Collateral Crisis

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Overview

Motivation:

Financial crisis is hard to explain without resorting to large shocks:

- The Financial Crisis Inquiry Commission (FCIC) Report (2011):
  - "Overall, for 2005 to 2007 vintage tranches of mortgage-backed securities originally rated triple-A, despite the mass downgrades, only about 10 percent of Alt-A and 4 percent of subprime securities had been "materially impaired"
- However, the crisis is large:
  - "13 of the most important financial institutions in the United States, 12 were at risk of failure within a period of a week or two."

Why Bond Market Froze After Credit Shocks?

- After credit shocks (e.g. MBS Crisis in US, Yongmei in China):
  - Inter-bank lending market froze with Repo rate rocket up;
  - Primary market froze for a longer time without bond issuance.
  - The effect is higher when there is a high-credit-rated bond default (e.g. Prime Level MBS before Crisis was AAA; Yongmei default at AAA; Baoshang Bank 'bankrupt' with rate AA+, etc.)

- **But why cannot banks and firms borrow at higher rate?**

Why Bond Market Froze After Credit Shocks?

- Like 'land' in Kiyotaki and Moore (1997), corporate bond also have dual functions: generate investment return and **be collateral for repurchase agreement borrowing (repo)**:
  - Corporate bond investors are highly leveraged relying on especially repo;
  - Investigate the real value of collateral is costly since repo usually matures less than 30 days (the most popular repo agreement is overnight).
- Therefore, corporate bond investors (e.g. trust funds) do not accept the bond that is consider 'bad' collateral, even the return is high.

A. Agents

- Two agents: entrepreneurs and households (each with mass 1):
  - Both agents are risk-neutral.
- Two goods: numeraire K and Land L.
  - Numeraire: represent productive capital and consumption goods.
  - Land: represent collateral (e.g. real estate, MBS, bond, etc.)

- Entrepreneur:
  - Endowed with nontransferable managers N^* and 1 unit of land.
  - Also endowed with $\bar{L} = 1$ unit of land.
- Households:
  - Endowed with $\bar{K} \geq N^*$ unit of numeraire.

B. Goods

- Numeraire (Capital & Consumption) K:
  - Can be used to produce more numeraire in the end of period.
  - Assume numeraire is consumable in the end of period.

$$K' = \begin{cases} A \min\{K, N^*\} & \text{with prob. } q \\ 0 & \text{with prob. } (1-q). \end{cases} \tag{1}$$

  - Technical assumption: $qA > 1$ (i.e. it is optimal to invest).

- Land (Collateral Goods):
  - $\hat{p}$ fraction of 'good' land is endowed oil worth C per unit of land;
  - $1 - \hat{p}$ fraction of 'bad' land have no oil therefore worth 0;
  - Before verification, people believe the probability is p.

  - γ **unit of numeraire need to be paid to verify whether the land is good or bad.**

C. Production

- Given production function, entrepreneur wants to borrow $K^* = N^*$:

$$K' = \begin{cases} A \min\{K, N^*\} & \text{with prob. } q \\ 0 & \text{with prob. } (1-q). \end{cases}$$

  - But how?

- If output is verifiable: state contingent claims.
- However, assume output is not verifiable:
  - Entrepreneurs have incentive to hide output and pay nothing;
  - Households have no incentive to lend.

- Entrepreneur can use x fraction of land as collateral:
  - Technical assumption: $C > K^* = N^*$ (i.e. land known to be good is enough to borrow at optimal K^*)

D. Financial Market

- Market Assumption:
  - Entrepreneurs does not know the land quality (no numeraire to verify);
  - Each entrepreneur randomly matches with a household and borrow;
  - Entrepreneur have bargaining power in writing debt contract;
  - A household may verify the land (after paying γ) and it keeps it as secret until the end fo the period unless it want to disclosure it.
- Entrepreneur choose between:
  - Information-Sensitive (IS) Debt: Verify and lend to good land;
  - Information-Insensitive (II) Debt: Lend without verify.
- Entrepreneur also write debt contract:
  - K: Amount of numeraire borrowing for production;
  - R: Repayment value if success;
  - $x < 1$: Amount of land used as collateral.

