Exercise 33.8

Romeo loves Juliet and Juliet loves Romeo. Besides love, they consume only one good, spaghetti. ...

$$
\begin{aligned}
U_R(S_R, S_J) &= S_R^a S_J^{1-a} \\
U_J(S_J, S_R) &= S_J^a S_R^{1-a}
\end{aligned}
$$

$$
S_R + S_J = 24
$$

Exercise 33.8.a

$$a = \frac{2}{3}$$

$$\max_{S_R, S_J} S_R^{\frac{2}{3}} S_J^{\frac{1}{3}}$$
$$\text{subject to } S_R + S_J = 24$$

$$S_R^* = 16$$
$$S_J^* = 8$$

Exercise 33.8.b

$$a = \frac{2}{3}$$

$$\max_{S_J, S_R} S_J^{\frac{2}{3}} S_R^{\frac{1}{3}}$$
$$\text{subject to } S_R + S_J = 24$$

$$S_J^* = 16$$
$$S_R^* = 8$$

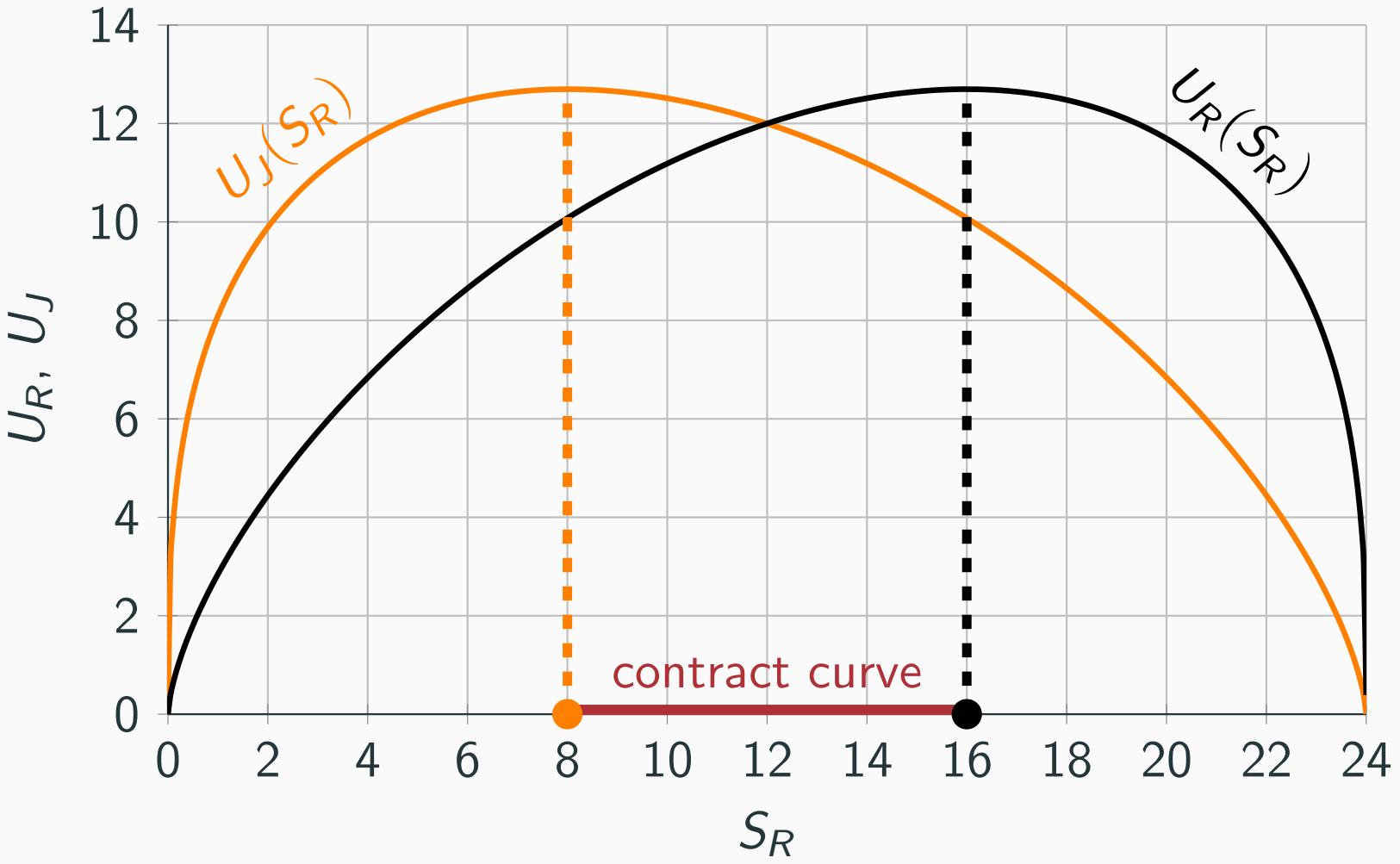
Exercise 33.8.c-e

$$S_R + S_J = 24 \quad \rightarrow \quad S_J = 24 - S_R$$

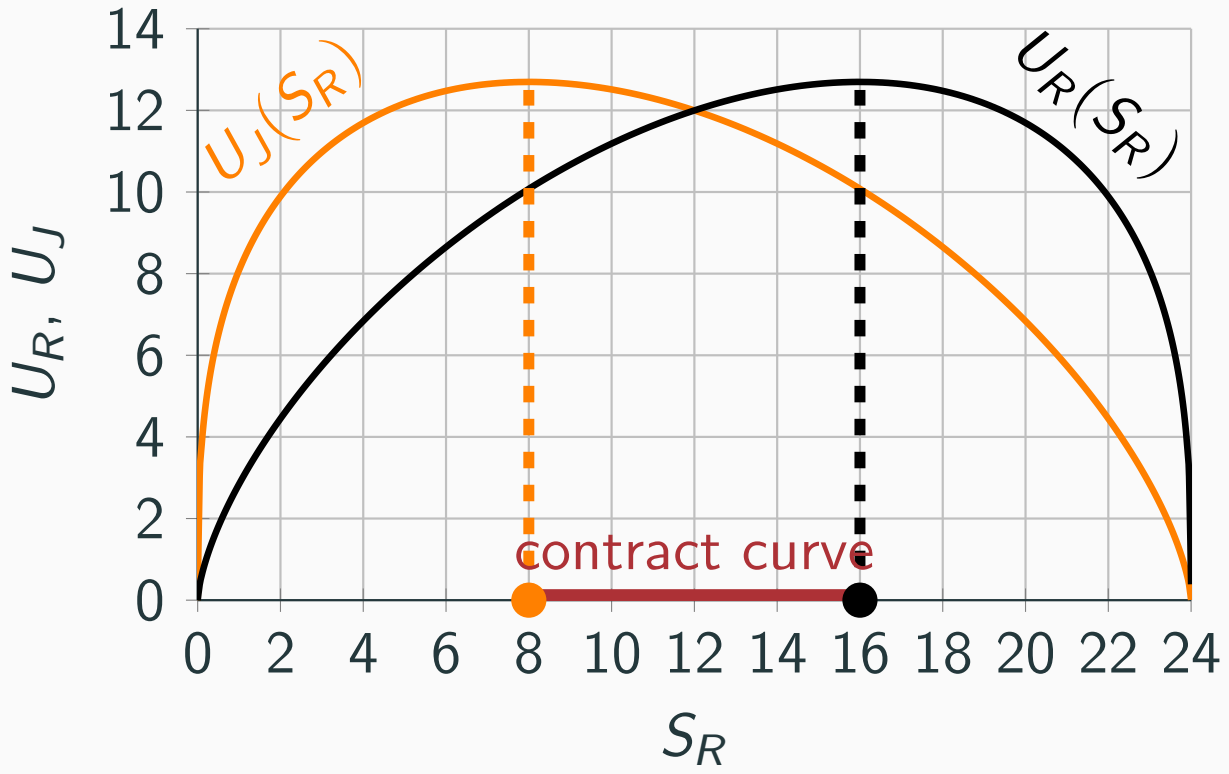
$$\begin{aligned} U_R(S_R, S_J) = S_R^a S_J^{1-a} &\quad \rightarrow \quad U_R(S_R) = S_R^a (24 - S_R)^{1-a} \\ U_J(S_J, S_R) = S_J^a S_R^{1-a} &\quad \rightarrow \quad U_J(S_R) = (24 - S_R)^a S_R^{1-a} \end{aligned}$$

$$\begin{aligned} a &= \frac{2}{3} \\ U_R(S_R) &= S_R^{\frac{2}{3}} (24 - S_R)^{\frac{1}{3}} \\ U_J(S_R) &= (24 - S_R)^{\frac{2}{3}} S_R^{\frac{1}{3}} \end{aligned}$$

