Appendix

A. List of Symbols

The following table provides a comprehensive list of symbols and their definitions as used in the various mathematical formulations and analyses throughout this manuscript:

TABLE I: List of symbols and their definitions.

Symbols	Definition
N	Set of all validators in the network
I	Total number of distinct validator types
H, M, L	High, medium, and low types of validators
n	A validator within the set N
i	Validator type, where $i \in \{1, 2, \ldots, I\}$
θ_i	Revenue-generating capability of validator type i
ε_i	Effort level of validator type i
a_i	Effort scale factor for validator type i
b_i	Rate at which effort adapts for validator type i
X_i	Dynamic factors influencing effort for validator type i
$\bar{\varepsilon}_i$	Expected effort level for type i validators
E	Maximum allowable deviation from expected effort level
$R_{i,\text{block}}$	Revenue from block rewards for validator i
$R_{i,\text{fees}}$	Revenue from transaction fees for validator i
$R_{i,\text{incentives}}$	Additional incentives for validator i
$\mathcal{C}_i$	Contract bundle for validator i
s_i	Stake of validator i
t_i	Block submission time for validator i
t_{max}	Maximum block submission time
p_i	Probability related to validator i's category assignment
α, β, γ	Constants in utility and reward functions
σ	Sensitivity of the penalty risk to effort level
U_i	Utility function for validator i
W	Utility function of the blockchain network
$B(\varepsilon_i, s_i)$	Successful blocks added function for validator i
υ_1, υ_2	Elasticities of block addition with respect to effort and stake
U_{SW}	Social welfare function of the system

B. Experimental Parameters for Setup

This section provides a detailed list of parameters used in the experimental setup of our study. These parameters are crucial for replicating the experimental conditions and understanding the framework within which the validators operate.

C. Detailed Proofs of the Optimization Problem Formulation

This appendix provides detailed derivations of the optimization problem formulation for designing contracts for validators as discussed in Section III-B.

The optimization problem can be stated as follows:

$$\underset{\mathcal{C}_i}{\text{maximize}} \quad U_{SW} \tag{32a}$$

$$\text{subject to} \quad U_i(s_i, \varepsilon_i, t_i, R_{i,\text{incentives}}) \geq 0, \tag{32b}$$

$$U_i(s_i, \varepsilon_i, t_i, R_{i,\text{incentives}}) \geq U_j(s_i, \varepsilon_i, t_i, R_{j,\text{incentives}}), \tag{32c}$$

$$0 \leq R_{1,\text{incentives}} < \cdots < R_{i,\text{incentives}} < \cdots < R_{I,\text{incentives}}. \tag{32d}$$

The Lagrangian must satisfy the Karush-Kuhn-Tucker (KKT) conditions for optimality. Given the optimization problem formulated in (15), we derive the KKT conditions for the optimality of solutions. The KKT conditions are composed of stationarity, primal feasibility, dual feasibility, and complementary slackness.

TABLE II: Parameter Values for Experiment Setup

Parameter	Value
N	210 validators
I	3 types (High, Medium, Low)
n_H	50
n_M	80
n_L	80
θ_H	1.0
θ_M	0.75
θ_L	0.5
ε_H	0.9
ε_M	0.7
ε_L	0.5
s_H	15,000 tokens
s_M	10,000 tokens
s_L	5,000 tokens
t_{max}	10 seconds
$R_{i,\text{block}}$	Variable based on validation frequency
$R_{i,\text{fees}}$	Variable based on transaction volume
$R_{i,\text{incentives}}$	Variable to ensure competitive behavior
α	0.04 (4%)
β	0.1 (10%)
σ	2
a_i	Variable based on type-specific performance metrics
b_i	0.5 (uniform across types)
X_i	Includes factors like network load, transaction size, etc.
$\bar{\varepsilon}_i$	Computed as the average of ε_i over time
E	0.1
p_H, p_M, p_L	0.4, 0.3, 0.3 respectively
$B(\varepsilon_i, s_i)$	Defined as $\gamma \varepsilon_i^{\upsilon_1} s_i^{\upsilon_2}$
γ	1 (normalization factor)
υ_1, υ_2	1.2, 1.1 (to model increasing returns to scale)

