Long-Run Productivity Risk (Croce, 2014)

Numerical Result Replication

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This project aim to replicate the result of Croce (2014) long-run productivity risk model. Croce (2014) builds the long-run risk framework proposed by Bansal & Yaron (2004) into general equilibrium model with production. The model manages solve the risk premium puzzle, the risk free rate puzzle, and the volatility puzzle in a RBC type of production economy. The numerical simulation result shows that the model is relatively successful in generating volatility in asset price and real variables including consumption, investment and output.

1 Model

1.1 Model Setup

Because there is no market imperfection, the economy can be summarized by a social planner problem according to the First Fundamental Theorem of Welfare Economics. The consumer maximize the following Epstein-Zin (1989) target function:

$$U_t = \left[(1-\delta)\widetilde{C}_t^{1-\frac{1}{\Psi}} + \delta \left(E_t \left[U_{t+1}^{1-\gamma} \right] \right)^{\frac{1-\frac{1}{\Psi}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\Psi}}}$$

With respect to the following constraints:

Consumption Leisure aggregation:

$$\widetilde{C}_t = \left[oC_t^{1-\frac{1}{\xi_l}} + (1-o)\left(A_{t-1}l_t\right)^{1-\frac{1}{\xi_l}} \right]^{\frac{1}{1-\frac{1}{\xi_l}}}$$

$$\Rightarrow \quad \widetilde{C}_t = C_t^o (A_{t-1}l_t)^{1-0} \quad \text{as } \xi_l = 0$$

Cobb-Douglas Production Function:

$$Y_t = K_t^{\alpha} \left[A_t N_t \right]^{1-\alpha}$$

Resources Allocation Constraint:

$$Y_t \geq C_t + I_t$$

Time Allocation Constraint:

$$1 \geq N_t + L_t$$

Dynamic of Capital:

$$K_{t+1} \leq (1 - \delta_k) K_t + I_t - G_t K_t$$

Adjustment Cost:

$$G_t = \frac{I_t}{K_t} - \left[\frac{\alpha_1}{1 - \frac{1}{\xi}} \left(\frac{I_t}{K_t} \right)^{1-\frac{1}{\xi}} + \alpha_0 \right]$$

The stochastic process of the technology is given by:

Productivity Process:

$$\log \frac{A_{t+1}}{A_t} \equiv \Delta a_{t+1} = \mu + \underbrace{x_t}_{LRR} + \sigma \underbrace{\epsilon_{a,t+1}}_{SRR}$$

Long-run Risk Process:

$$x_t = \rho x_{t-1} + \sigma_x \epsilon_{x,t}$$

Innovation Process:

$$\begin{bmatrix} \epsilon_{a,t+1} \\ \epsilon_{x,t+1} \end{bmatrix} \sim iidN \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_{xa} \\ \rho_{xa} & 1 \end{bmatrix} \right)$$

1.2 Detrend Variables and Log Growth Rates

On the balance growth path, all the real variables $\{c_t, \tilde{c}_t, i_t, y_t, k_t\}$ growth at same rate, so that these variables are not stationary. We can define the following log detrended variables as follows:

$$\{c_t, \tilde{c}_t, i_t, y_t, k_t\} \equiv \left\{\ln \frac{C_t}{A_{t-1}}, \ln \frac{\tilde{C}_t}{A_{t-1}}, \ln \frac{I_t}{A_{t-1}}, \ln \frac{Y_t}{A_{t-1}}, \ln \frac{K_t}{A_{t-1}}\right\}$$

Also, we can define the growth rate of variables as:

$$\{\Delta a_t, \Delta c_t, \Delta \tilde{c}_t, \Delta i_t, \Delta y_t, \} \equiv \left\{\ln \frac{A_t}{A_{t-1}}, \ln \frac{C_t}{C_{t-1}}, \ln \frac{\tilde{C}_t}{\tilde{C_{t-1}}}, \ln \frac{I_t}{I_{t-1}}, \ln \frac{Y_t}{Y_{t-1}}\right\}$$

1.3 System of Detrended Equations

We can write the system of equations in the model in **detrended variables** and **log growth rates**. To conciliate with the original dynare code provided by the author, I write all the equations in linearity form. However, it is usually recommended to write down equation in original form to avoid errors in the equations:

