# Uncertainty Shocks Are Aggregate Demand Shocks

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June 30, 2021

# Overview

1. Model Setup
2. System of Equations
3. Open Questions

# The Production Technology

- There are **4 types of agents** and **3 markets** in the economy.

# Agent A: Household

- Household utility maximization problem:

$$\max_{\{N_t, C_t, B_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln\left(C_t - hC_{t-1}\right) - \chi N_t\right]$$

$$s.t. \quad C_t + \frac{B_t}{P_t R_t} = \frac{B_{t-1}}{P_t} + w_t N_t + \phi\left(1 - N_t\right) + d_t - T_t, \quad \forall t \geq 0$$

  - $h$: Habit parameter;
  - $B_t$: Risk-free borrowing from $t$ to $t+1$;
  - $N_t$: Employment ($\chi N_t$: Disutility from employment);
  - $\phi$: Unemployment insurance payment per unit of unemployment;
  - $d$: Dividend income from retailer and firms;
  - $T_t$: Tax to finance the unemployment insurance.

# Agent A: Household

- FOC + envelop theorem for $B_t$:

$$1 = \mathrm{E}_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} \frac{R_t}{\pi_{t+1}} \tag{1}$$

  - $\pi_t \equiv \frac{P_t}{P_{t-1}}$ is the inflation rate.

- FOC + envelop theorem for $C_t$:

$$\Lambda_t = \frac{1}{C_t - hC_{t-1}} - \mathrm{E}_t \frac{\beta h}{C_{t+1} - hC_t} \tag{2}$$

  - $\Lambda_t$: Shadow price of budget constrain.

# Agent A: Household

- Two question is equivalent:
  - Household directly determine consumption on each good each period;
  - Household determine the total consumption each period, then allocate total expenditure on differentiated goods.
    - $\Rightarrow$ The second approach is easy to work with.
- The static consumption allocation problem:

$$\max_{Y_t^d(j)} Y_t = \left( \int_0^1 Y_t^d(j)^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}}$$

$$\int_0^1 Y_t^d(j) P_t(j) dj \leq E_t$$

  - $Y_t^d(j)$: Demand of differentiated goods; $Y_t^d$: Demand of bundle;
  - $\eta$: Elasticity of substitution;
  - $E_t$: Total expenditure.

# Agent A: Household

- Take FOC:

$$Y_t^d(j) = Y_t^d[\lambda_t P_t(j)]^{-\eta}$$

  - Take exponent of $\frac{\eta-1}{\eta}$, and aggregate across goods to cancel $Y_t$:

$$\frac{1}{\lambda} = \left(\int_0^1 P_t(j)^{1-\eta} dj\right)^{\frac{1}{1-\eta}}$$

  - We can further rewrite the FOC as:

$$Y_t^d(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\eta} Y_t \quad \text{where} \quad P_t \equiv \frac{1}{\lambda} = \left(\int_0^1 P_t(j)^{1-\eta} dj\right)^{\frac{1}{1-\eta}}$$

  - Footnote: I think equation (7) in the original paper is wrong.

# Agent B: Retailer

- A retailer:
  - Produce and supply $Y_t(j)$ **differentiated** retail goods to households;
  - Buy $X_t(j)$ **homogeneous** raw material from firms;
  - Production function is one for one:

$$Y_t(j) = X_t(j)$$

- Market power:
  - The retail market (retailer $\Leftrightarrow$ household): monopolistic competition;
  - The raw material market (firms $\Leftrightarrow$ retailer): perfect competition.

# Agent B: Retailer

- Retailer $j's$ profit maximization problem:

$$\max_{\{P_{t+i}\}_{i=0}^{\infty}} \mathrm{E}_t \sum_{i=0}^{\infty} \frac{\beta^i \Lambda_{t+i}}{\Lambda_t} \left[ \overbrace{\left( \frac{P_{t+i}(j)}{P_{t+i}} - q_{t+i} \right)}^{\text{Profit per unit}} Y_{t+i}^d(j) - \underbrace{\frac{\Omega_p}{2} \left( \frac{P_{t+i}(j)}{\pi P_{t+i-1}(j)} - 1 \right)^2 Y_{t+i}}_{\text{Price adjustment cost (Rotemberg, 1982)}} \right]$$

$$s.t. \quad Y_t^d(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\eta} Y_t$$