Stationarity conditions necessitate the derivatives of the Lagrangian with respect to all decision variables to be zero:

$$\nabla_{s_i} L = 0, \tag{33}$$

$$\nabla_{\varepsilon_i} L = 0, \tag{34}$$

$$\nabla_{t_i} L = 0, \tag{35}$$

$$\nabla_{R_{i,\text{incentives}}} L = 0. \tag{36}$$

These stationarity conditions imply that:

$$\frac{\partial U_{SW}}{\partial s_i} - \lambda_i \frac{\partial U_i}{\partial s_i} - \sum_j \mu_{ij} \left(\frac{\partial U_i}{\partial s_i} - \frac{\partial U_j}{\partial s_i} \right) = 0, \tag{37}$$

$$\frac{\partial U_{SW}}{\partial \varepsilon_i} - \lambda_i \frac{\partial U_i}{\partial \varepsilon_i} - \sum_j \mu_{ij} \left(\frac{\partial U_i}{\partial \varepsilon_i} - \frac{\partial U_j}{\partial \varepsilon_i} \right) = 0, \tag{38}$$

$$\frac{\partial U_{SW}}{\partial t_i} - \lambda_i \frac{\partial U_i}{\partial t_i} - \sum_j \mu_{ij} \left(\frac{\partial U_i}{\partial t_i} - \frac{\partial U_j}{\partial t_i} \right) = 0, \tag{39}$$

$$\frac{\partial U_{SW}}{\partial R_{i,\text{incentives}}} - \lambda_i \frac{\partial U_i}{\partial R_{i,\text{incentives}}} - \sum_j \mu_{ij} \left(\frac{\partial U_i}{\partial R_{i,\text{incentives}}} - \frac{\partial U_j}{\partial R_{i,\text{incentives}}} \right) - \nu_i + \nu_{i-1} = 0. \tag{40}$$

Primal feasibility requires that all constraints of the optimization problem are satisfied:

$$U_i(s_i, \varepsilon_i, t_i, R_{i,\text{incentives}}) \geq 0, \tag{41}$$

$$U_i(s_i, \varepsilon_i, t_i, R_{i,\text{incentives}}) \geq U_j(s_i, \varepsilon_i, t_i, R_{j,\text{incentives}}), \tag{42}$$

$$R_{1,\text{incentives}} < R_{2,\text{incentives}} < \ldots < R_{I,\text{incentives}}. \tag{43}$$

Dual feasibility ensures the non-negativity of the Lagrange multipliers:

$$\lambda_i \geq 0, \tag{44}$$
$$\mu_{ij} \geq 0, \tag{45}$$
$$\nu_i \geq 0. \tag{46}$$

Complementary slackness conditions dictate that each Lagrange multiplier times the associated constraint equals zero:

$$\lambda_i \left(U_i(s_i, \varepsilon_i, t_i, R_{i,\text{incentives}}) \right) = 0, \tag{47}$$
$$\mu_{ij} \left(U_i(s_i, \varepsilon_i, t_i, R_{i,\text{incentives}}) - U_j(s_i, \varepsilon_i, t_i, R_{j,\text{incentives}}) \right) = 0, \tag{48}$$
$$\nu_i \left(R_{i,\text{incentives}} - R_{i+1,\text{incentives}} \right) = 0. \tag{49}$$

Then, we solve the resulting system of equations to obtain the optimal values as follows: The optimal validator's stake s_i^* can be derived as

$$s_i^* = \frac{\theta_i \varepsilon_i}{1 + \lambda_i \varepsilon_i^{\sigma}}, \tag{50}$$

the optimal validator's effort ε_i^* can be derived as

$$\varepsilon_i^* = \frac{c + 2\theta_i s_i}{\lambda_i \theta_i - \mu_{ij} \theta_i}, \tag{51}$$

the validator's optimal time t_i^* can be derived as

$$t_i^* = \frac{\sum_{i,j} \mu_{ij} \theta_i}{\sum_i \lambda_i \theta_i}, \tag{52}$$

the optimal validator's incentive $R_{i,\text{incentives}}^*$ can be derived as

$$R_{i,\text{incentives}}^* = \frac{\mu_{ij} \mathbb{1}_{t_i \leq t_{\max}} \theta_i s_i - \gamma}{p_i + \mathbb{1}_{t_i \leq t_{\max}} \lambda_i \varepsilon_i}. \tag{53}$$