1. Process of Productivity:

$$\Delta a_t = \mu + x_{t-1} + \sigma_a \epsilon_{a,t} \tag{1}$$

2. Process of Long Run Risk:

$$x_t = \rho x_{t-1} + \sigma_x \epsilon_{x,t} \tag{2}$$

We can define **Log Utility-consumption Ratio** $uc_t \equiv \ln \frac{U_t}{\tilde{C}_t}$, define **Expected Utility-consumption Ratio:** $Q_t \equiv \frac{E_t(U_{t+1}^{1-\gamma})}{\tilde{C}_t^{1-\gamma}}$, and define **Log Expected Utility-consumption Ratio:** $logQ_t \equiv \ln(Q_t)$. In reality, there is no need to define the above variables. I define the above variables to be accordance with the author:

3. Expected Utility-consumption Ratio

$$Q_t \equiv \left(\frac{E_t(U_{t+1})}{\tilde{C}_t}\right)^{1-\gamma} = \left(\frac{E_t(U_{t+1})}{E_t(\tilde{C}_{t+1})}\frac{E_t(\tilde{C}_{t+1})}{\tilde{C}_t}\right)^{1-\gamma}$$

$$\Rightarrow Q_t = \exp\left[\left(uc_{t+1} + \Delta\tilde{c}_{t+1}\right)(1-\gamma)\right] \tag{3}$$

4. Log Expected Utility-consumption Ratio:

$$\Rightarrow logQ_t = \ln Q_t \tag{4}$$

5. Utility Function:

$$U_t = \left[(1-\delta)\widetilde{C}_t^{1-\frac{1}{\Psi}} + \delta\left(E_t\left[U_{t+1}^{1-\gamma}\right]\right)^{\theta}\right]^{\frac{1}{1-\frac{1}{\Psi}}} \quad \text{where } \theta = \frac{1-\frac{1}{\Psi}}{1-\gamma}$$

$$\Rightarrow \left(1-\frac{1}{\Psi}\right)\ln U_t = \ln\left[(1-\delta)\widetilde{C}_t^{1-\frac{1}{\Psi}} + \delta\left(E_t\left[U_{t+1}^{1-\gamma}\right]\right)^{\theta}\right]$$

$$\Rightarrow \left(1-\frac{1}{\Psi}\right)\underbrace{\ln\frac{U_t}{\widetilde{C}_t}}_{uc_t} = \ln\left[(1-\delta) + \delta\underbrace{\left(\frac{E_t\left[U_{t+1}^{1-\gamma}\right]}{\widetilde{C}_t^{1-\gamma}}\right)^{\theta}}_{Q_t}\right]$$

$$\Rightarrow \exp\left[\left(1-\frac{1}{\Psi}\right)uc_t\right] = (1-\delta) + \delta Q_t^{\theta} \tag{5}$$

6. Consumption Leisure Aggregation

$$\tilde{C}_t = C_t^{o}(A_{t-1}L_t)^{1-o}$$

$$\Rightarrow \ln\tilde{C}_t = o\ln C_t^{o} + (1-o)(\ln A_{t-1} + L_t)$$

$$\Rightarrow \ln\frac{\tilde{C}_t}{A_{t-1}} = o\ln\frac{C_t^{o}}{A_{t-1}} + (1-o)(L_t)$$

$$\Rightarrow \tilde{c}_t = c_t + (1-o)l_t \tag{6}$$

7. Production Function

$$Y_t = K_t^{\alpha}\left[A_tN_t\right]^{1-\alpha}$$

$$
\begin{aligned}
\Rightarrow \ln \frac{Y_t}{A_{t-1}} &= \alpha \ln \frac{K_t}{A_{t-1}} + (1-\alpha)\left(\ln N_t + \ln \frac{A_t}{A_{t-1}}\right) \\
\Rightarrow \exp ya_t &= \exp(\alpha \cdot ka_t) \cdot \exp(n_t + da_t) \qquad (7)
\end{aligned}
$$

