1. The village of Péteri, located just west of the town of Monor, has a population of 2 000 people. Péteri, very much like Monor, has a single public good, the *village park* and a single private good, *sör*. In Péteri, everyone's utility function is $U_i(X_i, Y) = X_i - 3\,000/Y$, where $X_i$ is the number of bottles of *sör* consumed by $i$ and $Y$ is the size of the *village park* measured in square meters. The price of *sör* is \$2 per bottle. The cost of the *village park* to the village is $\$0.10$ per square meter. Everyone has an income of at least $\$1\,000$.

   What is the Pareto efficient size for the *village park*?

2.5 points

The *village park* is a public good. We can use the following condition to find its Pareto efficient size.

$$\sum_{i=1}^{2\,000} |MRS_i| = \frac{MC(Y)}{p_x}$$

Note that $|MRS_i| = \frac{\partial U_i/\partial Y}{\partial U_i/\partial X_i} = \frac{-3\,000/Y^2 \cdot (-1)}{1} = \frac{3\,000}{Y^2}$, $MC(Y) = 0.1$ and $p_X = 2$. The above condition then leads to:

$$\begin{aligned} 2\,000 \cdot \frac{3\,000}{Y^2} &= 0.05 \\ Y &\approx 10\,954. \end{aligned}$$

Note that the inhabitants of the village all have enough money to contribute towards a *village park* with that size.

$\boxed{2.}$ Consider a pure exchange economy with two agents (let's say, Attila and Balázs) and two goods (let's say, *exes* and *whys*):

- $u_A(x_A, y_A) = 2x_A + y_A$,
- $u_B(x_B, y_B) = x_B^{\frac{1}{4}} \cdot y_B^{\frac{1}{4}}$,
- $w_A^x + w_B^x = 10$,
- $w_A^y + w_B^y = 20$.

(a) Find mathematically and represent graphically in an Edgeworth box the set of Pareto efficient allocation of this economy.

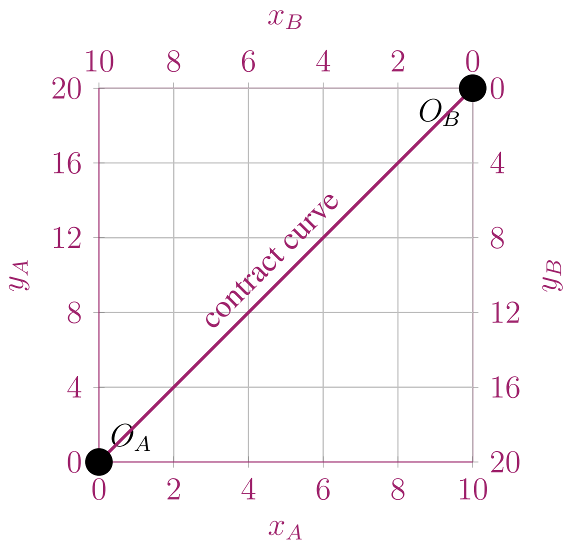
2.5 points

Both consumers have Cobb-Douglas type utility functions, therefore the set of Pareto efficient allocations is characterized by the following tangency condition:

$$\begin{aligned} MRS^A &= MRS^B \\ \frac{\partial u_A/\partial x_A}{\partial u_A/\partial y_A} &= \frac{\partial u_B/\partial x_B}{\partial u_B/\partial y_B} \\ \frac{2}{1} &= \frac{\frac{1}{4}\cdot x_B^{-\frac{3}{4}}\cdot y_B^{\frac{1}{4}}}{\frac{1}{4}\cdot x_B^{\frac{1}{4}}\cdot y_B^{-\frac{3}{4}}} \\ 2 &= \frac{y_B}{x_B} \\ 2x_B &= y_B \end{aligned}$$

The feasibility constraints in this exercise are $x_A + x_B = w_A^x + w_B^x = 10$ and $y_A + y_B = w_A^y + w_B^y = 20$. Therefore, we can write that $x_B = 10 - x_A$ and $y_B = 20 - y_A$.

$$\begin{aligned} MRS^A &= MRS^B \\ 2 &= \frac{y_B}{x_B} \\ 2 &= \frac{20 - y_A}{10 - x_A} \\ 20 - 2x_A &= 20 - y_A \\ 2x_A &= y_A \end{aligned}$$




(b) Plot the utility possibilities frontier for this economy.

