Independent Set Decision Problem

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*Abstract* - Must be in English.

# I. Introduction

In mathematics, graph theory is the study of graphs, which are mathematical structures used to model pairwise relations between objects. Graphs can be used to model many types of relations and processes in the real world, ranging from social networks and transportation systems to biological interactions [1][2][3].

One important concept in graph theory is that of an independent set. The independent set of a graphalso known as a stable set, coclique, or anticlique is a subset of vertices, such that no two vertices are adjacent to each other.

There are several computational problems related to independent sets that have been studied. The most famous of these might be the maximum independent set problem, an optimization problem that seeks to find the largest possible independent set in a given graph. The problem is known to be NP-hard, meaning that there is no known efficient algorithm that can solve it in polynomial time [4].

The independent set decision problem is essentially a differently formulated version of the maximum independent set problem. It arises from the observation that a graph possesses an independent set of at least k vertices if and only if it harbors an independent set of exactly k vertices. This observation underscores the equivalence of these two problems, as the formulation of the independent set decision problem retains the computational complexity inherent in the maximum independent set problem.

In this paper, we will focus on the independent set decision problem. Our primary goal is to investigate the problem, examining its theoretical foundations and providing a comprehensive analysis of existing algorithms.

# II. Related work

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# III. Preliminaries

# III. Algorithmics

## A. Exhaustive Search

In the context of the independent set decision problem, an exhaustive search refers to a brute-force approach where all possible combinations of vertices are examined to determine whether they form an independent set or not. This approach is conceptually simple, but impractical for large graphs due to the combinatorial explosion of possibilities. It involves generating all possible subsets of vertices from the graph. The time complexity of this operation is proportional to the binomial coefficient " choose ", denoted as . This coefficient represents the number of ways to pick elements from a set of elements and follows the formula . The worst-case time complexity of the algorithm is dominated by the generation of these subsets. For each subset, the algorithm checks whether it forms an independent set by examining all possible pairs of vertices within the subset. This additional check introduces a factor of in the overall time complexity. Therefore, the total time complexity of the algorithm can be expressed as . This renders the algorithm unsuitable for large-scale applications, where the sheer number of potential combinations renders the computational cost prohibitively high. Alternative algorithms, incorporating more efficient strategies or heuristic approaches, become imperative for addressing this problem at scale.

## B. Branching Algorithm

In contrast to exhaustive search, a branching algorithm for the independent set decision problem takes a more targeted approach to explore the solution space efficiently. The algorithm strategically makes decisions at each step, branching into subproblems and avoiding the need to examine all possible combinations.

The algorithm begins by examining the degrees of nodes in the graph. It strategically selects nodes with degrees satisfying specific conditions, introducing branching based on the degree distribution. The time complexity associated with degree-based branching involves examining the degrees of nodes in the graph, contributing a factor proportional to the number of nodes. Let represent the number of nodes in the graph. The time complexity for this step is .

## C. Greedy Algorithm

## D. Improved Greedy Algorithm

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# References

[1] Otte, E., & Rousseau, R. (2002). Social network analysis: a powerful strategy, also for the information sciences. Journal of information Science, 28(6), 441-453.

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[3] McNaught, A. D. (1997). Compendium of chemical terminology (Vol. 1669). Oxford: Blackwell Science.

[4] Garey, M. R., & Johnson, D. S. (1978). ``strong''np-completeness results: Motivation, examples, and implications. Journal of the ACM (JACM), 25(3), 499-508.

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