8. Resources Constraint

$$
\begin{aligned}
Y_t &\geq C_t + I_t \\
\Rightarrow \frac{Y_t}{A_{t-1}} &= \frac{C_t}{A_{t-1}} + \frac{I_t}{A_{t-1}} \\
\Rightarrow \exp(y_t) &= \exp(c_t) + \exp i_t \qquad (8)
\end{aligned}
$$

9. Time Constraint

$$
\begin{aligned}
1 &\geq N_t + L_t \\
\Rightarrow 1 &= \exp(n_t) + \exp(l_t) \qquad (9)
\end{aligned}
$$

To better write down the capital dynamic function, we can follow the original paper and define **Log Efficient Capital Investment** as $\tilde{i}_t = \ln(\frac{I_t}{K_t} - G_t)$

10. Efficient Investment

$$
\begin{aligned}
\tilde{i}_t &= \ln(\frac{I_t}{K_t} - G_t) \\
\Rightarrow \exp(\tilde{i}_t) &= \frac{I_t/A_{t-1}}{K_t/A_{t-1}} - G_t \\
\Rightarrow \exp(\tilde{i}_t) &= \exp(i_t - k_t) - G_t \qquad (10)
\end{aligned}
$$

11. Capital Dynamics

$$
\begin{aligned}
K_t &\leq (1-\delta_k)\, K_{t-1} + I_{t-1} - G_{t-1}K_{t-1} \\
\Rightarrow \frac{K_t}{A_{t-1}}\frac{A_{t-1}}{A_{t-2}} &= (1-\delta_k)\,\frac{K_{t-1}}{A_{t-2}} + \frac{I_{t-1}}{K_{t-1}}\frac{K_{t-1}}{A_{t-2}} - G_{t-1}\frac{K_{t-1}}{A_{t-2}} \\
\Rightarrow \frac{K_t}{A_{t-1}}\frac{A_{t-1}}{A_{t-2}} &= \left(1 - \delta_k + \underbrace{\frac{I_{t-1}}{K_{t-1}} - G_{t-1}}_{\exp(\tilde{i}_{t-1})}\right)\frac{K_{t-1}}{A_{t-2}} \\
\Rightarrow \exp\left(k_t + a_{t-1}\right) &= \big(1 - \delta_k + \exp(\tilde{i}_{t-1})\big)\exp(k_{t-1}) \qquad (11)
\end{aligned}
$$

12. Adjustment Cost

$$G_t = \frac{I_t}{K_t} - \left[\frac{\alpha_1}{1-\frac{1}{\xi}}\left(\frac{I_t}{K_t}\right)^{1-\frac{1}{\xi}} + \alpha_0\right]$$

$$\Rightarrow G_t = \frac{I_t/A_{t-1}}{K_t/A_{t-1}} - \left[\frac{\alpha_1}{1-\frac{1}{\xi}}\left(\frac{I_t/A_{t-1}}{K_t/A_{t-1}}\right)^{1-\frac{1}{\xi}} + \alpha_0\right]$$

$$\Rightarrow G_t = \exp(i_t - k_t) - \left[\frac{\alpha_1}{1-\frac{1}{\xi}}\exp\left((i_t-k_t)(1-\frac{1}{\xi})\right) + \alpha_0\right] \tag{12}$$

13. Adjustment Cost Derivative

$$G_t'(\frac{I_t}{K_t}) = 1 - \alpha_1\left(\frac{I_t}{K_t}\right)^{-\frac{1}{\xi}}$$

$$\Rightarrow G'(t) = 1 - \alpha_1 \exp\left[-\frac{1}{\xi}(i_t - k_t)\right] \tag{13}$$