**Hint:** Recall that, with the help of the utility possibilities frontier, we are simply considering the set of Pareto efficient allocations from a different perspective. Follow the three steps below to complete this part of the exercise.

i. Use the feasibility constraints and your answer to the previous question, and write $u_A$ and $u_B$ (along the contract curve) as a function of $x_A$ only.
ii. Given your answers to the previous point, write $u_B$ as a function of $u_A$.
iii. Now you should be able to plot the utility possibilities frontier for this economy.

2.5 points

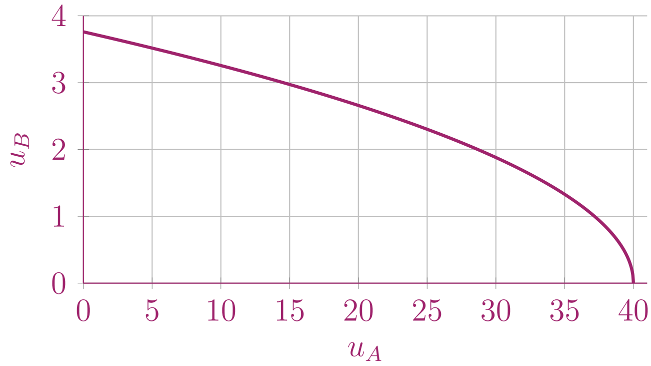
The utility functions are $u_A(x_A, y_A) = 2x_A + y_A$ and $u_B(x_B, y_B) = x_B^{\frac{1}{4}} \cdot y_B^{\frac{1}{4}}$. After substituting the feasibility constraints into them, we can rewrite the utility functions as:

$$\begin{aligned} u_A(x_A, y_A) &= 2x_A + y_A, \text{ and} \\ u_B(x_B, y_B) &= (10 - x_A)^{\frac{1}{4}} \cdot (20 - y_A)^{\frac{1}{4}}. \end{aligned}$$

Now let's impose the condition of Pareto efficiency, i.e. $y_A = 2x_A$.

$$\begin{aligned} u_A(x_A, y_A) &= 2x_A + 2x_A = 4x_A, \text{ and} \\ u_B(x_B, y_B) &= (10 - x_A)^{\frac{1}{4}} \cdot (20 - 2x_A)^{\frac{1}{4}} = 2^{\frac{1}{4}} \cdot (10 - x_A)^{\frac{1}{2}}. \end{aligned}$$

From the above results, we have that $x_A = \frac{1}{4} \cdot u_A$. Therefore, $u_B = 2^{\frac{1}{4}} \cdot (10 - \frac{1}{4} \cdot u_A)^{\frac{1}{2}}$.




(c) Consider the following social-welfare maximization problem:

$$\max_{x_A,y_A,x_B,y_B} \lambda_A \cdot u_A(x_A,y_A) + \lambda_B \cdot u_B(x_B,y_B)$$
$$\text{subject to } x_A + x_B = 10$$
$$y_A + y_B = 20$$

The function $\lambda_A \cdot u_A(x_A,y_A) + \lambda_B \cdot u_B(x_B,y_B)$ is called *social welfare function*. The parameters $\lambda_A$ and $\lambda_B$ are called *Pareto weights*.

i. Write the first-order conditions and argue that any solution to the social-welfare maximization problem is Pareto efficient. Remember that $u_A(x_A,y_A) = 2x_A + y_A$ and $u_B(x_B,y_B) = x_B^{\frac{1}{4}} \cdot y_B^{\frac{1}{4}}$.

