

Introduction to Computer Graphics main concepts and methods - II



(Wikipedia)

Topics

- Computer Graphics main tasks
- Geometric Primitives
- Geometric transformations
- 2D and 3D visualization
- Projections

CG Main Tasks

Modeling

- Construct individual models / objects
- Assemble them into a 2D or 3D scene (using transformations)

Rendering

- Generate final images:
- How is the scene illuminated?
- What are the materials of the objects?
- Where is the observer? How is he/she looking at the scene?

Animation

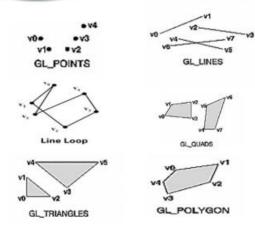
- Static vs. dynamic scenes
- Movement and / or deformation

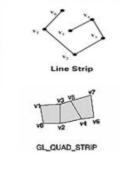
Geometric Primitives

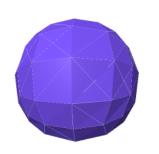
- Simple primitives
 - Points
 - Line segments
 - Polygons
- Geometric primitives
 - Parametric curves / surfaces
 - Cubes, spheres, cylinders, etc.

Examples:













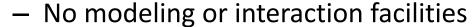




Computer Graphics APIs

- Create 2D / 3D scenes from simple primitives
- OpenGL and variants ...













- Direct 3D Microsoft
- VTK







Vulkan ...



Three.js

- Cross-browser JavaScript library/API used to create and display animated 3D computer graphics in a web browser.
- Uses WebGL

three.js ^{r87} featured projects submit project

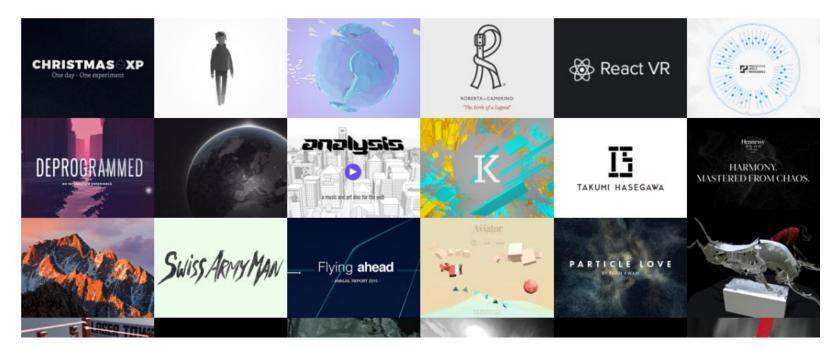
documentation examples

download

source code questions forum irc slack google+

editor





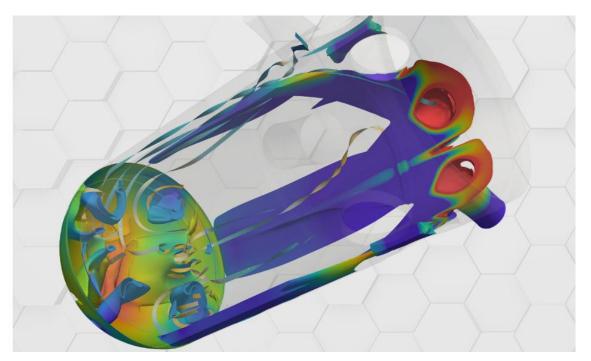
https://threejs.org/

Geometric Primitives – three.js examples

```
const width = 8; // ui: width
const height = 8; // ui: height
const depth = 8; // ui: depth
const geometry = new THREE.BoxGeometry(width, height, depth)
const radius = 6; // ui: radius
const height = 8; // ui: height
const radialSegments = 16; // ui: radialSegments
const geometry = new THREE.ConeGeometry(radius, height, radialSegments);
const radius = 7; // ui: radius
const widthSegments = 12; // ui: widthSegments
const heightSegments = 8; // ui: heightSegments
const geometry = new THREE.SphereGeometry(radius, widthSegments,
heightSegments);
```