14. Stochastic Discount Factor

$$M_{t+1} = \delta\left(\frac{C_{t+1}}{C_t}\right)^{-1/\xi_l}\left(\frac{\widetilde{C}_{t+1}}{\widetilde{C}_t}\right)^{\frac{1}{\xi}_l-\frac{1}{\Psi}}\left(\frac{U_{t+1}}{E_t\left[U_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\Psi}-\gamma}$$

$$\Rightarrow M_{t+1} = \delta\left(\frac{C_{t+1}}{C_t}\right)^{-1/\xi_l}\left(\frac{\widetilde{C}_{t+1}}{\widetilde{C}_t}\right)^{\frac{1}{\xi}_l-\frac{1}{\Psi}}\left(\frac{\frac{U_{t+1}}{\tilde{C}_{t+1}}\frac{\tilde{C}_{t+1}}{\tilde{C}_t}}{\left[\frac{E_t U_{t+1}^{1-\gamma}}{C_t^{1-\gamma}}\right]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\Psi}-\gamma}$$

$$\Rightarrow \exp(m_{t+1}) = \delta\exp\left(-\Delta c_{t+1}\frac{1}{\xi_l}\right)\exp\left(\Delta\tilde{c}_{t+1}(\frac{1}{\xi_l}-\frac{1}{\Psi})\right)\left(\frac{\exp(uc_{t+1}+\Delta\tilde{c}_{t+1})}{Q_t^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\Psi}-\gamma}$$

$$\Rightarrow \exp(m_{t+1}) = \delta\exp\left(-\Delta c_{t+1}\frac{1}{\xi_l}\right)\exp\left(\Delta\tilde{c}_{t+1}(\frac{1}{\xi_l}-\frac{1}{\Psi})\right)\frac{\exp\left((\frac{1}{\Psi}-\gamma)(uc_{t+1}+\Delta\tilde{c}_{t+1})\right)}{Q_t^{1-\theta}} \tag{14}$$

15. Labor Market Equilibrium Condition

$$\frac{\partial\widetilde{C_t}}{\partial l_t}/\frac{\partial\widetilde{C_t}}{\partial C_t} = (1-\alpha)\frac{Y_t}{n_t}$$

$$\Rightarrow (\frac{1}{o} - 1) \exp(c_t - n_t) = (1 - \alpha) \exp(y_t - n_t) \tag{15}$$

16. Log Dividend

$$d_{t+1} \equiv \alpha \frac{y_{t+1}}{k_{t+1}} - \delta_k q_{t+1} - \frac{i_{t+1}}{k_{t+1}} + q_{t+1} G\left(\frac{i_{t+1}}{k_{t+1}}\right)$$

$$\Rightarrow \exp(d_t) = \alpha \exp(y_t - k_t) + (\exp(\tilde{i}_t) - \delta_k) \exp(q_t) - \exp(i_t - k_t); \tag{16}$$

17. Capital Equilibrium Condition

$$Q_t = \frac{1}{1 - G'_t} \tag{17}$$

18. Log Price to Dividend Ratio

$$PD_t = \frac{Q_t}{D_t}$$

$$pd_t = q_t - d_t \tag{18}$$

19. Log Market Capital Return

$$R_t = \frac{Q_t + D_t}{Q_{t-1}}$$

$$\Rightarrow \exp(r_t) = \frac{\exp(q_t) + \exp(d_t)}{\exp(q_{t-1})} \tag{19}$$

20. Fundamental Theorem of Asset Pricing

$$1 = E_t(R_{t+1} M_{t+1})$$

$$\Rightarrow 1 = E_t[\exp(r_{t+1}) + \exp(m_{t+1})] \tag{20}$$

21. Risk Free Return

$$1 = E_t(M_{t+1})R_{f,t}$$
$$\Rightarrow \frac{1}{\exp(r_{f,t})} = E_t[\exp(m_{t+1})] \tag{21}$$

22. Risk Premium

$$rp_t = r_t - r_{f,t-1} \tag{22}$$

23. Leveraged Risk Premium

$$r_t^{LEV} = \phi_{lev}\left(r_t - r_{f,t-1}\right) + \epsilon_t^d \tag{23}$$

Finally, we include the definition of growth rates:

24. Definition of Log Output Growth

$$\Delta y_t \equiv \ln \frac{Y_t}{Y_{t-1}} = \ln \frac{Y_t}{A_{t-1}} - \ln \frac{Y_{t-1}}{A_{t-2}} + \ln \frac{Y_{t-1}}{A_{t-2}}$$
$$\Delta y_t = y_t - y_{t-1} + \Delta a_{t-1} \tag{24}$$