2.5 points

Before finding the first-order condition, let's incorporate the two conditions into the objective function by substitution.

$$\max_{x_A,y_A} \quad \lambda_A \cdot u_A(x_A,y_A) + \lambda_B \cdot u_B(10-x_A, 20-y_A)$$
$$\max_{x_A,y_A} \quad \lambda_A \cdot (2x_A + y_A) + \lambda_B \cdot (10-x_A)^{\frac{1}{4}} \cdot (20-y_A)^{\frac{1}{4}}$$

Now we compute the first-order conditions:

$$\lambda_A \cdot 2 + \lambda_B \cdot \frac{1}{4} \cdot (10-x_A)^{-\frac{3}{4}} \cdot (20-y_A)^{\frac{1}{4}} \cdot (-1) = 0$$
$$\lambda_A \cdot 1 + \lambda_B \cdot \frac{1}{4} \cdot (10-x_A)^{\frac{1}{4}} \cdot (20-y_A)^{-\frac{3}{4}} \cdot (-1) = 0$$

From the second one, we have that $\lambda_A = \lambda_B \cdot \frac{1}{4} \cdot (10-x_A)^{\frac{1}{4}} \cdot (20-y_A)^{-\frac{3}{4}}$. With that, we can rewrite the first condition:

$$\lambda_B \cdot \frac{1}{4} \cdot (10-x_A)^{\frac{1}{4}} \cdot (20-y_A)^{-\frac{3}{4}} \cdot 2 = \lambda_B \cdot \frac{1}{4} \cdot (10-x_A)^{-\frac{3}{4}} \cdot (20-y_A)^{\frac{1}{4}}$$
$$2 \cdot (10-x_A) = 20 - y_A$$
$$y_A = 2x_A$$

As we have seen before, this condition characterizes the Pareto-efficient allocations.

ii. Consider the allocation in which agent $A$ consumes $2$ units of *exes* and $4$ units of *whys*, and agent $B$ consumes the rest.

A. Argue that this is a Pareto efficient allocation.

1 point

We have seen before that as long as agent $A$ consumes twice as many units of *exes* as *whys* we have a Pareto efficient allocations. Now $y_A = 2x_A = 2 \cdot 2 = 4$.

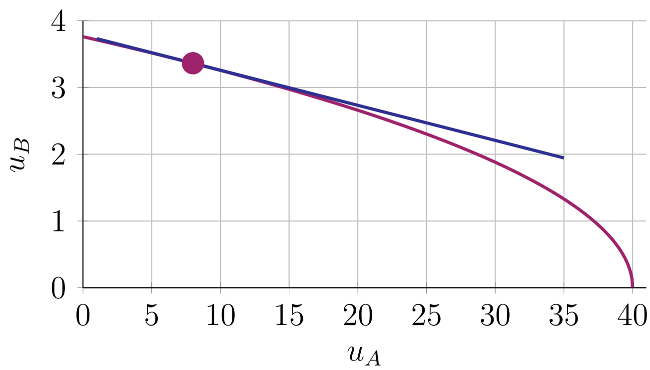
B. How much should $\lambda_A$ and $\lambda_B$ be so that the solution of the social-welfare maximization problem is exactly the above allocation?

1 point

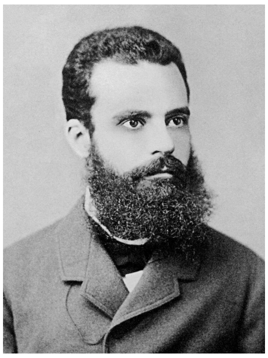
We have shown above that $\lambda_A = \lambda_B \cdot \frac{1}{4} \cdot (10 - x_A)^{\frac{1}{4}} \cdot (20 - y_A)^{-\frac{3}{4}}$. By substituting $x_A = 2$ and $y_A = 4$, we have that $\lambda_A = \lambda_B \cdot \frac{1}{4} \cdot 8^{\frac{1}{4}} \cdot 16^{-\frac{3}{4}} = \lambda_B \cdot 2^{-2} \cdot 2^{\frac{3}{4}} \cdot 2^{-3} = 2^{-\frac{17}{4}} \cdot \lambda_B$. That is, $\lambda_B = 2^{\frac{17}{4}} \cdot \lambda_A$.
If, for instance, $\lambda_A = 1$, then we must have $\lambda_B = 2^{\frac{17}{4}} \approx 19$.

iii. Use the graph with the utility possibilities frontier and represent the above social-welfare maximization problem.

1 point




The blue line represents an *isowelfare curve* when $\lambda_A = 1$ and $\lambda_B = 19$. Its slope is $-\frac{\lambda_A}{\lambda_B}$.



