Daniel Cu Sánchez – A01703613

Markov chains

For each of the following matrices determine:

a) If it represents a regular or non-regular Markov chain.
b) If it's an absorbing Markov chain.
c) The long trend or steady state of the matrix (if that's the case)

(a) $\mathbf{P} = \begin{pmatrix} .5 & .5 \\ .5 & .5 \end{pmatrix}$.

(b) $\mathbf{P} = \begin{pmatrix} .5 & .5 \\ 1 & 0 \end{pmatrix}$.

(c) $\mathbf{P} = \begin{pmatrix} 1/3 & 0 & 2/3 \\ 0 & 1 & 0 \\ 0 & 1/5 & 4/5 \end{pmatrix}$.

(d) $\mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

(e) $\mathbf{P} = \begin{pmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}$.

(f) $\mathbf{P} = \begin{pmatrix} 1 & 0 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 0 & 1 \end{pmatrix}$

A – Es regular

No es una cadena absorbente

$$\begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}^{5} = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}$$

B – Es regular

No es una cadena absorbente

Display decimals, number of fraction digits: 3

$$\begin{pmatrix} 0.5 & 0.5 \\ 1 & 0 \end{pmatrix}^{13} = \begin{pmatrix} 0.667 & 0.333 \\ 0.667 & 0.333 \end{pmatrix}$$

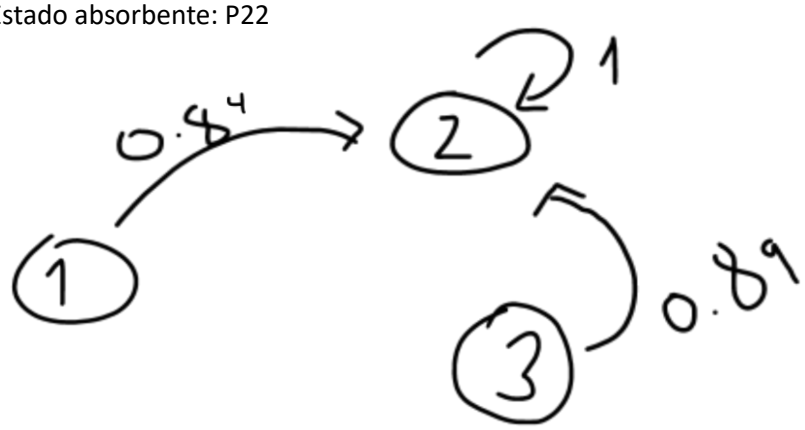
C – NO es regular

Display decimals, number of fraction digits: 3

$$\begin{pmatrix} \frac{1}{3} & 0 & \frac{2}{3} \\ 0 & 1 & 0 \\ 0 & \frac{1}{5} & \frac{4}{5} \end{pmatrix}^{10} = \begin{pmatrix} +0.000 & 0.847 & 0.153 \\ 0 & 1 & 0 \\ 0 & 0.893 & 0.107 \end{pmatrix}$$

