Problem Set 2

$\boxed{1.}$ The village of Péteri, located just west of the town of Monor, has a population of $2\,000$ people. Péteri, very much like Monor, has a single public good, the *village park* and a single private good, *sör*. In Péteri, everyone's utility function is $U_i(X_i, Y) = X_i - 3\,000/Y$, where X_i is the number of bottles of *sör* consumed by i and Y is the size of the *village park* measured in square meters. The price of *sör* is $\$2$ per bottle. The cost of the *village park* to the village is $\$0.10$ per square meter. Everyone has an income of at least $\$1\,000$.

What is the Pareto efficient size for the *village park*?

2.5 points

$\boxed{2.}$ Consider a pure exchange economy with two agents (let's say, Attila and Balázs) and two goods (let's say, *exes* and *whys*):

- $u_A(x_A, y_A) = 2x_A + y_A,$
- $u_B(x_B, y_B) = x_B^{\frac{1}{4}} \cdot y_B^{\frac{1}{4}},$
- $w_A^x + w_B^x = 10,$
- $w_A^y + w_B^y = 20.$

(a) Find mathematically and represent graphically in an Edgeworth box the set of Pareto efficient allocation of this economy.

2.5 points

(b) Plot the utility possibilities frontier for this economy.

Hint: Recall that, with the help of the utility possibilities frontier, we are simply considering the set of Pareto efficient allocations from a different perspective. Follow the three steps below to complete this part of the exercise.

i. Use the feasibility constraints and your answer to the previous question, and write u_A and u_B (along the contract curve) as a function of x_A only.

ii. Given your answers to the previous point, write u_B as a function of u_A.

iii. Now you should be able to plot the utility possibilities frontier for this economy.

2.5 points

(c) Consider the following social-welfare maximization problem:

$$\max_{x_A, y_A, x_B, y_B} \lambda_A \cdot u_A(x_A, y_A) + \lambda_B \cdot u_B(x_B, y_B)$$
$$\text{subject to } x_A + x_B = 10$$
$$y_A + y_B = 20$$

The function $\lambda_A \cdot u_A(x_A, y_A) + \lambda_B \cdot u_B(x_B, y_B)$ is called *social welfare function*. The parameters λ_A and λ_B are called *Pareto weights*.

i. Write the first-order conditions and argue that any solution to the social-welfare maximization problem is Pareto efficient. Remember that $u_A(x_A, y_A) = 2x_A + y_A$ and $u_B(x_B, y_B) = x_B^{\frac{1}{4}} \cdot y_B^{\frac{1}{4}}$.

2.5 points

ii. Consider the allocation in which agent A consumes 2 units of *exes* and 4 units of *whys*, and agent B consumes the rest.

A. Argue that this is a Pareto efficient allocation.

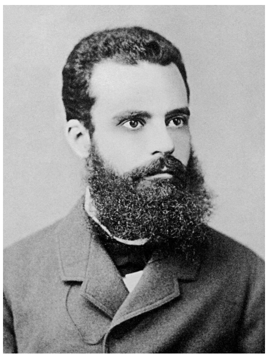
1 point

B. How much should λ_A and λ_B be so that the solution of the social-welfare maximization problem is exactly the above allocation?

1 point

iii. Use the graph with the utility possibilities frontier and represent the above social-welfare maximization problem.

1 point



$\boxed{3.}$ Attila and Balázs are two *homo economicus* engaged in exchange over *exes* (x) and *whys* (y).[1] Their utility functions are given by the following:

$$\begin{aligned} \text{Attila}: \quad & u_A(x_A, y_A) = x_A^{\frac{1}{5}} y_A^{\frac{1}{5}} \\ \text{Balázs}: \quad & u_B(x_B, y_B) = 0.05x_B + 0.05y_B \end{aligned}$$

Attila has an initial endowment of 5 *exes* and 15 *whys*. Balázs has an initial endowment of 15 *exes* and 5 *whys*.

(a) What is Attila's marginal rate of substitution of *exes* for *whys* (*MRS*$_A$)? What is Balázs's marginal rate of substitution of *exes* for *whys* (*MRS*$_B$)?

1 point

(b) Using the marginal rates of substitution, find the equation for the Pareto-efficient curve.

In this exercise, you are allowed to focus all allocations in which both of the decision-makers consumes strictly positives amounts of both goods. In other words, you can ignore the sides of the Edgeworth box.

1 point

(c) How much is Attila's utility at his initial allocation, (x_A^I, y_A^I)?

0.5 point

(d) How much is Balázs's utility at his initial allocation, (x_B^I, y_B^I)?

0.5 points

[1]This exercise has been adapted from the draft version of Bowles, S. & Halliday, S. (2022) *Microeconomics: Competition, Conflict and Coordination, Oxford University Press.*

(e) Is the initial allocation of goods Pareto efficient?

0.5 points

(f) Graph the Edgeworth box depicting the exchange between Attila and Balázs. Show their initial allocations, their initial indifference curves, the Pareto-efficient curve, and each consumer's most preferred allocation in the Pareto-improving lens (that is the set of Pareto improvements upon the initial allocation).

2.5 points

(g) Instead of his previous utility function, assume that Balázs's utility function is given by the following:

$$v_B\left(u_A(x_A, y_A), u_B(x_B, y_B)\right) = 0.05x_B + 0.05y_B + \frac{1}{2}x_A^{\frac{1}{5}}y_A^{\frac{1}{5}}$$

Explain how this function is different from Balázs's utility function when he was a *homo economicus*.

Hint: Explain how this function captures the idea of *altruism* and why we could call Balázs a *homo generosus*.

1 point

(h) What is Balázs's marginal rate of substitution of *exes* (x_B) for *whys* (y_B)?

Hint: Use the feasibility constraints (on x and y) to eliminate x_A and y_A from Balázs's utility function, and write v_B as a function of x_B and y_B.

2 points

(i) Graph a new Edgeworth box depicting the exchange between Attila and Balázs where Balázs is *homo generosus* and Attila is *homo economicus*. Show their initial allocations, the Pareto-efficient curve, and each consumer's most preferred allocation in the Pareto-improving lens.

Briefly explain the differences between this Edgeworth box and your previous Edgeworth box when both players were *homo economicus*.

Hint: This is the new, the most interesting, and also the most challenging part of the exercise. For that reason, you may want to consider the following pieces of advice.

- Balázs's indifference curves will look like circles around an ideal point. First, find this ideal point, by maximizing v_B in x_B and y_B. Then, you can compute the marginal rate of substitution for v_B.
- You are not required to compute anything else in this part of the exercise, but your graph should illustrate the new situation. It is enough is you create a rough sketch for the Edgeworth box. For a more precise picture, you may want to use Mathematica / WolframAlpha (or some other software / website) and the following two commands.
  - ContourPlot[0.05*(x + y) + 0.5*((20 - x)^ 0.2)*((20 - y)^ 0.2), {x, 0, 20}, {y, 0, 20}]
  - Maximize[{0.05*(x + y) + 0.5*((20 - x)^ 0.2)*((20 - y)^ 0.2), x >= 0, y >= 0, x <= 20, y <= 20}, {x, y}]

3 points

generosus (Latin)

Origin & history

From *genus* ("birth, origin")

Adjective

generōsus (*feminine* **generōsa**, *neuter* **generōsum**)

1. well-born, noble
2. superior, excellent
3. (*figuratively*) generous, magnanimous
4. (*figuratively*) dignified, honorable

Related words & phrases

- generātor
- generōsē
- generō

Descendants

French: *généreux*, généreuse