25. Definition of Detrended Log Tilde Consumption Growth

$$\Delta \tilde{c}a_t \equiv \ln \left(\frac{\tilde{C}_t}{A_{t-1}} / \frac{\tilde{C}_{t-1}}{A_{t-2}} \right)$$
$$\Delta \tilde{c}a_t = \tilde{c}a_t - \tilde{c}a_{t-1} \tag{25}$$

26. Definition of Log Tilde Consumption Growth

$$\Delta \tilde{c}_t \equiv \ln \frac{\tilde{C}_t}{\tilde{C}_{t-1}}$$
$$\Delta \tilde{c}_t = \Delta \tilde{c}a_t + \Delta a_{t-1} \tag{26}$$

27. Definition of Log Consumption Growth

$$\Delta c_t \equiv \ln \frac{C_t}{C_{t-1}} = \ln \frac{C_t}{A_{t-1}} - \ln \frac{C_{t-1}}{A_{t-2}} + \ln \frac{C_{t-1}}{A_{t-2}}$$

$$\Delta c_t = c_t - c_{t-1} + \Delta a_{t-1} \tag{27}$$

28. Definition of Log Investment Growth

$$\Delta i_t \equiv \ln \frac{I_t}{I_{t-1}} = \ln \frac{I_t}{A_{t-1}} - \ln \frac{I_{t-1}}{A_{t-2}} + \ln \frac{I_{t-1}}{A_{t-2}}$$

$$\Delta i_t = i_t - i_{t-1} + \Delta a_{t-1} \tag{28}$$

2 Equation System

The equation system consist of 28 endogenous variables as follow:

$$\left\{x, y, c, \tilde{c}, i, k, \Delta y, \Delta a, \Delta c, \Delta \tilde{c}, \Delta \tilde{ca}, \Delta i, uc, \tilde{i}, G, G', Q, logQ, n, l, d, q, pd, m, r, r_f, rp, r_{LEV}\right\}$$

These variables can be solved out using the above 28 equations.

There are 3 exogenous variables as follow:

$$\{\epsilon_a, \epsilon_x, \epsilon_d\}$$

3 Key Steady State Variables

We need to calculate key steady state variables to let dynare solve out the steady state, and calibrate the value a_1 and a_0. These key variables are:

1. Steady State Labor:

Table 1: Endogenous Variables

Variable	Code	Description
x	x	Long-run Productivity Risk
y	ya	Log Detrended Output
c	ca	Log Detrended Consumption
	tca	Log Detrended Tilde Consumption
i	ia	Log Detrended Investment
k	ka	Log Detrended Capital
Δy	dy	Log Output Growth
Δa	da	Log Productivity Growth
Δc	dc	Log Consumption Growth
$\Delta \tilde{c}$	dtc	Log Tilde Consumption Growth
$\Delta \tilde{ca}$	dtca	Log Detrended Tilde Consumption Growth
Δi	di	Log Investment Growth
uc	uc	Log Utility Consumption Ratio
	newk	Log Efficient Capital Investment
G	G	Adjustment Cost
G'	GP	Adjustment Cost Derivative
Q	Q	Expected Utility-Consumption Ratio
logQ	logQ	Log Expected Utility-Consumption Ratio
n	n	Log Labor
l	l	Log Leisure
d	d	Log Dividend
q	q	Log Tobin q
pd	pd	Log Price to Dividend Ratio
m	m	Log SDF
r	r	Log Capital Return
r_f	rf	Log Risk Free Return
rp	exr	Risk Premium
r_{LEV}	rlev	Leverged Risk Premium

Table 2: Exogenous Variables

Variable	**Code**	**Description**
ϵ_a	ea	Short-run Productivity Shock
ϵ_x	ex	Long-run Productivity Shock
ϵ_d	ed	Leveraged Risk-premium Shock

The steady state labor is a calibration target of the model. According to page 19 of the paper, the steady state labor is labor is 0.18.

$$\bar{N} = 0.18 \tag{29}$$

2. Steady State Log Detrended Capital:

To calculate the steady state capital, we start from the consumption Euler Equation:

$$\frac{1}{C_t^{\frac{1}{\Psi}}} = \delta E_t \frac{1}{C_{t+1}^{\frac{1}{\Psi}}} \left(A_{t+1}\alpha K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} + 1 - \delta\right)$$