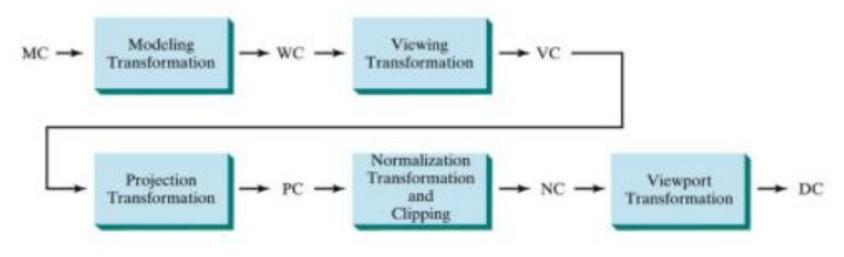
VTK

- Open-source, freely available software system for 3D computer graphics, modeling, image processing, volume rendering, scientific visualization.
- Is designed to be platform agnostic



https://vtk.org/

3D visualization pipeline (coordinate transformations)



MC - Modelling coordinates

(Hearn & Baker, 2004)

WC – World coordinates

VC – Viewing coordinates

PC – Projection coordinates

NC – Normalized coordinates

DC – Display coordinates

2D and 3D visualization pipeline

 We will start by 2D transformations and viewing a 2D scene and then generalize to 3D

- Main operations represented as point transformations
 - Basic transformation matrices
 - Homogeneous coordinates
 - Matrix multiplication and composed transformations

Basic 2D Transformations

$$p = (x, y) \rightarrow original point$$

$$p' = (x', y') \rightarrow transformed point$$

- Basic transformations:
 - Translation
 - Scaling
 - Rotation

$$P = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

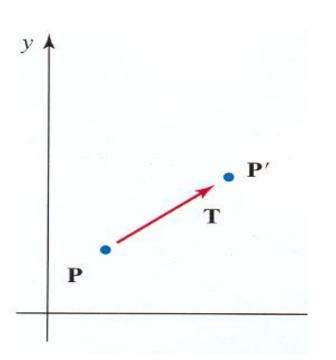
Vector notation

Complex transformations may be expressed as a composition of these

Translation

• It is necessary to specify translations in x and y

$$x' = x + tx$$
 $y' = y + ty$



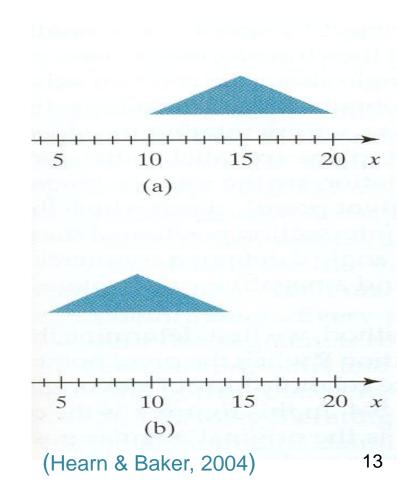
$$\mathbf{P} = \begin{bmatrix} x \\ y \end{bmatrix} \qquad \mathbf{P'} = \begin{bmatrix} x' \\ y' \end{bmatrix} \qquad \mathbf{T} = \begin{bmatrix} tx \\ ty \end{bmatrix}$$

$$P' = P + T$$
transformation matrix

Translation

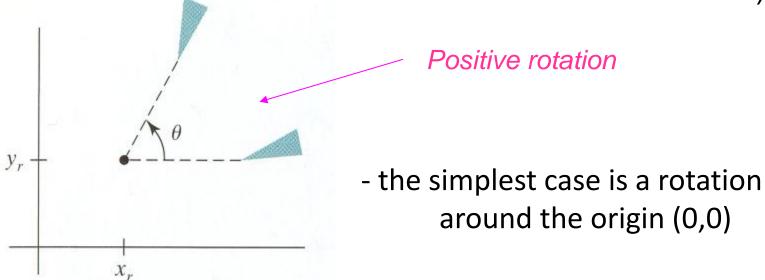
It is a rigid body transformation (it does not deform the object)

- To apply a translation to a line segment we need only to transform the end points
- To apply a translation to a polygon we need only to transform the vertices



