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Mining Large Scale Datasets

Locality-Sensitive Hashing

(Adapted from CS246@Starford.edu; http://www.mmds.org)

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Locality-Sensitive Hashing

- Family of related techniques
- Allows to only examine pairs that are likely to be similar

    Avoids quadratic growth in computation time

LSH: general idea

- Hash items into buckets using many different hash functions

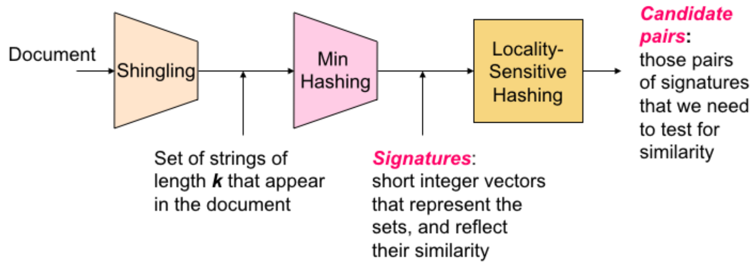
  ↪ Functions are designed so that similar items are more likely to hash into same bucket

- Only pairs that share a bucket for at least one of the hash functions need to be examined

  ↪ There may be false negatives – pairs of similar items may not be considered at all

  ↪ There may be false positives – pairs of items may be erroneously found as similar

Similar Documents: Steps



- **Shingling**: Converts a document into a set representation
- **Min-Hashing**: Convert large sets to short signatures, while preserving similarity
- **Locality-Sensitive Hashing**: Focus on pairs of signatures likely to be from similar documents

Min-Hashing

Permutations

2	4	3
3	2	4
7	1	7
6	3	2
1	6	6
5	7	1
4	5	5

Characteristic matrix

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

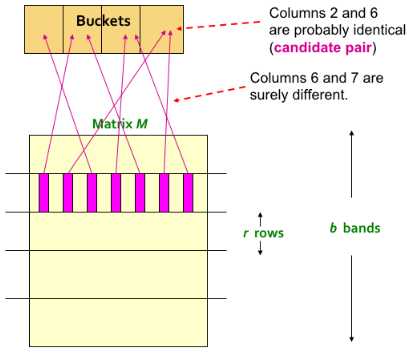
Signature matrix

2	1	2	1
2	1	4	1
1	2	1	2

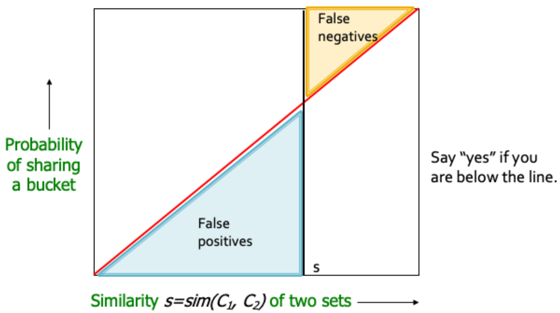
Similarities

1-2	1-3	2-4	3-4	
0	.75	.75	0	Original
0	.67	1	0	Signatures

Locality-Sensitive Hashing

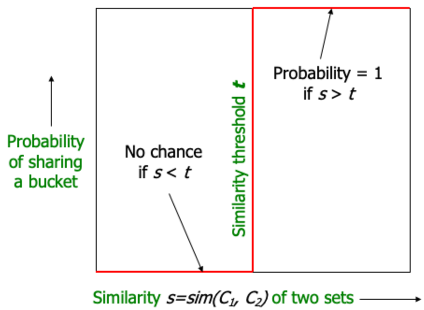


Locality-Sensitive Hashing



LSH with a single min hash function (one band of one row)

Locality-Sensitive Hashing

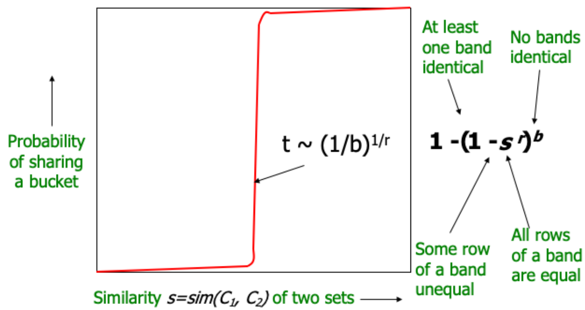


LSH – Optimal scenario: only pairs of sets with similarity $>$ t are selected as candidates

