# MILP Optimization Data-Driven Decision-Making

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## 1. Nurse planning

- Two types of nurses:
  - o Full-time: work five consecutive days followed by two days off.
  - o Part-time: work three consecutive days followed by four days off.
- Minimum number of nurses required per day:

Mon	Tue	Wed	Thu	Fri	Sat	Sun
17	13	15	19	14	16	11

• Wages:

Weekdays	Saturdays	Sundays	
€ 250 / day	€ 315 / day	€ 375 / day	
€ 150 / day	€ 185 / day	€ 225 / day	

• Part-time nurses can cover at most 25% of total shifts.

$$X = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ x_8 & x_9 & x_{10} & x_{11} & x_{12} & x_{13} & x_{14} \end{bmatrix}$$

 $x_i$ : Number of each kind of nurse that start working on each day of the week (Monday to Sunday).

 $i = 1, \dots, 7$ : Full time nurses.

i = 8, ..., 14: Part time nurses.

Calculating cost per nurse:  $c_i$  per  $x_i$ 

	 Mon	Tus	Wen	Thu	Fri	Sat	Sun	Mon	Tus	Wen	Thu	 $\sum_{i} w_{i}$	$c_i = c_i$
$x_1$	250	250	250	250	250							1250	$c_1$
$x_2$		250	250	250	250	315						1315	$c_2$
$x_3$			250	250	250	315	375					1440	$c_3$
$x_4$				250	250	315	375	250				1440	$c_4$
$x_5$					250	315	375	250	250			1440	$c_5$
<i>x</i> <sub>6</sub>						315	375	250	250	250		1440	c <sub>6</sub>
$x_7$							375	250	250	250	250	1375	c <sub>7</sub>
<i>x</i> <sub>8</sub>	150	150	150									450	c <sub>8</sub>
$x_9$		150	150	150								450	C <sub>9</sub>
<i>x</i> <sub>10</sub>			150	150	150							450	c <sub>10</sub>
<i>x</i> <sub>11</sub>				150	150	185						485	$c_{11}$
x <sub>12</sub>					150	185	225					560	c <sub>12</sub>
x <sub>13</sub>						185	225	150				560	c <sub>13</sub>
x <sub>14</sub>							225	150	150			525	c <sub>14</sub>

$$C = \begin{bmatrix} 1250 & 1315 & 1440 & 1440 & 1440 & 1375 \\ 450 & 450 & 450 & 485 & 560 & 560 & 525 \end{bmatrix}$$

#### 1.1 Model

Objective function:

$$Min\left(\sum_{i}c_{i}x_{i}\right)$$

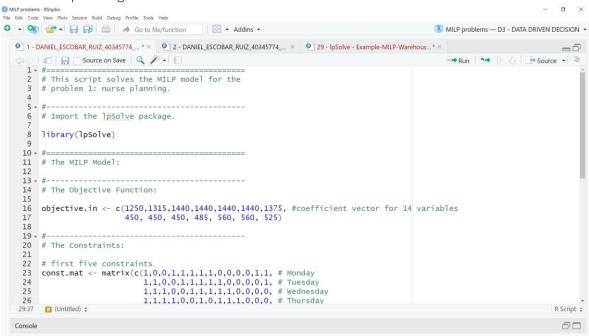
$$Min\left(\frac{1250x_{1} + 1315x_{2} + 1440x_{3} + 1440x_{4} + 1440x_{5} + 1440x_{6} + 1375x_{7}}{+450x_{8} + 450x_{9} + 450x_{10} + 485x_{11} + 560x_{12} + 560x_{13} + 525x_{14}}\right)$$

#### Subject to:

- $x_1 + x_4 + x_5 + x_6 + x_7 + x_8 + x_{13} + x_{14} \ge 17$ , for Monday
- $x_1 + x_2 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{14} \ge 13$ , for Tuesday
- $x_1 + x_2 + x_3 + x_6 + x_7 + x_8 + x_9 + x_{10} \ge 15$ , for Wednesday
- $x_1 + x_2 + x_3 + x_4 + x_7 + x_9 + x_{10} + x_{11} \ge 19$ , for Thursday
- $x_1 + x_2 + x_3 + x_4 + x_5 + x_{10} + x_{11} + x_{12} \ge 14$ , for Friday
- $x_2 + x_3 + x_4 + x_5 + x_6 + x_{11} + x_{12} + x_{13} \ge 16$ , for Saturday
- $x_3 + x_4 + x_5 + x_6 + x_7 + x_{12} + x_{13} + x_{14} \ge 11$ , for Sunday
- $\sum_{i=1}^{7} x_i 3 \sum_{i=1}^{14} x_i \ge 0$ , Part-time nurses at most 25% of total
- $x_i \in \mathbb{Z}^+, i = (1, ..., 14)$

The problem has 8 restrictions

## 1.2 R-lpSolving



I build the coefficient matrix for restrictions using a excel sheet by transforming the table of costs. (see Appendix 1).

