University of Strathclyde

Department of Electronic and Electrical Engineering

EE466 Coursework

Assessment

Daniel Dobiński

Student's number: 201685058

Glasgow, 2016

1. Q1

a) Separating the R_2/s term.

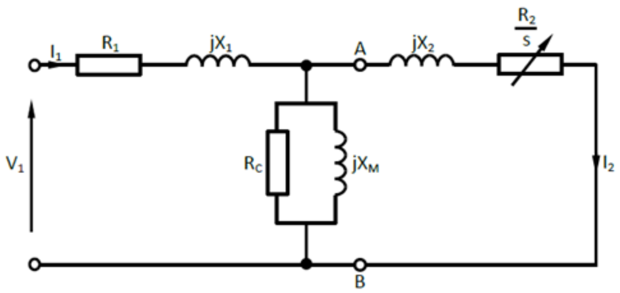

Fig. 1. Induction machine equivalent circuit.

General equation for the losses and power are following:

$$P_{AG} = I_2^2 \frac{R_2}{s}, \qquad \text{Air gap loss} \tag{1.1}$$

$$P_{CONV} = I_2^2\, R_2 \frac{1-s}{s}, \qquad \text{Mechanical output power} \tag{1.2}$$

$$P_{RCL} = I_2^2\, R_2, \qquad \text{Rotor copper loss} \tag{1.3}$$

Knowing that, and substituting P_{conv} and P_{rcl} from equations 1.2 and 1.3:

$$P_{AG} = P_{CONV} + P_{RLC} \tag{1.4}$$

$$\mathrm{I}_2^2 \frac{\mathrm{R}_2}{\mathrm{s}} = \mathrm{I}_2^2\, \mathrm{R}_2 \frac{1-\mathrm{s}}{\mathrm{s}} + \mathrm{I}_2^2\, \mathrm{R}_2 = \mathrm{I}_2^2 \mathrm{R}_2 (\frac{1-\mathrm{s}}{\mathrm{s}} + 1) \tag{1.5}$$

we can see by comparison of the left-hand side and right-hand side of the equation, that:

$$\frac{1-s}{s} + 1 = \frac{1}{s} \tag{1.6}$$

It means that it is possible to split $\frac{R_2}{s}$ term, to $R_2 \frac{1-s}{s}$ and R_2, where the $\frac{R_2}{s}$ will represent air gap losses, $R_2 \frac{1-s}{s}$ and R_2 mechanical output power and rotor copper loss respectively.

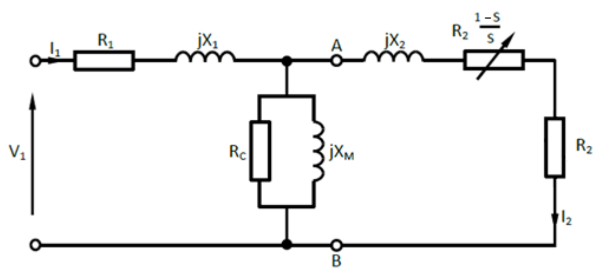

Fig.2. Induction machine equivalent circuit.

b) (i) Drawing the circuit.

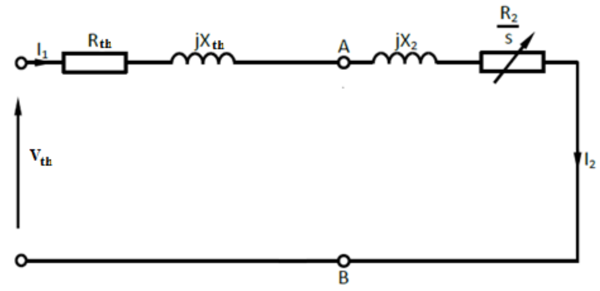

Fig. 3. Thevenin equivalent circuit.

(ii) Derivation of the Thevenin circuit parameters.

At first Thevenin equivalent impedance is calculated, in order to do that, the first step is calculation of equivalent impedance of parallel branch R_c, jX_m:

$$\frac{1}{Z_{eq1}} = \frac{1}{R_c} + \frac{1}{jX_m} \tag{1.7}$$

$$Z_{eq1} = \frac{jR_cX_m}{R_c+jX_m} \tag{1.8}$$

After calculation, of impedance of parallel branch, it is possible to calculate the $Z_{th.}$ In order to that, it is assumed that V_1 is short circuited, and the section to the right of terminals AB in fig. 1 is open circuited.

