

# Bank Runs, Deposit Insurance, and Liquidity

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# Overview

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# Key Concepts

- Bank Run:

- A bank run occurs when a large number of customers of a bank or other financial institution withdraw their deposits simultaneously over concerns of the bank's solvency.

- Deposit Insurance:

- A **statutory insurance** provided by FDIC in the US.
- Fund by assessing a premium on banks, to protect against bank's default on deposit up to \$250,000 per depositor, per insured bank.

- Liquidity:

- In finance: ability to quickly purchase or sell an asset without causing a drastic change in the asset price.
- In this paper: ability to have money to consume whenever you want.

# Bank Run and Deposit Insurance in China



Sheyang Bank Run (Apr 2014)



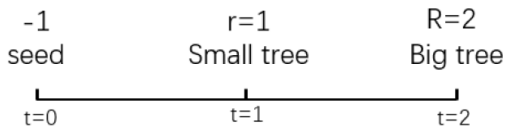
*Deposit Insurance Act (May 2015)*  
《存款保险条例》国务院令第660号

# Abstract

- Why banks exist?
  - **Consumption time risk:** people have no plan when to consume.
  - **Demand deposit:** creation of liquid claims on illiquid asset.
- Model feature: multiple equilibrium of demand deposit:
  - **Good equilibrium vs Bank run equilibrium**
  - Bank run equilibrium cause real economic problems:
    - Illiquid Asset are liquidate early: return ↓
    - Even 'healthy' bank can fail.
- How to rule out the bank run equilibrium:
  - Suspension of convertibility
  - Deposit insurance

# Model setup: Technology

- Technology:
  - $t = 0$ : there is an investment opportunity cost \$1.
  - $t = 1$ : you can liquidate the investment and get \$1 back.
  - $t = 2$ : you can get full return  $R > \$1$ .



- Liquidating investment causes economic loss.

# Model setup: Consumer

- Consumer:
  - Continuous with measure 1
  - People do not know when to consume at  $t = 0$ .
- $t = 1$ : Uninsurable liquidity risk:
  - $\pi$  proportion only care consumption at  $t = 1$ .
  - $1 - \pi$  proportion only care consumption at  $t = 2$ .

$$U(c_1, c_2; \Theta) = \begin{cases} u(c_1) & \text{if } j \text{ is of type 1 in state } \Theta \\ \rho u(c_2) & \text{if } j \text{ is of type 2 in state } \Theta \end{cases}$$

# Model setup: Consumer

- Consumer objective function at  $t = 0$ :
  - vN-M utility form:

$$U(c_1, c_2) = \pi u(c_1) + (1 - \pi)\rho u(c_2)$$

- $u(\cdot)$  satisfies Inada conditions.
  - Relative risk aversion coefficient  $-cu''(c)/u'(c) > 1$
- Information:
  - First, we assume  $\pi$  is constant and public information.
  - However,  $\Theta_j$  or whether a consumer is Type 1 or Type 2 is private information.



## Scenario 1: Autarky

- Type 1 consumer will liquidate the consumption at  $t = 1$  and type 2 consumer will keep the investment to  $t = 2$ .

$$U(c_1, c_2) = \pi u(1) + (1 - \pi)\rho u(R)$$

- Numerical example:
  - $u(c) = 1 - 1/c$
  - Percentage of Type 1 consumer  $\pi = 0.25$
  - $R = 2$  and utility discount rate  $\rho = 1$

$$U(c_1, c_2) = 0.25u(1) + 0.75u(2) = 0.375$$

- Not social optimal, because people are risk adverse, so they want to smooth consumption under two circumstances.