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    19 - #-----
    20 # The Constraints:
        # first five constraints
    22
    23
        const.mat \leftarrow matrix(c(1,0,0,1,1,1,1,1,0,0,0,0,1,1, # Monday
    24
25
                                1,1,0,0,1,1,1,1,1,0,0,0,0,1, # Tuesday
                                1,1,1,0,0,1,1,1,1,1,0,0,0,0, # Wednesday 1,1,1,1,0,0,1,0,1,1,1,1,0,0,0, # Thursday
    26
27
28
                                1,1,1,1,1,0,0,0,0,1,1,1,0,0, # Friday 0,1,1,1,1,1,0,0,0,0,1,1,1,0, # Saturday
                                0,0,1,1,1,1,1,0,0,0,0,1,1,1, # Sunday
1,1,1,1,1,1,1,-3,-3,-3,-3,-3,-3,-3 # Par-time <= 0.25 Total
    29
    30
                                ), nrow=8,byrow=TRUE)
    32
    33 + #-
       # The Equality/inequality Signs:
    35
    36 const.dir <- c(rep(">=",8))
    38 + #-
       # The Right Hand Side Parameters (Constants):
    40
    41 const.rhs <- c(17, 13, 15, 19, 14, 16, 11, 0)
    43 - #-----
  Console
                                                                                                                                80

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■ MILP problems — D3 - DATA DRIVEN DECISION ▼
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    38 • #
                                                                                                       → Run | → ↑ 🕘 | → Source 🗸 🗏
    39 # The Right Hand Side Parameters (Constants):
    41 const.rhs <- c(17, 13, 15, 19, 14, 16, 11, 0)
    43 - #-----
    44
       # Mathematical Programming Setting:
    46 model <- lp(direction="min",
                     objective.in = objective.in,
    48
                     const.mat = const.mat,
const.dir = const.dir,
    49
    50
                     const.rhs = const.rhs,
    51
52
                    all.int = TRUE)
        model
    53
        model$solution
    54
    55
  Console Terminal × Background Jobs ×
  R 4.2.1 · C:/Users/DANIEL/OneDrive - Queen's University Belfast/D3 - DATA DRIVEN DECISION/MILP problems/
  Success: the objective function is 28015
   [1] 6 7 0 4 0 0 2 0 0 0 0 0 5 0
  >|
```

#### 1.3 Decision

The hospital gets the minimum cost of €28015 when they hire:

- $x_1 = 6$ , full-time starting Mondays
- $x_2 = 7$ , full time starting Tuesdays
- $x_4 = 4$ , full time starting Thursdays
- $x_7 = 2$ , full time starting Sundays
- $x_{13} = 5$ , part time starting Saturdays

## 2. Chipset Logistics

Fabrics = 
$$(F1, F2, F3)$$

Distribution centres = (D1, D2)

Plants = 
$$(P1, P2)$$

26 relations as the next figure shows.

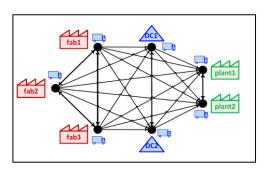


Table of costs: Per pack cost of shipment via the links (in €).

	F1	F2	F3	D1	D2	P1	P2
F1		100	60	100	100	400	400
F2	180		180	20	20	160	300
F3	8	160		20	10	200	240
D1					24	40	240
D2				16		40	240
P1							20
P2						140	

Since each link is a decision variable, the model has 26 decision variables. I keep double notation of letter and numbers for practical reasons.

Decision variables

	F1	F2	F3	D1	D2	P1	P2