$$\frac{1}{Z_{th}} = \frac{1}{R_1+jX_1} + \frac{R_c+jX_m}{jR_cX_m} = \frac{(jR_cX_m)}{(R_1+jX_1)(jR_cX_m)} + \frac{(R_1+jX_1)(R_c+jX_m)}{(jR_cX_m)(R_1+jX_1)} \tag{1.9}$$

$$\frac{1}{Z_{th}} = \frac{jR_cX_m+R_1R_c+jR_1X_m+jX_1R_c-X_1X_m}{jR_cX_mR_1-X_1X_m\mathrm{R_c}} \tag{1.10}$$

$$Z_{th} = \frac{jR_cX_mR_1-X_1X_m\mathrm{R_c}}{R_1R_c-X_1X_m+\mathrm{j}(R_cX_m+R_1X_m+X_1R_c)} \tag{1.11}$$

We can obtain the formula for the $Z_{th,}$ where the real part and imaginary part is separated, short multiplication formulas, treating (a+b)(a–b)=a^2–b^2, where $R_1R_c - X_1X_m$ = a, and j$(R_cX_m + R_1X_m + X_1R_c)$ = b.

$$Z_{th} = \frac{(jR_cX_mR_1-X_1X_m\mathrm{R_c})(R_1R_c-X_1X_m-\mathrm{j}\,[R_cX_m+R_1X_m+X_1R_c])}{(R_1R_c-X_1X_m)^2-(R_cX_m+R_1X_m+X_1R_c)^2} \tag{1.12}$$

$$\boldsymbol{Z_{th}} = \frac{\boldsymbol{R_1R_cX_m(R_cX_m+R_1X_m+X_1R_c)-X_1XR_c{}^2}}{\boldsymbol{(R_1R_c-X_1X_m)^2-(R_cX_m+R_1X_m+X_1R_c)^2}} + \boldsymbol{j}\frac{\boldsymbol{R_1{}^2R_c{}^2X_m-R_1R_cX_m{}^2X_1+X_1X_mR_c(R_cX_m+R_1X_m+X_1R_c)}}{\boldsymbol{(R_1R_c-X_1X_m)^2-(R_cX_m+R_1X_m+X_1R_c)^2}} \tag{1.13}$$

Thevenin equivalent voltage V_{th}, can be calculated, by open-circuiting terminals AB, and assuming that all the current is flowing through impedance Z_{eq1}. Then V_{th} is equal to the the voltage across the terminals AB.

$$V_{th} = \mathrm{I}\,Z_{eq1} = \frac{V_1}{Z_{th}}Z_{eq1} \tag{1.14}$$

$$V_{th} = V_1 \frac{jR_cX_m + R_1R_c + jR_1X_m + jX_1R_c - X_1X_m}{jR_cX_mR_1 - X_1X_m\mathrm{R_c}} * \frac{jR_cX_m}{R_c + jX_m} \tag{1.15}$$

It is possible to seperate the formula to obtain real and imaginary part, however, it is easier to calculate Z_{th} and Z_{eq1} at first.

c) Calculations for given Thevenin equivalent circuit parameters

(i) The no load speed.

When there is no load, the slip is equal to 0. It means that the rotor is operating at the same speed as synchronous speed.

$$s = \frac{n_{syn} - n_{rt}}{n_{syn}}, for\ s = 0, n_{syn} = n_{rt} \tag{1.16}$$

$$n_{syn} = \frac{60f_1}{p}\ rpm \tag{1.17}$$

n_{syn}-synchronous speed.

n_{rt} - rotor speed.

f_1 - frequency of voltage in stator.

p - number of pole pairs.

$$n_{rt} = \frac{60*50}{3} = 1000\ rpm. \tag{1.18}$$

(ii) The slip corresponding to 960 rpm.

Slip is the parameter associated with the difference between synchronous speed, and rotor speed, given by the following equation.

$$s = \frac{n_{syn} - n_{rt}}{n_{syn}} = \frac{1000-960}{1000} = 0.04 \tag{1.19}$$

(iii) The mechanical power at 960 rpm.

Mechanical power can be calculated using the equation 1.2 and multiplying it by 3 (3phase system). In order to that that it is necessary to calculate I_2. We can deduce that $I_2 = I_1 = I_{th}$ considering the equivalent thevenin circuit. It is possible to calculate the phasor of I_2, but it is needed to only find the absolute value of the phasor for equation 1.19.