Locality-Sensitive Hashing

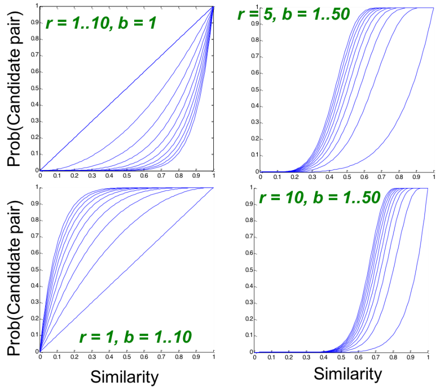
- b bands, r rows/band
- Consider columns C1 and C2 with similarity s
- For any band (r rows):
  - Probability that all rows in band are equal $= s^r$
  - Probability that some row in band is unequal $= 1 - s^r$
  - Probability that no band identical $= (1 - s^r)^b$
  - Probability that at least 1 band identical $= 1 - (1 - s^r)^b$

Locality-Sensitive Hashing: S-curve



LSH – b bands of r rows: S-curve

Locality-Sensitive Hashing: S-curve



Given a fixed threshold t, we want to choose r and b such that *Prob(Candidate pair)* has a "step" around t.

LSH Families of Hash Functions

- In the context of LSH families, a "hash function" is any function that allows us to say whether two elements are candidates for comparison
  - We use the notation

    $$h(x) = h(y)$$

    to mean "h says x and y are a candidate pair"

- A family of hash functions is any set of hash functions from which we can pick one at random efficiently

  For Min-Hashing signatures, each permutation of rows gives us a different Min-Hash function

LSH Families of Hash Functions

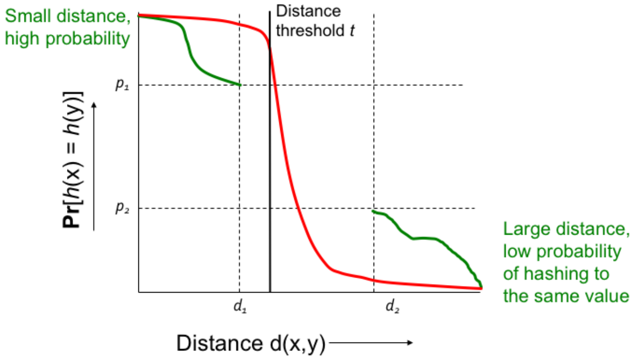
- Consider a space S of points with a distance measure $d(x, y)$
  - Can be Jaccard, Cosine, Euclidean, or other distance

(d_1, d_2, p_1, p_2)-sensitive family

- A family H of hash functions is said to be $(d1, d2, p1, p2)$-sensitive if for any x and y in S:
  - If $d(x, y) < d_1$, then the probability that $h(x) = h(y)$, over all $h \in H$, is at least p_1
  - If $d(x, y) > d_2$, then the probability that $h(x) = h(y)$, over all $h \in H$, is at most p_2

$\hookrightarrow$ With a LS Family we can do LSH!

(d_1, d_2, p_1, p_2)-sensitive family



For distances d_1 and below, the probability is at least p_1, and for distances d_2 and above, the probability is at most p_2. Between distances d_1 and d_2, we know nothing.

Goal: Minimize difference btw d_1 and d_2 and maximize distance btw p_1 and p_2.

Example of LS Family: Min-Hash

- Consider

  S = space of all sets
  d = Jaccard distance
  H : a family of Min-Hash functions for all permutations of rows

- Then for any hash function $h \in H$

$$Pr\left[h(x) = h(y)\right] = 1 - d(x, y)$$

Example of LS Family: Min-Hash

- For Jaccard distance, Min-Hashing gives a
  $(d_1, d_2, (1 - d_1), (1 - d_2))$-sensitive family for any $d_1 < d_2$

  Example:
  H is a ($\underline{1/3}$, $2/3$, $\underline{2/3}$, $1/3$)-sensitive family for S and d