F1		$x_{F1F2} = x_1$	$\begin{array}{c} x_{F1F3} \\ = x_2 \end{array}$	$\begin{array}{c} x_{F1D1} \\ = x_3 \end{array}$	$\begin{array}{c} x_{F1D2} \\ = x_4 \end{array}$	$\begin{array}{c} x_{F1P1} \\ = x_5 \end{array}$	$\begin{array}{c} x_{F1P2} \\ = x_6 \end{array}$
F2	$x_{F2F1} = x_7$		$\begin{array}{c} x_{F2F3} \\ = x_8 \end{array}$	$x_{F2D1} = x_9$	$x_{F2D2} = x_{10}$	$x_{F2P1} = x_{11}$	$x_{F2P2} = x_{12}$
F3	$x_{F3F1} = x_{13}$	$\begin{array}{c} x_{F3F2} \\ = x_{14} \end{array}$		$x_{F3D1} = x_{15}$	$\begin{array}{c} x_{F3D2} \\ = x_{16} \end{array}$	$x_{F3P1} = x_{17}$	$\begin{array}{c} x_{F3P2} \\ = x_{18} \end{array}$
D1					$x_{D1D2} = x_{19}$	$x_{D1P1} = x_{20}$	$x_{D1P2} = x_{21}$
D2				$x_{D2D1} = x_{22}$		$x_{D2P1} = x_{23}$	$x_{D2P2} = x_{24}$
P1							$\begin{array}{c} x_{P1P2} \\ = x_{25} \end{array}$
P2						$x_{P2P1} = x_{26}$	

From the previous tables, we get the following vectors:

$$C = [\ 100\,, \quad 60\,\,, \quad 100\,, \quad 100\,, \quad 400\,, \quad 400\,, \quad 180\,, \quad 180\,, \quad 20\,\,, \quad 20\,, \\ 300\,, \quad 8\,\,, \quad 160\,, \quad 20\,\,, \quad 10\,\,, \quad 200\,, 240\,, \quad 24\,\,, \quad 40\,\,, \\ 240\,, \quad 16\,\,, \quad 40\,\,, \quad 240\,, \quad 20\,\,, \quad 140\,\,]$$

$$X = [x_{F1F2}, \dots, x_{P2P1}] = [x_1, \dots, x_{26}]$$

#### 2.1 Model

Objective function:

$$Min\left(\sum_{i}c_{i}x_{i}\right)$$

$$\begin{array}{l} \mathit{Min}(100x_{F1F2} + 60x_{F1F3} + 100x_{F1D1} + 100x_{F1D2} + 400x_{F1P1} + 400x_{F1P2} + 180x_{F2F1} \\ &\quad + 180x_{F2F3} + 20x_{F2D1} + 20x_{F2D2} + 160x_{F2P1} + 300x_{F2P2} + 8x_{F3F1} \\ &\quad + 160x_{F3F2} + 20x_{F3D1} + 10x_{F3D2} + 200x_{F3P1} + 240x_{F3P2} + 24x_{D1D2} \\ &\quad + 40x_{D1P1} + 240x_{D1P2} + 16x_{D2D1} + 40x_{D2P1} + 240x_{D2P2} + 20x_{P1P2} \\ &\quad + 140x_{P2P1}) \end{array}$$

$$Min(100x_1 + 60x_2 + 100x_3 + 100x_4 + 400x_5 + 400x_6 + 180x_7 + 180x_8 + 20x_9 + 20x_{10} + 160x_{11} + 300x_{12} + 8x_{13} + 160x_{14} + 20x_{15} + 10x_{16} + 200x_{17} + 240x_{18} + 24x_{19} + 40x_{20} + 240x_{21} + 16x_{22} + 40x_{23} + 240x_{24} + 20x_{25} + 140x_{26})$$

Subject to:

- $x_{F1F2} + x_{F1F3} + x_{F1D1} + x_{F1D2} + x_{F1P1} + x_{F1P2} x_{F2F1} \le 400$ , Fabric 1 produce at most 400k
- $-x_{F1F2} + x_{F2F1} + x_{F2F3} + x_{F2D1} + x_{F2D2} + x_{F2P1} + x_{F2P2} x_{F3F2} \le 600$ , Fabric 2 produce at most 600K
- $-x_{F1F3} x_{F2F3} + x_{F3F1} + x_{F3F2} + x_{F3D1} + x_{F3D2} + x_{F3P1} + x_{F3P2} \le 200$ , Fabric 2 produce at most 200K
- $-x_{F1P1} x_{F2P1} x_{F3P1} x_{D1P1} x_{D2P1} + x_{P1P2} x_{P2P1} = 400$ , Plant 1 demand exactly 400K
- $-x_{F1P2} x_{F2P2} x_{F3P2} x_{D1P2} x_{D2P2} x_{P1P2} + x_{P2P1} = 180$ , Plant 1 demand exactly 180K
- $x_i \le 400$ , i = (1, ..., 26), each link can transport at most 400k
- $x_i \in \mathbb{Z}^+, i = (1, ..., 26)$

We have in total 31 restrictions, in addition positive integers limitation constrain.

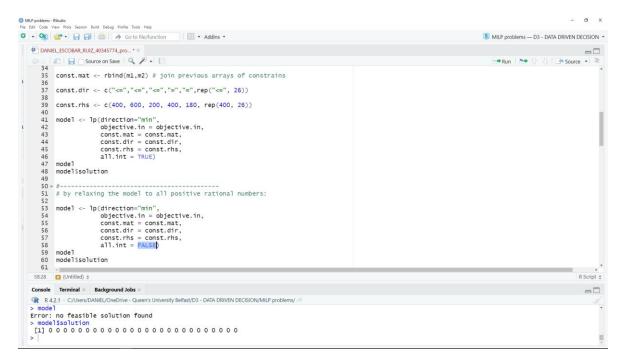
## 2.2 R-lpSolving

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### The Constraints:

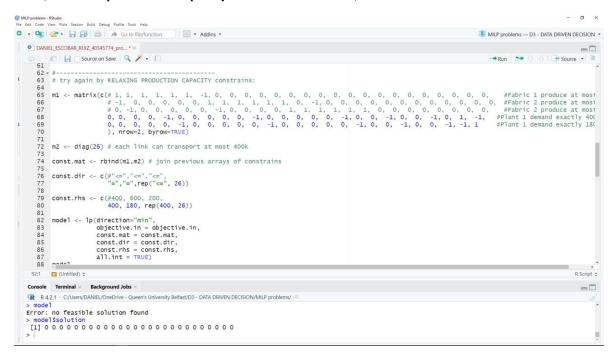
| **Ray | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | *** | ***
```

