1. A monopolist has a constant marginal cost, and it is facing a linear demand curve. If the government imposes a quantity tax of $\$4$ per unit of output, how much does the price rise?

Let the inverse market demand be $p = a - b \cdot q$, and let c denote the monopolist's (constant) marginal cost. Recall that the profit-maximizing quantity then is $q^* = \frac{a-c}{2b}$. If the government imposes a quantity tax of $\$4$ per unit of output, the marginal cost is going to be $c' = c + 4$ and the profit-maximizing quantity $q' = \frac{a-c-4}{2b}$. In other words, the quantity changes by $\Delta q = -\frac{4}{2b} = -\frac{2}{b}$ units.

The change in price is $\Delta p = -b \cdot \Delta q = 2$.

2. The village of Péteri, located just west of the town of Monor, has a population of $2\,000$ people. Péteri, very much like Monor, has a single public good, the *village park* and a single private good, *sör*. In Péteri, everyone's utility function is $U_i(X_i, Y) = X_i - 3\,000/Y$, where X_i is the number of bottles of *sör* consumed by i and Y is the size of the *village park* measured in square meters. The price of *sör* is $\$2$ per bottle. The cost of the *village park* to the village is $\$0.10$ per square meter. Everyone has an income of at least $\$1\,000$.

What is the Pareto efficient size for the *village park*?

The *village park* is a public good. We can use the following condition to find its Pareto efficient size.

$$\sum_{i=1}^{2\,000} |MRS_i| = \frac{MC(Y)}{p_x}$$

Note that $|MRS_i| = \frac{\partial U_i/\partial Y}{\partial U_i/\partial X_i} = \frac{-3\,000/Y^2 \cdot (-1)}{1} = \frac{3\,000}{Y^2}$, $MC(Y) = 0.1$ and $p_X = 2$. The above condition then leads to:

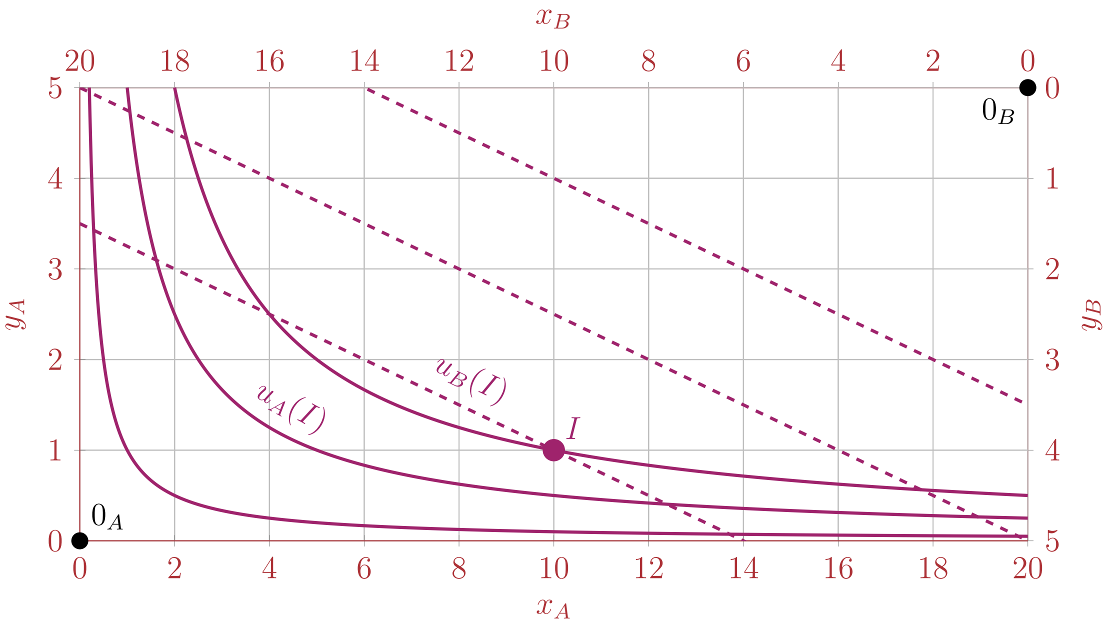
$$\begin{aligned} 2\,000 \cdot \frac{3\,000}{Y^2} &= 0.05 \\ Y &\approx 10\,954. \end{aligned}$$