3. Attila and Balázs are two *homo economicus* engaged in exchange over *exes* ($x$) and *whys* ($y$).[1] Their utility functions are given by the following:

$$\begin{aligned} \text{Attila}: \quad & u_A(x_A, y_A) = x_A^{\frac{1}{5}} y_A^{\frac{1}{5}} \\ \text{Balázs}: \quad & u_B(x_B, y_B) = 0.05x_B + 0.05y_B \end{aligned}$$

Attila has an initial endowment of 5 *exes* and 15 *whys*. Balázs has an initial endowment of 15 *exes* and 5 *whys*.

(a) What is Attila's marginal rate of substitution of *exes* for *whys* (*MRS*$_A$)? What is Balázs's marginal rate of substitution of *exes* for *whys* (*MRS*$_B$)?

1 point

$$MRS_A = -\frac{\partial u_A/\partial x_A}{\partial u_A/\partial y_A} = -\frac{\frac{1}{5}x_A^{-\frac{4}{5}} y_A^{\frac{1}{5}}}{\frac{1}{5}x_A^{\frac{1}{5}} y_A^{-\frac{4}{5}}} = -\frac{y_A}{x_A}$$

$$MRS_B = -\frac{\partial u_B/\partial x_B}{\partial u_B/\partial y_B} = -\frac{0.05}{0.05} = -1$$

(b) Using the marginal rates of substitution, find the equation for the Pareto-efficient curve.

In this exercise, you are allowed to focus all allocations in which both of the decision-makers consumes strictly positives amounts of both goods. In other words, you can ignore the sides of the Edgeworth box.

1 point

$$\begin{aligned} MRS_A &= MRS_B \\ -\frac{y_A}{x_A} &= -1 \\ y_A &= x_A \end{aligned}$$

(c) How much is Attila's utility at his initial allocation, $(x_A^I, y_A^I)$?

0.5 point

$$u_A(x_A^I, y_A^I) = (x_A^I)^{\frac{1}{5}} \cdot (y_A^I)^{\frac{1}{5}} = 5^{\frac{1}{5}} \cdot 15^{\frac{1}{5}} = 75^{\frac{1}{5}}$$

[1] This exercise has been adapted from the draft version of Bowles, S. & Halliday, S. (2022) *Microeconomics: Competition, Conflict and Coordination, Oxford University Press.*

(d) How much is Balázs's utility at his initial allocation, $(x_B^I, y_B^I)$?

0.5 points

$$u_B(x_B^I, y_B^I) \quad = \quad 0.05x_B^I + 0.05y_B^I = 0.05 \cdot 15 + 0.05 \cdot 5 = 1$$

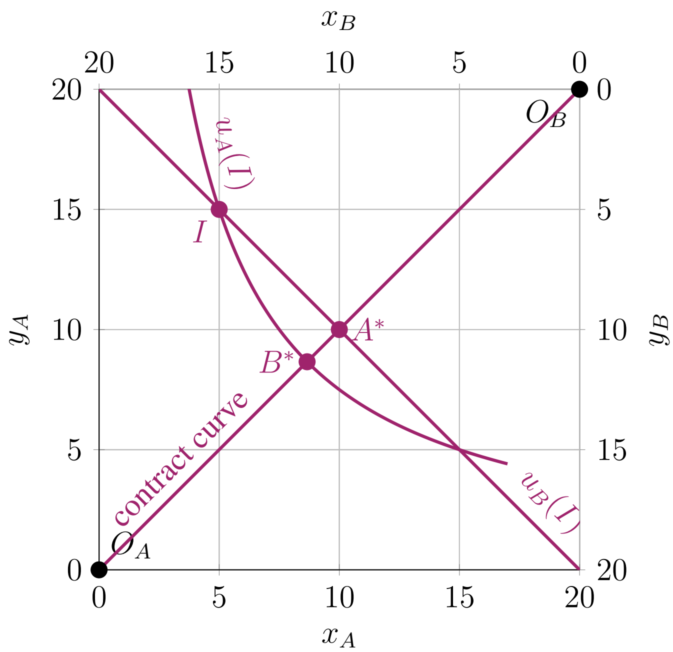
(e) Is the initial allocation of goods Pareto efficient?

0.5 points

As $y_A^I = 15 \neq x_A^I = 5$, the initial allocation is not Pareto efficient.