D1. Information-Sensitive (IS) Debt

- Verify or issue **Information-Sensitive (IS) Debt**:
  - After verify, households only lend to entrepreneurs with 'good' land;
  - Binding **Participation constrain** (the zero profit condition):

$$\overbrace{p\left(qR_{IS} + (1-q)x_{IS}C - K\right)}^{\text{Expected Revenue if Good Land}} + \overbrace{(1-p)0}^{\text{if Bad}} = \gamma \tag{2}$$

Lemma

For optimal IC Debt: $x_{IS}C = R_{IS}$

- i.e. Collateral liquidation value = repayment;

- Proof:
  - Suppose $x_{IS}C < R_{IS}$: If success, entrepreneur will not repay;
  - Suppose $x_{IS}C > R_{IS}$: If success, entrepreneur will sell the collateral and pay back R_{IS}.

D1. Information-Sensitive (IS) Debt

- Combine the lemma and the participation constrain:

$$x_{IS} = \frac{pK + \gamma}{pC} \leq 1 = \bar{L} \tag{3}$$

- There are three possible cases:
  - Case A: $\frac{pK^* + \gamma}{pC} \leq 1$ Borrow $K = K^*$;
  - Case B: $\frac{pK^* + \gamma}{pC} \geq 1$ and $pC > \gamma$ Borrow $K = (pC - \gamma)/p \leq K^*$;
  - Case C: $pC \leq \gamma$ Borrow $K = 0$.

- Expected profit of entrepreneur if choose IS debt:

$$E(\pi \mid p, IS) = p\left(qAK - x_{IS}C\right)$$

D1. Information-Sensitive (IS) Debt

- For simplicity, we make technical assumption:

$$\frac{\gamma}{K^*(qA-1)} > \frac{\gamma}{C-K^*} \quad \Leftrightarrow \quad qA < C/K^*$$

  - Which rule out case B and case C;
  - i.e. Entrepreneurs with land verify to be good borrow at optimal.

- Expected profit can be simplify to:

$$E(\pi \mid p, IS) = \begin{cases} pK^*(qA-1) - \gamma & \text{if} \quad p \geq p_{IS}^L \equiv \frac{\gamma}{K^*(qA-1)} \\ 0 & \text{if} \quad p < p_{IS}^L \equiv \frac{\gamma}{K^*(qA-1)} \end{cases} \tag{4}$$

  - Where we define cutoff subjective probability as p_{IS}^L.

D2. Information-Insensitive (II) Debt

- No Verify or issue **Information-Insensitive (II) Debt**:
  - Household lend to all the entrepreneur without verify.
  - Binding **participation constrain** (zero profit condition):

$$qR_{II} + (1-q)px_{II}C = K$$

  - $R_{II} = x_{II}pC$ also holds (similar to the lemma in IS Debt).

- Combine equations, we get **maximum collateral constrain:**

$$x_{II} = \frac{K}{pC} \leq 1 = \bar{L} \tag{5}$$

D2. Information-Insensitive (II) Debt

- The debt is also subject to a **incentive compatibility constrain**:
  - Household may secretly verify the land and only lend to good;
  - If this gives positive profit rather than zero profit if no check, then households will always check;
  - Entrepreneur have incentive to rule out this case:

$$\overbrace{p\left(qR_{II} + (1-q)x_{II}C - K\right) - \gamma}^{\text{Household's Profit if Secretly Check}} < 0$$

$$\Leftrightarrow \; K < \frac{\gamma}{(1-p)(1-q)} \tag{6}$$

- Intuition: Entrepreneur can disincentive secret verification by borrowing less.