$$|\mathrm{I_2}| = |\mathrm{I_{th}}| = \frac{\mathrm{V_{th}}}{|\mathrm{Z_{th}} + \frac{\mathrm{R_2}}{\mathrm{s}} + \mathrm{X_2}|} = \frac{\mathrm{V_{th}}}{|\mathrm{R_t} + \mathrm{jX_t} + \frac{\mathrm{R_2}}{\mathrm{s}} + \mathrm{jX_2}|} \tag{1.20}$$

$$|\mathrm{I_2}| = \frac{245}{\left|1.2 + 2\mathrm{j} + \frac{0.9}{0.04} + 3.2\mathrm{j}\right|} = 10.097\ \mathrm{A} \tag{1.21}$$

$$\mathrm{P_{CONV}} = 3\mathrm{I_2^2}\ \mathrm{R_2}\frac{1-\mathrm{s}}{\mathrm{s}} = 3 * (10.097)^2 * 0.9 * \frac{1-0.04}{0.04} = 6606.322\ \mathrm{W} \tag{1.22}$$

(iv) The torque at 960 rpm.

After converting the speed of the rotor to the SI units, one can calculate the torque:

$$\omega_{rt} = n_{rt}\frac{2\pi}{60} = 32\pi \ \frac{rad}{s} \quad (1.23)$$

$$T = \frac{P_{CONV}}{w_{rt}} = \frac{6606.322}{32\pi} = 65.71\ Nm \quad (1.24)$$

(v) The rotor power loss at 960 rpm.

Using the equation 1.3 and multiplying it by 3 (3 phase system), rotor power loss can be calculated:

$$P_{RCL} = 3\mathrm{I}_2^2\,\mathrm{R}_2 = 3 * (10.097)^2 * 0.9 = 275.26\ W \quad (1.25)$$

2. Q2

a) (i) Sketch of the equivalent circuit and the electrical equations the armature circuit.

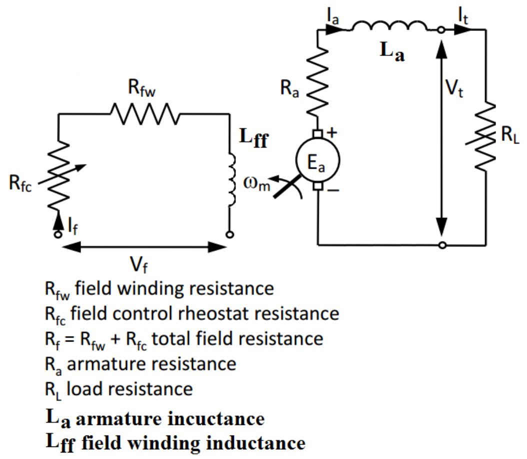

*Fig. 4. Separately excited DC machine.**[from lecture slides]***

Voltage equation, taking into account the mutual inductance. ($V_t \equiv V_a$)

$$\boldsymbol{V_t = R_a I_a + I_f \omega_m L_{ad} + L_a \frac{dI_a}{dt}}\text{, **armature equation**} \quad \textbf{(2.1)}$$

$$V_f = R_f I_f + L_{ff}\frac{dI_f}{dt}\text{, field winding equation} \quad (2.2)$$

$$E_a = I_f \omega_m L_{ad} \quad (2.3)$$

(ii) Steady state voltage and Torque equations.

For steady state, the derivative with respect to time = 0, hence equation 2.1. and 2.2. becomes:

$$\boldsymbol{V_t = R_a I_a + I_f \omega_m L_{ad}}\text{, **armature equation, steady state**} \quad \textbf{(2.4)}$$

$$V_f = R_f I_f\text{, field winding equation} \quad (2.5)$$

Knowing that electrical torque is given by the following equation:

$$T_e = \frac{P}{\omega_m} = \frac{E_a I_a}{\omega_m} \quad (2.6)$$

substituting E_a from equation 2.3.

$$T_e = \frac{I_f \omega_m L_{ad} I_a}{\omega_m} = I_f L_{ad} I_a \tag{2.7}$$

(iii) Equation linking speed and torque.

Deriving equations 2.4 for I_a:

$$I_a = \frac{Vt}{R_a} - \frac{I_f \omega_m L_{ad}}{R_a} \tag{2.8}$$

Substituting I_a from 2.8 into equation 2.7:

$$\boldsymbol{T_e = I_f L_{ad} I_a = I_f L_{ad}\left(\frac{V_t}{R_{a_t}} - \frac{I_f \omega_m L_{ad}}{R_a}\right) = L_{ad}(I_f \frac{I_a}{R_a} - I_f{}^2 \frac{\omega_m L_{ad}}{R_a})} \tag{2.9}$$

(iv) Torque-speed graph.