# Agent B: Retailer

- Take FOC on $P_t$:
  - further suppose it is symmetric equilibrium so that $P_t = P_t(j)$:

$$q_t = \frac{\eta - 1}{\eta} + \frac{\Omega_p}{\eta}\left[\frac{\pi_t}{\pi}\left(\frac{\pi_t}{\pi} - 1\right) - \mathrm{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t}\frac{Y_{t+1}}{Y_t}\frac{\pi_{t+1}}{\pi}\left(\frac{\pi_{t+1}}{\pi} - 1\right)\right] \quad (3)$$

# Market 3: The Labor Market

- Suppose the **matching function** is given by:

$$m_t = \mu u_t^{\alpha} v_t^{1-\alpha} \tag{4}$$

  - $m_t$: Measure of labor find new job in period t;
  - $u_t$: Measure of labor in need of a job in period t;
  - $v_t$: Measure of firms in need of a labor in period t;
  - $\mu$: Efficiency parameter of labor market;
  - $\alpha$: Share parameter of labor market.

- Question:
  - Intuitively, $m_t < u_t$ and $m_t < v_t$ must hold.
  - How can the author rule out possibility of $m_t > u_t$ and $m_t > v_t$?

# Market 3: The Labor Market

- Define the following probability:
  - **Job filling rate**: Probability of a firm find a labor

$$q_t^v = \frac{m_t}{v_t} \tag{5}$$

  - **Job finding rate**: Probability of a labor find a job

$$q_t^u = \frac{m_t}{u_t} \tag{6}$$

# Market 3: The Labor Market

- Evolution of employment:

## Market 3: The Labor Market

- Evolution of employment:
    - Employed labor:

$$N_t = \overbrace{(1-\rho)N_{t-1}}^{\text{'Veteran'}} + \underbrace{m_t}_{\text{'Novice'}} \tag{7}$$

    - Labor in job market:

$$u_t = 1 - (1-\rho)N_{t-1} \tag{8}$$

    - Unemployed labor:

$$U_t = 1 - N_t \tag{9}$$

# Agent C: Firms

- One measure of firm:
  - Produce and supply homogeneous $X_t$ raw material to retailer;
  - Each firm can hire 1 unit of labor to produce;
  - If no labor is hired, no raw material is produced.

  - Production function:

$$X_t = \begin{cases} Z_t & (1 \text{ labor hired}) \\ 0 & (0 \text{ labor hired}) \end{cases}$$

- $Z_t$: The productivity of firms.
  - It follow a stochastic process described in the next slide.

# Agent C: Firms

- The process of productivity:

$$\ln Z_t = \rho_z \ln Z_{t-1} + \sigma_{zt}\varepsilon_{zt}$$

$$\ln \sigma_{zt} = (1-\rho_{\sigma_z}) \ln \sigma_z + \rho_{\sigma_z} \ln \sigma_{z,t-1} + \sigma_{\sigma_z}\varepsilon_{\sigma_z,t}$$

  - $\varepsilon_{z,t} \sim N(0,1)$: Productivity shocks;
  - $\varepsilon_{\sigma_z,t} \sim N(0,1)$: Productivity uncertainty shocks.

# Agent C: Firms

- Evolution of firm value:
  - Firm with worker employed:

$$J_t^F = q_t Z_t - w_t + E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left[ (1-\rho) J_{t+1}^F + \rho V_{t+1} \right] \tag{13}$$

  - Firm without worker employed:

$$V_t = -\kappa + q_t^v J_t^F + E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left( 1 - q_t^v \right) V_{t+1}$$

- Free entry condition imply $V_t = 0 \quad \forall t = 0, 1, 2 \ldots$:
  - Take into the above equation:

$$\Leftrightarrow \quad \frac{\kappa}{q_t^v} = J_t^F \tag{14}$$

# Agent D: Workers

- Evolution of workers' 'value':
  - Marginal 'value' of employed worker:

$$J_t^W = w_t - \frac{\chi}{\Lambda_t} + E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left\{ \left[ 1 - \rho \left( 1 - q_{t+1}^u \right) \right] J_{t+1}^W + \rho \left( 1 - q_{t+1}^u \right) J_{t+1}^U \right\}$$

  - Marginal 'value' of unemployed worker:

$$J_t^U = \phi + E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left[ q_{t+1}^u J_{t+1}^W + \left( 1 - q_{t+1}^u \right) J_{t+1}^U \right]$$

  - These two equation is little bit hard to understand;
  - But we can show its intuition from the following way.