## Scenario 2: Social planner

- Social planner problem:

$$\max_{(c_1, c_2) \in R_+^2} \pi u(c_1) + (1 - \pi) \rho u(c_2)$$

$$s.t. \pi c_1 + (1 - \pi) \frac{c_2}{R} = 1 \quad (1)$$

- FOC:

$$u'(c_1^*) = \rho R u'(c_2^*) \quad (2)$$

- We can prove that  $1 < c_1^* < c_2^* < R$ . (See footnote 3)
- Consider the numerical example before, the optimal consumption bundle is  $(c_1^*, c_2^*) = (1.28, 1.813)$

## Scenario 2: Social planner

- Consider numerical example before:
  - $u(c) = 1 - 1/c$
  - Percentage of Type 1 consumer  $\pi = 0.25$
  - $R = 2$  and utility discount rate  $\rho = 1$
- The optimal consumption bundle is  $(c_1^*, c_2^*) = (1.28, 1.813)$
- $U^* = 0.25u(1.28) + 0.75u(1.813) = 0.391 > 0.375$
- So if there is a financial contract that cost 1 at  $t = 0$  and pays  $(1.28, 1.813)$  for type 1 and type 2 consumer at  $t = 1$  and  $t = 2$ . Consumer will be willing to buy it.
  - Require private information  $\Theta$ !
  - But we can let customer select themselves.
  - Check **self-selection constraints**.

## Scenario 3: Bank

- Bank: Demand deposit, and investment.
- Bank promises  $r_1$  interest rate if withdraw at  $t = 1$ , and rest of the money left for people withdraw at  $t = 2$ .
- **Sequential service constrain:**
  - $f$ : the proportion of  $t = 1$  withdraw
  - $f_j$ : number of withdraw before  $j$  as fraction of total population

$$V_1(f_j; r_1) = \begin{cases} r_1 & \text{if } f_j < r_1^{-1} \\ 0 & \text{if } f_j \geq r_1^{-1} \end{cases}$$

$$V_2(f; r_1) = \max \left[ R \frac{1 - r_1 f}{1 - f}, 0 \right]$$

- Where  $V_1$  is the money received by people withdraw at  $t = 1$  and  $V_2$  is the money received by people withdraw at  $t = 2$ .

## Scenario 3: Bank (Good Equilibrium)

### Good Equilibrium

When  $r_1 = c_1^*$  and only type 1 consumer withdraw at  $t = 1$  ( $f = \pi$ ):

- The demand deposit contract achieve social optimal.
  - It is a Nash equilibrium.
- 
- Consider numerical example before:
    - $u(c) = 1 - 1/c$
    - Percentage of Type 1 consumer  $\pi = 0.25$
    - $R = 2$  and utility discount rate  $\rho = 1$
  - $(V_1, V_2) = (c_1^*, c_2^*) = (1.28, 1.813)$  achieves social optimal.
    - Type 1 will not betray: consumption at  $t = 2$  is useless.
    - Type 2 will not betray:  $V_2 > V_1$
    - $\Rightarrow$  **Nash equilibrium**

## Scenario 3: Bank (Bank Run Equilibrium)

### Bank Run Equilibrium

When  $r_1 > 1$  and all consumers withdraw at  $t = 1$  ( $f = 1$ ):

- It is a Nash equilibrium.
- At  $t = 1$ , random  $r_1^{-1}$  proportion get  $r_1$ , and the remaining get 0.
  - Type 1 will not betray: consumption at  $t = 2$  is useless.
  - Type 2 will not betray: there is nothing left if not withdraw.
  - $\Rightarrow$  **Nash equilibrium**
- **Social welfare:**

Good Equilibrium  $>$  Autarky (No Bank)  $>$  Bank Run Equilibrium

## Scenario 4: Bank (Suspension of Convertibility)

- So how to protect against bank run (rule out the bad equilibrium)?
  - Suspension of convertibility
  - Government deposit insurance
- **Suspension of convertibility**
  - Bank stops return money at  $t = 1$  after  $\hat{f}$  of withdraw.

$$V_1(f_j; r_1) = \begin{cases} r_1 & \text{if } f_j \leq \hat{f} \\ 0 & \text{if } f_j > \hat{f} \end{cases}$$

$$V_2(f; r_1) = \max \left[ R \frac{1 - r_1 f}{1 - f}, R \frac{1 - r_1 \hat{f}}{1 - \hat{f}} \right]$$

## Scenario 4: Bank (Suspension of Convertibility)

### Suspension of Convertibility When $\pi$ is Constant

When  $\hat{f} \in \{\pi, (R - r_1)/r_1(R - 1)\}$ :

- Good equilibrium is the only (dominate strategies) Nash equilibrium.
  - It achieves social optimal.
- 
- Check  $\hat{f} = \pi$  as example:
    - Type 1 will not betray: consumption at  $t = 2$  is useless
    - Type 2 will not betray: at  $t = 2$  they can get at least as much as withdraw at  $t = 1$ .
  - Suspension of convertibility during Greek Debt Crisis.