(f) Graph the Edgeworth box depicting the exchange between Attila and Balázs. Show their initial allocations, their initial indifference curves, the Pareto-efficient curve, and each consumer's most preferred allocation in the Pareto-improving lens (that is the set of Pareto improvements upon the initial allocation).

2.5 points




The allocations $A^*$ and $B^*$ represent Attila's and Balázs's most preferred allocation, respectively, in the set of all Pareto improvements with respect to the initial allocation.

(g) Instead of his previous utility function, assume that Balázs's utility function is given by the following:

$$v_B\left(u_A(x_A, y_A), u_B(x_B, y_B)\right) = 0.05x_B + 0.05y_B + \frac{1}{2}x_A^{\frac{1}{5}}y_A^{\frac{1}{5}}$$

Explain how this function is different from Balázs's utility function when he was a *homo economicus*.

**Hint**: Explain how this function captures the idea of *altruism* and why we could call Balázs a *homo generosus*.

1 point

We could write that $v_B = u_B + \frac{1}{2}u_A$. Note that now Balázs cares about how much utility Attila derives from consuming *exes* and *whys*. The larger Attila's utility level, the larger Balázs's, as well.

In other words, Balázs is not completely selfish in this case, although he still cares about his own consumption (and "original" utility) level twice as much Attila's.

(h) What is Balázs's marginal rate of substitution of *exes* ($x_B$) for *whys* ($y_B$)?

**Hint**: Use the feasibility constraints (on $x$ and $y$) to eliminate $x_A$ and $y_A$ from Balázs's utility function, and write $v_B$ as a function of $x_B$ and $y_B$.

2 points

$$\begin{aligned} v_B\left(u_A(x_A, y_A), u_B(x_B, y_B)\right) &= v_B\left(u_A(20 - x_B, 20 - y_B), u_B(x_B, y_B)\right) = \\ &= 0.05x_B + 0.05y_B + \frac{1}{2}(20 - x_B)^{\frac{1}{5}}(20 - y_B)^{\frac{1}{5}} \end{aligned}$$

$$\begin{aligned} MRS_B &= -\frac{\partial v_B / \partial x_B}{\partial v_B / \partial y_B} = -\frac{0.05 + \frac{1}{2} \cdot \frac{1}{5} \cdot (20 - x_B)^{-\frac{4}{5}} \cdot (-1) \cdot (20 - y_B)^{\frac{1}{5}}}{0.05 + \frac{1}{2} \cdot \frac{1}{5} \cdot (20 - x_B)^{\frac{1}{5}} \cdot (20 - y_B)^{-\frac{4}{5}} \cdot (-1)} = \\ &= -\frac{0.05 - \frac{1}{10}(20 - x_B)^{-\frac{4}{5}}(20 - y_B)^{\frac{1}{5}}}{0.05 - \frac{1}{10}(20 - x_B)^{\frac{1}{5}}(20 - y_B)^{-\frac{4}{5}}} \end{aligned}$$

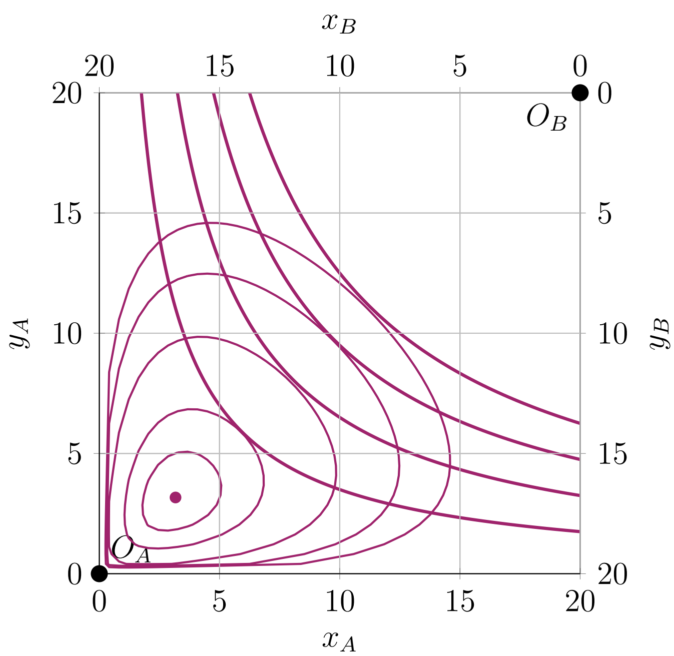
(i) Graph a new Edgeworth box depicting the exchange between Attila and Balázs where Balázs is *homo generosus* and Attila is *homo economicus*. Show their initial allocations, the Pareto-efficient curve, and each consumer's most preferred allocation in the Pareto-improving lens.

