1. Suppose that the inverse demand curve for a *very mysterious good* is given by $P = 20 - 0.5 \cdot Q$, where P is the price and Q is the total industry output.

Suppose that the industry has two firms. One firm has a constant marginal cost of $\$1$ per unit of output, while the other firm has a constant marginal cost of $\$3$ per unit of output. If the two firms compete à la Cournot, how much output will each firm produce? How much will the unit price of this *very mysterious good* be?

3 points

Let q_1 and q_2 denote the output level for firm 1 and firm 2, respectively. In order to find the firms' reaction function, we solve their profit-maximization problems.

firm 1:

$$\begin{aligned}
&\max_{q_1} P \cdot q_1 - 1 \cdot q_1 \\
&\max_{q_1} [20 - 0.5 \cdot (q_1 + q_2)] \cdot q_1 - q_1 \\
FOC &: 20 - q_1 - 0.5 \cdot q_2 - 1 = 0 \\
q_1 &= 19 - 0.5 q_2
\end{aligned}$$

firm 2:

$$\begin{aligned}
&\max_{q_2} P \cdot q_2 - 3 \cdot q_2 \\
&\max_{q_2} [20 - 0.5 \cdot (q_1 + q_2)] \cdot q_2 - 3 \cdot q_2 \\
FOC &: 20 - 0.5 \cdot q_1 - q_2 - 3 = 0 \\
q_2 &= 17 - 0.5 \cdot q_1
\end{aligned}$$

For the Cournot equilibrium we have to solve the following system of equations:

$$\begin{aligned}
q_1 &= 19 - \frac{1}{2} q_2 \\
q_2 &= 17 - \frac{1}{2} q_1
\end{aligned}$$

$$\begin{aligned}
q_1 &= 19 - \frac{1}{2}(17 - \frac{1}{2} q_1) = 10.5 + \frac{1}{4} q_1 \\
\frac{3}{4} q_1 &= 10.5 \\
q_1^* &= \frac{42}{3} = 14 \\
q_2^* &= 17 - \frac{1}{2} \cdot q_1^* = 17 - \frac{1}{2} \cdot 14 = 10
\end{aligned}$$

$$P^*(q_1^* + q_2^*) = 20 - 0.5 \cdot (14 + 10) = 8$$

15%

2. An airport is located next to a housing development. Where X is the number of planes that land per day and Y is the number of houses in the housing development, profits of the airport are $36X - X^2$ and profits of the developer are $42Y - Y^2 - XY$. For simplicity, X and Y are expressed in 10s of units (i.e., 10s of planes and houses, respectively), while profits are to be interpreted in 1000s of monetary units.

(a) How many houses are going to be built and how many planes are going to land if a single profit-maximizing company owns the airport and the housing development?

1 points

$$\max_{X,Y}\,(36X - X^2) + (42Y - Y^2 - XY)$$
$$FOC: 36 - 2X - Y = 0$$
$$42 - 2Y - X = 0$$

$$72 - 4X - 2Y = 0$$
$$42 - (72 - 4X) - X = 0$$
$$-30 - 3X = 0$$

$$X^* = 10$$
$$Y^* = 16$$

(b) How many houses are going to be built and how many planes are going to land if the airport and the housing development are operated independently?

1 point

$$\max_{X}\,36X - X^2$$
$$FOC: 36 - 2X = 0$$

$$X^{**} = 18$$

$$\max_{Y}\,42Y - Y^2 - XY$$
$$FOC: 42 - 2Y - X = 0$$

$$Y^{**} = 12$$

25%

(c) How should the social planner tax or subsidize the airport's activity so that it produces the socially-optimal level of output.