Rotation

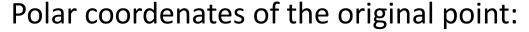
- To apply a 2D rotation we need to specify:
 - a point (center of rotation) (x_r, y_r)
 - a rotation angle *θ* (the convention is: positive -> counterclockwise)



Rotation around the origin

The simplest case:

$$x' = r \cos (\Phi + \Theta) = r \cos \Phi \cos \Theta - r \sin \Phi \sin \Theta$$
$$y' = r \sin (\Phi + \Theta) = r \cos \Phi \sin \Theta + r \sin \Phi \cos \Theta$$

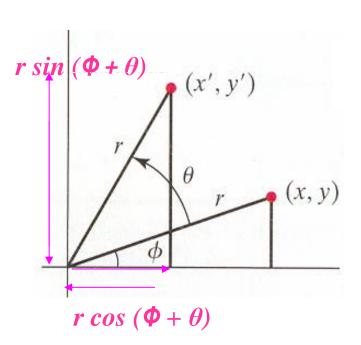


$$x = r \cos \Phi$$
$$y = r \sin \Phi$$

Replacing:

$$x' = x \cos \Theta - y \sin \Theta$$

 $y' = x \sin \Theta + y \cos \Theta$



2D Rotation in matrix notation

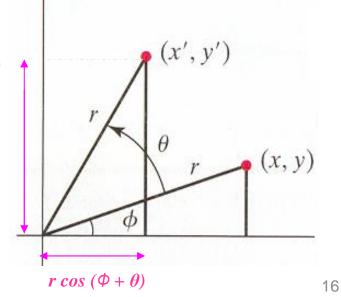
$$x' = r \cos (\Phi + \Theta) = r \cos \Phi \cos \Theta - r \sin \Phi \sin \Theta$$

 $y' = r \sin (\Phi + \Theta) = r \cos \Phi \sin \Theta + r \sin \Phi \cos \Theta$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$
$$\mathbf{P}' = \mathbf{R}(\theta) \cdot \mathbf{P}$$

Reminder:

 $cos(\alpha+\beta) = cos \alpha cos \beta - sin \alpha sin \beta$ $cos (\alpha-\beta) = cos \alpha cos \beta + sin \alpha sin \beta$



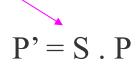
Scaling

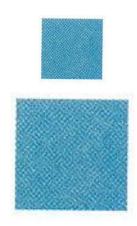
• Modifies the size of an object; we need to specify scaling factors: s_x and s_y

$$x' = x \cdot s_x$$
$$y' = y \cdot s_y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & o \\ o & s_y \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

Trasformation matrix





Transforming a square into a larger square applying a scaling s_{χ} =2, s_{χ} =2

(Hearn & Baker, 2004)

2D Transformations (composed)

- Matrix representation
 - Homogeneous coordinates !!
 - Concatenation = Matrix products

- Complex transformations ?
 - Decompose into a sequence of basic transformations

Homogeneous coordinates

- Most applications involve sequences of transformations
- For instance:
- visualization transformations involve a sequence of translations and rotations to render an image of a scene
- animations may imply that an object is rotated and translated between two consecutive frames

 Homogeneous coordinates provide an efficient way to represent and apply sequences of transformations

- It is possible to combine in a matrix the multiplying and additive terms if we use 3x3 matrices
- All transformations may be represented by multiplying matrices in homogenous coordinates
- Each point is now represented by 3 coordinates

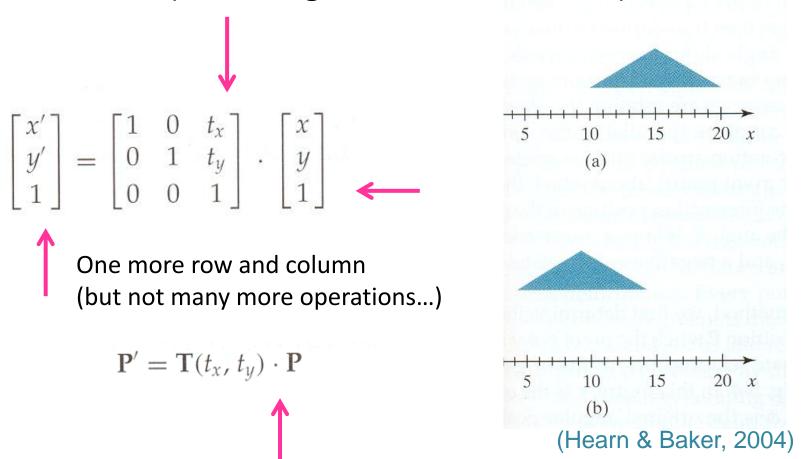
$$(x, y) \rightarrow (x_h, y_h, h), \qquad h \neq 0$$

