%$? If you're presented a rate of $k = 5\%$, then you take it as the fixed rate receiver since it's higher than the theoretical fixed rate we just calculated. In this scenario you're effectively agreeing to invest F at $k = 5\%$ in 6 months time for a 6 month period. Borrow $P = \dfrac{F}{1 + r_{12}}$ for 1 year at $r_{12} = 4.59608\%$, immediately invest it for 6 months at $r_6 = 5.00763\%$, and agree to the FRA as the fixed rate receiver at $k = 5\%$. No matter what the actual spot

6 month Term SOFR rate is in 6 months, you lock in $k = 5\%$ in 6 months time. Hence, your investment proceeds are

$$\text{proceeds} = P(1 + r_6T_6)(1 + kT) = F\frac{(1 + r_6T_6)(1 + kT)}{1 + r_{12}}.$$

The loan amount you pay off is F. Let $f = 4.08232\%$, the theoretically correct FRA fixed rate. We have that

$$(1 + r_6T_6)(1 + kT) > 1 + r_{12} = (1 + r_6T_6)(1 + fT)$$

since $k > f$, so your investment proceeds are larger than F, the loan amount you have to pay back. *Remark*: Note that this process is often used as an arbitrage argument for pricing FRA (calculating the correct fixed rate k), instead of using the insight that FRA are priced so that they have 0 value to each party when originated.

Question 5. 1. Use similar reasoning as for FRA to derive a formula for the fixed rate k in an Australian 90 day bank accepted bill futures contract. Here we use the key insight again that many derivative securities are priced so that its value is 0 to each party when entered into. The fixed rate receiver of a BAB futures contract hypothetically agrees to buy a 90 day bank accepted bill at maturity, time T_1, for $\dfrac{F}{1 + kT}$ (cash outflow), and then receive the notional principal or face value F (cash inflow) at time T_2, 90 days (3 months) after maturity. Hence, the value to the fixed rate receiver at time $t = 0$ is

$$V = -\frac{F/(1 + kT)}{1 + r_1T_1} + \frac{F}{1 + r_2T_2}.$$

Setting this to 0, we therefore want to solve for k satisfying

$$\frac{F}{(1 + r_1T_1)(1 + kT)} = \frac{F}{1 + r_2T_2}.$$

Of course this rearranges to

$$1 + r_2T_2 = (1 + r_1T_1)(1 + kT),$$

so the situation is no different to a FRA, except that we note that the day count convention in Australian markets is time is quoted as time $= \dfrac{d}{365}$, where d is the number of days in the time period.

2. Using the mid BBSW rates above, calculate the theoretically correct fixed rate k for BAB futures contracts maturing in 1, 2 and 3 months time. It turns out that things are no different to pricing FRA, except for the day count convention, so I'm sure you can do it yourself by now.

Optimal hedging

Question 6. You're a corn farmer whose grade of corn does not match the CME Group corn futures specifications. You plan on selling 500,000 bushels of corn in December and want to use the CME Group corn futures to hedge your risk of corn prices falling.

1. Download the daily historical CME Group corn futures data from yahoo!finance and use them to calculate the standard deviation $\bar{\sigma}_K$ of the CME Group corn futures contract daily returns.
2. Suppose your corn prices have a daily standard deviation of $\bar{\sigma}_A = 1.25\%$ and correlation with the CME Group corn futures price of $\bar{\rho} = 0.8$. Also let the current price of your corn be $A = 380$ cents per bushel. How many contracts should you short to hedge your corn price risk?

The below Python code automates both 1. and 2. for me, but you're welcome to download the prices manually and use say Excel :-(for the calculations:

```python
import numpy as np
import yfinance as yf
K_corn=yf.download("ZC=F")["Adj Close"].dropna()
K_returns=np.log(K_corn).diff(1).dropna()
K=K_corn.iloc[-1]
sigma_K=np.std(K_returns)
sigma_A=0.0125
rho=0.8
m=5000
F=K*m
A=380
Q=500000
V=A*Q
h=rho*(sigma_A/sigma_K)*(V/F)
```

This code yields $\sigma_K = 0.0182$ (1.82%) and $h = 54.023 \approx 54$ corn futures contracts. Note that the "naive" hedge quantity is $\dfrac{V}{F} = 98.32 \approx 98$. The optimal hedge *ratio* is $\rho\dfrac{\sigma_A}{\sigma_K} = 54.95$, the reason being that we assumed your corn price is less volatile than the corn futures price.

Question 7. In the lecture notes we presented optimal hedging with the objective of reducing the beta β of a share portfolio to 0. We can generalise this to modifying the portfolio's beta to some **target beta** of $\hat{\beta}$. The following formula tells us how many futures contracts $\hat{h}$ to short in order to modify the portfolio's beta to the target beta:

$$\hat{h} = (\beta - \hat{\beta})\frac{V}{F},$$

where β is the beta of our share portfolio, V is the current value of our share portfolio (is given by $V = QA$ if we hold Q shares of one company with share price A) and $F = Km$ is the notional or face value of 1 index futures contract.

1. Using the Tesla optimal hedging example in the lecture notes, how many NYSE FANG+ index futures contracts should we short if we want to reduce Tesla's beta relative to the NYSE FANG+ index to $\hat{\beta} = 0.5$? In the first example in the lecture notes we had $V = \$997,556$, $F = \$54,686$, and $\beta = 1.248$. Hence, we want to short

$$\hat{h} = (\beta - \hat{\beta})\frac{V}{F} = (1.248 - 0.5)\frac{997556}{54686} = 13.65 \approx 14$$

contracts. Recall that to achieve an optimal hedge targeted at reducing our Tesla portfolio's beta to $\hat{\beta} = 0$, in the lecture notes we got $h = 23$.

2. Using the NYSE FANG+ index portfolio example in the lecture notes, how many NYSE FANG+ index futures contracts should we short if we want to reduce our portfolio beta to $\hat{\beta} = 0.7$? In this example we had $V = \$2,710,180$, F is as above, and $\beta = 0.922$. Hence, we calculate that

$$\hat{h} = (0.922 - 0.7)\frac{2710180}{54686} = 11,$$

recalling that in the lecture notes we calculated $h = 46$.

3. How many futures would you need to "short" in each case here in order to increase the beta of your Tesla holding and your index holding to $\hat{\beta} = 2$? *Remark*: Hedge funds often use futures like this as a **market timing** strategy: They use futures to reduce their portfolio betas (deleverage or risk off) when they believe the market will fall and increase their portfolio beta (leverage up or risk on) if they believe the market will rise. I'll do the calculation for the Tesla holding:

$$\hat{h} = (1.248 - 2)\frac{997556}{54686} = -13.72 \approx -14,$$

meaning that we would go *long* $\hat{h} = 14$ futures contracts (risk on!).