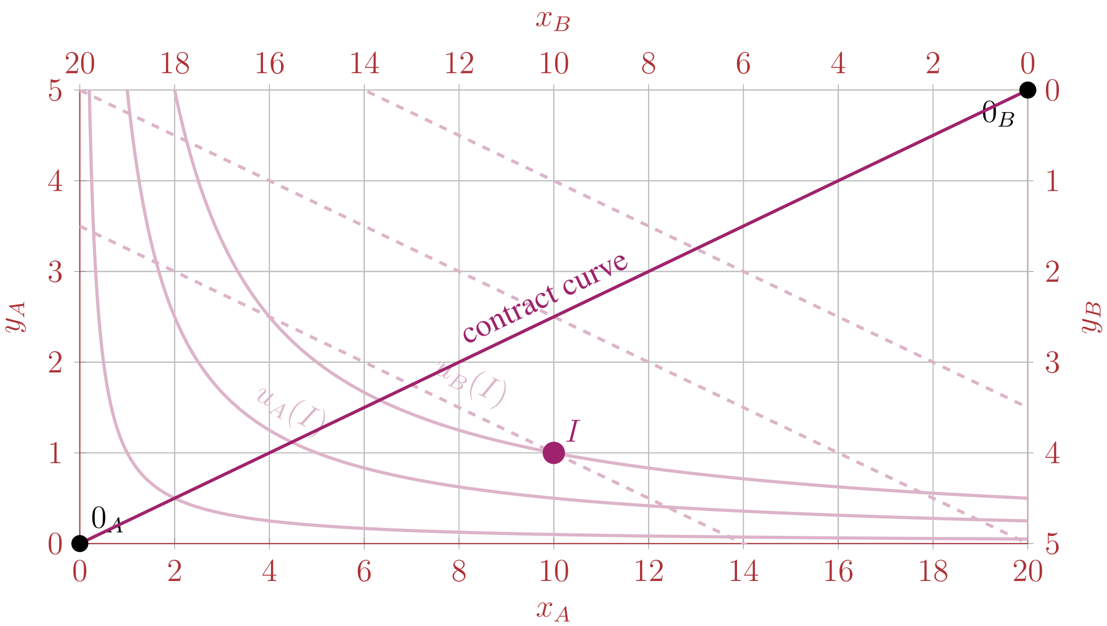
Note that the inhabitants of the village all have enough money to contribute towards a *village park* with that size.

$\boxed{3.}$ Consider a pure exchange economy with two consumers (let's say, *Attila* and *Balázs*) and two goods (let's say, *exes* and *whys*):

- $u_A(x_A, y_A) = x_A^{\frac{1}{2}} y_A^{\frac{1}{2}}$,
- $u_B(x_B, y_B) = x_B + 4y_B$,
- Agent A initially owns 10 units of *exes* and 1 unit of *whys*: $\omega_A^x = 10$, $\omega_A^y = 1$.
- Agent B initially owns 10 units of *exes* and 4 units of *whys*: $\omega_B^x = 10$, $\omega_B^y = 4$.

(a) Represent this pure exchange economy with the help of an Edgeworth box (a sketch will suffice!), and find (mathematically) the contract curve.





In the interior of the Edgeworth box, Pareto-efficient allocations can be characterized by the following tangency condition:

$$MRS_A = MRS_B,$$

$$-\frac{\frac{1}{2}x_A^{-\frac{1}{2}}y_A^{\frac{1}{2}}}{\frac{1}{2}x_A^{\frac{1}{2}}y_A^{-\frac{1}{2}}} = -\frac{y_A}{x_A} = -\frac{1}{4},$$

$$y_A = \frac{x_A}{4}.$$

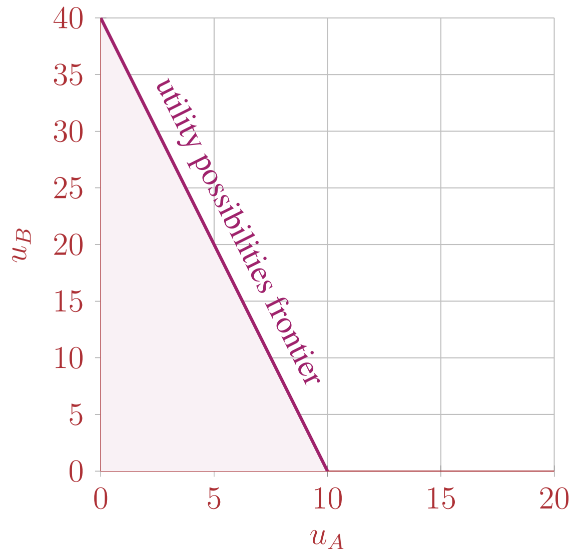
(b) Find (mathematically) and represent graphically the utility possibilities set for this pure exchange economy.