$$x = x_h/h \qquad y = y_h/h$$

$$(x, h, y, h, h)$$

The simplest way: h=1!

2D Translation (in homogeneous coordinates)



Now, translation is applied by matrix multiplication

2D Rotation around (0,0) (in homogeneous coordinates)

$$x' = r \cos (\Phi + \Theta) = r \cos \Phi \cos \Theta - r \sin \Phi \sin \Theta$$

 $y' = r \sin (\Phi + \Theta) = r \cos \Phi \sin \Theta + r \sin \Phi \cos \Theta$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\mathbf{P}' = \mathbf{R}(\theta) \cdot \mathbf{P}$$

2D Scaling (in homogeneous coordinates)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\mathbf{P}' = \mathbf{S}(s_x, s_y) \cdot \mathbf{P}$$

- Now all basic transformations are applied by multiplication!
- We may concatenate multiple matrices





$$(Sx = Sy)$$





$$(Sx \neq Sy)$$

Concatenation of two translations

$$\mathbf{P}' = \mathbf{T}(t_{2x}, t_{2y}) \cdot \{ \mathbf{T}(t_{1x}, t_{1y}) \cdot \mathbf{P} \}$$
$$= \{ \mathbf{T}(t_{2x}, t_{2y}) \cdot \mathbf{T}(t_{1x}, t_{1y}) \} \cdot \mathbf{P}$$

$$\begin{bmatrix} 1 & 0 & t_{2x} \\ 0 & 1 & t_{2y} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & t_{1x} \\ 0 & 1 & t_{1y} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_{1x} + t_{2x} \\ 0 & 1 & t_{1y} + t_{2y} \\ 0 & 0 & 1 \end{bmatrix}$$

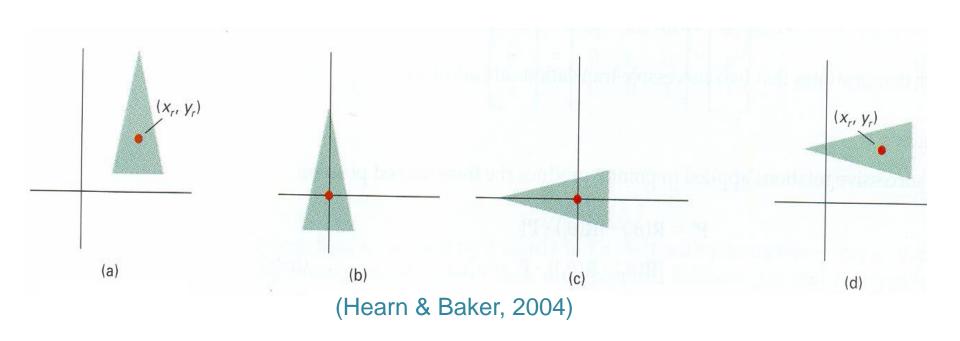
$$\mathbf{T}(t_{2x}, t_{2y}) \cdot \mathbf{T}(t_{1x}, t_{1y}) = \mathbf{T}(t_{1x} + t_{2x}, t_{1y} + t_{2y})$$

Concatenation of two scaling transformations

$$\begin{bmatrix} s_{2x} & 0 & 0 \\ 0 & s_{2y} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} s_{1x} & 0 & 0 \\ 0 & s_{1y} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} s_{1x} \cdot s_{2x} & 0 & 0 \\ 0 & s_{1y} \cdot s_{2y} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