I created m1 and m2, then merged them to define the coefficient matrix for restrictions. Where m1 contains coefficients for the first five restrictions, and m2 contains the coefficients of restrictions regarding link capacities. Furthermore, m1 was created in excel table and simple copied and pasted on the R script (see Appendix 2).

Solution is unfeasible, even if I relax the model by accepting all positives rational numbers as follows.



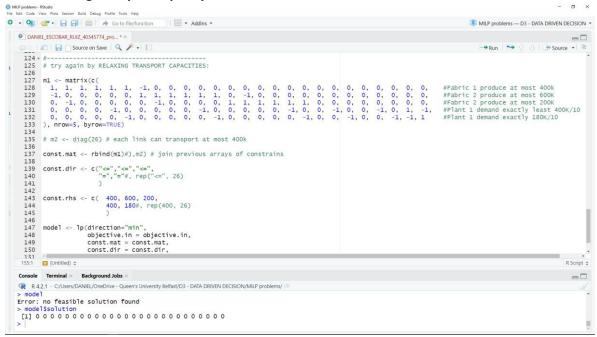
Then, I relaxed production capacity constraints. However, the solution still unfeasible.



Then I **relaxed demand constrains**. Despite plants decreasing plant demands by 10% of original, solution still unfeasible.

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                             # try again by RELAXING DEMAND CONSTRAINS. Plants now demand only 10% of original demands:
                           The state of the s
                                                                                                                                                                                                                                                                                                                                                                                                #Fabric 1 produce at most 400k
#Fabric 2 produce at most 600k
#Fabric 2 produce at most 200K
#Plant 1 demand exactly least 4
                             m2 <- diag(26) # each link can transport at most 400k
                            const.mat <- rbind(m1,m2) # join previous arrays of constrains
                           const.dir <- c("<=","<=","<=",
"=","=", rep("<=", 26))
               110 const.rhs <- c( 400, 600, 200, 111 400/10, 180/10, 122 rep(400, 26))
                           118
              119
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         Error: no feasible solution found
```

When relaxing transport capacity constrains solution is still unfeasible.



#### 2.3 Decision

The company need to change the cost structure. As it is currently, the logistic model is nonviable, and the solution is unfeasible even when there are no limitations regarding production or transport capacity.

#### 3. Ocean Internet Cables

Cables i = (A, B)

Plants j = (1,2)

Months t = (1,2,3) = (Jan, Feb, Mar)

The problem request to devise a production schedule for each cable at each plant for each month.

 $P_{ijt}$ : production (in km) of cable i from plant j in month t.

month	cable	Α	Α	В	В
month	plant	1	2	1	2
1		$P_{A11} = ?$	$P_{A21} = ?$	$P_{B11} = ?$	$P_{B21} = ?$
2		$P_{A12} = ?$	$P_{A22} = ?$	$P_{B12} = ?$	$P_{B22} = ?$
3		$P_{A12} = ?$	$P_{A22} = ?$	$P_{P12} = ?$	$P_{P22} = ?$

We are provided with a demand that must be fulfilled by each month for each cable.  $D_{it}$ : deman (in km) of cable i in month t.

month\cable	Α	В
1	$D_{A1} = 8000$	$D_{B1} = 2000$
2	$D_{A2} = 16000$	$D_{B2} = 10000$
3	$D_{A3} = 6000$	$D_{B3} = 10000$
	$\sum_{t} D_{At} = 30000$	$\sum_{t} D_{Bt} = 22000$

As the plants can produce more than demand required, at the end of the month we may have an excess of production for each cable for each plant.

 $E_{ijt}$ : excess of production for cable i in plant j for month t.

The excess production of the month enters inventory at the end of that month. Thus, production of February depends on excess of January, and so on for March. Furthermore, there are no holding inventory by the beginning and ending of the analysis.

$$E_{it} = \sum_{i} P_{ijt} - D_{it}$$

month\cable	Α	В
0	$E_{A0} = 0$	$E_{B0} = 0$
1	$E_{A1} = ?$	$E_{B1} = ?$
2	$E_{A2} = ?$	$E_{B2} = ?$
3	$E_{A3} = 0$	$E_{B3} = 0$

The cost of holding inventory is 20.20

Each plant has different production rates per cable.

 $R_{ij}$ : production rate per hour per km for cable i in plant t.

Plant\cable	Α	В
1	$R_{A1} = 0.3$	$R_{B1} = 0.24$
2	$R_{A2} = 0.32$	$R_{B2} = 0.28$

The cost per hour per plant for either cable or plant is  $\mathcal{Z}$  10. Then,  $10R_{ij}$  is the cost per hour required to produced one km of cable i in plant j for every month.

The production requires hours of work and raw material.