D2. Information-Insensitive (II) Debt

- To summarize, the II debt must subject to three constrains:
  - Maximum collateral constrain (5);
  - IC constrain (6);
  - Technology constrain $K \leq K^* \equiv N^*$.

$$K(p \mid II) = \min\left\{K^*, \frac{\gamma}{(1-p)(1-q)}, pC\right\} \tag{7}$$

- Expected profit of entrepreneur if choose II debt:

$$E(\pi \mid p, II) = qAK - x_{II}pC = K(p \mid II)(qA - 1)$$

D2. Information-Insensitive (II) Debt

- Plug in equation (7), we have:

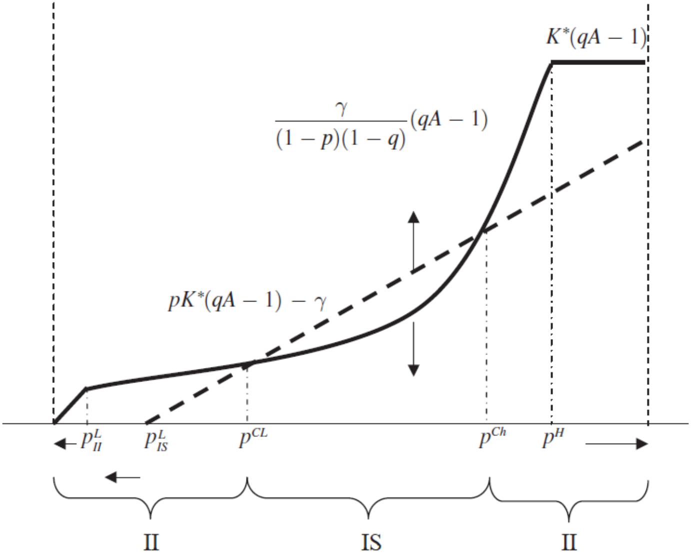
$$E(\pi \mid p, II) = \begin{cases} K^*(qA-1) & \text{if} \quad K^* \leq \frac{\gamma}{(1-p)(1-q)} \\ \frac{\gamma}{(1-p)(1-q)}(qA-1) & \text{if} \quad K^* > \frac{\gamma}{(1-p)(1-q)} \\ pC(qA-1) & \text{if} \quad pC < \frac{\gamma}{(1-p)(1-q)} \end{cases} \tag{8}$$

- If $\frac{\gamma}{(1-p)(1-q)} = p$ have solution ($\Leftrightarrow C(1-q) > \gamma$):

$$E(\pi \mid p, II) = \begin{cases} K^*(qA-1) & \text{if} \quad p \geq p^H \equiv 1 - \frac{\gamma}{K^*(1-q)} \\ \frac{\gamma}{(1-p)(1-q)}(qA-1) & \text{if} \quad p^L_{II} \leq p < p^H \\ pC(qA-1) & \text{if} \quad p < p^L_{II} \equiv \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{\gamma}{C(1-q)}} \end{cases} \tag{9}$$

  - Notice that we define the higher cutoff as p^H and the lower as p^L_{II}.

Choice of Information Regime: IS or II



Choice of Information Regime: IS or II

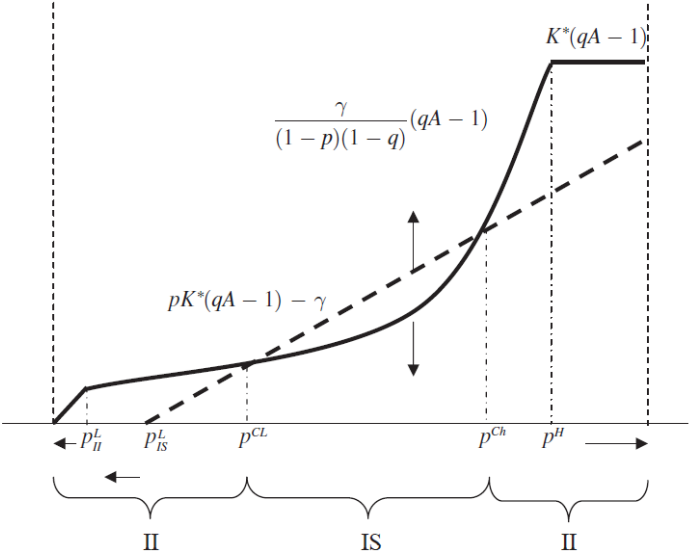
- Entrepreneur choose IS or II debt to maximize his expected profit:
  - Entrepreneur compare equation (4) and equation (8), given every household's belief on p (proportion of good land).