1 point

Let's suppose that the airport must pay t units of money to the social planner for each plane that lands. Now its profit-maximization problem (and its solution) can be written as

$$\max_X 36X - X^2 - t \cdot X$$
$$FOC : 36 - 2X - t = 0,$$
$$X(t) = \frac{36 - t}{2}.$$

For the airport to operate at the socially-optimal level, the social planner should set t so that

$$X(t) = X^*,$$
$$\frac{36 - t}{2} = 10,$$
$$t = 16.$$

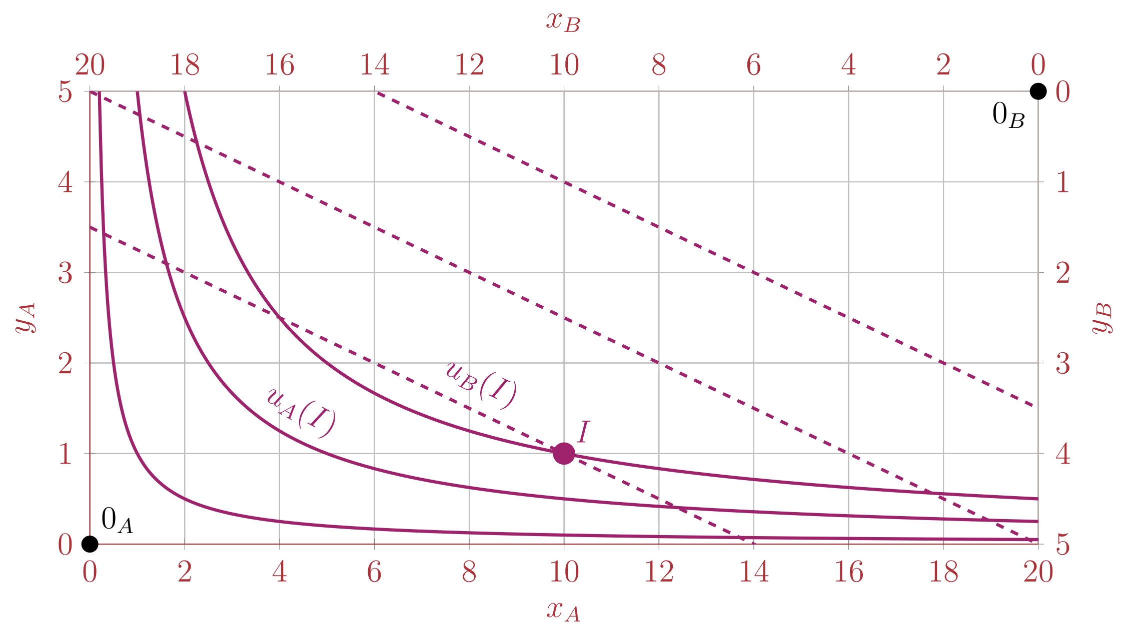
30%

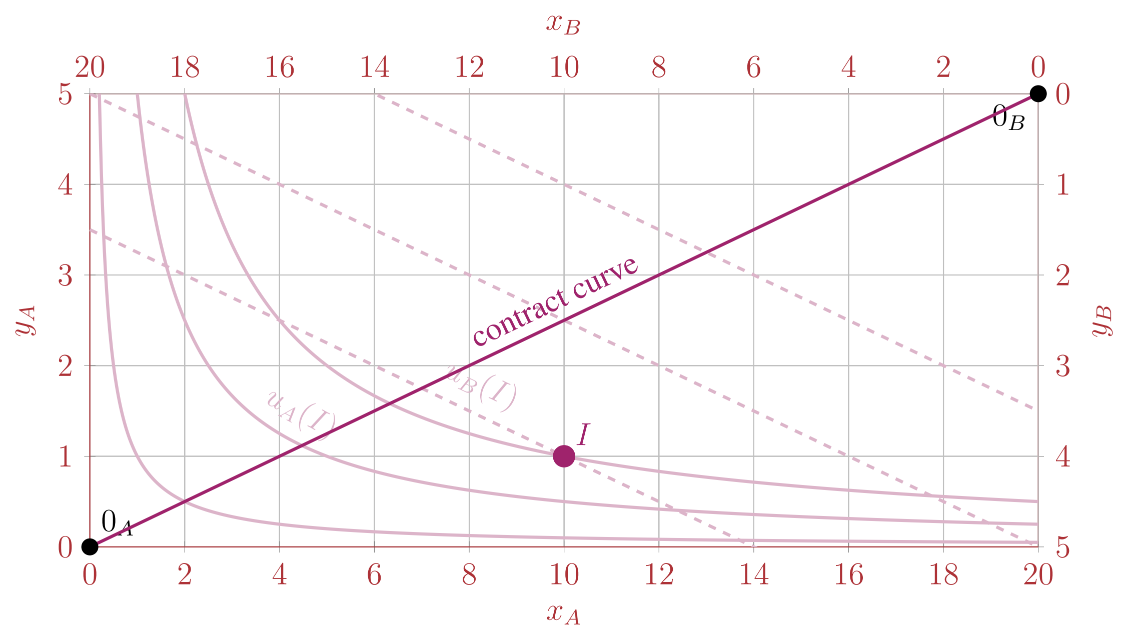
$\boxed{3.}$ Consider a pure exchange economy with two consumers (let's say, *Attila* and *Balázs*) and two goods (let's say, *exes* and *whys*):