The Pareto efficient allocations must satisfy that $y_A = \frac{x_A}{4}$. With this restriction, we can rewrite the agents utility function.

$$u_A = x_A^{\frac{1}{2}}y_A^{\frac{1}{2}} = \frac{1}{2}x_A$$
$$x_A = 2u_A$$

$$u_B = x_B + 4y_B = (20 - x_A) + 4 \cdot (5 - y_A) = 40 - 2x_A$$

$$u_B = 40 - 4u_A$$



(c) Assume that the two consumers are allowed and able to trade with each other, and that *exes* are the numeraire. Also assume that both consumers act as price-takers.

i. Find the competitive equilibrium of this pure exchange economy. In other words, find the equilibrium price of *whys*.

The unit price of *exes* is equal to 1. Let p denote the unit price of *whys*.

Note that trade should be mutually beneficial and lead to an allocation on the contract curve.

For consumer B to consume positive amounts of both goods, we must have that $MRS_B = \frac{p_x}{p_y} = \frac{1}{p}$. Given that $MRS_B = \frac{1}{4}$, $p = 4$.

That means that the competitive equilibrium of this pure exchange economy is $(1, 4)$.

ii. Find the allocation of *exes* and *whys* in the competitive equilibrium.

Consumer A would want to split his income equally between *exes* and *whys*. His initial endowment is worth $1 \cdot 10 + 4 \cdot 1 = 14$. Therefore, he would want to consume 7 units of *exes* and $\frac{7}{4} = 1.75$ units of *whys*.

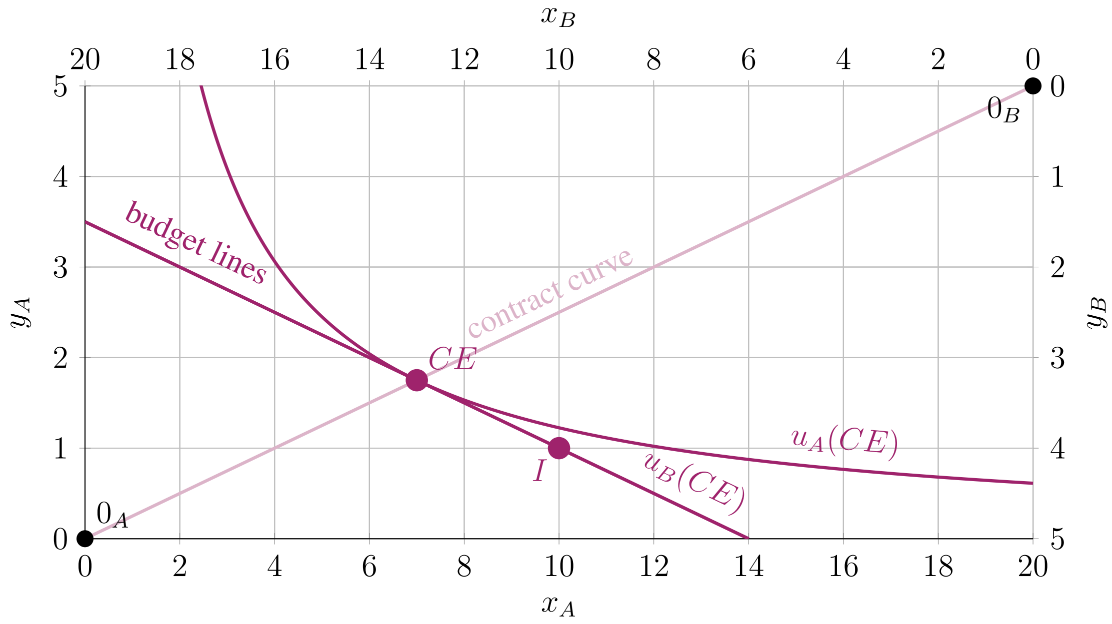
Consumer B would then consume $20 - 7 = 13$ units of *exes* and $5 - \frac{7}{4} = \frac{13}{4} = 3.25$ units of *whys*.

