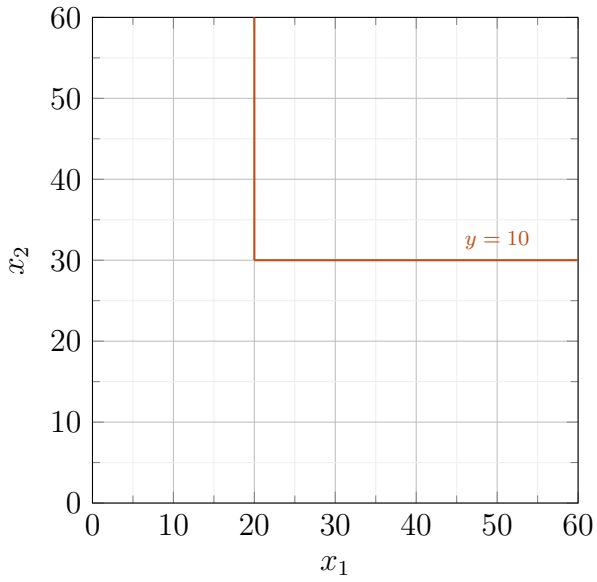
Quiz 9 - SOLUTION

You probably have been wondering how Attila earns the income that later he spends on *exes* and *whys*. Well, he is the proud owner of a highly profitable business. His secret lies in the technology: he essentially combines two (really cheap) inputs, x_1 and x_2, and sells the output y (at an exorbitant price).

Attila's factory is able to produce each unit of output, y, by combining two units of x_1 with three units of x_2. That is, Attila uses two inputs in fixed proportions.

- In the graph below, draw an isoquant showing the combinations of input that give exactly ten units of output.



- Write the Mathematical formula that describes Attila's technology.

  $y = F(x_1, x_2) = \min\{\frac{1}{2}x_1, \frac{1}{3}x_2\}$

- Is Attila's technology monotonic? Why?

  Take (x_1, x_2) and (x_1', x_2') such that $x_1' \geq x_1$ and $x_2' \geq x_2$. Then we have that $\min\{\frac{1}{2}x_1', \frac{1}{3}x_2'\} \geq \min\{\frac{1}{2}x_1, \frac{1}{3}x_2\}$, that is $F(x_1', x_2') \geq F(x_1, x_2)$. This means that Attila's technology is monotonic.

- Is Attila's technology convex? Why?

  Given that the isoquants that describe Attila's technology are convex curves, Attila's technology is convex.

- Does Attila's technology exhibit decreasing, constant, or increasing returns to scale? Why?

  Take $t > 1$ and consider $F(t \cdot x_1, t \cdot x_2)$.

  $$F(t \cdot x_1, t \cdot x_2) = \min\{t \cdot \tfrac{1}{2}x_1, t \cdot \tfrac{1}{3}x_2\} = t \cdot \min\{\tfrac{1}{2}x_1, \tfrac{1}{3}x_2\} = t \cdot F(x_1, x_2)$$

  This means that Attila's technology exhibits constant returns to scale.