  If distance $\leq 1/3$, similarity $\geq 2/3$

  Then probability that min-hash values
  are the same is $\geq 2/3$

Amplifying a LS-Family

- Reproduce the "S-curve" effect for any LS family
- The "bands" technique we learned for signature matrices carries over to this more general setting

- Two constructions

  AND construction ~ "rows in a band"
  OR construction ~ "many bands"

AND of Hash Functions

- Given family H, construct family H' consisting of r functions from H

  For $h = [h_1, \ldots, h_r]$ in H',

  $h(x) = h(y)$ if and only if $h_i(x) = h_i(y)$ for <u>all i</u>

  $\hookrightarrow$ corresponds to creating a band of size r

Theorem

If H is (d_1, d_2, p_1, p_2)-sensitive, H' is $(d_1, d_2, {p_1}^r, {p_2}^r)$-sensitive

Lowers probability for large distances (Good)
Also lowers probability for small distances (Bad)

OR of Hash Functions

- Given family H, construct family H' consisting of b functions from H

  For $h = [h_1, \ldots, h_b]$ in H',

  $h(x) = h(y)$ if and only if $h_i(x) = h_i(y)$ for <u>at least one i</u>

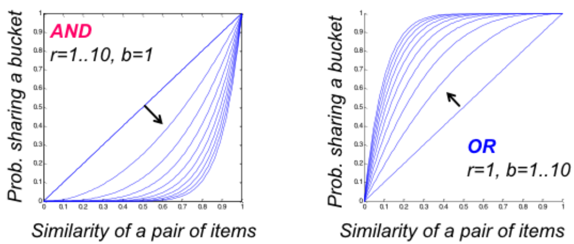
Theorem

If H is (d_1, d_2, p_1, p_2)-sensitive,
H' is $(d_1, d_2, 1-(1-p_1)^b, 1-(1-p_2)^b)$-sensitive

Raises probability for small distances (Good))
Raises probability for large distances (Bad)

Effect of AND and OR constructions

- **AND** makes all probabilities shrink
  - by choosing *r* correctly, we can make the lower probability approach 0 while the higher does not
- **OR** makes all probabilities grow
  - by choosing *b* correctly, we can make the upper probability approach 1 while the lower does not



Combining AND and OR Constructions

- r-way **AND** followed by b-way **OR** construction
- Same as in Min-Hashing
  - **AND**: If bands match in **all r** (values hash to same bucket)
  - **OR**: Columns that have **at least one** common bucket form a candidate pair
- If for points x and y $Pr[h(x) = h(y)] = s$
  - H will make (x, y) a candidate pair with probability s
  - r***AND** b***OR** construction makes (x, y) a candidate pair with probability $1 - (1 - s^r)^b$

    $\hookrightarrow$ **S-curve**
- Can use OR followed by AND; can combine sequences

AND-OR construction: example

s	$p = 1 - (1 - s^4)^4$
0.2	0.0064
0.3	0.0320
0.4	0.0985
0.5	0.2275
0.6	0.4260
0.7	0.6666
0.8	0.8785
0.9	0.9860

r = 4, b = 4 transforms a $(d_1, d_2, 0.8, 0.2)$-sensitive family into a $(d_1, d_2, 0.8785, 0.0064)$-sensitive family.

OR-AND construction: example

s	$p = \left(1 - (1 - s)^4\right)^4$
0.1	0.0140
0.2	0.1215
0.3	0.3334
0.4	0.5740
0.5	0.7725
0.6	0.9015
0.7	0.9680
0.8	0.9936

r = 4, b = 4 transforms a $(d_1, d_2, 0.8, 0.2)$-sensitive family into a $(d_1, d_2, 0.9936, 0.1215)$-sensitive family.