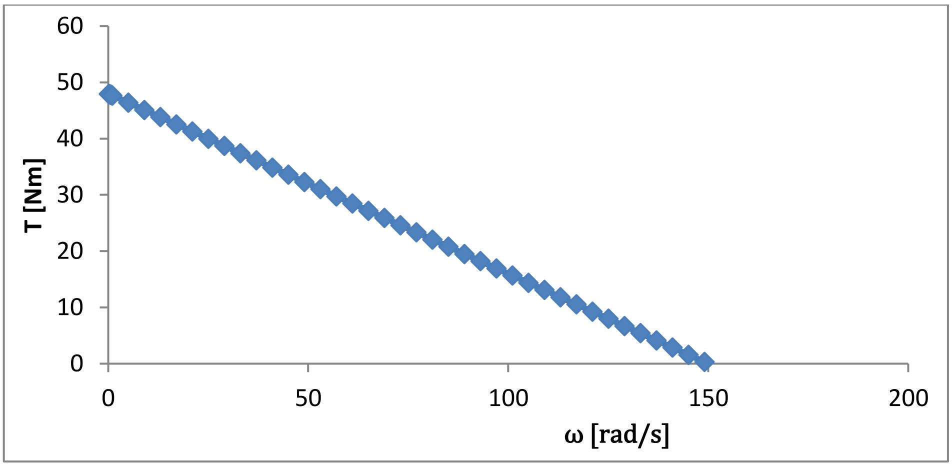

Fig. 5. Torque-speed graph.

b)(i) Free body diagram

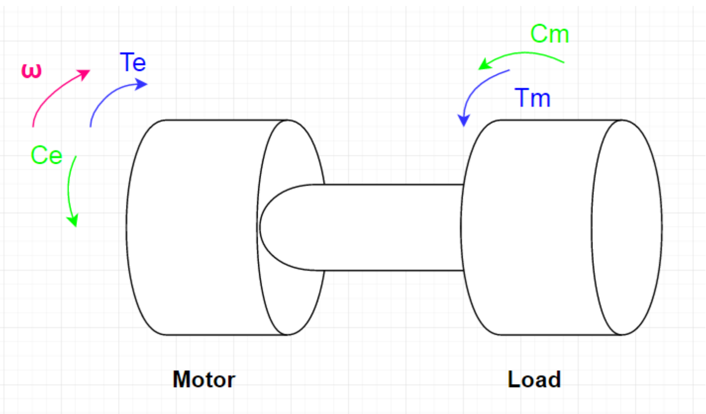

Fig. 6. Free body diagram.

Electrical torque has the same direction of as the rotational speed. The rest of torques and drag coefficients are opposing the rotational speed, that is why they have different signs.

(ii) Mechanical system equations

$$T_e - (J_e + J_m)\frac{d\omega_m}{dt} - T_m - B_m \omega_m - C_e - C_m = 0 \tag{2.10}$$

In that case we can treat it as the control system having input u(t)= T_e and output ω=y(t), and use standard formula for first order differential equation, we can assume that $B_m = 0$.

$$\tau \frac{dy(t)}{dt} + y(t) = Ku(t) \tag{2.11}$$

It is possible to calculate its Laplace transform in order to get a transfer function, assuming that initial conditions = 0.

$$Y(s) = H(s)U(s) = \frac{K}{\tau s+1} U(s) \tag{2.12}$$

Rearranging equation 2.8 in order to be able to compare to the standard equation, gives following results:

$$(J_e + J_m) \frac{d\omega_m}{dt} + C_e + C_m + T_m = T_e \tag{2.13}$$

$$\frac{(J_e + J_m)}{C_e + C_e + T_m} \frac{d\omega_m}{dt} + 1 = \frac{1}{C_e + C_m + T_m} T_e \tag{2.14}$$

$$\frac{(J_e + J_m)}{C_e/\omega_m + C_m/\omega_m + T_m/\omega_m} \frac{d\omega_m}{dt} + \omega_m = \frac{1}{C_e/\omega_m + C_m/\omega_m + T_m/\omega_m} T_e \tag{2.15}$$

Then by comparison:

$$K_{general} = \frac{(J_e + J_m)}{C_e/\omega_m + C_m/\omega_m + T_m/\omega_m} = 9.996 \tag{2.16}$$

$$\tau_{general} = \frac{1}{C_e/\omega_m + C_m/\omega_m + T_m/\omega_m} = 1.249\ s \tag{2.17}$$

Having the transfer function, it is possible to calculate any response for any input of T_e. The response would be the inverse transform of H(s) U(s).

(iii) Steady state speed and the mechanical time constant.