Consumer B's initial endowment is worth $1 \cdot 10 + 4 \cdot 4 = 26$. Note that that is exactly the value of the bundle that he would like to consume: $1 \cdot 13 + 4 \cdot \frac{13}{4} = 26$.

iii. Is the allocation of *exes* and *whys* in the competitive equilibrium Pareto efficient? (Hint: Check whether it is located on the contract curve or not.)

For Pareto efficiency, we need $y_A = \frac{x_A}{4}$. Note that this is satisfied when $y_A = \frac{7}{4}$ and $x_A = 7$.

iv. Represent the competitive equilibrium in an Edgeworth box. A sketch will suffice as long as it shows the initial endowment, the consumers' budget constraint, the equilibrium allocation, and the indifference curves going through the equilibrium allocation.



(d) The social planner looking over this pure exchange economy would like to maximize the following social welfare function: $SWF(u_A, u_B) = \min\{4u_A, u_B\}$.

i. Find the allocation in the utility possibilities set for this pure exchange economy that maximizes the social welfare function.

The social planner solves $max_{4u_A,u_B} \min\{u_A, u_B\}$ subject to $4u_A + u_B = 40$.

Note that at the optimal point $4u_A = u_B$. Therefore $4u_A + u_B = 8u_A = 40$ and $u_A^{ideal} = 5$ and $u_B^{ideal} = 20$.

ii. How should the social planner rearrange the initial endowment of *exes* (*exes* only!) so that this pure exchange economy reaches the allocation that maximizes the social welfare function through the two consumers trading with each other?

Remember that on the contract curve $x_A = 2u_A$ and $y_A = \frac{x_A}{4}$. Therefore, $x_A^{ideal} = 2u_A^{ideal} = 10$ and $y_A^{ideal} = \frac{10}{4} = 2.5$.

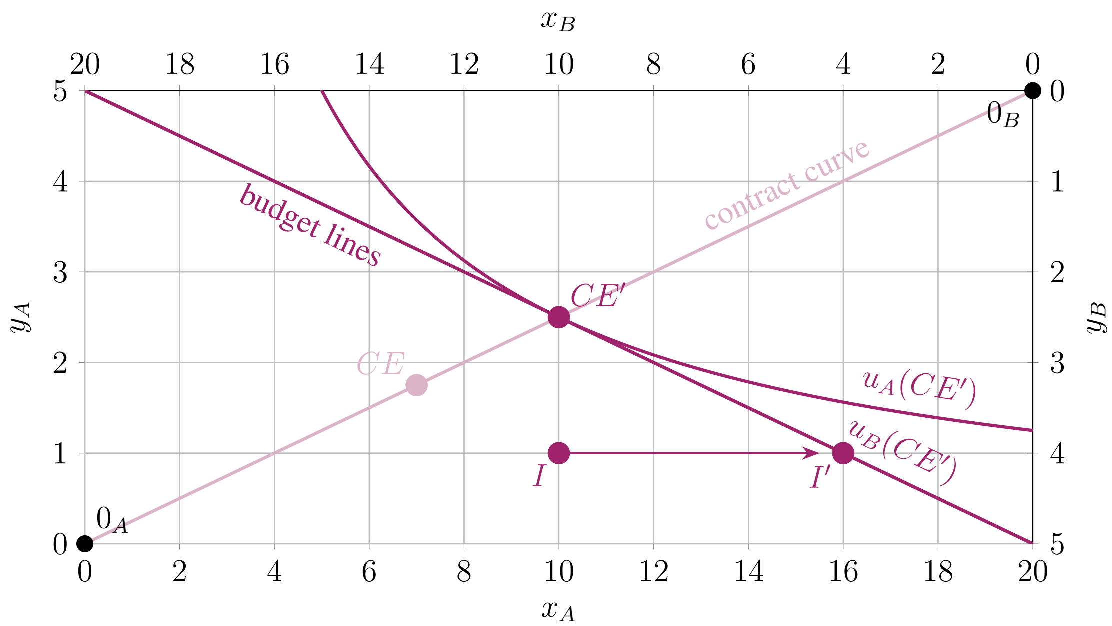
For consumer B to consume positive amounts of both goods, we still must have that $p = 4$.