D. Lagrangian Formulation for Stage 1: Leader's Game

The Lagrangian for Stage 1 of the optimization problem, which involves the Leader's Game in setting rewards, is expressed as follows:

$$\mathcal{L}(R_{i,\text{incentives}}, \lambda, \mu) = W(R_{i,\text{incentives}}) - \sum_{i}^{I} \lambda_i (U_i) - \sum_{i \neq j} \mu_{ij} (U_i - U_j) \tag{54}$$

The KKT conditions, which include setting partial derivatives of the Lagrangian with respect to $R_{i,\text{incentives}}$, λ, and μ to zero, are thoroughly detailed, highlighting the primal feasibility, dual feasibility, complementary slackness, and stationarity. The differential of the Lagrangian with respect to $R_{i,\text{incentives}}$ is calculated as:

$$\frac{\partial \mathcal{L}}{\partial R_{i,\text{incentives}}} = \frac{\partial W}{\partial R_{i,\text{incentives}}} + \sum_i \lambda_i \frac{\partial U_i}{\partial R_{i,\text{incentives}}} + \sum_{i,j} \mu_{ij} \left(\frac{\partial U_i}{\partial R_{i,\text{incentives}}} - \frac{\partial U_j}{\partial R_{i,\text{incentives}}} \right). \tag{55}$$

E. Analysis of Acceptance Rates

The acceptance rates for Stake, Effort, and Reward were analyzed to substantiate the effectiveness of the normalization process implemented in our study. The following figure illustrates the comparison between the acceptance rates derived from our initial model and those following the proposed normalization process.

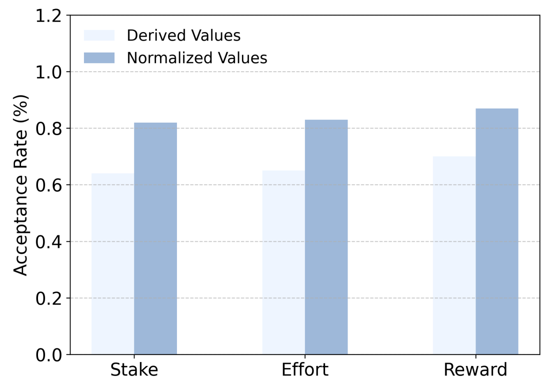

Fig. 3: Comparison of Acceptance Rates between Derived and Proposed Approaches for Stake, Effort, and Reward, illustrating the effectiveness of normalization in increasing acceptance rates.

The derived values show acceptance rates of 64%, 65%, and 70% for Stake, Effort, and Reward, respectively. Following the implementation of normalized measures, these rates improved significantly to 82%, 83%, and 87%. This increase highlights the effectiveness of our normalization process, ensuring that each validator's contributions are evaluated on a consistent and comparable basis across the network. The consistency and comparability introduced by normalization not only align rewards more accurately with the validators' proportional contributions but also enhance participation and satisfaction among validators. This analysis serves as a crucial validation of our normalization approach, demonstrating its practical impact on participant behavior and system performance.

