NAME OF THE STUDENT IN FRONT OF YOU ⇑

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YOUR NAME

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YOUR STUDENT ID NUMBER

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⇐ NAME OF THE STUDENT ON YOUR LEFT

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NAME OF THE STUDENT ON YOUR RIGHT ⇒

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! You have 100 minutes to solve the following 10 exercises.

Answering the exam questions is a strictly individual task. You are not allowed to use electronic devices (including phones, calculators, computers) and to consult books or notes.

All exercises require some justification. You will not get full score unless you show your working and briefly justify each of your answers. Read all the questions very carefully.

Please write your answers in the space provided right below the exam questions.

! Mark the correct answer to each of the following five exercises, and **briefly justify your solution.**

1. Georgina consumes only grapefruits and pineapples. Her utility function is $U(x, y) = x^2 \cdot y^8$, where x is the number of grapefruits consumed and y is the number of pineapples consumed. Georgina's income is $\$105$, and the prices of grapefruits and pineapples are $\$1$ and $\$3$, respectively. How many grapefruits will she consume?

   (a) 10.5
   (b) 7
   (c) 63
   (d) 21
   (e) None of the above.

1 point

Given that Georgina has Cobb-Douglas type preferences, she is going to split her income between the two goods according to the exponents in her utility function. She is going to spend $\frac{2}{2+8} = 0.2$ part of her income on grapefruits.

That is, $p_x \cdot x = 0.2 \cdot 105$. Therefore, she will consume $x = \frac{0.2 \cdot 105}{1} = 21$ units of grapefruit.

2. Cindy consumes goods x and y. Her demand for x is given by $x(p_x, m) = 0.01m - \frac{1}{2}p_x$. Now her income is $\$300$, the price of x is $\$2$, and the price of y is $\$1$. If the price of x rises to $\$4$ and if we denote the income effect on her demand for x by Δx^N and the substitution effect on her demand for x by Δx^S, then

   (a) $\Delta x^N = 0$ and $\Delta x^S = -1.00$.
   (b) $\Delta x^N = -1.00$ and $\Delta x^S = 0$.
   (c) $\Delta x^N = -0.96$ and $\Delta x^S = -0.04$.
   (d) $\Delta x^N = -0.04$ and $\Delta x^S = -0.96$.
   (e) None of the above.

1 point

Original consumption: $x_A(p_x = 2, m = 300) = 0.01 \cdot 300 - \frac{1}{2} \cdot 2 = 2$. New consumption: $x_C(p'_x = 4, m = 300) = 0.01 \cdot 300 - \frac{1}{2} \cdot 4 = 1$.

Compensated income: $m' = m + \Delta m = m + \Delta p_x \cdot x_A = 300 + (4 - 2) \cdot 2 = 304$.

Intermediate bundle: $x_B(p'_x = 4, m' = 304) = 0.01 \cdot 304 - \frac{1}{2} \cdot 4 = 1.04$.

In summary, $\Delta x^T = x_C - x_A = 1 - 2 = -1$, with $\Delta x^S = x_B - x_A = 1.04 - 2 = -0.96$, and $\Delta x^N = x_C - x_B = 1 - 1.04 = -0.04$.

3. The production function for good y is $y = \min\{2x_1, x_2\}$, where x_1 and x_2 are the amounts of factors 1 and 2. Let w_1 and w_2 denote the unit price of factor 1 and factor 2 respectively. Find the cost function for good y.

(a) $c(y) = \min\{2w_1, w_2\} \cdot y$
(b) $c(y) = \min\{w_1, 2w_2\} \cdot y$
(c) $c(y) = (\frac{w_1}{2} + w_2) \cdot y$
(d) $c(y) = (2w_1 + w_2) \cdot y$
(e) None of the above.

1 point

According to this productions function, the firm uses the two factors in fixed proportions: half a unit of factor 1 with one unit of factor 2 for each unit of output.

Therefore, the least costly way to produce y units of output is by combining $\frac{y}{2}$ units of factor 1 with y units of factor 2 (these are the conditional factor demand functions). The cost of doing that is $\frac{w_1}{2} \cdot y + w_2 \cdot y$.