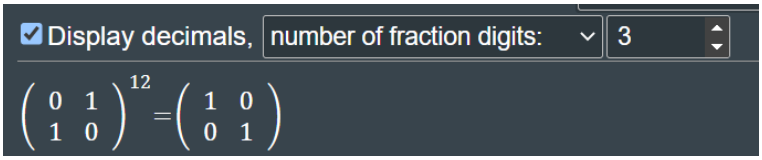
Daniel Cu Sánchez – A01703613

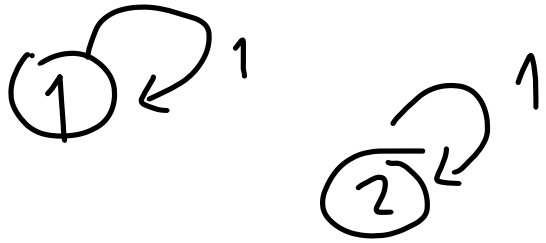
Estado absorbente: P22



Daniel Cu Sánchez – A01703613

D – No es regular

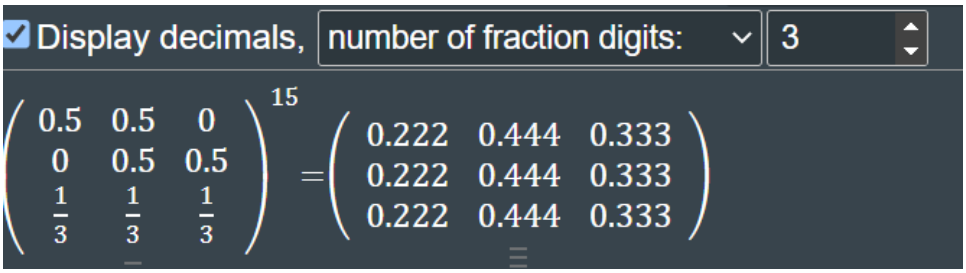




Estados absorbentes: P11, P21

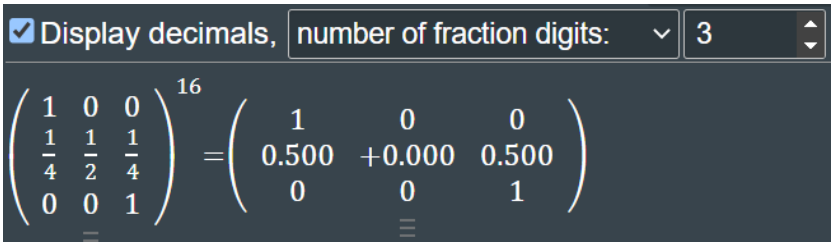
No es una matriz absorbente

E) Es regular



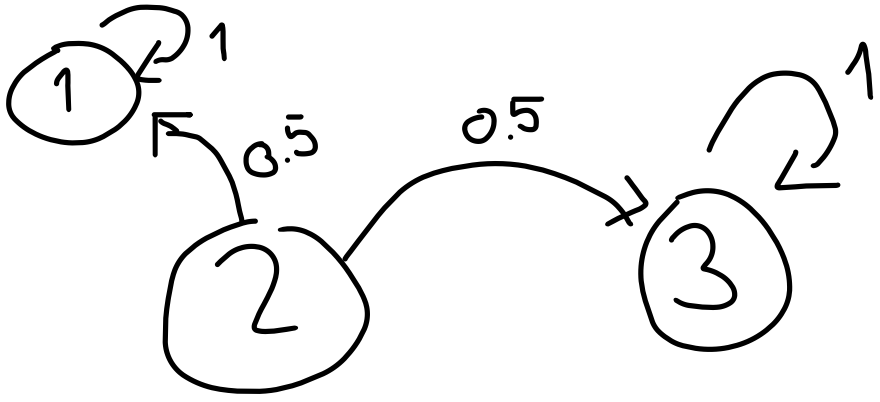
No tiene estados absorbentes

F) No es regular



Estados absorbentes: P11, P33

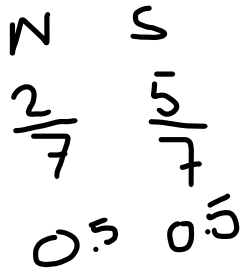
No es una matriz absorbente



Example 11.1 According to Kemeny, Snell, and Thompson,[2] the Land of Oz is blessed by many things, but not by good weather. They never have two nice days in a row. If they have a nice day, they are just as likely to have snow as rain the next day. If they have snow or rain, they have an even chance of having the same the next day. If there is change from snow or rain, only half of the time is this a change to a nice day.

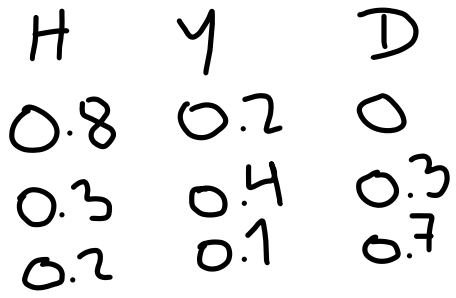
[2] J. G. Kemeny, J. L. Snell, G. L. Thompson, *Introduction to Finite Mathematics*, 3rd ed. (Englewood Cliffs, NJ: Prentice-Hall, 1974).

a) Draw the diagram state and the transition matrix (call the states N, S, R)
b) If today (monday) is snowy, what is the probability of being rainy next Thursday?
c) Calculate the long term probabilities for all three types of weather.



Example 11.6 In the Dark Ages, Harvard, Dartmouth, and Yale admitted only male students. Assume that, at that time, 80 percent of the sons of Harvard men went to Harvard and the rest went to Yale, 40 percent of the sons of Yale men went to Yale, and the rest split evenly between Harvard and Dartmouth; and of the sons of Dartmouth men, 70 percent went to Dartmouth, 20 percent to Harvard, and 10 percent to Yale.

a) Draw the diagram state and the transition matrix (call the states H, D, Y)
b) Suppose you went at Yale. What is the probability that your grandson will also go to Yale?
c) Download the following Family Tree Suppose that guy with the blue arrow went to Harvard. What is the probability that the girl with the pink arrow went to Dartmouth?



b)

$$\begin{pmatrix} 0.8 & 0.2 & 0 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.1 & 0.7 \end{pmatrix}^2 = \begin{pmatrix} 0.7 & 0.24 & 0.06 \\ 0.42 & 0.25 & 0.33 \\ 0.33 & 0.15 & 0.52 \end{pmatrix}$$

P = 0.25