- We can plot $E(\pi \mid p, IS)$ and $E(\pi \mid p, II)$ in different p:
  - Expected IS profit in solid line and Expected II profit in dash line;
  - Arrows denotes the direction of change when $\gamma \Downarrow$

- If γ is low enough, IS debt is chosen in the mid between p^{CL} and p^{Ch}.
  - Cutoff p^{CL} and p^{Ch} are two solution to:

$$\gamma = \left[pK^* - \frac{\gamma}{(1-p)(1-q)} \right] (qA - 1) \tag{10}$$

Choice of Information Regime: IS or II



Aggregate: Productive Numeraire (Capital)

- The productive numeraire or capital (excluding numeraire used to verify):

$$K(p) = \begin{cases} K^* & \text{if } p^H < p \\ \frac{\gamma}{(1-p)(1-q)} & \text{if } p^{Ch} < p < p^H \\ pK^* - \frac{\gamma}{(qA-1)} & \text{if } p^{Cl} < p < p^{Ch} \\ \frac{\gamma}{(1-p)(1-q)} & \text{if } p^L_{II} < p < p^{Cl} \\ pC & \text{if } p < p^L_{II} \end{cases}$$

Aggregate: Wealth of Economy

- Household's wealth is

$$\bar{K} - K(p) + E(\text{repay} \mid p)$$

- Entrepreneur's wealth is

$$E(K' \mid p) - E(\text{repay} \mid p) = qAK(p) - E(\text{repay} \mid p)$$

- Add together we get **total wealth of economy**:

$$W_t = \bar{K} + \int_0^1 K(p)(qA - 1)dF(p) \tag{11}$$

  - where we assume household's belief of land quality $p \sim^{CDF} F(p)$;
  - First best wealth (everybody borrow): $W_{fb}^* = \bar{K} + K^*(qA - 1)$.

Dynamic OLG Model: Overview

- Purpose: Study the evolution and influence of collateral belief p
- OLG setup: Two cohort of population:
  - Young household: born with **non-storable** $\bar{K}$ no management skill;
  - Old entrepreneur: Management skill N^* and purchased land $L = 1$.
  - Each period beginning young and old randomly matched and borrow.
- Land market:
  - Land is storable, sold to young when old is dying (period end);
  - We want to rule out multiple equilibrium (i.e. asset bubble equilibrium):
  - Why? We want to separate the effect of information regime.
  - **We fixed land prices at** $Q(p) = pC$,which holds under assumption:
    - 1, Young have bargaining power: 'take-it-or-leave-it offers';
    - 2, Only possible to verify at period beginning, not at end.

Timeline: Life of an Agent

- At $t = T$: A household is born with $\bar{K}$;
- At beginning of period $[T, T+1]$: K_t is lent to the old
- At the end of period $[T, T+1]$:
    - households gets repayment with prob q (no repayment otherwise);
    - Purchase the 1 unit of land from the the old at fundamental price (if the old default, only need to purchase $(1-x)$ fraction of land);
    - Consume all the numeraire left (because numeraire is non-storable).

- At $t = T+1$:
    - Household transform to entrepreneur acquiring N^*;
    - The land acquired exposed to idiosyncratic (and systematic) shock;
- At beginning of period $[T+1, T+2]$: K_{t+1} is borrowed.
- At the end of period $[T+1, T+2]$:
    - Produces $A\min\{K_{t+1}, N^*\}$ repay with prob q (default otherwise);
    - Sell the land at fundamental price and consume all numeraire.

- At $t = T+2$: The old entrepreneur passes away.

Idiosyncratic Shocks

- Mean reversion idiosyncratic shocks at each period start:
  - Shock is observable: people know whether the land is shocked;
  - Realized value of shock is unknown unless pay γ to verify;
  - Fraction of land face the shock is independent of land types;
  - Probability of being good after shock is independent of land types.

- For simplicity we assume:
  - λ fraction of land remain unchanged;
  - $1-\lambda$ fraction of land changes: in which $\hat{p}$ to good land and $1-\hat{p}$ to bad. ($\hat{p}$ is the real probability of good land)

- This means that after shock, distribution of collateral belief p have three point support $\{0, \hat{p}, 1\}$.

