Endogenous Economic Growth

出所: http://blogs.worldbank.org/psd/why-china-ahead-india-fascinating-analysis-amartya-sen

Harrod-Domar Model

- Capital accumulation and growth
- Assumptions

① Close economy (No trade)

② Capital and Labor used in fixed proportion (No substitution)

③ Capital is the limited factor but Labor is unlimited supply (Population growth does not matter)

④ Constant return to scale for two factors

⑤ Technology: Fixed quantity of additional capital leads to fixed proportional increase in output

($k = \Delta K / \Delta Y$=Incremental Capital Output Ratio: ICOR)

→No marginal decrease in capital

Review: H-D Model

- ICOR: $k=\Delta K/\Delta Y$, *Then* $\Delta Y=1/k\,\Delta K$
- Saving function: $S=sY$ (*s:Saving ratio,* $s=S/Y$
- Investment function: $I=\Delta K=S$
- Then Growth rate of $y=\Delta Y/Y$

  So, $y=\Delta Y/Y=1/k^{*}\Delta K/Y=1/k^{*}S/Y=s/k$
- Therefore, y(Growth rage) ↑ if s (Saving ratio) ↑ or ICOR (k) ↓
- If there is depreciation of capital, $I\equiv\Delta K+\delta$(depreciation rate)$K=S$
- Then, $y=s/k-\delta$ (Zero or negative growth happens if s is insufficient to cover depreciation

2008 Country Name	Gross fixed capital formation (% of GDP)	GDP growth (annual %)	ICOR
India	32.9	4.9	6.7
South Africa	22.6	3.6	6.3
China	40.8	9.6	4.2
Brazil	19.1	5.2	3.7

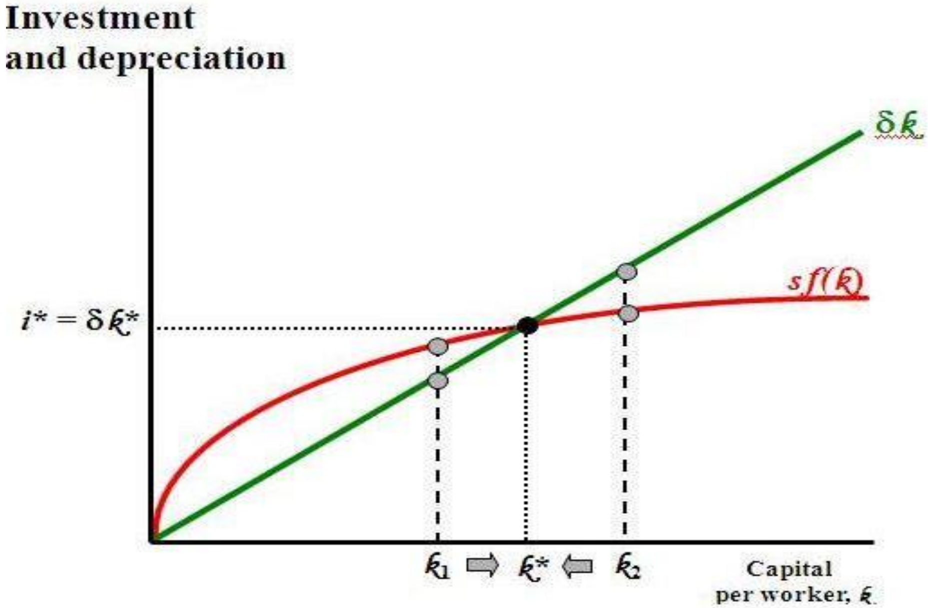
Source: World Bank, World Development Indicators.

Solow Model

- Long-term equilibrium and Income convergence
- Assumptions：
    ① Closed economy (No trade)
    ② Income＝C:Consumption ＋S:Saving
    ③ Population=Laborers（Increases at constant rate)
    ④ Technology= Constant (No innovation)
- Gross production=Gross income
- ***Diminishing Marginal product of capital*** (If capital stock per capita *k* is smaller, the production increases faster but if larger *k* then slower growth
- Capital stock per capita k ↓
  1) By capital wastage
  2) By L (Population) ↑

Solow model

4. The Steady State (k^*)
Long-run equilibrium of the economy



- At **k***:
Investment=depreciation, capital won't change
- Below k* (**k_1**):
investment >depreciation, the capital stock grows.
- Above k* (**k_2**):
depreciation > investment, the capital stock shrinks.