Exercise 33.8.c-e



Exercise 33.8.c-e

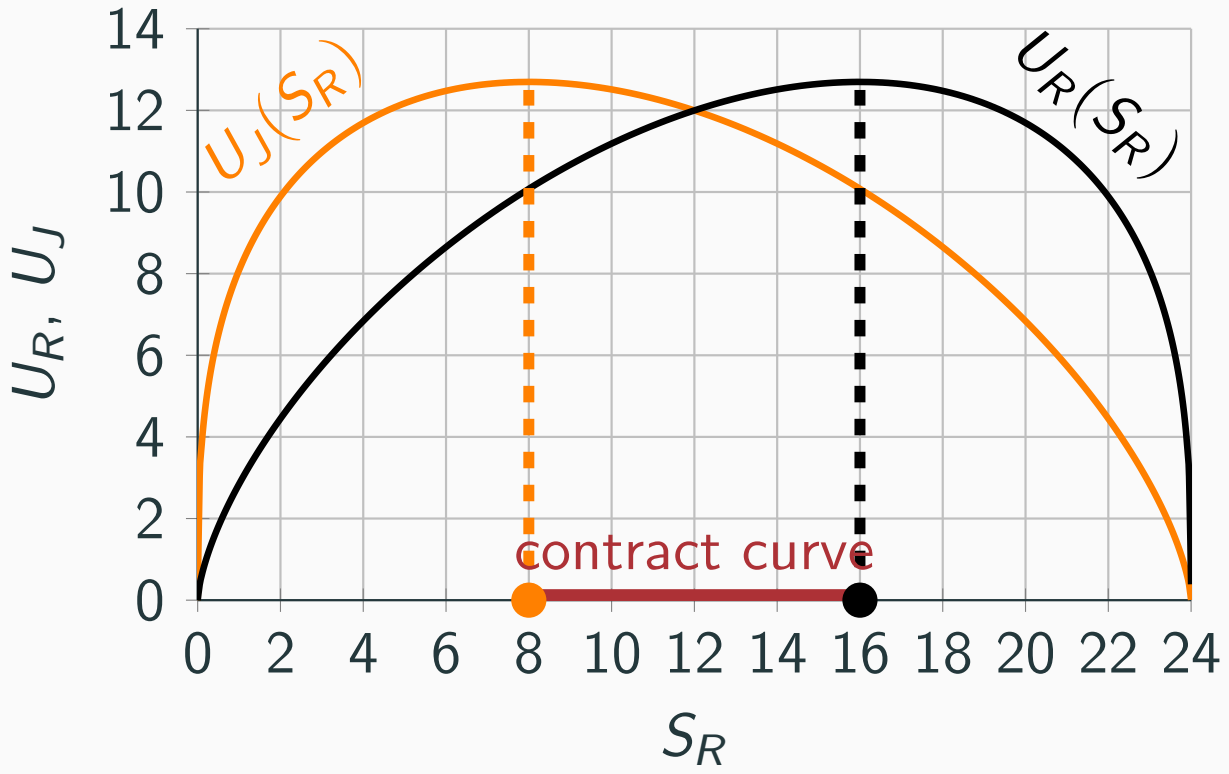


$$\frac{dU_R}{dS_R}(S_R) > 0 \text{ if } S_R < 16 \qquad \frac{dU_J}{dS_R}(S_R) > 0 \text{ if } S_R < 8$$

$$\frac{dU_R}{dS_R}(S_R) = 0 \text{ if } S_R = 16 \qquad \frac{dU_J}{dS_R}(S_R) > 0 \text{ if } S_R = 8$$

$$\frac{dU_R}{dS_R}(S_R) < 0 \text{ if } S_R > 16 \qquad \frac{dU_J}{dS_R}(S_R) > 0 \text{ if } S_R > 8$$

Exercise 33.8.c-e



$$\text{if } S_R \in [0,8]\,, \quad \frac{dU_R}{dS_R}(S_R) > 0 \text{ and } \frac{dU_J}{dS_R}(S_R) \geq 0$$

$$\text{if } S_R \in [8,16]\,, \quad \frac{dU_R}{dS_R}(S_R) \geq 0 \text{ and } \frac{dU_J}{dS_R}(S_R) \leq 0$$

$$\text{if } S_R \in [16,24]\,, \quad \frac{dU_R}{dS_R}(S_R) \leq 0 \text{ and } \frac{dU_J}{dS_R}(S_R) < 0$$