A Breif Description of the Pigeon Hole Principle

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The PigeonHole Theorem

$$f : X- > Y, |X| > |Y| \Rightarrow \exists\ x_1, x_2 \in X : x_1 \neq x_2\ \wedge\ f(x_1) = f(x_2)$$

The Pigeonhole Principle is a fundamental discrete mathametics principle defining the characteristics of two sets of seperate cardinalities. The theorem begins with two claims, that the set X is mapped to y $\big(f : X- > Y\big)$ and that the cardinality of X is greater than the cardinality of Y. $\big(|X| > |Y|\big)$ This is used to imply the next part of the equation.

The next part is implied based off of the first two claims. It is imlied that there exists 2 members of X that have seperate indicies.Yet are mapped to the same Y.

Extended Pigeon Hole Theorem

$$f : X \to Y, |X| > k \ * \ |Y|, \Rightarrow y_1 = f(x_1)...f(x_{k+1}) =$$

The extended Pigeon Hole Principle speaks of the multiplicity of sets. The theorem uses k as a variable to multiply the cardinality of Y. $(k \ * \ |Y|)$

The theorem proposes that if the cardinality of X, is greater than k times the cardinality of Y.$(|X| > k \ * \ |Y|)$ That there is at least one member of Y that is mapped to k + 1 members of X. $(y_1 = f(x_1)...f(x_{k+1}))$

To put in terms of pigeons, the Extended Pigeon Hole Theorem states that if there are more pigeons than k times the number of Pigeon Holes, $(|Pigeons| > k \ * \ |PigeonHoles|)$ Than there must be k + 1 pigeons in 1 hole. The logic behind this is that since there are more pigeons than k times the number of pigeon holes than mathamatically each pigeon hole must have k pigeons, and one pigeon hole must hvae k + 1 pigeons. To assign variables to make it more readable, there will be 32 pigeons and 6 pigeon holes, and a (k) value of 5. Then we would be saying

$32 > 6 * 5, \Rightarrow Pigeonhole_1 = Pigeon_1...Pigeon_{5+1}$ at least one PigeonHole will have 6 pigeons, or $\lceil 32/6 \rceil$ the ceiling of 32 over 6, (which means the smallest whole number representation of 32 over 6.