LINEAR ALGEBRA:
EXERCISES: SET 04

1. Numerical exercises

1. Let

$$A = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 5 & 1 & -1 \\ 2 & 1 & -1 \end{pmatrix}.$$

   (1) Write down the row vectors and the column vectors of A and B.
   (2) Find $A + B$, $3B$, $-2B$, $A + 2B$, $2A + B$, $A - B$, $A - 2B$, $B - A$.
   (3) Find A^T and B^T.

2. Let

$$A = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -1 & 1 \\ 0 & -3 \end{pmatrix}.$$

   (1) Find A^T and B^T.
   (2) Find $(A + B)^T$ and $A^T + B^T$.
   (3) Find $A + A^T$ and $B + B^T$.
   (4) Find $A - A^T$ and $B - B^T$.

2. Problems

3. Let A and B be two matrices of the same size. Prove that $(A + B)^T = A^T + B^T$.

4. Let c be a number and A be a matrix. Prove that $(cA)^T = cA^T$.

5. Let A be a square matrix. Prove that the diagonal components of A and A^T are the same.

6. Let A be a square matrix. Prove that the matrix $A + A^T$ is symmetric.

7. Let A be a square matrix. Prove that the matrix $A - A^T$ is skew-symmetric.

8. Let A be a skew-symmetric matrix. What can be said about its diagonal components?