Let $\omega_A^{x,new}$ denote consumer A's endowment of *exes* after the rearrangement. Consumer A then solves the following constrained utility-maximization problem.

$$\max_{x_A, y_A} x_A^{\frac{1}{2}} y_A^{\frac{1}{2}}$$
$$\text{subject to } 1 \cdot x_A + 4 \cdot y_A = 1 \cdot \omega_A^{x,new} + 4 \cdot 1$$

Consumer A would want to split his income equally between *exes* and *whys*, that means that $x_A = \frac{1}{2} \cdot (\omega_A^{x,new} + 4)$ and $y_A = \frac{\frac{1}{2} \cdot (\omega_A^{x,new}+4)}{4} = \frac{1}{8} \cdot (\omega_A^{x,new} + 4)$.
For consumer A to consume the *ideal* bundle $(10, 2.5)$, we need $\omega_A^{x,new} = 16$. In other words, the social planner would have to allocate a total of 16 units of *exes* available to consumer A before trade takes place.

iii. Represent the new competitive equilibrium (after the social planner has rearranged the initial allocation) in an Edgeworth box. A sketch will suffice as long as it shows the original and the rearranged initial endowments, the consumers' budget constraint, the equilibrium allocation, and the indifference curves going through the equilibrium allocation.



Note that the shaded area lies outside the Edgeworth box. It represents those missing 8 units of *exes* that would be necessary for the arrangement to work.

(e) Assume that the two consumers are allowed and able to trade with each other, and that *exes* is the numeraire. Also assume that Balázs has market power and acts as price-maker, while Attila acts as price-taker. Use the *original* initial endowment when answering this question.

i. Find the equilibrium of this pure exchange economy. In other words, find the price that Balázs would set for *whys*.

Consumer A solves the following constrained utility-maximization problem.

$$\max_{x_A, y_A} x_A^{\frac{1}{2}} y_A^{\frac{1}{2}}$$
$$\text{subject to } 1 \cdot x_A + p \cdot y_A = 1 \cdot 10 + p \cdot 1$$

Consumer A would want to split his income equally between *exes* and *whys*, that is $x_A = \frac{1}{2} \cdot (10 + p)$ and $y_A = \frac{\frac{1}{2} \cdot (10+p)}{p} = \frac{1}{2p} \cdot (10 + p)$.

Consumer B would want to maximize his utility that can be written in terms of p as follows:

$$\begin{aligned} u_B(x_B, y_B) = x_B + 4y_B &= (20 - x_A) + 4 \cdot (5 - y_A) = \\ &= \left(20 - \frac{1}{2} \cdot (10 + p)\right) + 4 \cdot \left(5 - \frac{1}{2p} \cdot (10 + p)\right) = \\ &= 20 - \frac{1}{2} \cdot (10 + p) + 20 - \frac{2}{p} \cdot (10 + p) = 33 - \frac{p}{2} - \frac{20}{p} \end{aligned}$$

$$\max_p \left(33 - \frac{p}{2} - \frac{20}{p}\right)$$
$$\text{FOC: } -\frac{1}{2} + \frac{20}{p^2} = 0$$
$$p^2 = 40 \rightarrow p = \sqrt{40} = 2\sqrt{10} \approx 6.32$$

ii. Find the allocation of *exes* and *whys* in this equilibrium.

Using the formulae from before, $x_A = \frac{1}{2} \cdot (10 + 2\sqrt{10}) = 5 + \sqrt{10} \approx 8.16$ and $y_A = \frac{\frac{1}{2} \cdot (10+p)}{p} = \frac{1}{4\sqrt{10}} \cdot (10 + 2\sqrt{10}) = \frac{\sqrt{10}}{4} + \frac{1}{2} \approx 1.29$. Consumer B consumes the rest: $x_B = 20 - x_A = 15 - \sqrt{10}$ and $y_B = 5 - y_A = \frac{18 - \sqrt{10}}{4}$.

iii. Is the allocation of *exes* and *whys* in this equilibrium Pareto efficient? (Hint: Check whether it is located on the contract curve or not.)

For Pareto efficiency, we need $y_A = \frac{x_A}{4}$. Note that this is not satisfied when $y_A = 1.29$ and $x_A = 8.16$, as $4 \cdot 1.29 = 5.16$.