1. Show that a monopolist would never choose a price-quantity combination on the inelastic part of the market demand.

2.5 points

Recall that for finding the profit-maximizing quantity the monopolist would have to set $MR = MC$, and that we can write $MR = p \cdot (1 - \frac{1}{|\epsilon|})$.

Note that on the inelastic part of the market demand marginal revenue is negative ($MR < 0$), because there $|\epsilon| < 1$ by definition. Given that MC is typically non-negative, the above equality (the first-oder condition for profit maximization) would not hold. In other words, on the inelastic part of the market demand the monopolist's marginal profit is negative. It means that by producing less, the monopolist's revenue would increase. Note that producing less would also decrease costs and ultimately would increase profits.

In conclusion, it can not be the case that the monopolist is maximizing its profits at a price-quantity combination on the inelastic part of the market demand.

2. Can the leader ever get a lower profit in a Stackelberg equilibrium than it would get in the Cournot equilibrium? Explain!

2.5 points

Note that the follower's reaction function is the same in the two models, Stackelberg and Cournot. For that reason, if the Stackelberg leader chooses the Cournot-equilibrium quantity, it will earn the exact same profits as in the Cournot equilibrium.

If the Stackelberg leader chooses a quantity different from the Cournot-equilibrium quantity, it is because it can earn higher profits with it. In conclusion, the Stackelberg leader never gets a lower profit than it would get in the Cournot equilibrium.

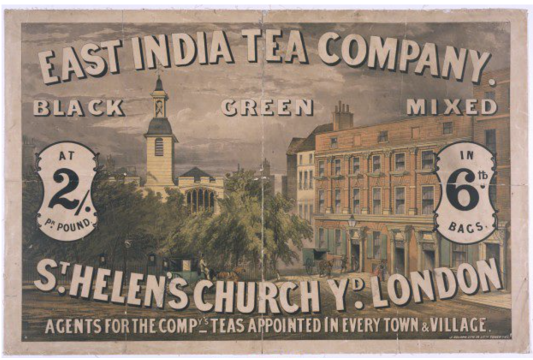
3. In 1784 the British import tariff on tea was 119% and 5 million pounds of it were imported. The accountant of the East India Company reckoned that no more than a third of the whole British consumption was imported legally, the rest being smuggled. He was apparently correct, for tea was inelastically demanded, and in 1785 after the reduction of the tariff to a mere 12.5%, the amount of legal tea imported increased to 16 million pounds.

   (a) In light of the implied elasticity of demand for *legal* tea, and using a straight-line "demand curve" (after smuggling), what was the revenue-maximizing tariff rate?

   2.5 points

As reading this story and this question in a microeconomics problem set might have surprised you, let me guide you through the problem with the following questions.

- First of all, note that setting an import tariff by a state can be considered as setting a price by a monopolist. An import tariff of 119% is like a unit price collected for each unit (imported and) sold. For simplicity, let us assume that the direct costs associated with import tariffs are negligible. For that reason, the goal of this monopolist is to maximize revenue.
- Based on the information provided by the exercise above, write an equation that describes the demand for tea. You can assume that demand is linear, so all you need to do is to "connect" two points mentioned above.
- Use the demand curve to find the revenue-maximizing price.



- The two points on the demand curve are 119% and 5 million pounds, and 12.5% and 16 million pounds. The general form of a linear demand curve is $Q = a - b \cdot P$. Therefore, we can write that

$$\begin{aligned} 5 &= a - b \cdot 119, \text{ and} \\ 16 &= a - b \cdot 12.5. \end{aligned}$$

  Solving this system of equations gives $a = 17.29$ and $b = 0.10$. The demand curve for tea is given by $Q = 17.29 - 0.10 \cdot P$.

- The marginal revenue can be derived from the inverse demand, $P = 172.9 - 10 \cdot Q$.

$$MR = 172.9 - 20 \cdot Q$$

The revenue-maximizing quantity is where $MR = 0$, that is $Q^* = \frac{172.9}{20} = 8.645$. The corresponding price is $P^* = 172.9 - 10 \cdot 8.645 = 86.45$. The revenue-maximizing tariff therefore is around 86.5%.

$\boxed{4.}$ Consider an oligopolistic market with $N \geq 2$ identical firms that compete *à la Cournot*. The cost function of each firm can be written as $c(q_i) = 20q_i - 2.5q_i^2$, where q_i denotes the firm's output level.

The inverse demand function of the market is $p = 100 - 4Q$, where p is the unit price of the output and Q is the aggregate output level produced by the N firms together.

(a) Find the typical firm's reaction function.
(Hint: It will be a function of the other firms' production level.)

2.5 points

Consider the profit-maximization problem for firm i.

$$\max_{q_i} \quad p \cdot q_i - c(q_i)$$

$$\max_{q_i} \quad [100 - 4 \cdot (q_i + \textstyle\sum_{j \neq i} q_j)] \cdot q_i - 20q_i + 2.5q_i^2$$

$$FOC: \quad 100 - 8q_i - 4\textstyle\sum_{j \neq i} q_j - 20 + 5q_i = 0$$

$$q_i = \tfrac{1}{3}(80 - 4\textstyle\sum_{j \neq i} q_j) = \tfrac{80}{3} - \tfrac{4}{3}\textstyle\sum_{j \neq i} q_j$$

$$SOC: \quad -8 + 5 = -3 < 0$$

(b) Find the equilibrium level of aggregate output, Q, as a function of the number of firms, N.