Briefly explain the differences between this Edgeworth box and your previous Edgeworth box when both players were *homo economicus*.

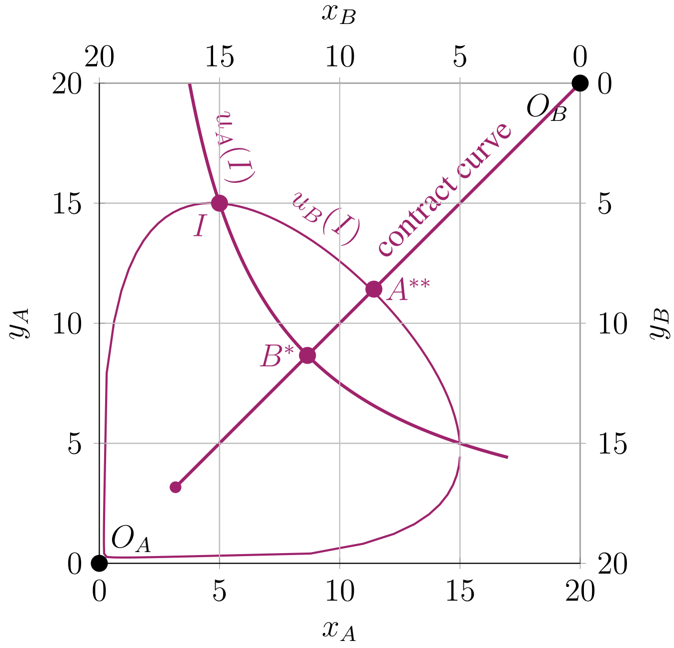
**Hint**: This is the new, the most interesting, and also the most challenging part of the exercise. For that reason, you may want to consider the following pieces of advice.

- Balázs's indifference curves will look like circles around an ideal point. First, find this ideal point, by maximizing $v_B$ in $x_B$ and $y_B$. Then, you can compute the marginal rate of substitution for $v_B$.
- You are not required to compute anything else in this part of the exercise, but your graph should illustrate the new situation. It is enough is you create a rough sketch for the Edgeworth box. For a more precise picture, you may want to use Mathematica / WolframAlpha (or some other software / website) and the following two commands.
  - ContourPlot[0.05*(x + y) + 0.5*((20 - x)ˆ 0.2)*((20 - y)ˆ 0.2), {x, 0, 20}, {y, 0, 20}]
  - Maximize[{0.05*(x + y) + 0.5*((20 - x)ˆ 0.2)*((20 - y)ˆ 0.2), x >= 0, y >= 0, x <= 20, y <= 20}, {x, y}]

3 points




Balázs obtains his highest possible utility level at $(x_B^{max}, y_B^{max}) = (16.83, 16.83)$. "Moving" in any direction from that ideal point results in a utility loss for him.




When Balázs cares about Attila, the contract curve (i.e., the set of Pareto-efficient allocations) becomes shorter. It is still the case that at any Pareto-efficient allocation $y_A = x_A$, but extreme allocations in which Attila consumes small quantities (less than 3.17 units of *exes* and less than 3.17 units of *whys*) are not Pareto efficient any longer. Balázs's favorite point in the Pareto-improving lens is still the same ($B^*$), but Attila's moves to $A^{**}$, an allocation where he can consume somewhat more of each good than at $A^*$. This is because Attila might be able to take advantage of the fact that Balázs cares about him.

## generosus (Latin)

### Origin & history

From *genus* ("birth, origin")

### Adjective

**generōsus** (*feminine* **generōsa**, *neuter* **generōsum**)

1. well-born, noble
2. superior, excellent
3. (*figuratively*) generous, magnanimous
4. (*figuratively*) dignified, honorable

### Related words & phrases

generātor
generōsē
generō

### Descendants

French: *généreux*, généreuse