F. Lagrangian Formulation for Stage 2: Follower's Game

The Lagrangian for Stage 2 involves each validator's optimization problem, incorporating the incentive compatibility (IC) and individual rationality (IR) constraints through Lagrange multipliers. The Lagrangian is formulated as:

$$\mathcal{L}(s_i, \varepsilon_i, \lambda_i, \mu_{ij}) = U_i(s_i, \varepsilon_i, t_i, R_{i,\text{incentives}}) - \lambda_i \cdot (U_i - 0) - \mu_{ij} \cdot (U_i - U_j). \tag{56}$$

Taking partial derivatives of $\mathcal{L}$ with respect to s_i and ε_i, and setting these derivatives to zero provides the conditions to find the optimal s_i^* and ε_i^*, detailed as follows:

$$\frac{\partial \mathcal{L}}{\partial s_i} = \frac{\partial U_i}{\partial s_i} - \lambda_i \cdot \frac{\partial U_i}{\partial s_i} - \mu_{ij} \cdot \left(\frac{\partial U_i}{\partial s_i} - \frac{\partial U_j}{\partial s_i} \right) = 0, \tag{57}$$

$$\frac{\partial \mathcal{L}}{\partial \varepsilon_i} = \frac{\partial U_i}{\partial \varepsilon_i} - \lambda_i \cdot \frac{\partial U_i}{\partial \varepsilon_i} - \mu_{ij} \cdot \left(\frac{\partial U_i}{\partial \varepsilon_i} - \frac{\partial U_j}{\partial \varepsilon_i} \right) = 0. \tag{58}$$

G. Proof of Stackelberg Equilibrium Uniqueness

This appendix section provides a detailed proof of the uniqueness of the Stackelberg Equilibrium in our Contract-Stackelberg Game. The analysis begins with the assumptions underpinning our mathematical model:

- Validators' efforts are positive and bounded, $\varepsilon_i \in (0, \varepsilon_{\text{max}})$.
- The cost function associated with effort is convex and twice differentiable.
- The reward function is concave with respect to the effort and incentive parameters.

Consider the utility function of a follower i, U_i, given by

$$U_i = \mathbb{1}_{t_i \leq t_{\text{max}}} \theta_i \varepsilon_i R_{i,\text{incentives}} - \frac{c}{2}\varepsilon_i^2 - \alpha s_i \times \frac{1}{1+\varepsilon_i^\sigma}. \quad (59)$$

Taking the first derivative with respect to ε_i and setting it to zero, we find the condition for the best response:

$$\frac{\partial U_i}{\partial \varepsilon_i} = \mathbb{1}_{t_i \leq t_{\text{max}}} \theta_i R_{i,\text{incentives}} - c\varepsilon_i - \alpha s_i \sigma \varepsilon_i^{\sigma-1} \frac{1}{(1+\varepsilon_i^\sigma)^2} = 0. \quad (60)$$

To ensure this represents a maximum, we check the second derivative:

$$\frac{\partial^2 U_i}{\partial \varepsilon_i^2} = -c - \alpha s_i \sigma(\sigma-1)\varepsilon_i^{\sigma-2}\frac{1}{(1+\varepsilon_i^\sigma)^2} + 2\alpha s_i \sigma \varepsilon_i^{2\sigma-2}\frac{1}{(1+\varepsilon_i^\sigma)^3}. \quad (61)$$

Assuming that the parameters and the forms of c, α, σ ensure this is negative, we conclude that U_i is strictly concave in ε_i. The leader's utility W is expressed as

$$W = p_i \sum_{i=1}^{I} B(\varepsilon_i, s_i) - \sum_{i=1}^{I} R_{i,\text{incentives}}, \quad (62)$$

where $B(\varepsilon_i, s_i) = \gamma \varepsilon_i^{\upsilon_1} s_i^{\upsilon_2}$. Maximizing W involves ensuring concavity with respect to $R_{i,\text{incentives}}$. We check this by analyzing the second derivative:

$$\frac{\partial^2 W}{\partial R_{i,\text{incentives}}^2} < 0. \quad (63)$$

We assume the negative signs of the derivatives arise due to the decreasing marginal utility with increasing incentive, which satisfies our concavity condition. Combining the strict concavity of U_i in ε_i and W in $R_{i,\text{incentives}}$, and using the Fixed Point Theorem [32], we demonstrate that the game admits a unique SE. This unique SE arises because the mapping from strategies to responses and incentives forms a contraction mapping under the given assumptions, leading to a unique fixed point.