2.5 points

$$\begin{aligned}
\sum_{i=1}^{N} q_i &= \frac{80}{3}N - \frac{4}{3}\sum_{i=1}^{N}\sum_{j\neq i} q_j = \\
&= \frac{80}{3}N - \frac{4}{3}(N-1)\sum_{i=1}^{N} q_i \\
4(N-1)\sum_{i=1}^{N} q_i + 3\sum_{i=1}^{N} q_i &= 80N \\
(4N-1)\sum_{i=1}^{N} q_i &= 80N \\
Q = \sum_{i=1}^{N} q_i &= \frac{80N}{4N-1} = \frac{80}{4-\frac{1}{N}}
\end{aligned}$$

5. Read the following quotes.

> "A number of misunderstandings have long accompanied the static Nash equilibrium concept. Many commentators on the Cournot duopoly solution – the first and most famous example of a Nash equilibrium – criticized the supposedly myopic and rather irrational behaviour of firms. However, Cournot-Nash behaviour only appears irrational if one says that each firm chooses its best output given its rival's output and if, at the same time, one adds to the static model a kind of dynamic adjustment process (as is too often done in intermediate microeconomic textbooks). In fact it is quite consistent with rational behaviour."
>
> Montet, C. (2014) "Game theory and strategic behaviour," in: (Eds.) Bleaney, M., Greenaway, D., Stewart, I. Dr., Stewart, I., *Companion to Contemporary Economic Thought,* Routledge, 1st edition (p.348)

> "Were the number of firms small, [...] any manager would be crazy to make a Cournot assumption. This fact rather spoils the prettiness of Cournotesque arguments. The manager is supposed to treat the actions of others as given. But the actions of others change."
>
> McCloskey, D.M. (1985) *The applied theory of price*, Macmillan Publishing Company (p.428)

> "The Cournot solution, then is irrational. A Cournot duopolist can always do better by not acting like one, and by taking advantage of the simplemindedness of his competitor who goes on acting like one. But his competitor, too, is no fool. He too will recognize that his competitor does *not* keep selling the same quantity regardless of the quantity sold by his competitor."
>
> McCloskey, D.M. (1985) *The applied theory of price*, Macmillan Publishing Company (p.433)

Now construct a simple numerical (!) example based on a linear (!) demand function and two (!) firms with identical, constant (!) marginal costs.

(a) Use your model to illustrate that a Cournot duopolist can do "better" by producing the monopoly output, assuming that its competitor reacts and adjusts its output optimally.

5 points

Let's suppose that market (inverse) demand is given by $P = a - b \cdot Q$ and the firms' marginal cost is c, where a, b, and c are positive constants. I present the general solution with these parameters, so everyone can check his/her answers by looking at this material.

Firm 1's reaction function is $q_1(q_2) = \frac{a-c-bq_2}{2b}$, that is the solution to the following profit-maximization problem: $\max_{q_1} \Pi_1(q_1, q_2) = [a - b \cdot (q_1 + q_2)] \cdot q_1 - c \cdot q_1$. Similarly, firm 2's reaction function is $q_2(q_1) = \frac{a-c-bq_1}{2b}$, that is the solution to the following profit-maximization problem: $\max_{q_2} \Pi_2(q_2, q_1) = [a - b \cdot (q_2 + q_1)] \cdot q_2 - c \cdot q_2$.
The Cournot equilibrium then is $q_1^* = q_2^* = \frac{a-c}{3b}$, with a market price of $P^* = a - b \cdot 2 \cdot \frac{a-c}{3b} = \frac{a+2c}{3}$ and each firm making a profit of $\Pi_1^* = \Pi_2^* = (\frac{a+2c}{3} - c) \cdot \frac{a-c}{3b} = \frac{(a-c)^2}{9b}$.

Under the same circumstances, a monopoly would produce where $MR = a - 2b \cdot Q = MC = c$, that is $Q_M^* = \frac{a-c}{2b}$.

Now if firm 1 chose $q_1^{**} = Q_M^* = \frac{a-c}{2b}$ and firm 2 adjusted to that, firm 2 would produce $q_2^{**} = \frac{1}{2b} \cdot (a - c - b \cdot \frac{a-c}{2b}) = \frac{a-c}{4b}$. With that, $P^{**} = a - b \cdot (\frac{a-c}{2b} + \frac{a-c}{4b}) = \frac{a+3c}{4}$ and $\Pi_1^{**} = (\frac{a+3c}{4} - c) \cdot \frac{a-c}{2b} = \frac{(a-c)^2}{8b}$. Note that $\Pi_1^{**} = \frac{(a-c)^2}{8b} >= \frac{(a-c)^2}{9b} = \Pi_1^*$.

(b) How does this outcome compare to the one predicted by the Stackelberg model?

2.5 points

Let's consider now the problem that firm 1 is facing as the Stackelberg leader when firm 2 reacts according to the following rule: $q_2(q_1) = \frac{a-c-bq_1}{2b}$.

$$
\begin{aligned}
\max_{q_1} \quad & [a - b \cdot (q_1 + q_2(q_1))] \cdot q_1 - c \cdot q_1 = \\
= & \left[a - b \cdot \left(q_1 + \frac{a-c-bq_1}{2b}\right)\right] \cdot q_1 - c \cdot q_1 = \\
= & \frac{1}{2}(a - c - bq_1) \cdot q_1
\end{aligned}
$$

We get that $q_1^{***} = \frac{a-c}{2b}$. Note that this quantity is equal to what the monopoly would have chosen, $q_1^{***} = Q_M^*$.