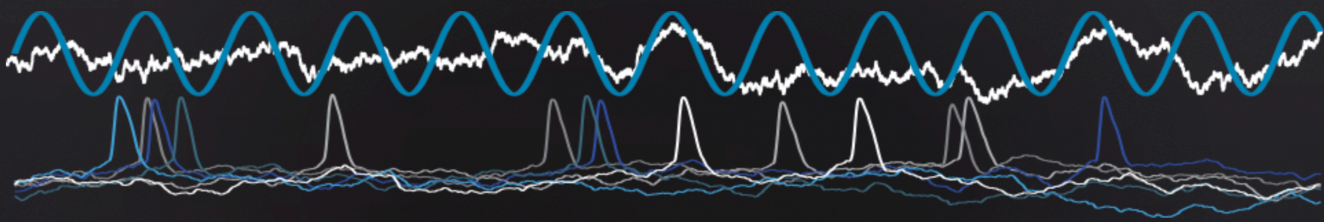
PRINCIPLES OF COMPUTATIONAL NEUROSCIENCE



AN INTRODUCTORY COURSE OFFERED TO
(UNDER)GRAD NEUROSCIENCES STUDENTS AT UNITS AND SISSA, TRIESTE (ITALY).

Michele GIUGLIANO

Mean field description of synaptic transmission

Even simpler models
of synaptic transmission:

“mean-field” descriptions
(*firing* *rate* models)

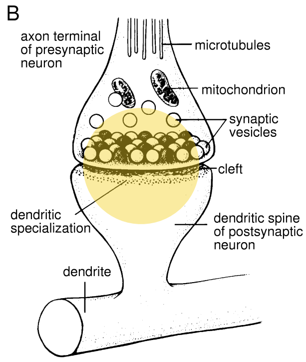
current-driven model synapse

$$I_{syn} \approx \bar{I}\ O(t)$$

$$\boxed{\bar{I} = \bar{g}_{syn}\ (E_{syn} - <V_m>)}$$

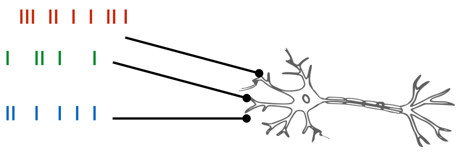
$$\frac{dI_{syn}}{dt} \approx -\beta I_{syn} + W \sum_k \delta\left(t - t_k\right)$$

$$W = \bar{g}\alpha T_{max}$$



TOTAL [current-driven] synaptic input

$$C\frac{dV}{dt} = ... + I_{syn_1} + I_{syn_2} + I_{syn_3}$$

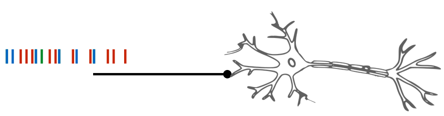


$$\frac{dI_{syn_1}}{dt} \approx -\beta I_{syn_1} + W_1 \sum_{k_1} \delta(t - t_{k_1})$$

$$\frac{dI_{syn_2}}{dt} \approx -\beta I_{syn_2} + W_2 \sum_{k_2} \delta(t - t_{k_2})$$

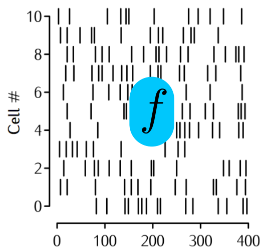
$$\frac{dI_{syn_3}}{dt} \approx -\beta I_{syn_3} + W_3 \sum_{k_3} \delta(t - t_{k_3})$$

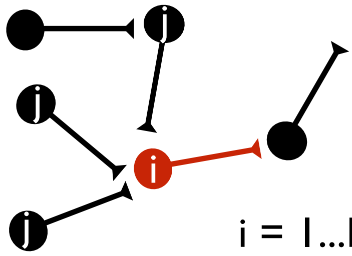
$$\frac{d(\qquad I_{syn} \qquad)}{dt} \approx -\beta(\qquad I_{syn} \qquad)+$$



$$+\sum_{m=1}^{3} W_m \sum_{k_m} \delta(t - t_{k_m})$$

Large [feed-forward] network of neurons





i = 1...N

$$\frac{dI_{syn\ i}}{dt} \approx -\beta I_{syn\ i} + \sum_j C_{ij}\, W_{ij} \sum_{k_j} \delta\left(t - t_{k_j}\right)$$

- each neuron has identical [intrinsic] parameters...
- each neuron fires independently from each other...
- each one firing asynchronously, irregularly [Poisson], ~f

"Mean-field" approximation:

neurons are indistinguishable and
*share the same **average** synaptic input*

$$\frac{dI_{syn\ i}}{dt} \approx -\beta I_{syn\ i} + \sum_j C_{ij}\, W_{ij} \sum_{k_j} \delta\left(t - t_{k_j}\right)$$

$$\frac{d}{dt} < I_{syn} > \approx -\beta < I_{syn} > + \sum_j \sum_j \ldots >$$

$$< C_{ij} W_{ij} \sum_{k_j} \delta(t - t_{k_j}) > = ??$$

If feed-forward, easy!

If recurrent, … ??!?