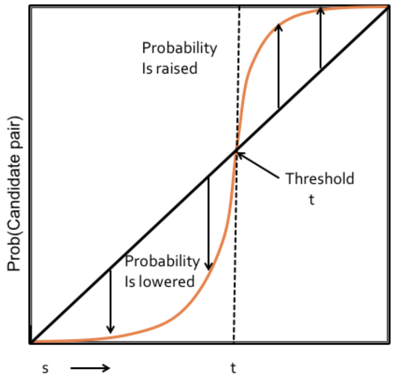
Cascading constructions

- Example: Apply the (4,4) OR-AND construction followed by the (4,4) AND-OR construction

- Transforms a $(d_1, d_2, 0.8, 0.2)$-sensitive family into a $(d_1, d_2, 0.9999996, 0.0008715)$-sensitive family

  Note that this family uses 256 (=4*4*4*4) of the original hash functions

Constructions: visualization of threshold t



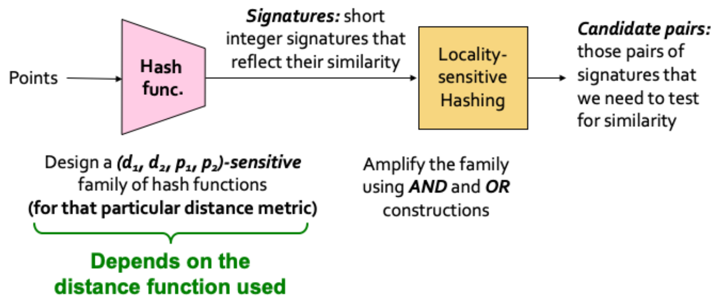
For an AND-OR S-curve $1 - (1 - s^r)^b$, the threshold t is where $1 - (1 - s^r)^b = t$

Probabilities p_1 and p_2 should be at opposite sides of t

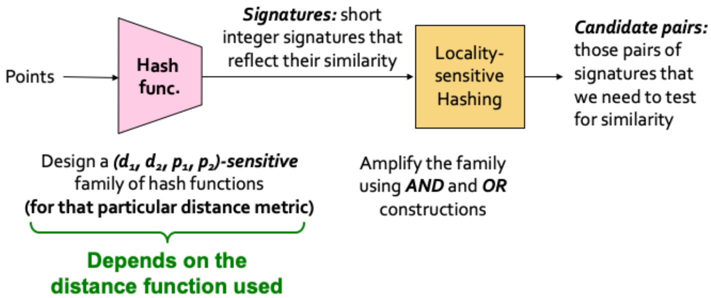
Summary

- Pick any two distances $d_1 < d_2$
- Start with a (d_1, d_2, p_1, p_2)-sensitive family
- Apply constructions to amplify (d_1, d_2, p_1^*, p_2^*)-sensitive family, where p_1^* is almost 1 and p_2^* is almost 0
- The closer to 0 and 1 we want to get, the more hash functions must be used!

LSH for other distance metrics



LSH for other distance metrics



- Cosine distance
  - **Random hyperplanes**
- Euclidean distance
  - **Random projections**

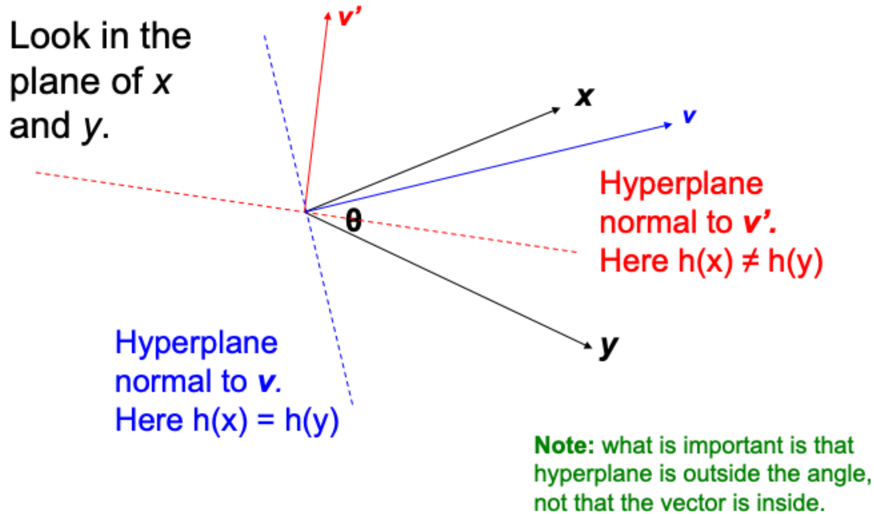
LSH for cosine distance

- For cosine distance, there is a technique called **Random Hyperplanes**
  - Technique similar to Min-Hashing