Evolution with Only Idiosyncratic Shocks

Proposition 3:
Evolution of Aggregate Consumption in the Absence of Aggregate Shocks

Assume there is perfect information about land types in the initial period.

- If $\hat{p}$ is in the information-sensitive region ($\hat{p} \in [p^{Cl}, P^{Ch}]$), consumption is constant over time and is lower than the unconstrained first-best.
- If $\hat{p}$ is in the information-insensitive region, consumption grows over time if $\hat{p} > \hat{p}_h^*$ or $\hat{p} < \hat{p}_l^*$, where $\hat{p}_h^* > \hat{p}_l^*$ are the two solutions to the quadratic equation $\hat{p}^* K^* = \frac{\gamma}{(1-\hat{p}^*)(1-q)}$.

Evolution with Only Idiosyncratic Shocks

- We particularly focus on $p > p^H > \hat{p}_h^*$:
  - Information decays overtime (At period t, $(1 - \lambda^t)$ fraction of land is of unknown quality);
  - Wealth approach unconstrained first-best W_{fb}^*.
- **'Blissful Ignorance'**: Always producing information is not optimal if there is no (or only small) aggregate shock.
  - No information, everybody can borrow.
  - Otherwise, agents with land that is known to be bad cannot borrow and produce.
- But 'Blissful Ignorance' is costly: **fragility to aggregate shock.**

Evolution with Aggregate Shocks

- Negative aggregate shock:
  - Transform $(1-\eta)$ fraction of good land to bad land;
  - Whether shock happens is observable, but which piece good land changes to bad land is not;
  - Belief $p = 1$ becomes $p' = \eta$, and $p = \hat{p}$ becomes $p' = \eta p$.

Proposition 4:
The Larger Boom and Shock, the Larger Crisis

Assume $p > p^H > \hat{p}_h^*$, and a negative aggregate shock η, hits after t periods of no aggregate shocks. The reduction in wealth $\Delta(t \mid \eta) = W_t - W_{t|\eta}$ is non-decreasing in the size of the shock and non-decreasing in the time t elapsed previously without a shock.

Recovery Speed

Proposition 5:
Information and Recoveries

Assume $p > p^H > \hat{p}_h^*$, and that a negative aggregate shock generates a crisis in period t. The recovery from the crisis is faster if information is generated after the shock when $\eta\hat{p} < \overline{\eta\hat{p}} \equiv \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\gamma}{C(1-q)}}$, where $p^{Ch} < \overline{\eta\hat{p}} < p^H$. That is $W_{t+1}^{IS} > W_{t+1}^{II}$ for all $\eta\hat{p} < \overline{\eta\hat{p}}$.

Corollary 2:
Potential Gain from Regulator Intervention

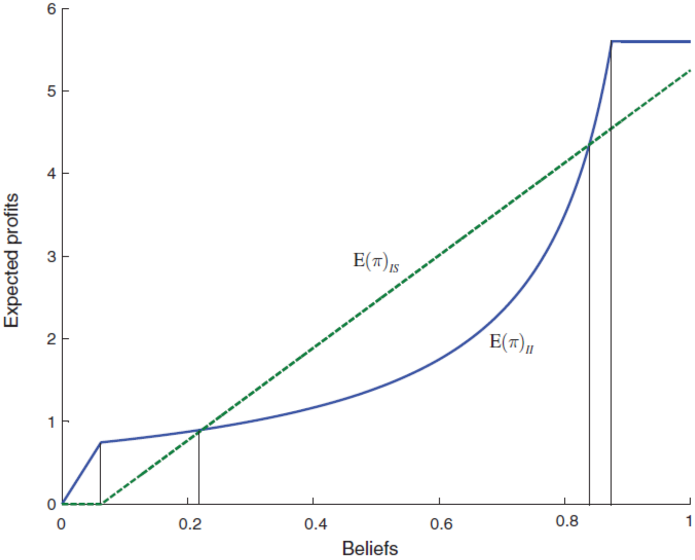
There exists a range of negative aggregate shocks (η such that $\eta\hat{p} \in [p^{Ch}, \overline{\eta\hat{p}}]$) in which agents do not acquire information, but recovery would be faster if they did.