- $u_A(x_A, y_A) = x_A^{\frac{1}{2}} y_A^{\frac{1}{2}}$,
- $u_B(x_B, y_B) = x_B + 4y_B$,
- Agent A initially owns 10 units of *exes* and 1 unit of *whys*: $\omega_A^x = 10$, $\omega_A^y = 1$.
- Agent B initially owns 10 units of *exes* and 4 units of *whys*: $\omega_B^x = 10$, $\omega_B^y = 4$.

(a) Represent this pure exchange economy with the help of an Edgeworth box (a sketch will suffice!), and find (mathematically) the contract curve.

2 points





40%

In the interior of the Edgeworth box, Pareto-efficient allocations can be characterized by the following tangency condition:

$$MRS_A = MRS_B,$$
$$-\frac{\frac{1}{2}x_A^{-\frac{1}{2}}y_A^{\frac{1}{2}}}{\frac{1}{2}x_A^{\frac{1}{2}}y_A^{-\frac{1}{2}}} = -\frac{y_A}{x_A} = -\frac{1}{4},$$

$$y_A = \frac{x_A}{4}.$$

(b) Find (mathematically) and represent graphically the utility possibilities set for this pure exchange economy.

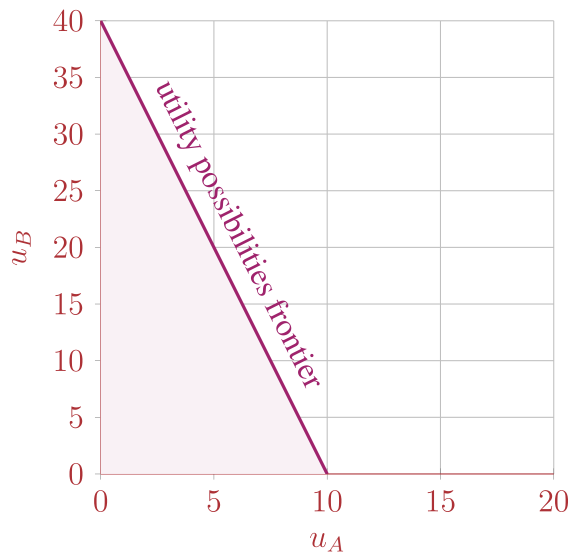
2 points

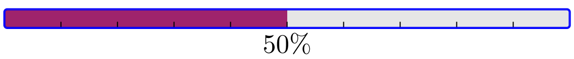
The Pareto efficient allocations must satisfy that $y_A = \frac{x_A}{4}$. With this restriction, we can rewrite the agents utility function.

$$u_A = x_A^{\frac{1}{2}}y_A^{\frac{1}{2}} = \frac{1}{2}x_A$$
$$x_A = 2u_A$$

$$u_B = x_B + 4y_B = (20 - x_A) + 4 \cdot (5 - y_A) = 40 - 2x_A$$

$$u_B = 40 - 4u_A$$





(c) Assume that the two consumers are allowed and able to trade with each other, and that *exes* are the numeraire. Also assume that both consumers act as price-takers.

i. Find the competitive equilibrium of this pure exchange economy. In other words, find the equilibrium price of *whys*.