Due to the fact that it is a difficult mathematical task to calculate the inverse Laplace transform of H(s)U(s) in that case, one can treat the input and transfer function as one bigger system, directly substituting the input signal into equation.

$$(J_e + J_m) \frac{d\omega_m}{dt} + C_e + C_m + T_m = T_e \tag{2.18}$$

$$(0.025 + 0.1) \frac{d\omega_m}{dt} + 0.00003\omega_m + 0.00001\omega_m + 0.1\omega_m = 48 - 0.32\omega_m \tag{2.19}$$

$$0.125 \frac{d\omega_m}{dt} + 0.42004\omega_m = 48 \tag{2.20}$$

$$0.125 \frac{d\omega_m}{dt} + 0.42004\omega_m = 48 \qquad /:0.42004 \tag{2.21}$$

$$0.2976 \frac{d\omega_m}{dt} + \omega_m = 114.27 \tag{2.22}$$

One can obtain the value of τ and K by comparison with equation 2.9.:

$$K = 114.27 \tag{2.23}$$

$$\boldsymbol{\tau = 0.2976\ s} \tag{2.24}$$

The τ for that system is calculated above, in order to calculate the one can use the equation 2.15, and set the derivative to zero.

$$0.00003\omega_m + 0.00001\omega_m + 0.1\omega_m = 48 - 0.32\omega_m \tag{2.25}$$

$$\boldsymbol{\omega_{steady-state} = 114.27\ rad/s} \tag{2.26}$$

The result is the same as the gain for the system, which is not surprising, because the steady state value should be equal to K, if the step signal is applied, in according to final value theorem. When the complex input signal is modeled as the part of the system, input can be treated as step.

(iv) Equation relating speed and time.

In order to do that, Laplace inverse transform should be applied to the equation 2.19.

$$\tau\big(sY(s) - Y(0)\big) + Y(s) = \frac{K}{s} \tag{2.27}$$

$$Y(s) = \frac{K}{(s\tau+1)s} \tag{2.28}$$

$$Y(s) = \frac{K/\tau}{(s+1/\tau)s} \tag{2.29}$$

using the Laplace tables:

$$\frac{1}{(s+a)a} => \frac{1}{a}(1 - e^{-at}) \tag{2.30}$$

$$y(t) = K(1 - e^{-\frac{t}{\tau}}) \tag{2.31}$$

Knowing that y(t) is the rotational speed, and substituting for K and τ:

$$\boldsymbol{\omega_m(t) = 114.27(1 - e^{-\frac{t}{0.2976}})} \tag{2.32}$$

(v) The time taken for the machine to reach 50% of its steady state speed.

$$0.5 = 1 - e^{-\frac{t}{\tau}} \tag{2.33}$$

$$1 - 0.5 = e^{-\frac{t}{\tau}} \tag{2.34}$$

$$-\tau \ln(0.5) = t \tag{2.35}$$

$$\boldsymbol{t = -0.2976 * \ln(0.5) = 0.206\ s} \tag{2.36}$$

3. Q3

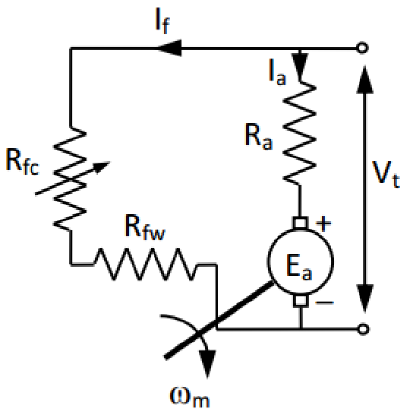

Fig. 7. Shunt DC motor ***[from lecture slides]***

a) Variable resistance in series with the field winding

At first it is important to calculate induced EMF, E_a.

$$E_a = V_t - I_a R_a = 100 - 6 * 0.5 = 97\ V \tag{3.1}$$

From magnetization curve, for $E_a = 97$ V, $I_f = 0.93$ A.

$$R_f = R_{fw} + R_{fc} = \frac{V_t}{I_f} = \frac{100}{0.93} = 107.53\ \Omega \tag{3.2}$$

$$R_{fc} = R_f - R_{fw} = 107.53 - 80 = 27.53\ \Omega \tag{3.3}$$

b) Rotational losses at 1000 rpm.

Rotational losses are the losses required to rotate the machine against windage and friction (rotor core losses are also included). At no-load condition, electromagnetic power is equal to rotational power loss.

$$P_{rot} = I_a E_a = 6 * 97 = 582\ \text{W} \tag{3.4}$$

c) Speed, electromagnetic torque and efficiency.

(i) No armature reaction.

