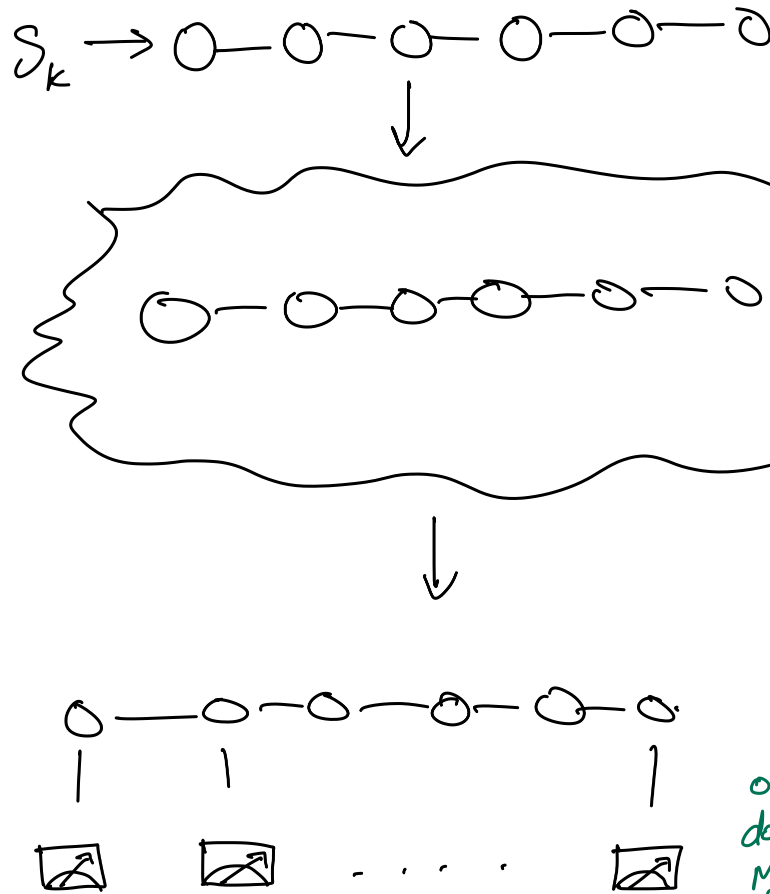
Changes to code

In code, need to switch order

Inject
↓
Measure
↓
Evolve

Inject
↓
Evolve
↓
Measure

inject



$U(t) = e^{-iHt}$

unitary evolution

Measurement
+
Measurement backaction

order doesn't matter here

In code need to change this

Dephasing channel:

$$\mathcal{M}[P] = (1-2g)^{\chi(P)} P$$

where $\chi(P)$ counts $\#\{Z, Y\}$ in P

$$\mathcal{M}[P] = \left(e^{-g^2/2}\right)^{\chi(P)} P$$

Difference between Schrödinger / Heisenberg

states evolving

true even if the state is represented by a density matrix

$$|\psi(t)\rangle = U(t)|\psi_0\rangle$$
$$\rho(t) = U(t)\rho U(t)^\dagger$$

observable evolving

$$\frac{dO(t)}{dt} = i[H, O]$$

$$O(t) = U(t)^\dagger O U(t)$$

dual

double check that your code does this

$$\frac{d\rho(t)}{dt} = -i[H, \rho]$$

To replicate the model in the paper, need to sample the couplings J_i uniformly from $[-J_s/2, J_s/2]$, where J_s is some reference (just set it equal to 1).

Initial state: Before any idea of injection/evolution/etc, the system needs to be initialized in a state ρ_0.
For the moment let's try the all zero state:

$$\rho_0 = |000\ldots 0\rangle\langle 000\ldots 0| = |0\rangle^{\otimes N}\langle 0|^{\otimes N}$$

Pure states are generally quite spread in the Pauli basis which is annoying, so when we move to larger systems we might move to something else.

To find representation for ρ_0 in Pauli strings. Use:

$$\rho = \sum c_P P \quad \text{where} \quad c_P = \mathrm{Tr}(\rho P)/2^N$$

All zero state only has overlap with Pauli strings that only contain 'Z' or '$\mathbb{1}$'. In fact it is a +1 eigenstate of all of these strings...

$$c_P = \begin{cases} 1/2^N & \text{if } P \text{ only contains 'Z' or '}\mathbb{1}\text{'} \\ 0 & \text{otherwise} \end{cases}$$

functionality in PauliStrings.jl : ρ = all_zeros(N)

Other notes

Projective Measurements

Weak measurements are a generalization

Projectors:

$\Pi_j^\dagger = \Pi_j$, $\Pi_j^2 = \mathbb{1}$

Measure observable $\hat{O}$

Projectors

$$\hat{O} = \sum \lambda_j \Pi_j$$

$\Pi_j = |v_j\rangle\langle v_j|$

eigenvectors of $\hat{O}$

don't have to be 1-dim

Krauss operators given by these projectors

$$K_j = \Pi_j \qquad \{K_j\} \longrightarrow \sum K_j^\dagger K_j = \mathbb{I}$$

P Pauli string $\rightarrow$ My projectors are given by

$$\Pi_+ = \frac{\mathbb{1} + P}{2} \quad , \quad \Pi_- = \frac{\mathbb{1} - P}{2}$$

eval $+1$ eval -1

"Backaction Map" $\mathcal{M}_p[A] = \sum K_j A K_j^\dagger$

$\hookrightarrow$ See overleaf notes for specific form when A is Pauli string.

Short term memory test

Can I retrieve signal s_k at τ steps later?
→ Implementing delay τ done at training level - not at reservoir dynamics level, so don't worry for now.

Just need to generate signal list $\{s_k\}$ of 0's, 1's and inject s_k @ step k via the following encoding on site 1:

to indicate only on 1st qubit

$$\rho_k^{(1)} = |\psi_k\rangle\langle\psi_k| \quad , \quad |\psi_k\rangle = \sqrt{1-s_k}\,|0\rangle + \sqrt{s_k}\,|1\rangle$$

→ Need to find Bloch representation of $\rho_k^{(1)}$:

$$r_k^x = \mathrm{Tr}(\rho X), \quad r_k^y = \mathrm{Tr}(\rho Y), \quad r_k^z = \mathrm{Tr}(\rho Z)$$

Once have these, can use them in inject code.