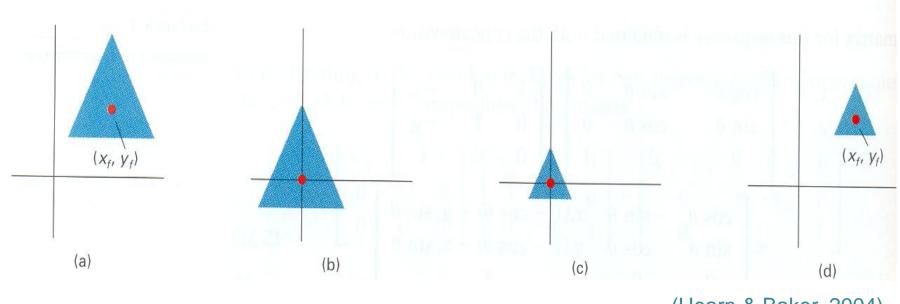
$$\mathbf{S}(s_{2x}, s_{2y}) \cdot \mathbf{S}(s_{1x}, s_{1y}) = \mathbf{S}(s_{1x} \cdot s_{2x}, s_{1y} \cdot s_{2y})$$

Arbitrary Rotation (around any point)



Translation + Rotation + Inverse Translation (to the origin) (around the origin) (to the initial position)

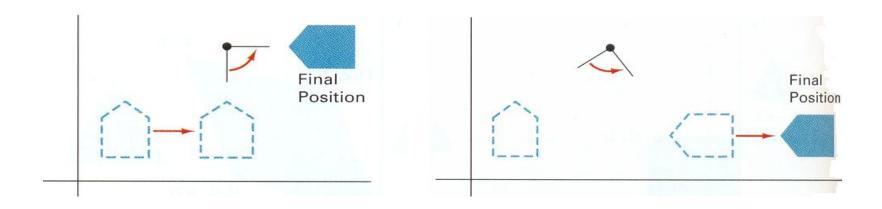
Arbitrary Scaling



(Hearn & Baker, 2004)

Translation + Scaling + Inverse Translation

Order is important!



a) Translation + rotation

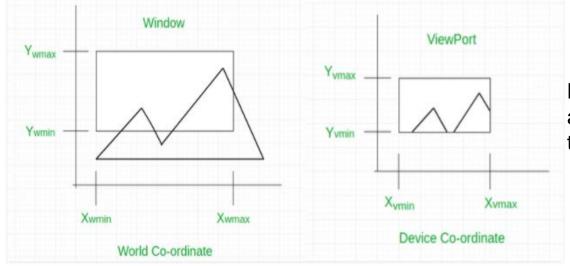
b) Rotation + translation

(Hearn & Baker, 2004)

Results may be different if transformations are applied in a different order!

The case of viewing 2D scenes

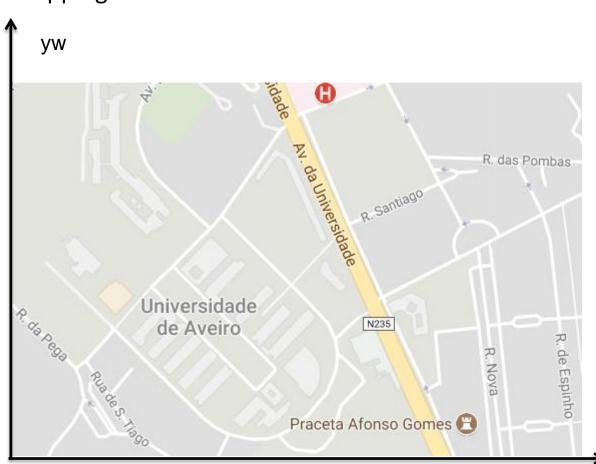
- Define a 2D scene in the world coordinate system
- Select a clipping window in the XOY plane
 - The window contents will be displayed
- Select a viewport in the display
- The viewport displays the contents of the clipping window