# Agent D: Workers

- Take the marginal 'value' of employed worker as example:

$$J_t^W = w_t - \frac{\chi}{\Lambda_t} + E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left\{ \left[1 - \rho\left(1 - q_{t+1}^u\right)\right] J_{t+1}^W + \rho\left(1 - q_{t+1}^u\right) J_{t+1}^U \right\}$$

  - Multiply both side by $\Lambda_t$:

$$\overbrace{J_t^W \Lambda_t}^{\text{MU of unit work}} = \overbrace{w_t \Lambda_t - \chi}^{\textbf{Intra-temporal MU}} + E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left\{ \underbrace{\left[1 - \rho\left(1 - q_{t+1}^u\right)\right]}_{\text{Prob. Continue or New Job}} J_{t+1}^W \Lambda_t + \rho \underbrace{\left(1 - q_{t+1}^u\right)}_{\text{Prob. find no job}} J_{t+1}^U \Lambda_t \right\}$$

# Market 3: The Labor Market (Nash Bargaining Problem)

- Worker and firms bargaining on wage:

$$\max_{w_t} \left(J_t^W - J_t^U\right)^b \left(J_t^F - V_t\right)^{1-b} = \left(J_t^W - J_t^U\right)^b \left(J_t^F\right)^{1-b}$$

$$S_t = J_t^F + J_t^W - J_t^U$$

  - $S_t$: 'Total surplus' of firm;
  - $b$: Parameter of bargaining power:
    - $b = 1$: Firm has full bargaining power (Monopoly of capitalist);
    - $b = 0$: Labor has full bargaining power (Monopoly of labor union).
  - Value of firms and worker:

$$J_t^F = (1-b)S_t, \quad J_t^W - J_t^U = bS_t$$

# Market 3: The Labor Market (Nash Bargaining Problem)

- Combine the above equations:

$$bS_t = w_t^N - \phi - \frac{\chi}{\Lambda_t} + E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left[ (1-\rho) \left(1 - q_{t+1}^u\right) bS_{t+1} \right]$$

$$\frac{b}{1-b} J_t^F = w_t^N - \phi - \frac{\chi}{\Lambda_t} + (1-\rho) \mathrm{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left(1 - q_{t+1}^u\right) \frac{b}{1-b} J_{t+1}^F \quad (15)$$

Footnote: I think the equation (30) in the original paper is also wrong.

- Wage rigidity

$$w_t = w_{t-1}^{\gamma} \left(w_t^N\right)^{1-\gamma} \quad (16)$$

  - $\gamma \in (0,1)$: Wage rigidity parameter.

## Government Policy

- Budget balances:

$$\phi(1 - N_t) = T_t$$

- The monetary authority follows the Taylor rule:

$$R_t = r\pi^* \left(\frac{\pi_t}{\pi^*}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y} \tag{11}$$

  - $r = \frac{R}{\pi}$: Steady-state real interest rate;
  - $\pi^*$: Target inflation rate;
  - $\phi_\pi$: How monetary policy accommodates inflation;
  - $\phi_y$: How monetary policy accommodates output fluctuations;

# Market Clearing

- Bond market-clearing condition:

$$B_t = 0$$

- Aggregate resources constrain:

$$C_t + \kappa v_t + \frac{\Omega_p}{2}\left(\frac{\pi_t}{\pi} - 1\right)^2 Y_t = Y_t \tag{12}$$