$\Delta k = sf(k) - \delta k$; In the steady state capital is not changing→
$\Delta k=0$ → $sf(k^*) - \delta k^* = 0$ → $\boldsymbol{sf(k^*) = \delta k^*}$

Solow equilibrium

- Pace of *k* increase falls, while the loss for *k* goes up with depreciation and population size
- Equilibrium *k**: *k* increases until *k** but after *k* loss for k surpasses then goes back to k**
- However, then per capita income of *y* converges into *k*, then no y increases*
- Change of technology is important (Exogenous)

How to measure technological progress?: Total Factor Productivity (TFP)

- GDP=Constant*Technology level*Capital stock $(K)^{1/3}$*Labor$(L)^{2/3}$

⇒Growth rate=Technology progress rate
+1/3*Capital stock growth rate + 2/3 Labor* growth rate, THEN,

Technology progress (Total Factor Productivity: TFP) =Growth rate- 1/3*Capital stock growth rate- 2/3 Labor*growth rate, OR,

Technology level=GDP/$K^{1/3}$*$L^{2/3}$

AK Model

- Y:GDP, K: Capital Stock, C:Consumption, I:Investment, S: Saving, $\bar{s}$: Saving rate, A:Productivity of Capital（Yt/Kt:Constant）、$\bar{s}$=St/Yt（t: time）
- No trade (export, import)
- ***Production Function*** $:Y_t=AK_t$ （How much input in K will get how much Production Y）

⇨ If A (Constant) is larger, better technology/ efficiency

1/A: Incremental Capital Output Ratio: ICOR)

From Demand side, $Yt=Ct+It$（Disregarding the government expenditure and net trade）

Investment adds capital stock：

Capital accumulation $\Delta K_{t+1}=K_{t+1}-K_t=I_t$

From Distribution side, $Yt=Ct+St$, Saving is decided by $S_t=\bar{s}Y_t$

（Saving rate: $0\leq \bar{s}\leq 1$）

⇨ $\Delta K_{t+1}=K_{t+1}-K_t=I_t=S_t=\bar{s}Y_t=\bar{s}AK_t$ ⇨ $\Delta K_{t+1}/K_t=\bar{s}A$

Liner model and Solow model: Marginal product of capital

- If we consider AK model by per capita Y：y_t,
  $y_t = Y_t/L_t = AK_t/L_t = Ak_t$ ⇨Linear
- ≠Solow model： *Marginal product of capital diminishes* (Without labor input increase, *k* increase will diminish contribution for *Y*)

⇨ Capital deepening: *k increase, then y increase, but* A(y/k) drops

⇨AK model：Growth rate is by $\bar{s}A$

Implications from AK model

- $\triangle K_{t+1} / K_t = \bar{s}A$ $Y_t = AK_t$ where A is constant

 Then, Growth rate of Y is the growth rate of K:

 $\triangle Y_{t+1}/Y_t$ (growth rate) $= \bar{s}A$ ($\triangle Y_{t+1} = A \triangle K_{t+1}$, then,

 $\triangle Y_{t+1}/Y_t = \triangle K_{t+1}/K_{Kt} = \bar{s}A$)

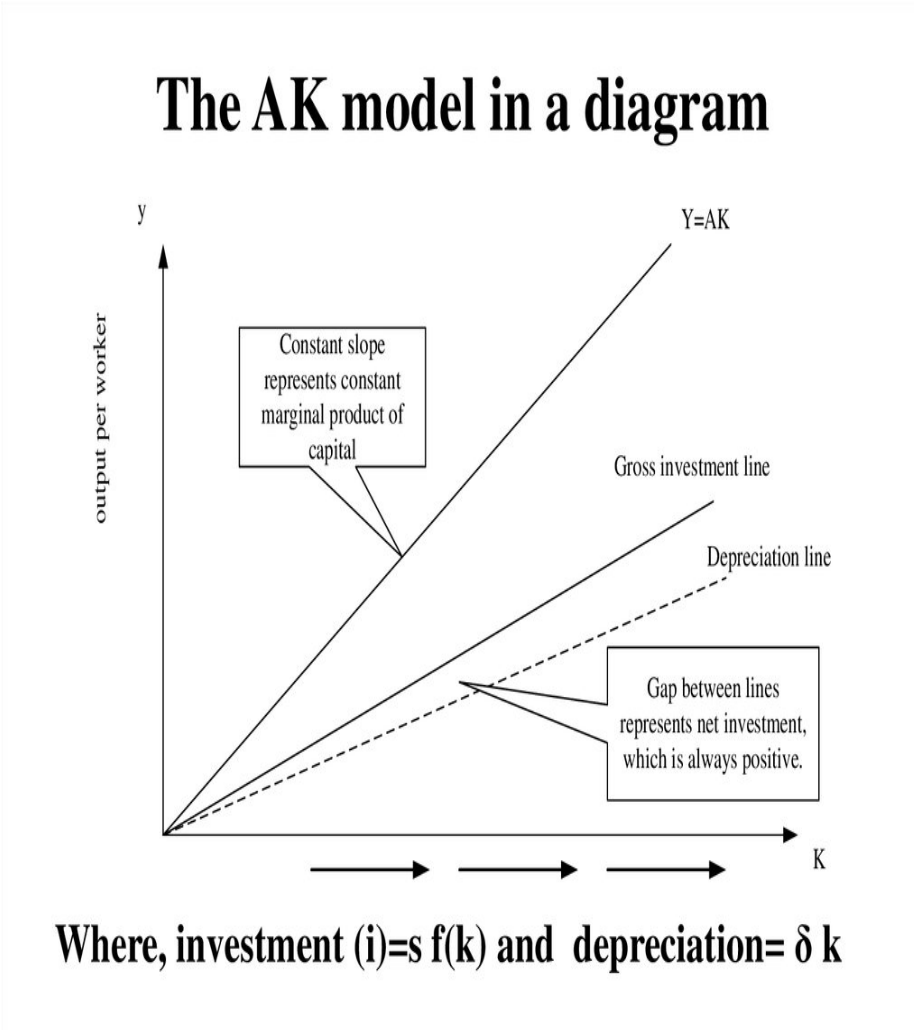
⇨Larger $\bar{s}$ realizes higher growth