4. A competitive firm has a long-run total cost function $c(y) = 3y^2 + 675$ for $y > 0$ and $c(0) = 0$. Its long-run supply function is described as

(a) $y = \frac{p}{6}$ if $p > 90$, $y = 0$ if $p < 90$.
(b) $y = \frac{p}{3}$ if $p > 88$, $y = 0$ if $p < 88$.
(c) $y = \frac{p}{3}$ if $p > 93$, $y = 0$ if $p < 99$.
(d) $y = \frac{p}{6}$ if $p > 93$, $y = 0$ if $p < 93$.
(e) $y = \frac{p}{3}$ if $p > 95$, $y = 0$ if $p < 85$.

1 point

According to the first-order condition of the firm's profit-maximization problem, $p = MC(y)$, that is $p = 6y$ or $y = \frac{p}{6}$. Given that the marginal cost function is strictly increasing, the second-order condition is satisfied.

In the long run, profit should not be negative (otherwise, the firm would exit). In other words, the price should be at least as large as the average cost. Mathematically: $p \geq AC(y) = 3y + \frac{675}{y}$.

Remember that the marginal cost and average cost functions cross at the minimum point of the average cost function. $MC(y) = AC(y) \to 6y = 3y + \frac{675}{y} \to y^2 = 225 \to 15$. At that point the firm's marginal (and average) cost is 90.

5. Consider a competitive industry with several firms all of which have the same cost function, $c(y) = y^2 + 4$ for $y > 0$ and $c(0) = 0$. The demand curve for this industry is $D(p) = 50 - p$, where p is the price. The long-run equilibrium number of firms in this industry is

(a) 4.

(b) 23.

(c) 25.

(d) 46.

(e) 2.

1 point

$$\min_y AC(y) = \frac{y^2+4}{y} = y + \frac{4}{y} \rightarrow 1 - \frac{4}{y^2} = 0 \rightarrow y = 2$$
$$p* = AC(2) = 4$$

short-run supply for a single firm:

$$MC(y) = 2y$$
$$p = MC(y) = 2y \rightarrow y = \frac{p}{2}$$

short-run supply for n firms

$$S(p) = \frac{np}{2}$$

short-run equilibrium with n firms:

$$D(p) = S(p)$$
$$50 - p = \frac{np}{2} \rightarrow p = \frac{100}{n+2}$$

long-run equilibrium:

$$p = \frac{100}{n+2} > p* = 4 \rightarrow n \leq 23 \rightarrow n* = 23$$

! Solve the following five exercises. **Show your working in all of them.**

6. When the prices were $(\$5, \$1)$, Vanessa chose the bundle $(x, y) = (6, 3)$. Now at the new prices, (p_x, p_y), she chooses the bundle $(x, y) = (5, 7)$. Write a restriction for the new prices, (p_x, p_y), that must be true for Vanessa's behavior to be consistent with the weak axiom of revealed preference.

1 point

When the prices were $(\$5, \$1)$, the $(6, 3)$ bundle that Vanessa chose cost $5 \cdot 6 + 1 \cdot 3 = 33$ dollars. The new bundle $(5, 7)$ would have cost her $5 \cdot 7 + 1 \cdot 7 = 32$ dollars in that situation. This means that both bundles were affordable, therefore bundle $(6, 3)$ was revealed to be preferred to bundle $(5, 7)$ by her choice.

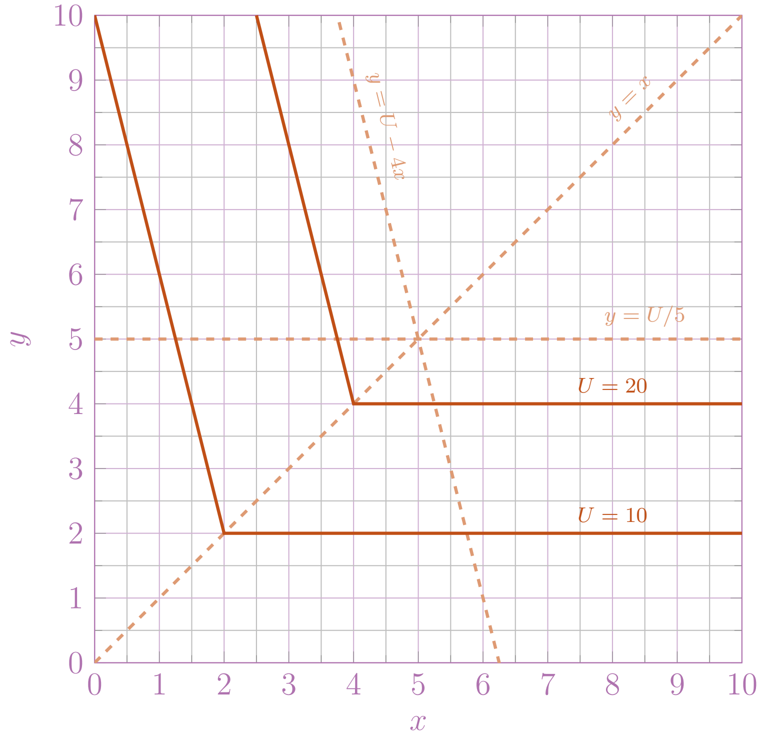
For her to be consistent with the weak axiom of revealed preference, it must be the case that bundle $(6, 3)$ is not affordable in the new situation (otherwise she should have chosen it again over bundle $(5, 7)$.

