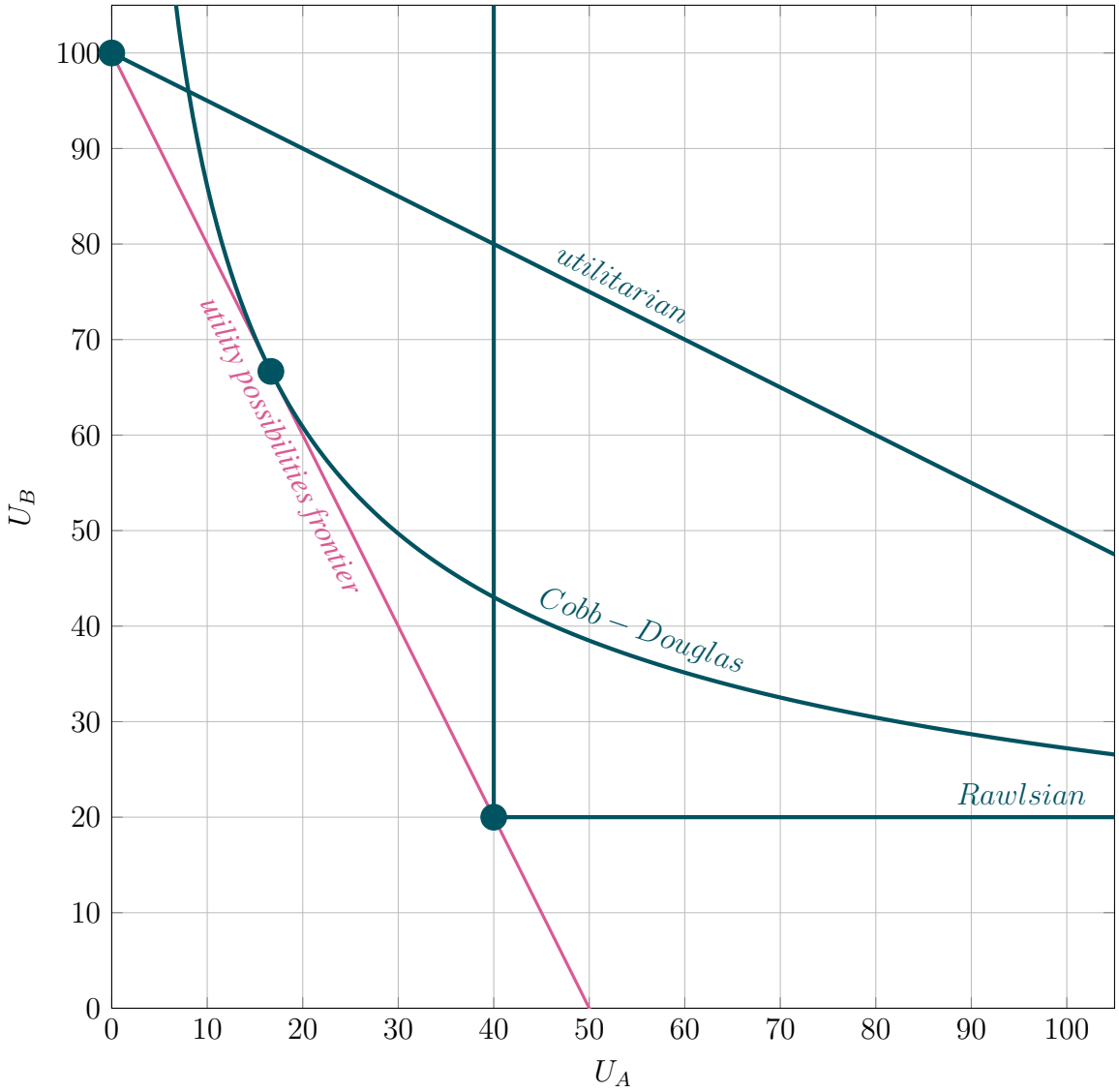
Social welfare functions

Suppose that the *utility possibilities frontier* for two individuals, Attila and Balázs, is given by $2U_A + U_B = 100$.



1. In the graph above, plot the utility possibilities frontier.

2. Consider the following four social welfare functions. How much would U_A and U_B have to be in order to maximize each of these social welfare functions? On the graph above, represent each social welfare function with an *isowelfare curve* and mark the social maximum. Do not forget to label them.

   (a) *Rawlsian social welfare function*: $W_{Rawlsian}(U_A, U_B) = \min\{U_A, 2U_B\} \rightarrow U_A = 40, U_B = 20$.

   (b) *Utilitarian social welfare function*: $W_{utilitarian}(U_A, U_B) = U_A + 2U_B \rightarrow U_A = 0, U_B = 100$.

   (c) *Cobb-Douglas type social welfare function*: $W_{Cobb-Douglas}(U_A, U_B) = U_A \cdot U_B^2 \rightarrow U_A = \frac{50}{3}, U_B = \frac{200}{3}$.

$$\max_{U_A, U_B} \quad U_A \cdot U_B^2 \text{ subject to } 2U_A + U_B = 100$$

$$FOC: \quad \frac{\partial W}{\partial U_A} \Big/ \frac{\partial W}{\partial U_B} = \frac{2}{1}$$

$$\frac{U_B^2}{U_A \cdot 2U_B} = \frac{U_B}{2U_A} = 2 \implies U_B = 4U_A \implies 2U_A + 4U_A = 100 \implies U_A = \frac{100}{6}$$