 $W_i$ : cost of raw material for cable i.

cable	Α	В
	$W_A = 6.2$	$W_B = 7.8$

Packing cost per km of either cable = 20.46. I assume that each km of cable produced is immediately packed whether is to deliver or keep it as holding inventory.

From the info above, the value of production per km depends on cost of hours required, cost of raw material, and packing cost.

 $V_{ij}$ : value (in  $\mathcal{Z}$ ) per km produced of cable i from plant j for every month. These values are constant in time. (Matrix calculations in appendix 3)

$$\frac{V_{ij}}{V_{ij}} = 10R_{ij} + W_i + 0.46$$

Plant\cable	A	В
1	$V_{A1} = 9.66$	$V_{B1}=10.66$
2	$V_{A2} = 9.86$	$V_{B2} = 11.06$

Production is limited by hours availability for plants each month.

 $A_{it}$ : availability (in hours) of plant j in month t.

Month\Plant	1	2
1	$A_{11} = 1400$	$A_{21} = 3000$
2	$A_{12} = 600$	$A_{22} = 800$
3	$A_{13} = 2000$	$A_{23} = 600$

Since hour resources are limited and the entire demand must be fulfilled, we have the following constrains:

• The sum of hours used to produce both cables must be at most the available hours for each plant in each month.

$$\sum_{i} R_{ij} P_{ijt} \le A_{jt}$$

• The sum of production and difference of excess in current and following period of both plants must be equal to the demand for each cable and each month.

$$\sum_{i} (P_{ijt} + E_{ij(t-1)} - E_{ijt}) = D_{it}$$

• There should be no inventory at the end of March.

$$E_{ii(3)} = 0$$

equivalent to

$$\sum_{i} \left( P_{ij3} + E_{ij2} \right) = D_{i3}$$

Thus, the total production is equal to the total demand for each cable.

$$\sum_{t} \sum_{j} P_{ijt} = \sum_{t} D_{it}$$

 $S_i$ : selling price for cable i.

cable	Α	В
	$S_4 = 14$	$S_{\rm P} = 18$

The profit is equal to the difference of the selling price and the value (cost) of all productions. The matrixial expression:

$$Profit = SP - VP$$

Equivalent to:

$$Profit = \sum_{i} \sum_{i} \sum_{t} \left( S_{i} P_{ijt} - V_{ij} P_{ijt} - 0.2 E_{i(t-1)} \right)$$

#### 3.1 Model

The problem requires to devise a production schedule for the maximum possible total profit. As the selling prices are fixed, the only way to maximize profit is by minimizing production costs.

#### The Generalized Model

#### Objective function:

$$\min\left(\sum_{i}\sum_{j}\sum_{t}\left(V_{ijt}P_{ijt}+0.2E_{i(t-1)}\right)\right)$$

#### Subject to:

•  $\sum_{i} R_{ii} P_{iit} \le A_{it}$  Available hours

•  $\sum_{j} P_{ijt} + E_{i(t-1)} - E_{it} = D_{it}$  Fulfil demands

•  $E_{i0} = E_{i3} = 0$  No excesses by the beginning and ending

•  $P_{i,it}$ ,  $E_{it} \in \mathbb{Z}^+$  Problem delimitation

## Specified model

#### Objective function:

$$\min \begin{pmatrix} 9.66P_{A11} + 9.86P_{A21} + 10.66P_{B11} + 11.06P_{B21} + \\ 9.66P_{A12} + 9.86P_{A22} + 10.66P_{B12} + 11.06P_{B22} + \\ 9.66P_{A13} + 9.86P_{A23} + 10.66P_{B13} + 11.06P_{B23} + \\ 0.2E_{A0} + 0.2E_{B0} + \\ 0.2E_{A1} + 0.2E_{B1} + \\ 0.2E_{A2} + 0.2E_{B2} + \\ 0.2E_{A3} + 0.2E_{B3} \end{pmatrix}$$

The model has 20 decision variables

#### Subject to:

- Available hours of production A<sub>jt</sub>
  - $0.3P_{A11} + 0.24P_{B11} \le 1400$  plant 1, month 1
  - $0.32P_{A21} + 0.28P_{B21} \le 3000$  plant 2, month 1
  - $0.3P_{A12} + 0.24P_{B12} \le 600$  plant 1, month 2
  - $0.32P_{A22} + 0.28P_{B22} \le 800$  plant 2, month 2
  - $0.3P_{A13} + 0.24P_{B13} \le 2000$  plant 1, month 3
  - $0.32P_{A23} + 0.28P_{B23} \le 600$  plant 2, month 3
- Demands *D\_it* 
  - $P_{A11} + P_{A21} + E_{A0} E_{A1} = 8000$  cable A, month 1
  - o  $P_{B11} + P_{B21} + E_{B0} E_{B1} = 2000$  cable B, month 1
  - o  $P_{A12} + P_{A22} + E_{A1} E_{A2} = 16000$  cable A, month 2
  - $\circ P_{B12} + P_{B22} + E_{B1} E_{B2} = 10000$  cable B, month 2
  - o  $P_{A13} + P_{A23} + E_{A2} E_{A3} = 6000$  cable A, month 3
  - o  $P_{B13} + P_{B23} + E_{B2} E_{B3} = 10000$  cable B, month 3
- No excesses by the beginning and ending
  - $\circ$   $E_{A0}=0$
  - o  $E_{B0} = 0$
  - $\circ$   $E_{A3}=0$

$$\circ$$
  $E_{B3}=0$ 

• Problem delimitation

o 
$$P_{iit}$$
,  $E_{it} \in \mathbb{Z}^+$ ,  $i = (A, B)$ ,  $j = (1,2)$ ,  $t = (1,2,3)$ 

The model has 16 constrains.

#### 3.2 R - lpSolve