Dispersion of Beliefs During Booms and Crises

- Parametric setting:
  - Idiosyncratic shock probability: $1-\lambda = 0.1$;
  - Real fraction of good land: $\hat{p} = 0.92$;
  - Probability of project success: $q = 0.6$;
  - Productivity parameter: $A = 3$ (Investment return is $qA - 1 = 80\%$);
  - Management skill and optimal input $N^* = K^* = 7$;
  - Numeraire endowment $\bar{K} = 20$;
  - Good land provide numeraire $C = 15$.

- Cutoff value:
  - Optimal capital cutoff: $p^H = 0.88 > \hat{p} = 0.92$;
  - Information sensitive region $[p^{Cl}, p^{Ch}] = [0.22, 0.84]$.

Choice of Information Regime: IS or II



Aggregate Shocks

- Three different aggregate shocks at periods 5 and 50:
  - **Small:** $\eta = 0.97$, so $\eta\hat{p} > p^h$, still at optimal borrowing;
  - **Medium:** $\eta = 0.91$, so $\eta\hat{p} \in [p^{Ch}, \bar{\eta}\hat{p}]$ (as described in Proposition 5). It will not trigger verification and therefore slow recovery.
  - **Large:** $\eta = 0.90$, so $\eta\hat{p} \in [p^{Cl}, p^{Ch}]$. It trigger verification, so recovery speed is higher even if the shock is larger than the medium one.

Aggregate Shocks

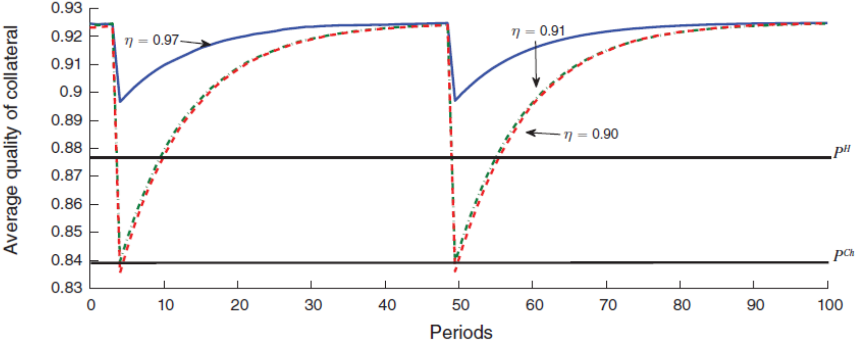


Figure: Average Value of Collateral

Wealth (Productivity) Evolution

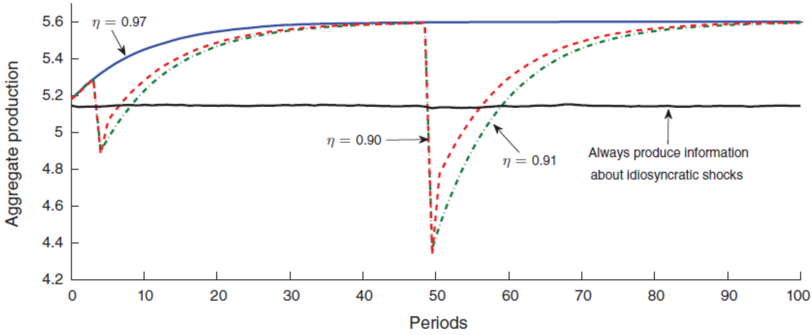


Figure: Wealth Evolution after Crisis

Wealth (Productivity) Evolution

- The evolution is consistent with analytical results:
  - As demonstrated in Proposition 3: If there is no shock or only small shock $\eta = 0.97$, the wealth will gradually approaching the first best, which is higher than always perfect information.
  - As demonstrated in Proposition 4: The longer time there is no shock, the larger crisis there will be;
  - As demonstrated in Proposition 5: Economy recover quicker from the large shock $\eta = 0.91$ that trigger verification quicker than after the medium shock that does not trigger verification.

- **If regulator can promote verification after medium shock $\eta = 0.91$, social welfare can potentially be larger.**

Thanks for listening!