At first, rated armature current has to be calculated:

$$I_{a(rated)} = \frac{P_{(rated)}}{V_{t(rated)}} = \frac{12\ 000}{100} = 120\ A \tag{3.5}$$

Then, one can calculate the EMF at rated load.

$$E_{a\ (loaded)} = V_t - I_{a(rated)} R_a = 100 - 120 * 0.5 = 40\ V \tag{3.6}$$

$E_{a\ (no\ load)} = 97\ V$, from eq. 3.1.

Assuming that $\Phi_{(loaded)} = \Phi_{(no\ load)}$, and dividing the equations for EMF:

$$\frac{E_{a\ (loaded)}}{E_{a\ (no\ load)}} = \frac{K_a \Phi_{(loaded)} \omega_{(loaded)}}{K_a \Phi_{(no\ load)} \omega_{(no\ load)}} = \frac{\omega_{(loaded)}}{\omega_{(no\ load)}} \tag{3.7}$$

$$\omega_{(loaded)} = \frac{E_{a\,(loaded)}}{E_{a\,(no\ load)}}\omega_{(no\ load)} = \frac{40}{97} * 1000 = 412.371\ rpm \quad (3.8)$$

$$\omega_{(loaded)} = 412.371 * \frac{2*\pi}{60} = 43.18\ rad/s \quad (3.9)$$

$$\boldsymbol{T = \frac{E_a I_a}{\omega_{(loaded)}} = \frac{40*120}{43.18} = 111.163\ Nm} \quad (3.10)$$

$$P_{out} = E_{a\,(loaded)} I_{a(rated)} - \mathrm{P_{rot}} = 40 * 120 - 582 = 4218\ \mathrm{W} \quad (3.11)$$

$$P_{in} = \mathrm{V_t I_t} = \mathrm{V_t}\left(I_{a(rated)} + \mathrm{I_f}\right) = 100 * (120 + 0.93) = 12093\ \mathrm{W} \quad (3.12)$$

$$\boldsymbol{\eta = \frac{P_{in}}{P_{out}} * 100 = \frac{4218}{12093} * 100 = 34.87}\ \% \quad (3.13)$$

(ii) The armature reaction that reduces air gap flux by 5 %.

$I_{a(rated)}$ and $E_{a(loaded)}$ is the same as in equations 3.4 and 3.5.

Assuming that $\Phi_{(loaded)} = 0.95\ \Phi_{(no\ load)}$, and dividing the equations for EMF:

$$\frac{E_{a\,(loaded)}}{E_{a\,(no\ load)}} = \frac{K_a \Phi_{(loaded)} \omega_{(loaded)}}{K_a \Phi_{(no\ load)} \omega_{(no\ load)}} = \frac{K_a 0.95 \Phi_{(no\ load)} \omega_{(loaded)}}{K_a \Phi_{(no\ load)} \omega_{(no\ load)}} = 0.95 \frac{\omega_{(loaded)}}{\omega_{(no\ load)}} \quad (3.14)$$

$$\omega_{(loaded)} = \frac{1}{0.95} \frac{E_{a\,(loaded)}}{E_{a\,(no\ load)}} \omega_{(no\ load)} = \frac{1}{0.95} * \frac{40}{97} * 1000 = 434.075\ rpm \quad (3.15)$$

$$\omega_{(loaded)} = 434.075 * \frac{2*\pi}{60} = 45.456\ rad/s \quad (3.16)$$

$$\boldsymbol{T = \frac{E_a I_a}{\omega_{(loaded)}} = \frac{40*120}{45.456} = 105.597\ Nm} \quad (3.17)$$

Assuming that the rotational losses remain constant, as the speed of the motor is changing equations 3.10, 3.11, 3.12 will be the same as equations 3.17, 3.18, 3.19.

$$P_{out} = E_{a\,(loaded)} I_{a(rated)} - \mathrm{P_{rot}} = 40 * 120 - 582 = 4218\ \mathrm{W} \quad (3.18)$$

$$P_{in} = \mathrm{V_t I_t} = \mathrm{V_t}\left(I_{a(rated)} + \mathrm{I_f}\right) = 100 * (120 + 0.93) = 12093\ \mathrm{W} \quad (3.19)$$

$$\boldsymbol{\eta = \frac{P_{in}}{P_{out}} * 100 = \frac{4218}{12093} * 100 = 34.87}\ \% \quad (3.20)$$

c) Starting torque, if armature current at start-up is limited to 150% of its rated value.

In both cases following equation for Torque is used:

$$\boldsymbol{T = K_a \Phi \omega} \quad (3.21)$$

Maximal armature start-up current:

$$\boldsymbol{I_{a(\mathrm{max})} = 1.5 * I_{a(\mathrm{rated})} = 1.5 * 120 = 180\ A} \quad (3.22)$$

(i) No armature reaction.