2 points

The unit price of *exes* is equal to 1. Let p denote the unit price of *whys*.

Note that trade should be mutually beneficial and lead to an allocation on the contract curve.

For consumer B to consume positive amounts of both goods, we must have that $MRS_B = \frac{p_x}{p_y} = \frac{1}{p}$. Given that $MRS_B = \frac{1}{4}$, $p = 4$.

That means that the competitive equilibrium of this pure exchange economy is $(1, 4)$.

ii. Find the allocation of *exes* and *whys* in the competitive equilibrium.

1 point

Consumer A would want to split his income equally between *exes* and *whys*. His initial endowment is worth $1 \cdot 10 + 4 \cdot 1 = 14$. Therefore, he would want to consume 7 units of *exes* and $\frac{7}{4} = 1.75$ units of *whys*.

Consumer B would then consume $20 - 7 = 13$ units of *exes* and $5 - \frac{7}{4} = \frac{13}{4} = 3.25$ units of *whys*.

Consumer B's initial endowment is worth $1 \cdot 10 + 4 \cdot 4 = 26$. Note that that is exactly the value of the bundle that he would like to consume: $1 \cdot 13 + 4 \cdot \frac{13}{4} = 26$.

iii. Is the allocation of *exes* and *whys* in the competitive equilibrium Pareto efficient? (Hint: Check whether it is located on the contract curve or not.)

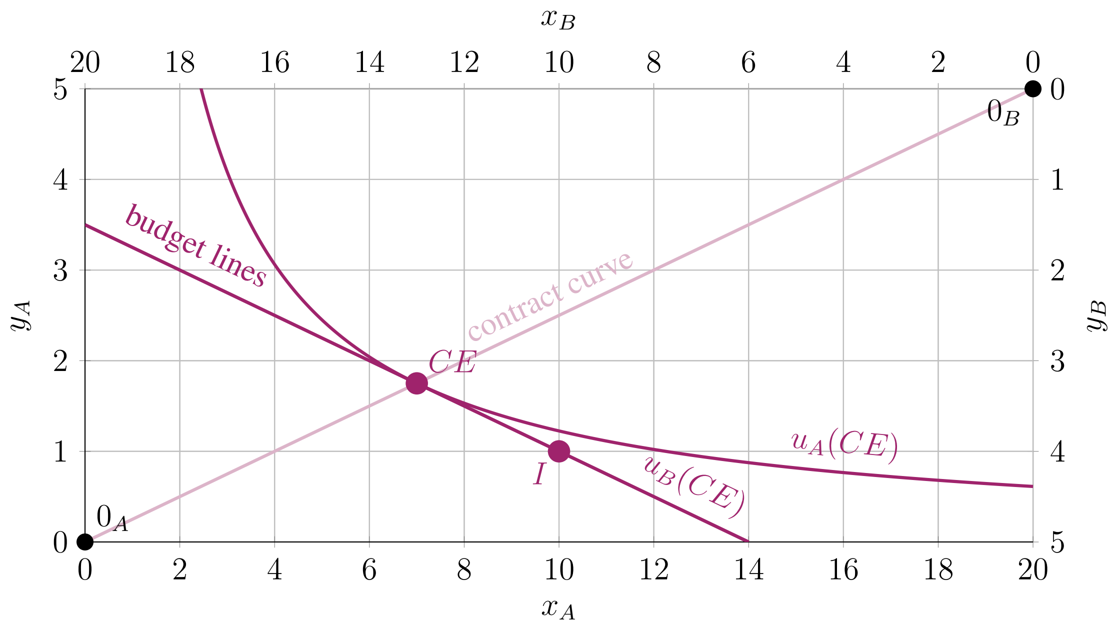
1 point

For Pareto efficiency, we need $y_A = \frac{x_A}{4}$. Note that this is satisfied when $y_A = \frac{7}{4}$ and $x_A = 7$.

70%

iv. Represent the competitive equilibrium in an Edgeworth box. A sketch will suffice as long as it shows the initial endowment, the consumers' budget constraint, the equilibrium allocation, and the indifference curves going through the equilibrium allocation.

