Net and gross demands

Attila only cares about two goods and his preferences can be represented by the utility function $u(x, y) = \min\{4x, 2y\}$, where x is the amount of *exes* that he consumes, and y is the amount of *whys* that he consumes.

Let p_x denote the unit price of *exes* and let us assume that the unit price of *whys* is 2. Attila currently owns 4 units of *exes* and 10 units of *whys*.

$\boxed{1.}$ Write a mathematical equation representing Attila's budget constraint.

$$\begin{aligned} p_x \cdot x + p_y \cdot y &= p_x \cdot w_x + p_y \cdot w_y \\ p_x \cdot x + 2 \cdot y &= p_x \cdot 4 + 2 \cdot 10 \end{aligned}$$

$\boxed{2.}$ Solve Attila's constrained utility-maximization problem and write his gross demand function for *exes*, that is $x(p_x)$.

Given Attila's utility function, we must have $4x = 2y$ (that is, $y = 2x$) in the optimal bundle. By substituting this into the budget constraint, we have that $p_x \cdot x + 2 \cdot 2x = p_x \cdot 4 + 2 \cdot 10$, that is $(p_x + 4) \cdot x = 4 \cdot p_x + 20$. Therefore $x(p_x) = \frac{4p_x + 20}{p_x + 4} = 4 + \frac{4}{p_x + 4}$.

$\boxed{3.}$ Write Attila's net demand function for *exes*, that is $d_x(p_x)$.

$d_x(p_x) = x(p_x) - w_x = 4 + \frac{4}{p_x + 4} - 4 = \frac{4}{p_x + 4}$

$\boxed{4.}$ Assume that the initial endowment and the price of *whys* do not change. Will Attila ever consume more than his initial 4 units of *exes*? Justify your answer.

Given that $p_x \geq 0$, we have that $d_x(p_x) = \frac{4}{p_x + 4} > 0$. This means that Attila is always going to be a net demander of x and will always consume more than his initial 4 units of it.