At first it is necessary to calculate the $\boldsymbol{K_a \Phi}$ for no load condition:

$$E_{a\,(no\ load)} = K_a \Phi_{(no\ load)} \omega_{(no\ load)} \quad (3.23)$$

$$K_a\Phi_{(no\ load)} = \frac{E_{a\ (no\ load)}}{\omega_{(no\ load)}} = \frac{97}{1000*\frac{2*\pi}{60}} = 0.926\ \frac{Vs}{rad} \tag{3.24}$$

Then, it is possible to calculate the starting torque.

$$\boldsymbol{T_{start} = K_a\Phi I_{a(\mathrm{max})} = 0.926 * 180 = 166.68\ Nm} \tag{3.25}$$

(ii) Armature reaction equivalent to a reduction in field current of 0.16 A.

Assuming that I_f = 0.93 A.

$$I_{f(actual)} = I_f - I_{f(armature\ reaction)} = 0.93 - 0.16 = 0.77\ A \tag{3.26}$$

From magnetization curve, for $I_{f(actual)}$ = 0.77, E_a= 88 V.

$$K_a\Phi_{(no\ load)} = \frac{E_{a\ (no\ load)}}{\omega_{(no\ load)}} = \frac{88}{1000*\frac{2*\pi}{60}} = 0.840\ \frac{Vs}{rad} \tag{3.27}$$

$$\boldsymbol{T_{start} = K_a\Phi I_{a(\mathrm{max})} = 0.840 * 180 = 151.2\ Nm} \tag{3.28}$$

<u>EE466 Coursework Assessment</u>

Introduction
The objective of this coursework exercise is to consolidate the lecture material that you have received to date, and to bring together associated engineering topics that relate to the successful operation and control of an electrical machine.

Another important objective is to assist you in the development of transferable skills, such as report writing, presentation of technical material, and examination technique.

The Assessment
The assessment consists of 3 questions where all of the material has been covered to greater or lesser degrees during lectures. Whilst some questions are straightforward, others are designed to test your comprehension beyond what has been explicitly covered in the class.

Submission

- You may submit either word processed or hand-written solutions.
- Your submission should be suitably bound and must include a cover sheet.
- When including your name on the coversheet, it <u>**must**</u> be presented in the order:

First Name Family Name

- If your solutions require graph paper, then it is your responsibility to provide this.
- Your solutions should resemble 'specimen solutions', i.e. the type of solution that you may expect to see in a textbook. Solutions must be clearly presented, legible, and concise. You should show all of your mathematical working. Diagrams and graphs must be clear and contain all of the relevant information.
- **Your completed assessment must be submitted to the Resource Centre by 12 noon on Friday 9th December**.

Q1 The per-phase equivalent circuit of an induction motor is shown in Figure Q1.

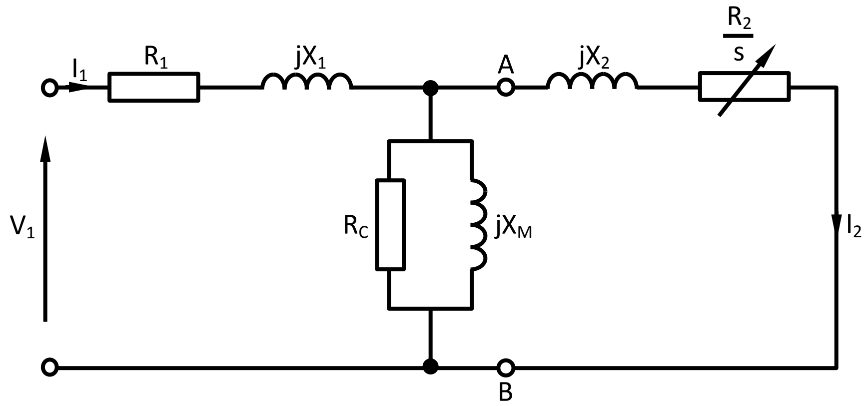

Figure Q1 Induction machine equivalent circuit

(a) Show that, by splitting the R_2/s term into two components, expressions can be developed for
  (i) Rotor copper loss.
  (ii) Mechanical output power.

(b) Calculations can be simplified if part of the circuit shown in Figure Q1 is replaced by a Thevenin equivalent circuit.

  (i) Draw this circuit which replaces the section to the left of terminals AB in Figure Q1.
  (ii) Derive the Thevenin circuit parameters from the parameters in the equivalent circuit of Figure Q1.