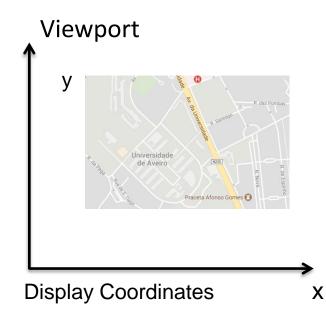


Note: Clipping window and viewport are the traditional terms in CG

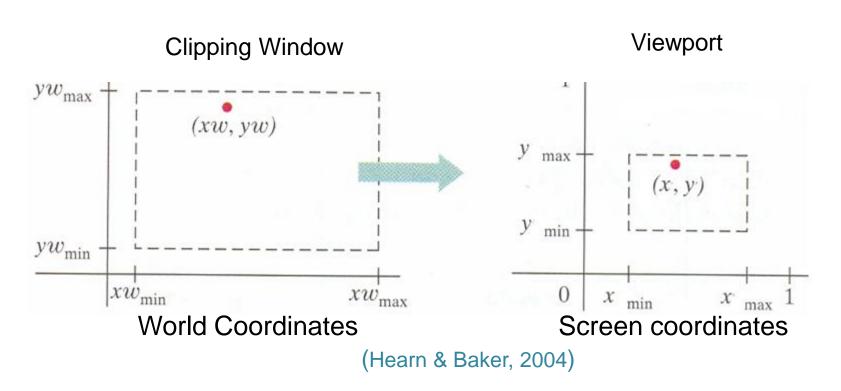
World ---> display

Clipping Window





Coordinate mapping

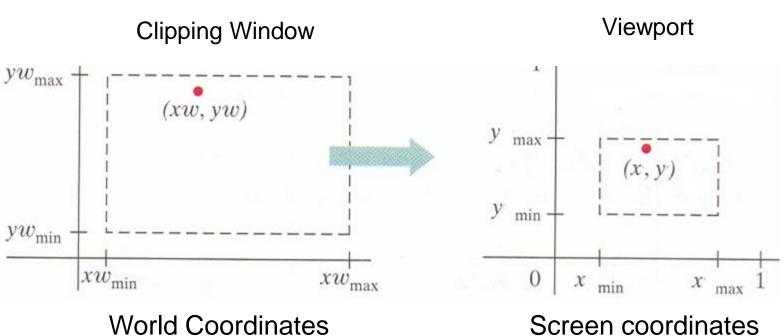


Objects inside the clipping window are mapped to the viewport (the area on the screen where they will be displayed).

Home work:

Compute (x,y) given (xw, yw)

Coordinate mapping



Screen coordinates

(x,y) given (xw, yw):

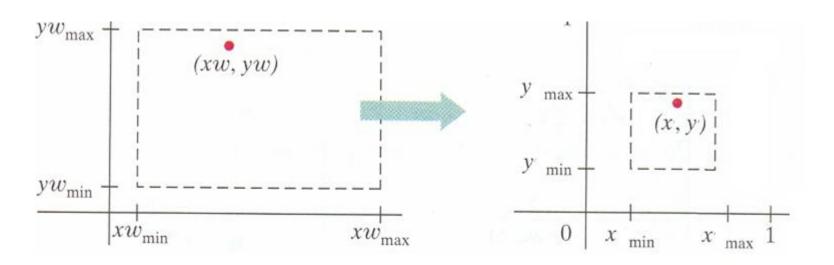
$$\frac{xw - xwmin}{xwmax - xwmin} = \frac{x - xmin}{xmax - xmin}$$