Mathematically, $p_x \cdot 6 + p_y \cdot 3 > p_x \cdot 5 + p_y \cdot 7 \rightarrow p_x > 4p_y$.

7. Phil Rupp has a sister Ethel who has the utility function $U(x, y) = \min\{4x + y, 5y\}$. Draw at least two of her indifference curves in the graph below.

1 point

Note that if $4x + y \leq 5y$ (i.e., if $x \leq y$), then $U(x, y) = 4x + y$. Otherwise, if $4x + y \geq 5y$ (i.e., if $x \geq y$), then $U(x, y) = 5y$.



$\boxed{8.}$ Pablo's utility function is $U(x,y) = 10x - \frac{x^2}{2} + y$, where x is the number of x's he consumes per week and y is the number of y's he consumes per week. Pablo has \$200 a week to spend. The price of y is \$1. The price of x is currently \$5 per unit. Pablo has received an invitation to join a club devoted to the consumption of x. If he joins the club, Pablo can get a discount on the purchase of x. If he belonged to the club, he could buy x for \$1 a unit.

(a) Find Pablo's optimal bundle if he decides not to join the club.

(b) What would be his optimal bundle if he joined the club?

(c) How much is the most Pablo would be willing to pay to join this club?

1 point

(a) In this case, Pablo solves the following utility-maximization problem:

$$\max_{x,y} \quad 10x - \frac{x^2}{2} + y$$
$$\text{subject to } 5 \cdot x + 1 \cdot y = 200$$

$$|MRS| = 10 - x = \frac{p_x}{p_y} = 5 \rightarrow x = 5$$

Note that Pablo has enough income to purchase this much of x, $y = 175$.

(b) In this case, Pablo solves the following utility-maximization problem:

$$\max_{x,y} \quad 10x - \frac{x^2}{2} + y$$
$$\text{subject to } 1 \cdot x + 1 \cdot y = 200$$

$$|MRS| = 10 - x = \frac{p'_x}{p_y} = 1 \rightarrow x = 9$$

Note that Pablo has enough income to purchase this much of x, $y = 191$.