Given the fact that on steady state $c_t = \ln \frac{C_t}{A_{t-1}}$ is constant, and $\Delta a_t = \ln \frac{A_t}{A_{t-1}} = \mu$. The above equation can be rewrite to:

$$\bar{k} = \ln\left(\frac{\alpha}{\frac{1}{\delta}\exp(\frac{\mu}{\Psi}) - 1 + \delta}\right)^{\frac{1}{1-\alpha}} + \ln(\bar{N}) + \mu \tag{30}$$

3. Steady State Log Detrended Output:

The steady state log detrended output is calculated using the production function, given the above steady state labor and detrended capital:

$$\bar{y} = (1-\alpha)(\mu + log(\bar{N})) + \alpha\bar{k} \tag{31}$$

4. Steady State Log Detrended Investment:

The steady state log detrended investment is calculated using the capital dynamic equation. On steady state, the adjustment cost $G = 0$, and detrended capital $k_t = \ln \frac{K_t}{A_{t-1}}$ is constant:

$$\bar{i} = \ln[\exp(\bar{k})(\exp(\mu) + \delta_k - 1)]$$

5. Steady State Log Utility Tilde Consumption Ratio:

Combine the utility function and consumption Leisure aggregation, we can compute the following steady state log utility tilde consumption ratio

$$\bar{uc} = \ln \left[\frac{1-\delta}{1-\delta \exp\left((1-\frac{1}{\Psi})\mu\right)} \right]^{\frac{\Psi}{\Psi-1}}$$

Using the above steady state variables and parameters provided in Table 3 Panel A, we can calibrate the adjustment cost parameter α_1 and α_0 using equation (12) and equation (13). Other steady state variables are provided in the in the *initval* part of the dynare ++ program.

4 Replication Program

The replication program consists of the following files:

- *Run_me.m*: Main MATLAB program that gives the results;
- *simulation_psi_2.mod*: Dynare++ program that compute the model with Intertemporal Elasticity of Substitution (IES) equals to 2;
- *simulation_psi_09.mod*: Dynare++ program that compute the model with IES equals to 0.9;
- *simulation_no_lrr.mod*: Dynare++ program that compute the model with IES equals to 2, but no long run risk ($\sigma_x = 0$);
- *moments_psi_2.m*: MATLAB program that compute the moments of the model with IES equals to 2 (Table 3 Column 2). Results store at *Moment_psi_2.xls*;
- *moments_psi_09.m*: MATLAB program that compute the moments of the model with IES equals to 0.9 (Table 3 Column 3). Results store at *Moment_psi_09.xls*;
- *moments_no_lrr.m*: MATLAB program that compute the moments of the model with IES equals to 2 but no long run risk (Table 3 Column 4). Results store at *Moment_no_lrr.xls*;
- *plotIRF.m*: MATLAB program that plot the Impulse Response Function (Figure 2).

5 Replication Result

Table 3: Replication of Table 3

	Data	IES=2	IES=0.9	No LRR
$\sigma(\Delta y)(\%)$	3.56 (0.65)	5.64	5.69	2.79
$\sigma(\Delta c)/\sigma(\Delta y)$	0.71 (0.05)	0.82	0.75	0.68
$\sigma(\Delta i)/\sigma(\Delta y)$	4.49 (0.61)	2.73	3.87	2.35
E[I/Y](%)	20.00 (0.97)	29.40	29.52	22.16
$\rho(\Delta c, \Delta i)$	0.39 (0.28)	0.31	0.03	0.82
$\rho(\Delta c, r_{ex}^{LEV})$	0.25 (0.12)	0.12	-0.29	0.43
$\mathrm{E}(\mathrm{r}_{ex}^{LEV})(\%)$	4.71 (2.25)	3.91	-5.06	1.05
$\sigma(r_{ex}^{LEV})(\%)$	20.89 (2.21)	7.81	8.75	7.23
$\sigma(q)$	0.29 (0.05)	0.06	0.07	0.01
$\mathrm{E}(\mathrm{r}_t^f)$	0.65 (0.38)	1.14	5.56	5.41
$\sigma(r_t^f)$	1.86 (0.32)	1.98	3.53	0.81
$\mathrm{ACF}_1[r_{ex}^{LEV}]$	0.09 (0.12)	-0.01	-0.02	0.00
$\mathrm{ACF}_1[r_t^f]$	0.64 (0.06)	0.73	0.90	-0.07
$\mathrm{ACF}_1[q]$	0.86 (0.08)	0.97	0.94	0.87
$\mathrm{ACF}_1[\Delta c]$	0.50 (0.15)	0.67	0.54	0.10