```
MILP problems — D3 - DATA DRIVEN DECISION *
             DANIEL_ESCOBAR_RUIZ_40345774_pro... ×
                            6 # Import the lpSolve package.
                               8 library(lpSolve)
                         15 # 26 variables, one for each link

tobjective.in <- c( 9,66,  9.86, 10.66,  11.06,  9.66,  9.86, 10.66,  11.06,  19  9.66,  9.86, 10.66,  11.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  10.06,  1
                                       # The Constraints:
                                           # the coefficient matrix for constrains
                                      0, 0, # available hours plant 1, month 1
0, 0, # available hours plant 2, month 1
0, 0, # available hours plant 1, month 2
0, 0, # available hours plant 1, month 2
0, 0, # available hours plant 1, month 3
0, 0, # available hours plant 2, month 3
0. # Demand to be fulfill cable A. month 1
             Console

MIR Pyroblems - Ribudio
File Edit Code View Plots Session Build Debug Profile Tools Help

V V Code View Plots Session Build Debug Profile Tools Help

A Go to file/function

A Addins •
               # The Constraints:

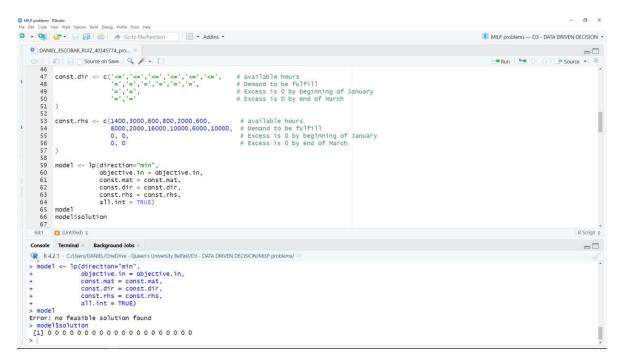
# The Constraints:

# the coefficiate const.m.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      III MILP problems — D3 - DATA DRIVEN DECISION •
              O DANIEL_ESCOBAR_RUIZ_40345774_pro... ×
                                      0, 0, 0, 0, 0, 0, 0, # available hours plant 1, month 1
0, 0, 0, 0, 0, 0, 0, # available hours plant 2, month 1
0, 0, 0, 0, 0, 0, 0, # available hours plant 1, month 2
0, 0, 0, 0, 0, 0, 0, # available hours plant 1, month 2
0, 0, 0, 0, 0, 0, 0, # available hours plant 1, month 3
0, 0, 0, 0, 0, 0, 0, # available hours plant 1, month 3
-1, 0, 0, 0, 0, # available hours plant 1, month 1
-1, 0, 0, 0, 0, # Bemand to be fulfill cable A, month 1
1, 0, -1, 0, 0, 0, # Demand to be fulfill cable B, month 1
1, 0, -1, 0, 0, 0, # Demand to be fulfill cable B, month 2
0, 1, 0, -1, 0, # Demand to be fulfill cable B, month 2
0, 1, 0, -1, 0, # Demand to be fulfill cable B, month 2
0, 0, 1, 0, -1, # Demand to be fulfill cable B, month 3
0, 0, 0, 1, 0, -1, # Demand to be fulfill cable B, month 3
0, 0, 0, 1, 0, -1, # Demand to be fulfill cable B, month 3
0, 0, 0, 0, 1, 0, -1, # Excess is 0 by beginning of January for cable A
0, 0, 0, 0, 0, 0, # Excess is 0 by beginning of January for cable B, 0, 0, 0, 0, 0, 1, 0, # Excess is 0 by beginning of January for cable A
0, 0, 0, 0, 0, 1, 0, # Excess is 0 by end of March for cable A
0, 0, 0, 0, 0, 1, 0, # Excess is 0 by end of March for cable A
                          29
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                                                                                                              (1400,3000,600,800,2000,600,  # available hours

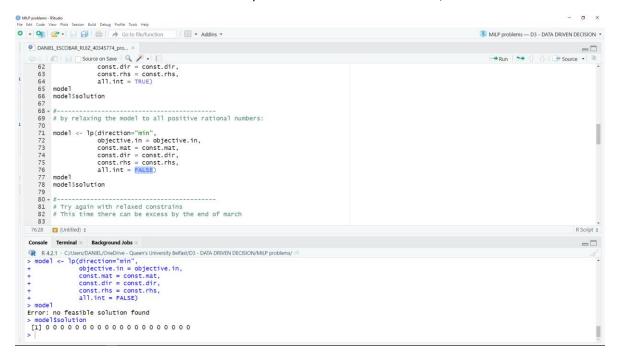
8000,2000,16000,10000,6000,10000,  # Demand to be fulfill

0, 0, 0,  # Excess is 0 by beginning of January

0, 0  # Excess is 0 by end of March
                          53 const.rhs <- c(1400.3000.600.800.2000.600.
                                     (Untitled) :
```

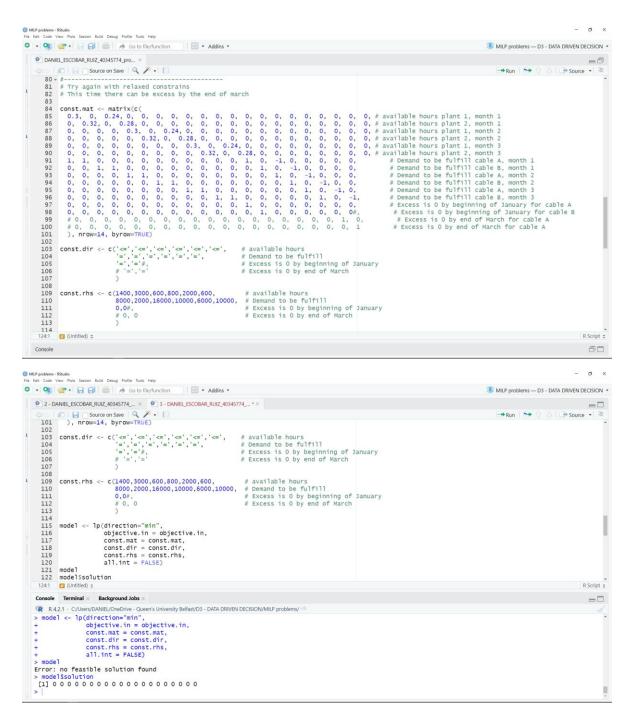


No feasible solution. Then tried with all positive real numbers. However, solution is still unfeasible.



Then I relaxed constrains regarding excess of production at the end of march. So now

$$E_{i3} \ge 0$$



Solution is still unfeasible. Then Try again by relaxing constrain regarding available hours for production. This time, we get a solution.