1 point



(d) The social planner looking over this pure exchange economy would like to maximize the following social welfare function: $SWF(u_A, u_B) = \min\{4u_A, u_B\}$.

i. Find the allocation in the utility possibilities set for this pure exchange economy that maximizes the social welfare function.

2 points

The social planner solves $max_{4u_A,u_B} \min\{u_A, u_B\}$ subject to $4u_A + u_B = 40$.

Note that at the optimal point $4u_A = u_B$. Therefore $4u_A + u_B = 8u_A = 40$ and $u_A^{ideal} = 5$ and $u_B^{ideal} = 20$.

85%

ii. How should the social planner rearrange the initial endowment of *exes* (*exes* only!) so that this pure exchange economy reaches the allocation that maximizes the social welfare function through the two consumers trading with each other?

2 points

Remember that on the contract curve $x_A = 2u_A$ and $y_A = \frac{x_A}{4}$. Therefore, $x_A^{ideal} = 2u_A^{ideal} = 10$ and $y_A^{ideal} = \frac{10}{4} = 2.5$.

For consumer B to consume positive amounts of both goods, we still must have that $p = 4$.

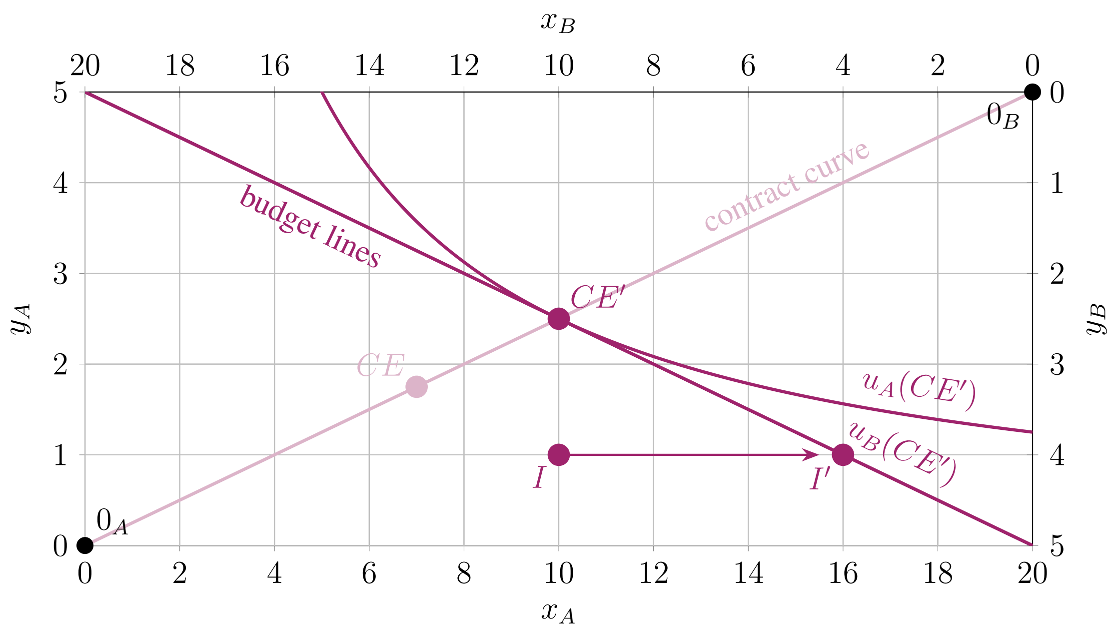
Let $\omega_A^{x,new}$ denote consumer A's endowment of *exes* after the rearrangement. Consumer A then solves the following constrained utility-maximization problem.

$$\max_{x_A, y_A} x_A^{\frac{1}{2}} y_A^{\frac{1}{2}}$$
$$\text{subject to } 1 \cdot x_A + 4 \cdot y_A = 1 \cdot \omega_A^{x,new} + 4 \cdot 1$$