⇨larger A boots growth (*Productivity←Technology*)

- Capital stock: K includes not only physical capital but others like human capital, knowledge)
- AK model: Saving rate and Capital productivity are to decides growth

⇨ Then how we explain economic catch-up?

AK model



- Production Function : $Y_t=AK_t$ （How much input in K will get how much Production Y）

⇨ If A (Constant) is larger, better technology/ efficiency (1/A: Incremental Capital Output Ratio: ICOR)

- From Demand side, $Yt=Ct+It$
- From Distribution side, $Yt=Ct+St$, and Saving is decided by $S_t=\bar{s}Yt$

（Saving rate: $0\leq \bar{s}\leq 1$）

⇨ $\triangle K_{t+1}=K_{t+1}-K_t=I_t=S_t=\bar{s}Y_t=\bar{s}AK_t$ ⇨ $\triangle K_{t+1}/K_t=\bar{s}A$

How we see “technology”: AK model

- Solow model : Technology is *Exogenous* $\neq$ *Endogenous*), but is it so?
- What, if *K* includes technological knowledge
- AK model: If Capital (*K*) includes *technological knowledge, skills, human capitals*, *K does not follow declining “marginal productivity of capital” in Solow model*)

$\Rightarrow$Linear growth, without equilibrium

How we see “technology”: Romer model

- Romer model: Increasing knowledge, skill (=Innovation) promotes growth
- Newly added knowledge: Constant*number of researchers*present knowledge
- Technology progress rate (equilibrium per capita GDP growth)=Constant(Newly added knowledge/ Present knowledge stock: R&D efficiency rate) *Researchers

→ Long-run per capita GDP (y) growth depends on *R&D efficiency rate and Population (larger researchers by larger population)*

Implications and Discussions from Romer model

- Nature of Knowledge: No competition and exclusiveness (IPR protection/ transaction is limited) ≠ Goods

⇨Externality exists, Market failure (Social optimum unattained by the market)

⇨Rationales for government intervention

(Human capital growth, Innovation subsidies/ supports)

⇨Innovation in developing economy includes copying/ reverse engineering (government supports?)

⇨Human capital volume: Large population economies for scale effects?

Implications and Discussions from Romer model(2)

- However, easy copying time may be over as R&D level goes higher, making additional knowledge becomes difficult：Diminishing growth rate (Technology progress rate)

⇨Precisely not R&D population but the *growth rate of R&D population* may be more important（Jones）

Solow model and Endogenous model

(Source: Textbook P.239 in the second edition)

Model structure	Solow model	Endogenous growth model
Production function	$Y = Af(K, L)$	$Y = f(K, A(L_A)L_Y)$
Technological change ΔA	Hicks neutral, exogenous	Labor-saving, endogenous
Origin of technological change	International public good	Firm investment of L_Y in R&D
Market structure	Perfect competition	Monopoly power (patents)
Returns to scale	Constant	Increasing due to $\Delta A = A\delta L_A$
Model predictions		
Income growth across countries	Convergence	Divergence
Predicted international labor flows	From MDCs to LDCs	From LDCs to MDCs

Suggested textbooks

- Timothy Taylor（2012）*The Instant Economist: Everything You Need to Know About How the Economy Works*、Plume original version
- Timothy Taylor, Steven A. Greenlaw, David Shapiro(2017*) Principles of Macroeconomics 2e),* OpenStax
- Dilip Mookherjee, Debraj Ray (2001) *Readings in the Theory of Economic Development,* John Wiley & Sons
- Debraj Ray (1998) *Development Economics*, Princeton University

Suggested Readings

- Romer, Paul (1990) "Endogenous Technological Change", Journal of Political Economy 98(5):s.71-102.
- Akerlof, George and Robert Shiller (2009) "Animal Spirits: How human psychology drives the economy, and Why it matters for global capitalism", Princeton University Press.
- Paul Krugman (1994) "The Myth of Asia's Miracle," Foreign Affairs 73 (November/December 1994) : 6, 62-79.