Exercise 24.5

In Gomorrah, New Jersey, there is only one newspaper, the *Daily Calumny*. The demand for the paper depends on the price and the amount of scandal reported. The demand function is $Q = 15S^{\frac{1}{2}}P^{-3}$, where Q is the number of issues sold per day, S is the number of column inches of scandal reported in the paper, and P is the price. Scandals are not a scarce commodity in Gomorrah. However, it takes resources to write, edit, and print stories of scandal. The cost of reporting S units of scandal is $\$10S$. These costs are independent of the number of papers sold. In addition it costs money to print and deliver the paper. These cost $\$0.10$ per copy and the cost per unit is independent of the amount of scandal reported in the paper. Therefore the total cost of printing Q copies of the paper with S column inches of scandal is $\$10S + 0.10Q$.

(a) Calculate the price elasticity of demand for the Daily Calumny.

$$\epsilon_P = \frac{dQ}{dP} \cdot \frac{P}{Q} = 15S^{\frac{1}{2}}(-3)P^{-4} \cdot \frac{P}{15S^{\frac{1}{2}}P^{-3}} = -3.$$

Does the price elasticity depend on the amount of scandal reported? No.

Is the price elasticity constant over all prices? Yes.

(b) Remember that $MR = P \cdot (1 + \frac{1}{\epsilon_P})$. To maximize profits, the *Daily Calumny* will set marginal revenue ϵ_P equal to marginal cost. Solve for the profit-maximizing price for the *Calumny* to charge per newspaper.

$$MR = P \cdot (1 + \frac{1}{-3}) = \frac{2}{3}P$$
$$MC = 0.10$$
$$MR = MC \rightarrow \frac{2}{3}P = 0.10 \rightarrow P^* = 0.15$$

When the newspaper charges this price, the difference between the price and the marginal cost of printing and delivering each newspaper is $P - MC = 0.15 - 0.10 = 0.05$.

(c) If the *Daily Calumny* charges the profit-maximizing price and prints 100 column inches of scandal, how many copies would it sell? (Round to the nearest integer.)

$$Q(S = 100, P = 0.15) = 15 \cdot 100^{\frac{1}{2}} \cdot 0.15^{-3} = 44\,444.$$

Write a general expression for the number of copies sold as a function of S:

$$Q(S) = Q(S, P = 0.15) = 15 \cdot S^{\frac{1}{2}} \cdot 0.15^{-3} = 4\,444.44S^{\frac{1}{2}}.$$

(d) Assuming that the paper charges the profit-maximizing price, write an expression for profits as a function of Q and S.

$$\Pi(Q, S) = \Pi(Q, S, P = 0.15) = 0.15Q - 0.10Q - 10S = 0.05Q - 10S.$$

Using the solution for $Q(S)$ that you found in the last section, substitute $Q(S)$ for Q to write an expression for profits as a function of S alone.

$$\Pi(S) = \Pi(Q(S), S, P = 0.15) = 0.05 \cdot (4\,444.44S^{\frac{1}{2}}) - 10S = 222.22S^{\frac{1}{2}} - 10S.$$

(e) If the *Daily Calumny* charges its profit-maximizing price, and prints the profit-maximizing amount of scandal, how many column inches of scandal should it print?

$$\max_S \Pi(S)$$
$$\frac{d}{dS}\Pi(S) = 222.22 \cdot \frac{1}{2} \cdot S^{-\frac{1}{2}} - 10 = 0$$
$$S^* = 123.456.$$

How many copies are sold?

$$Q^* = Q(S = 123.456, P = 0.15) = 15 \cdot 123.456^{\frac{1}{2}} \cdot 0.15^{-3} \approx 49382.$$

And what is the amount of profit for the *Daily Calumny* if it maximizes its profits?

$$\Pi^* = Q(S = 123.456) = 222.22 \cdot 123.456^{\frac{1}{2}} - 10 \cdot 123.456 \approx 1234.5.$$