- Intermediate goods market clearing:

$$Y_t = Z_t N_t \tag{10}$$

# System of Equations

- There are 16 endogenous variables, and 2 productivity variables;

$$\left[C_t, \Lambda_t, \pi_t, m_t, q_t^u, q_t^v, N_t, u_t, U_t, Y_t, R_t, v_t, q_t, J_t^F, w_t^N, w_t\right]$$

$$[Z_t, \sigma_{Z\,t}]$$

# System of Equations

$$1 = \mathrm{E}_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} \frac{R_t}{\pi_{t+1}} \tag{1}$$

$$\Lambda_t = \frac{1}{C_t - hC_{t-1}} - \mathrm{E}_t \frac{\beta h}{C_{t+1} - hC_t} \tag{2}$$

$$q_t = \frac{\eta - 1}{\eta} + \frac{\Omega_p}{\eta} \left[ \frac{\pi_t}{\pi} \left( \frac{\pi_t}{\pi} - 1 \right) - \mathrm{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \frac{Y_{t+1}}{Y_t} \frac{\pi_{t+1}}{\pi} \left( \frac{\pi_{t+1}}{\pi} - 1 \right) \right] \tag{3}$$

$$m_t = \mu u_t^{\alpha} v_t^{1-\alpha} \tag{4}$$

$$q_t^v = \frac{m_t}{v_t} \tag{5}$$

$$q_t^u = \frac{m_t}{u_t} \tag{6}$$

$$N_t = (1 - \rho) N_{t-1} + m_t \tag{7}$$

$$u_t = 1 - (1 - \rho) N_{t-1} \tag{8}$$

# System of Equations

$$U_t = 1 - N_t \tag{9}$$

$$Y_t = Z_t N_t \tag{10}$$

$$R_t = r\pi^* \left(\frac{\pi_t}{\pi^*}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y} \tag{11}$$

$$C_t + \kappa v_t + \frac{\Omega_p}{2}\left(\frac{\pi_t}{\pi} - 1\right)^2 Y_t = Y_t \tag{12}$$

$$J_t^F = q_t Z_t - w_t + E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t}\left[(1-\rho) J_{t+1}^F + \rho V_{t+1}\right] \tag{13}$$

$$\frac{\kappa}{q_t^v} = J_t^F \tag{14}$$

$$\frac{b}{1-b} J_t^F = w_t^N - \phi - \frac{\chi}{\Lambda_t} + (1-\rho)\mathrm{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t}\left(1 - q_{t+1}^u\right) \frac{b}{1-b} J_{t+1}^F \tag{15}$$

$$w_t = w_{t-1}^{\gamma}\left(w_t^N\right)^{1-\gamma} \tag{16}$$

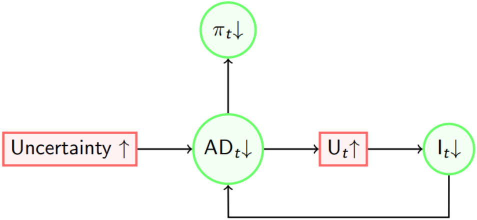
# System of Equations

$$\ln Z_t = \rho_z \ln Z_{t-1} + \sigma_{zt}\varepsilon_{zt} \tag{P1}$$

$$\ln \sigma_{zt} = (1 - \rho_{\sigma_z}) \ln \sigma_z + \rho_{\sigma_z} \ln \sigma_{z,t-1} + \sigma_{\sigma_z}\varepsilon_{\sigma_z,t} \tag{P2}$$

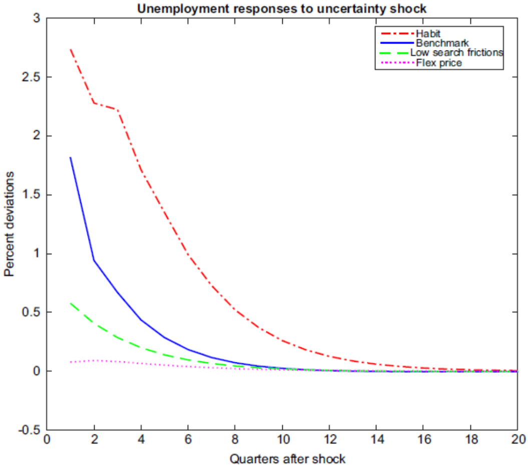
# Open Questions

- Higher uncertainty:
  - Option value channel: Option value of waiting $\uparrow$, $J_t^f \downarrow$, $U_t \uparrow$
  - Aggregate demand channel:




  - Question 1: How can we see from the model setup and equations that option value of waiting $\uparrow$?
  - Question 2: How can we see from equations that aggregate demand $\downarrow$?

# Open Questions




- Question 3: Why does habit lead to higher unemployment?

# Thanks for listening!