(c) A 440V, 3-phase, 50 Hz, 6-pole induction motor has the following Thevenin equivalent circuit parameters:

$$V_T = 245\text{V},\ R_T = 1.2\Omega \text{ and } X_T = 2\Omega$$

The rotor paramneters are $X_2 = 3.2\Omega$, $R_2 = 0.9\Omega$

Calculate
  (i) The no-load speed.
  (ii) The slip corresponding to a speed of 960 rpm.
  (iii) The mechanical power at 960 rpm.
  (iv) The torque at 960 rpm.
  (v) The rotor power loss at 960 rpm.

Q2 (a) A separately excited DC machine is used to drive a conveyor belt. The armature has the electrical parameters V_a=120V, R_a=2Ω, L_a=10mH, the field current I_f=10A and the mutual inductance L_{ad}=80mH.

(i) Sketch the equivalent circuit and derive the electrical equation describing the armature circuit.

(ii) Derive the steady-state voltage and torque equations.

(iii) Use your equations from part (ii) to derive an equation linking torque and speed.

(iv) Sketch the torque-speed profile of this machine on a graph. You are not required to use graph paper, but your sketch should be accurately and carefully drawn.

(b) An electrical machine is connected to a mechanical load. The system has the following mechanical parameters:

Electrical torque developed	T_e = 48-0.32ω Nm
Machine inertia	J_e=0.025 kgm^2
Machine drag coefficient	C_e=3x10^{-5} ω Nm
Load torque	T_m=0.1ω Nm
Load inertia	J_m=0.1 kgm^2
Load drag coefficient	C_m=1x10^{-5} ω Nm

The connecting shaft between the motor and load can be assumed to be stiff with zero inertia.

(i) Sketch the free body diagram of the motor and load system.

(ii) Derive the mechanical system equation including all torques, inertias and losses acting upon the system.

(iii) Calculate the steady-state speed and the mechanical time constant of the system.

(iv) Derive the equation relating speed and time.

(v) Calculate the time taken for the machine to reach 50% of its steady-state speed.

Q3 A 100V, 12kW, 1000rpm shunt DC motor has armature resistance of 0.5Ω and shunt field winding resistance of 80Ω. Field current is controlled using a variable resistance in series with the field winding. The magnetisation curve for the machine at 1000rpm is shown in Figure Q3. When connected to a 100V supply, under no-load conditions, the motor runs at 1000rpm and the armature takes 6A.

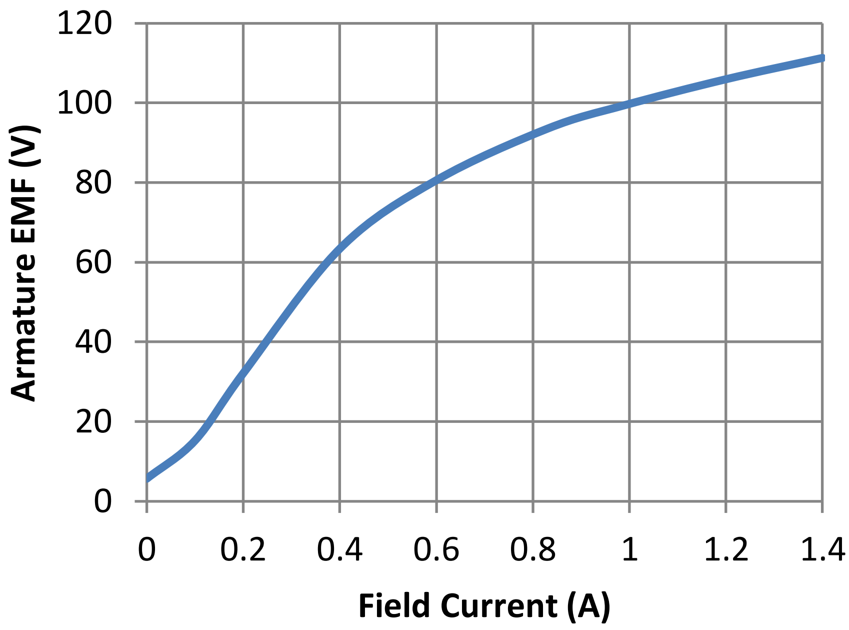

Figure Q3 Magnetising curve

(a) Find the value of the variable resistance in series with the field winding.

(b) Find the rotational losses at 1000rpm.

(c) Find the speed, electromagnetic torque and efficiency of the machine when rated current flows in the armature with
   (i) No armature reaction.
   (ii) Armature reaction that reduces airgap flux by 5%.

(d) Find the starting torque if the armature current at start-up is limited to 150% of its rated value with
   (i) No armature reaction.
   (ii) Armature reaction equivalent to a reduction in field current of 0.16A.