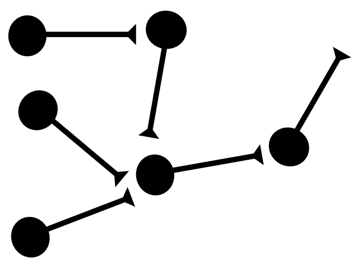
“Mean-field” approximation:

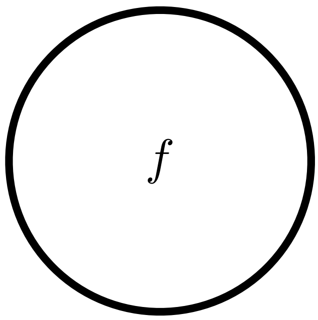
neurons are indistinguishable and
share the same average synaptic input

$$< C_{ij} >< W_{ij} >< \sum_{k_j} \delta(t - t_{k_j}) >= c\ w\ f$$

$$\frac{d}{dt} < I_{syn} > \approx -\beta < I_{syn} > + N\ c\ w\ f$$

- each neuron has identical [intrinsic] parameters...
- each neuron fires independently from each other...
- each one firing asynchronously, irregularly [Poisson], ~f



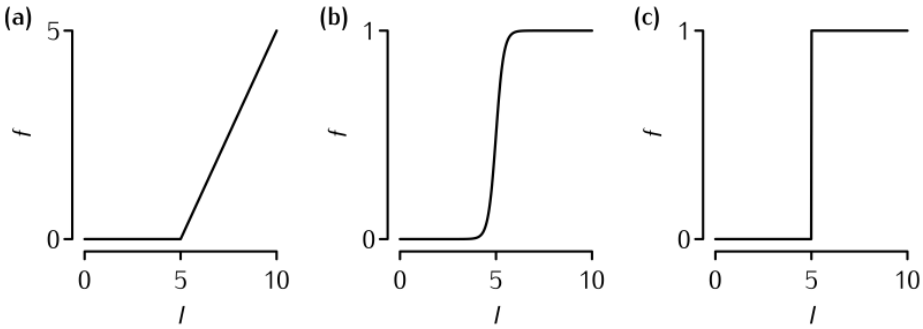


$$\frac{d}{dt} < I_{syn} > \approx -\beta < I_{syn} > + N\ c\ w\ f$$

$$f = F(I_{syn})$$

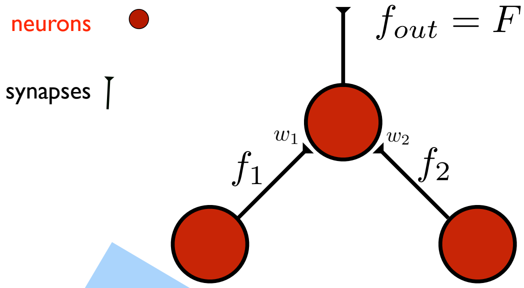
Rate models
[large populations]

$$f_i = F(I_{syn_i})$$

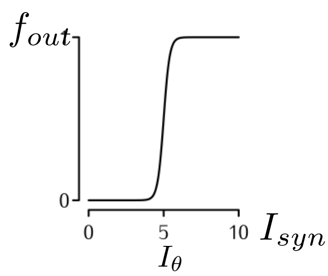


from Sterratt et al., 2011

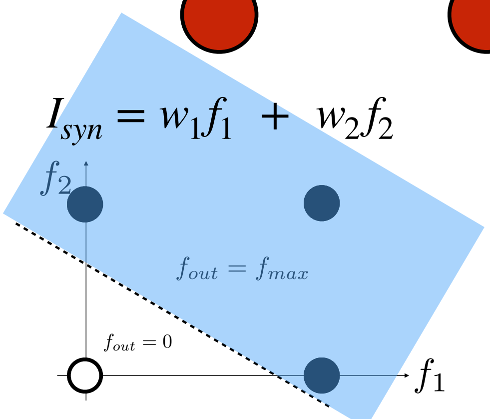
Rate models for the description of a **feedforward** cortical circuit



$$f_{out} = F(I_{syn})$$



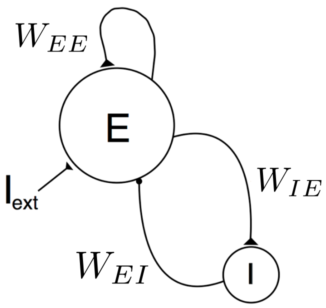
$$I_{syn} = w_1 f_1 + w_2 f_2$$

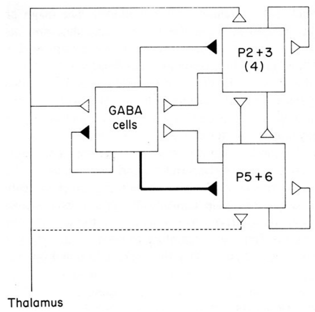


$$f_{out} = 0, \quad I_{syn} < I_\theta$$
$$f_{out} = f_{max}, \quad I_{syn} > I_\theta$$

Hertz, Krogh, Palmer, 1991, Addison-Wesley
Amit, 1989, Cambridge Univ. Press
Rolls & Treves, 1998, Oxford Univ. Press

Rate models for the description of a (cortical column) **recurrent** circuit?





$$f_E = F(< I_{syn\ E} >)$$

$$f_I = F(< I_{syn\ I} >)$$

$$\frac{d}{dt} < I_{syn\ E} > \approx -\beta_E < I_{syn\ E} > + N_E\ c_{EE}\ w_{EE} f_E - N_I\ c_{EI}\ w_{EI} f_I\ +\ I_{ext}$$

$$\frac{d}{dt} < I_{syn\ I} > \approx -\beta_I < I_{syn\ I} > + N_E\ c_{IE}\ w_{IE} f_E$$

Douglas & Martin, 1991