## Scenario 4: Bank (Suspension of Convertibility)

### Optimal Contract with Stochastic Withdraw

Any bank contracts which obey the sequential service constrain (including suspension of convertibility) **CANNOT** achieve social optimal with proportion of type 1 consumer  $\pi$  is stochastic and has a non-degenerate distribution.

- We can prove this by contradiction (P412-P413).
- Although it cannot be optimal, suspension of convertibility can also rule out bank run equilibrium when  $\pi$  is stochastic  $\Rightarrow$  welfare gain.

## Scenario 5: Bank (Deposit Insurance)

- Government can impose tax **after** withdraw realized.
- **Deposit insurance**
  - Funded by wealth tax on all wealth on  $t = 1$ .
  - $\Rightarrow$  Tax is determined after the government know how much withdraw  $f$  realized at  $t = 1$ .
  - $\Rightarrow$  Proportionate tax as a function of  $f$ :

$$\tau(f) = \begin{cases} 1 - \frac{c_1^*(f)}{r_1} & \text{if } f \leq \bar{\pi} \\ 1 - r_1^{-1} & \text{if } f > \bar{\pi} \end{cases}$$

- Where  $\bar{\pi}$  is the maximum possible realization of  $\pi$ .
- Tax is plowed back into bank for consumer withdraw at  $t = 2$ .

## Scenario 5: Bank (Deposit Insurance)

- After-tax proceeds, per dollar of deposit of withdrawal at  $t = 1$ , for  $f_j \leq f$ :

$$\hat{V}_1(f) = \begin{cases} c_1^*(f) & \text{if } f \leq \bar{\pi} \\ 1 & \text{if } f > \bar{\pi} \end{cases}$$

- After-tax proceeds, per dollar of deposit of withdrawal at  $t = 2$ :

$$\hat{V}_2(f) = \begin{cases} R \frac{1 - c_1^*(f)f}{1 - f} = c_2^*(f) & \text{if } f \leq \bar{\pi} \\ R \frac{1 - f}{1 - f} = R & \text{if } f > \bar{\pi} \end{cases}$$

- Notice that  $\hat{V}_2(f) > \hat{V}_1(f)$  for all  $f \in [0, 1]$ :
  - It means type 2 will not withdraw at  $t = 1$
  - Exist a unique (dominate strategies) Nash equilibrium.**

## Scenario 5: Bank (Deposit Insurance)

### Demand Deposit Contracts with Deposit Insurance

Demand deposit contracts with government deposit insurance achieve the unconstrained **optimum** as a **unique** Nash equilibrium if the government imposes an optimal tax to finance the deposit insurance.

- At Nash equilibrium:
  - Only type 1 withdraw at  $t = 1$ , so  $f = t$ :

$$\hat{V}_1(f = t) = c_1^*(\pi)$$

$$\hat{V}_2(f = t) = R \frac{1 - \pi c_1^*}{1 - \pi} = c_2^*(\pi)$$

# Discussion

- Them model provides insights in:
  - Risk sharing and liquidity provision function of banks
  - The strategic interaction between depositors
  - The cause, welfare loss of bank run, and how to protect against it
- This model assume single bank and riskless technology:
  - When there are multiple risky technology:
  - **Moral hazard:** Deposit insurance may encourage risk-taking investment.
- Government deposit insurance vs commercial deposit insurance
- Other application:
  - Discount window lending of central bank and inflation tax.
  - Bankruptcy protection  $\Leftrightarrow$  Suspension of convertibility.

# Extensions

- Secondary market (Dong, P54.)
  - Two types of technologies
  - Trade between two types of investors
- Debt vs. equity contract (Dong, P57.)
  - Financial intermediate is a mutual fund rather than a bank.
  - Mutual fund is free from bank run equilibrium.
- Inter-bank asset market and multiple equilibrium (Dong, P59.).

# The End! Thanks for listening!