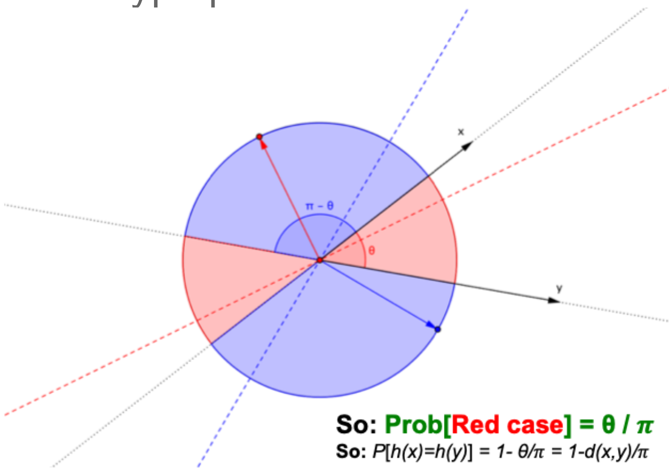
- Random Hyperplanes method is a $(d_1, d_2, (180 - d_1)/180, (180 - d_2)/180)$ - sensitive family for any d_1 and d

Reminder: (d_1, d_2, p_1, p_2)-sensitive
If $d(x, y) < d_1$, then probability that h(x) = h(y) is at least p_1
If $d(x, y) > d_2$, then probability that h(x) = h(y) is at most p_2

Random hyperplanes visualization



Random hyperplanes visualization



Signatures for Cosine Distance

- Pick some number of random vectors v_i, and hash your data for each vector

  $h_{v_i}(x) = +1$ if $v_i \cdot x \geq 0$
  $h_{v_i}(x) = -1$ if $v_i \cdot x < 0$

- Result is a signature (sketch) of $+1$'s and -1's for each data point
- Can be used for LSH, in same way as Min-Hash signatures are used for Jaccard distance
- Amplify using AND/OR constructions

How to select random vectors

- Expensive to pick random vectors in M dimensions for large M

  Would require generating M random numbers

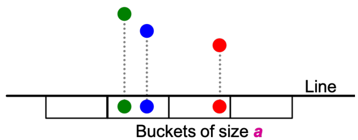
- It suffices to consider only vectors v_i consisting of $+1$ and –1 components

  Assuming data is random, then vectors of $+/$-1 cover the entire space evenly (no bias)

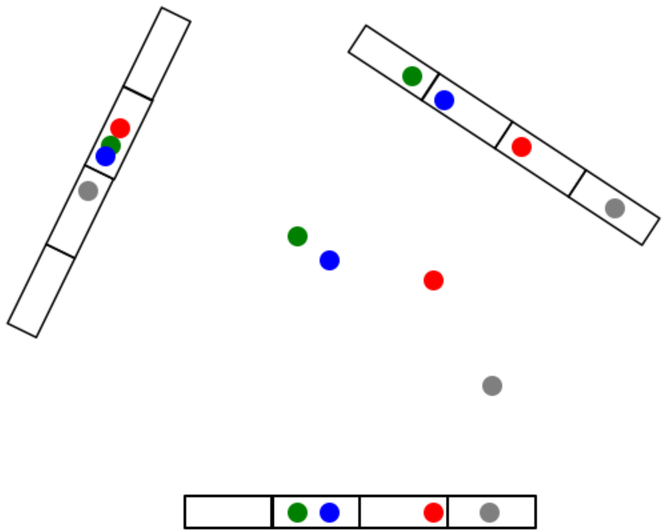
LSH for Euclidean distance

- Hash functions correspond to lines
- Partition the line into buckets (line segments) of size a
- Hash each point to the bucket containing its projection onto the line
- An element of the "signature" is a bucket id for that given projection line

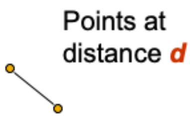
- Nearby points are always close; distant points are rarely in same bucket