References

Bansal, R., & Yaron, A. (2004). Risks for the long run: A potential resolution of asset pricing puzzles. *The journal of Finance*, *59*(4), 1481–1509.

Croce, M. M. (2014). Long-run productivity risk: A new hope for production-based asset pricing? *Journal of Monetary Economics*, *66*, 13–31.

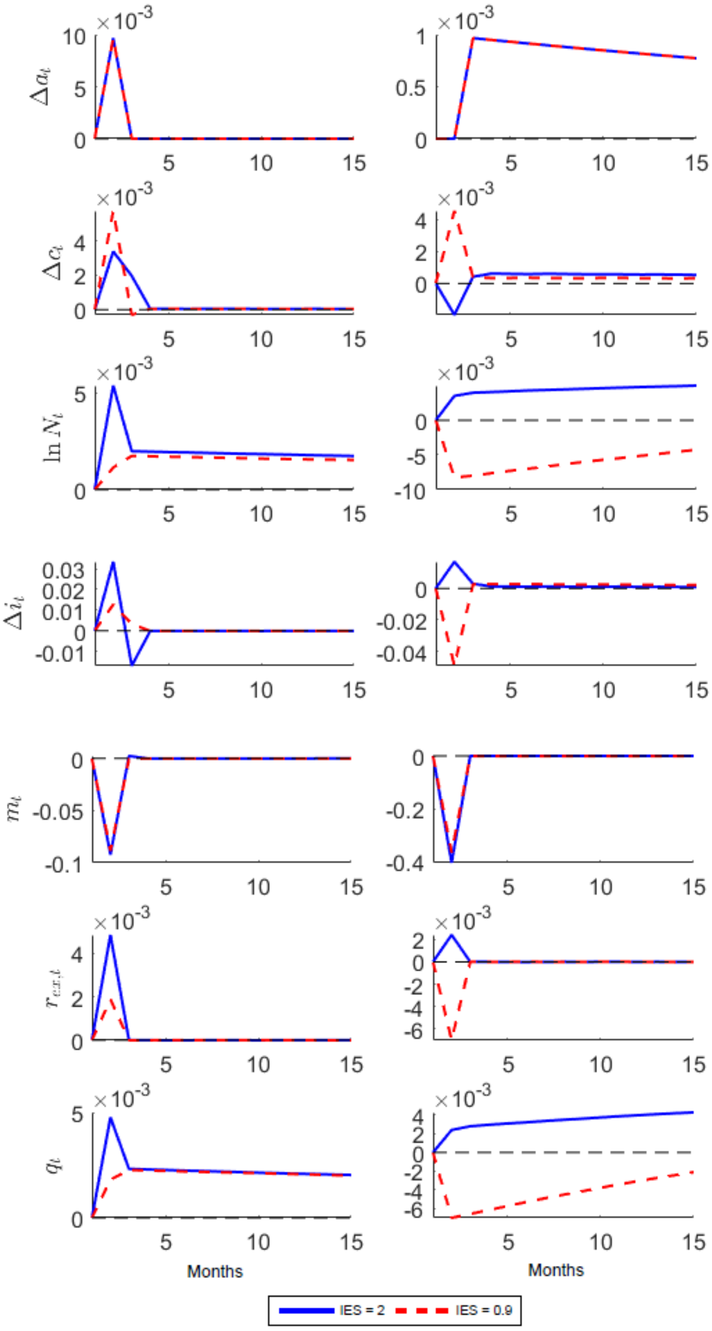

Figure 1: Replication of Figure 2: The Role of IES (Ψ)