Consumer A would want to split his income equally between *exes* and *whys*, that means that $x_A = \frac{1}{2} \cdot (\omega_A^{x,new} + 4)$ and $y_A = \frac{\frac{1}{2} \cdot (\omega_A^{x,new} + 4)}{4} = \frac{1}{8} \cdot (\omega_A^{x,new} + 4)$.
For consumer A to consume the *ideal* bundle $(10, 2.5)$, we need $\omega_A^{x,new} = 16$. In other words, the social planner would have to allocate a total of 16 units of *exes* available to consumer A before trade takes place.

iii. Represent the new competitive equilibrium (after the social planner has rearranged the initial allocation) in an Edgeworth box. A sketch will suffice as long as it shows the original and the rearranged initial endowments, the consumers' budget constraint, the equilibrium allocation, and the indifference curves going through the equilibrium allocation.

1 point



Note that the shaded area lies outside the Edgeworth box. It represents those missing 8 units of *exes* that would be necessary for the arrangement to work.

100%

You have reached the end of the exam.

100%

In case you have some extra time, consider the remaining questions for extra credit.

(e) Assume that the two consumers are allowed and able to trade with each other, and that *exes* is the numeraire. Also assume that Balázs has market power and acts as price-maker, while Attila acts as price-taker. Use the *original* initial endowment when answering this question.

i. Find the equilibrium of this pure exchange economy. In other words, find the price that Balázs would set for *whys*.

1 point

Consumer A solves the following constrained utility-maximization problem.

$$\max_{x_A,y_A} x_A^{\frac{1}{2}} y_A^{\frac{1}{2}}$$
$$\text{subject to } 1\cdot x_A + p\cdot y_A = 1\cdot 10 + p\cdot 1$$

Consumer A would want to split his income equally between *exes* and *whys*, that is $x_A = \frac{1}{2}\cdot(10+p)$ and $y_A = \frac{\frac{1}{2}\cdot(10+p)}{p} = \frac{1}{2p}\cdot(10+p)$.

Consumer B would want to maximize his utility that can be written in terms of p as follows:

$$\begin{aligned} u_B(x_B,y_B) &= x_B + 4y_B = (20-x_A) + 4\cdot(5-y_A) = \\ &= \left(20 - \frac{1}{2}\cdot(10+p)\right) + 4\cdot\left(5 - \frac{1}{2p}\cdot(10+p)\right) = \\ &= 20 - \frac{1}{2}\cdot(10+p) + 20 - \frac{2}{p}\cdot(10+p) = 33 - \frac{p}{2} - \frac{20}{p} \end{aligned}$$

$$\max_p \left(33 - \frac{p}{2} - \frac{20}{p}\right)$$
$$\text{FOC: } -\frac{1}{2} + \frac{20}{p^2} = 0$$
$$p^2 = 40 \to p = \sqrt{40} = 2\sqrt{10} \approx 6.32$$

ii. Find the allocation of *exes* and *whys* in this equilibrium.

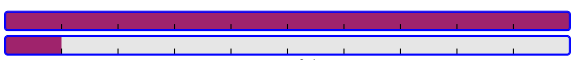
0.5 points

Using the formulae from before, $x_A = \frac{1}{2}\cdot(10+2\sqrt{10}) = 5+\sqrt{10} \approx 8.16$ and $y_A = \frac{\frac{1}{2}\cdot(10+p)}{p} = \frac{1}{4\sqrt{10}}\cdot(10+2\sqrt{10}) = \frac{\sqrt{10}}{4} + \frac{1}{2} \approx 1.29$. Consumer B consumes the rest: $x_B = 20 - x_A = 15 - \sqrt{10}$ and $y_B = 5 - y_A = \frac{18-\sqrt{10}}{4}$.

iii. Is the allocation of *exes* and *whys* in this equilibrium Pareto efficient? (Hint: Check whether it is located on the contract curve or not.)

0.5 points

For Pareto efficiency, we need $y_A = \frac{x_A}{4}$. Note that this is not satisfied when $y_A = 1.29$ and $x_A = 8.16$, as $4\cdot 1.29 = 5.16$.



110%