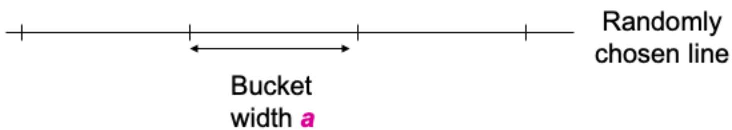
Projections



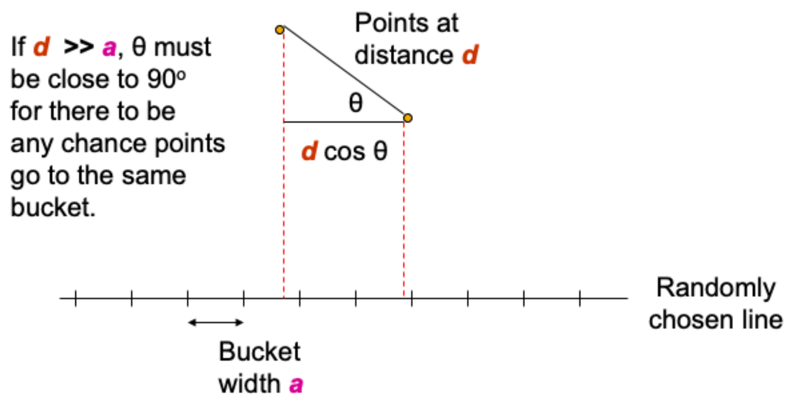
Projections



If *d* << *a*, then the chance the points are in the same bucket is at least **1 – *d*/*a***.



Projections



LS-Family for Euclidean Distance

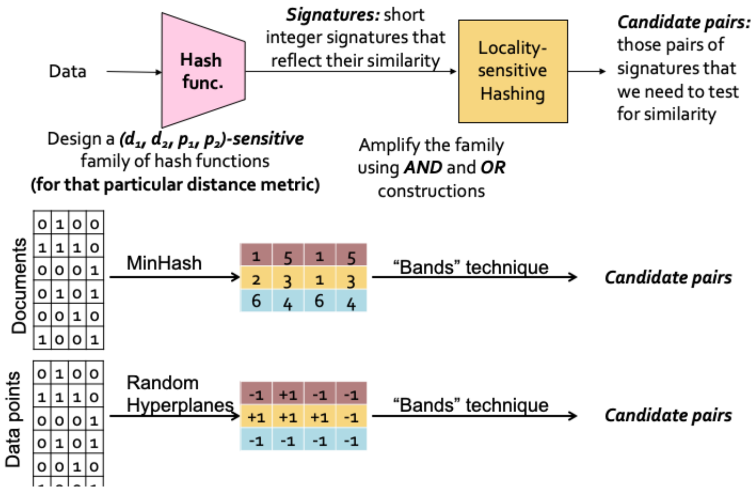
- If points are at distance $d \leq a/2$, then the probability of falling in same bucket is at least $1 - d/a$, or 1/2
- If points are at distance $d \geq 2a$, then they can fall in the same bucket only if $d \cdot cos\theta \leq a$, which is only true if

  $cos\theta \leq 1/2$

  $60 < \theta < 90$, i.e., at most 1/3 probability

- Yields a (a/2, 2a, 1/2, 1/3)-sensitive family of hash functions for any a
- Amplify using AND-OR cascades

LS families


Data
Hash func.
Signatures: short integer signatures that reflect their similarity
Locality-sensitive Hashing
Candidate pairs: those pairs of signatures that we need to test for similarity
Design a (d1, d2, p1, p2)-sensitive family of hash functions (for that particular distance metric)
Amplify the family using AND and OR constructions
Documents
MinHash
"Bands" technique
Candidate pairs
Data points
Random Hyperplanes
"Bands" technique
Candidate pairs

Important points

- Property P(h(C1)=h(C2)) = sim(C1,C2) of hash function *h* is the essential part of LSH, without it we can't do anything
- LS-hash functions transform data to signatures so that the bands technique (AND, OR constructions) can then be applied

Further fun... [MMDS 3.8, 3.9]

- Application examples

  Entity resolution

    Idea: Score 100 for each full match on name, address, or phone
    Sort on name, score matching lines; repeat for address, phone

  Fingerprint matching

    Fingerprints represented by set of grid squares containing minutiae: can use Jaccard similarity
    LSH family: each hash function is a set of grid squares

  Similar news articles

    Shingles defined by a stopword followed by the next two words

- Sets as strings
  - Length-based filtering
  - Prefix indexing