```
O - O Go to file/function
                                                                                                         Addins •
                                                                                                                                                                                                                                                                                                                     ■ MILP problems — D3 - DATA DRIVEN DECISION •
       DANIEL_ESCOBAR_RUIZ_40345774_pro... ×
          2 Source on Save Q ✓ • □
          124 * #------
125 # Try again with relaxing hour availability constrains
           127 const.mat <- matrix(c(
                        # available hours
# Demand to be fulfill
# Excess is 0 by beginning of January
# Excess is 0 by end of March
           149
150
                      )
         166:1 (Untitled) $
      Console
O - O Go to file/function
                                                                                                       - Addins -

■ MILP problems — D3 - DATA DRIVEN DECISION •
      2 - DANIEL_ESCOBAR_RUIZ_40345774_... * 9 3 - DANIEL_ESCOBAR_RUIZ_40345774_... * *
          161 cor
162 cor
163 al'
164 model
165 model$solution
          167
         166:1 (Untitled) :
       Console Terminal × Background Jobs ×
        R 4.2.1 · C:/Users/DANIEL/OneDrive - Queen's University Belfast/D3 - DATA DRIVEN DECISION/MILP problems/
           model <- local property of the control of the contr
       > model
Success: the objective function is 524320
```

#### 3.3 Decision

The company must increase the hours available for production for each cable if they pretend to fulfil the demand each month.

When the available hours are not restrictions. All the production is assigned to Plant 1, which make sense since the rate of required hours is lower for both cables.

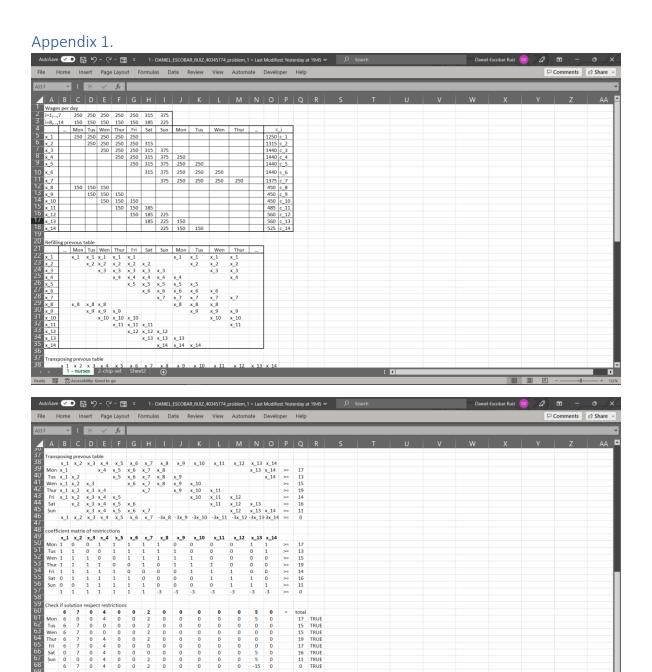
The production plan goes as follows:

- $\circ \quad P_{A11} = 8000$
- o  $P_{B11} = 2000$
- o  $P_{A12} = 16000$
- $\circ \quad P_{B12}=10000$
- $\circ \quad P_{A13}=6000$
- o  $P_{B13} = 10000$

## This production plan ensures:

- o the demands each month,
- o no excess of production and the end of march,
- o Minimum cost of production ₹524,320, and
- o Maximum profit  $\sum_i \sum_j \sum_t (S_i P_{ijt} V_{ij} P_{ijt} 0.2 E_{i(t-1)}) = 2379,680$

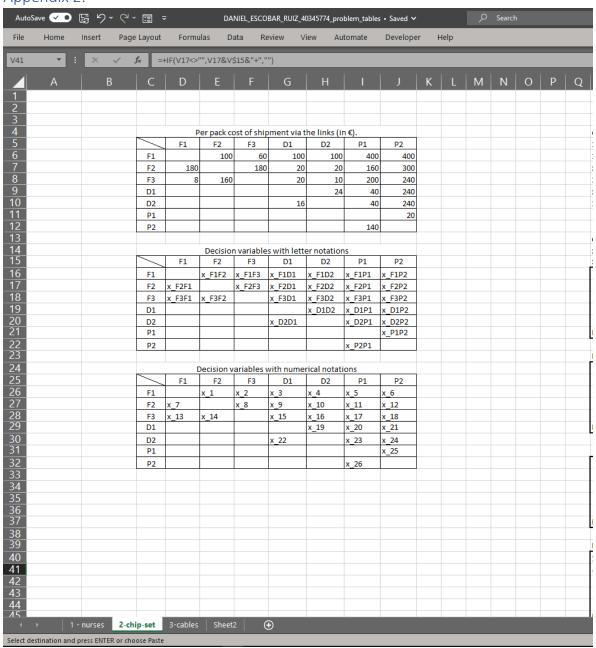
The company must increase production capacity (available hours), mainly in plant 1 where productivity if higher.



: 4

1 - nurses 2-chip-set Sheet2 🕦

## Appendix 2.



						AN		AM																w	V				
							П																					e.in	bjeti
			140	20	240	40		16	240	40	24	240	200	10	20	160	8	300	160	20	20	180	180	400	400	100	100	60	00
			140,				40	16,	240,	40,	24,	240,	200,	10,			8,	300,	160,	20,	20,	180,	180,	400,	400,	100,	100,	60,	00,
			x_26					x_22	x_21							x_14 160x 14	x_13			x_10	x_9	x_8	x_7	x_6	x_5	x_4	x_3	x_2	1
																x_F3F2													
		1+														160x_F31													