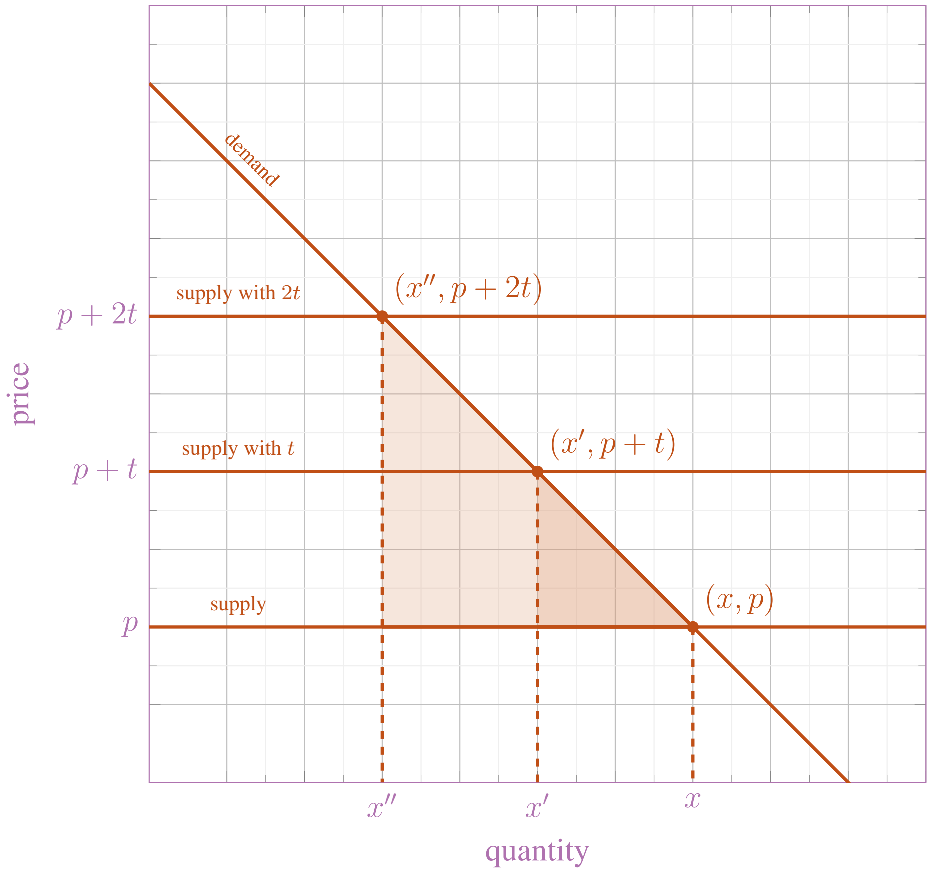
(c) Let's denote Pablo's willingness to pay to join this club by P.
Without joining, Pablo's utility is $U(5, 175) = 10 \cdot 5 - \frac{5^2}{2} + 175 = 50 - 12.5 + 175 = 212.5$. If he joins, he enjoy $U(9, 191 - P) = 10 \cdot 9 - \frac{9^2}{2} + 191 - P = 90 - 40.5 + 191 - P = 240.5 - P$. For him to prefer to join, it must be that $240.5 - P \geq 212.5 \rightarrow P \leq 28$.

$\boxed{9.}$ The market for tennis shoes has a horizontal supply curve and a linear, downward-sloping demand curve. Currently the government imposes a tax of t on every pair of tennis shoes sold and does not tax other goods. The government is considering a plan to double the tax on tennis shoes, while leaving other goods untaxed. Show that if the tax is doubled, then the total deadweight loss caused by the doubled tax will be more than twice the original deadweight loss.

1 point

Consider the figure below. The "original" deadweight loss is $\mathrm{DWL}' = \frac{1}{2} \cdot (x - x') \cdot t$, while the "new" deadweight loss is $\mathrm{DWL}'' = \frac{1}{2} \cdot (x - x'') \cdot 2t$.

Given that $(x - x'') > (x - x')$, DWL'' is more than twice as large as DWL'.



10. Hildegard, an intelligent and charming Holstein cow, grazes in a very large, mostly barren pasture with a few patches of lush grass. When she finds a new grassy area, in order to get y units of grass from it, she has to graze for y^2 hours. Finding a new patch of grass on which to graze takes her 1 hour. Since Hildegard does not have pockets, the currency in which her costs are measured is time.

(a) What is the total cost to Hildegard of finding a new plot of grass and getting y units of grass from it?

$c(y) = y^2 + 1$

(b) Find an expression for her marginal costs and her average cost per patch of grass as a function of the amount of grass she gets from each patch.

$MC(y) = 2y$
$AC(y) = y + \frac{1}{y}$

(c) How much time would she spend in each plot if she wanted to maximize her food intake? (Hint: Maximizing her hourly food intake is equivalent to minimizing average costs per unit of grass eaten.)

$MC(y) = AC(y) \rightarrow 2y = y + \frac{1}{y} \rightarrow y^* = 1$
Therefore, she would spend $(y^*)^2 = 1$ hour in each plot.

1 point

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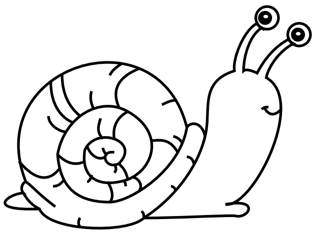

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