	const.rhs	const.dir																										nat	onst.
	Amount given		x 26	x 25	24	23	x	x 22	x 21	x 20	x 19	x 18	x 17	x 16	x 15	x 14	× 13	x 12	× 11	× 10	x 9	x 8	x 7	x 6	x 5	x 4	x 3	x 2	1
																x_F3F2										x_F1D2			
1 produce at most 400k	400	<11					Т																-1	1	1	1	1	1	1
2 produce at most 600k		<=														-1		1	1	1	1	1	1						-1
3 produce at most 200k		<=										1	1	1	1	1	1					-1						-1	
1 demand exactly 400k		=	-1	1		-1				-1			-1						-1						-1				
2 demand exactly 180k each link can sent at mos		= <=	1	-1	-1				-1			-1						-1						-1			0.76	natrix siz	ndov
activities carried at the	400						-	_								_										_	- 20	III III III	IGCA
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	400	<=	0	0	0	0		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	1	1	1	1	1
	600	<=	0	0	0	0		0	0	0	0	0	0	0	0	-1	0	1	1	1	1	1	1	0	0	0	0	0	-1
	200	<=	0	0	0	0		0	0	0	0	1	1	1	1	1	1	0	0	0	0	-1	0	0	0	0	0	-1	0
	400	=	-1	1	0	-1		0	0	-1	0	0	-1	0	0	0	0	0	-1	0	0	0	0	0	-1	0	0	0	0
	180		1	-1	-1	0		0	-1	0	0	-1	0	0	0	0	0	-1	0	0	0	0	0	-1	0	0	0	0	0
	400	<=					$\pm$																				e 26	natrix siz	ndex
	400	<=	0,	0,	0,	0,		0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	-1,	1,	1,	1,	1,	1,	1,
	600	<=	0,	0,	0,	0,		0,	0,	0,	0,	0,	0,	0,	0,	-1,	0,	1,	1,	1,	1,	1,	1,	0,	0,	0,	0,	0,	-1,
	200	<=	0,	0,	0,	0,		0,	0,	0,	0,	1,	1,	1,	1,	1,	1,	0,	0,	0,	0,	-1,	0,	0,	0,	0,	0,	-1,	0,
	400 180	- :	-1,	1,	0, -1.	-1, 0.		0,	0, -1,	-1, 0,	0,	0,	-1, 0.	0,	0,	0,	0,	0,	-1, 0.	0,	0,	0,	0,	0,	-1, 0,	0,	0,	0,	0,
	400	<u></u>	1,	-1,	-1,	0,	-	0,	-1,	J,	J,	-1,	u,	0,	J,	J,	0,	-1,	0,	0,	0,	0,	0,	-1,	0,	0,	e 26	natrix siz	
							$\mp$																					ument	
	400	<=					$\neg$															1	2 -1x_F2F	1-1x F1P	2 1x F1P	1 1x F1D	3-1x F1D		
	600	<=													į.	-1x_F3F2		1x_F2P2	1x F2P1	1 1x F2D	1x F2D			7	1				1x F1
	200	<=										1x_F3P2	1x F3P1	1x_F3D2		1x F3F2						-1x_F2F					FB	-1x_F1	
	400		-1x_P2P	1x_P1P2-		_D2P1	-1			-1x_D1P			-1x_F3P						-1x_F2P					p <sub>1</sub>	-1x_F18				
	180 400	-	1x_P2P1	-1x_P1P2	1x_D2P2-			ig .	-1x_D1P			-1x_F3P2					2	-1x_F2P2					P2	-1x_F16					
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# Appendix 3.

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Α	В	С	D	Е	F	G	н	
			P_ijt					
month	cable	A	Α	В	В			
month	plant	1	2	1	2			
	1	P_A11=?	P_A21=?	P_B11=?	P_B21=?			
	2	P_A12=?	P_A22=?	P_B12=?	P_B22=?			
	3	P_A13=?	P_A23=?	P_B13=?	P_B23=?			
		D_it				D_it		
	month\cable		В		month\cable		В	
						D_A1=8000		
	1	8,000	2,000				D_B1=2000	
	2	16,000	10,000			D_A2=16000		
	3	6,000	10,000		3	D_A3=6000	D_B3=10000	
	Total	30,000	22,000			SUM(D_At)=3	(SUM(D_Bt)=22	2000
		R_ji				A_ij		
	Plant\cable	Α	В		Plant\cable	Α	В	
	1	0.30	0.24		1	R_A1=0.3	R_B1=0.24	
	2	0.32	0.28		2	R_A2=0.32	R_B2=0.28	
		W_i				W_i		
	plant	Α	В		cable	Α	В	
		6.2	7.8			W_A=6.2	W_B=7.8	
						_	_	
	V_ij =	= 10R_ji + W_i +	+ <b>0.4</b> 6		V_ij =	- = 10R_ji + W_i -	+ 0.46	
	Plant\cable	A	В		Plant\cable	A	В	
	1	9.66	10.66			V_A1=9.66	V_B1=10.66	
	2	9.86				V_A2=9.86	V_B2=11.06	
<b>→</b>	1 - nurses	2-chip-set 3-c	ables Sheet2					