$$yw - ywmin = y - ymin$$

 $ywmax - ywmin = y - ymin$
 $ymax - ymin$

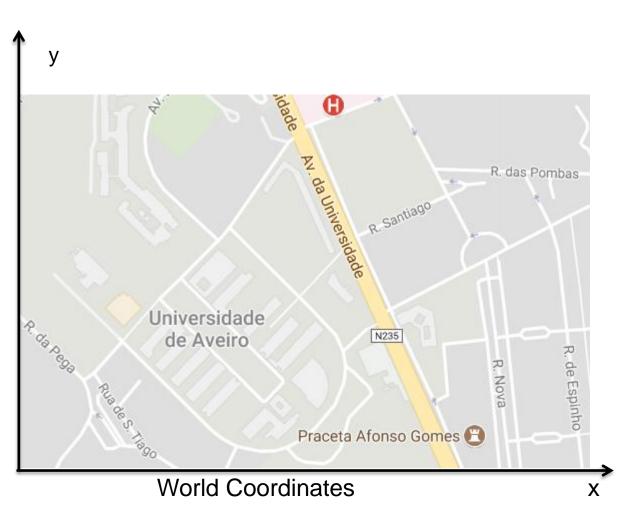
Coordinate mapping

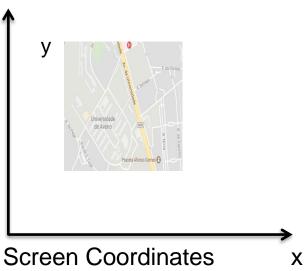
If the **aspect ratio** is not the same in both situations the result is distortion



(Hearn & Baker, 2004)

World -> screen





The **aspect ratio** is not the same in x and y: distortion!

3D Transformations (in homogeneous coordinates)

Translation

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$P = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Scaling

$$\mathbf{S} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

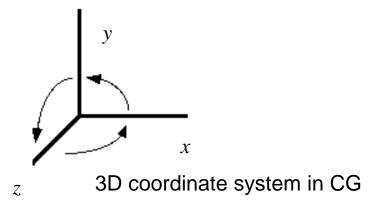
$$\mathbf{P'} = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

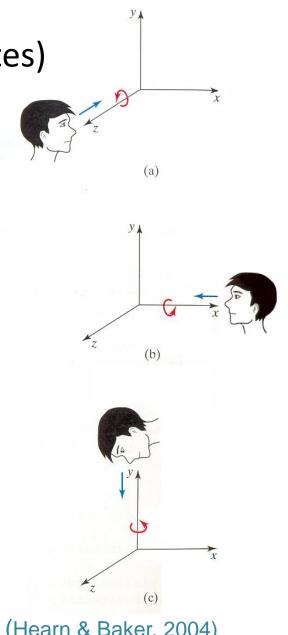
A 3D point in vector notation

3D Rotation (in homogeneous coordinates)

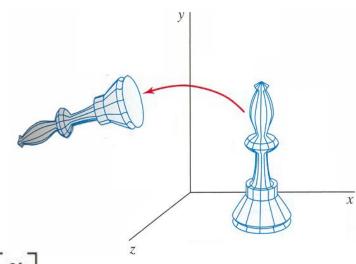
Rotation around each one of the coordinate axis

Positive rotations are CCW (counter clock wise)!!



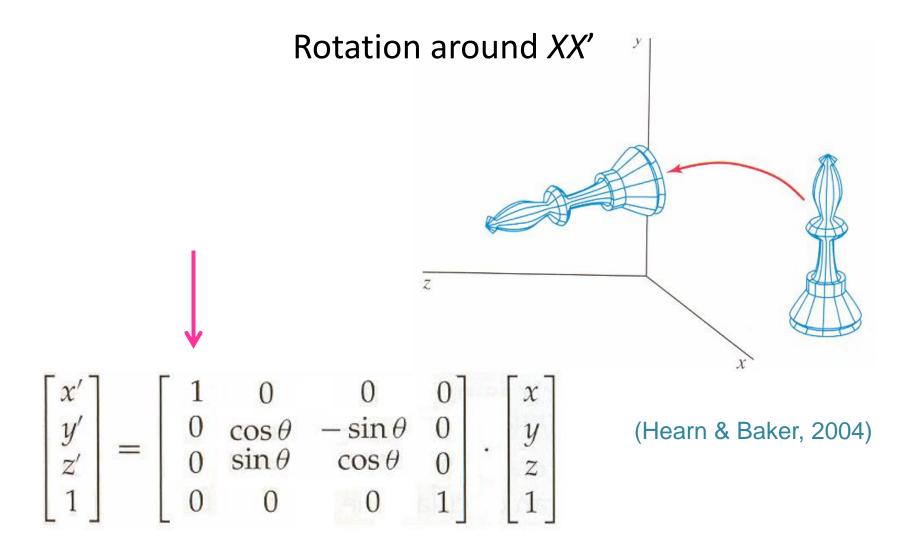


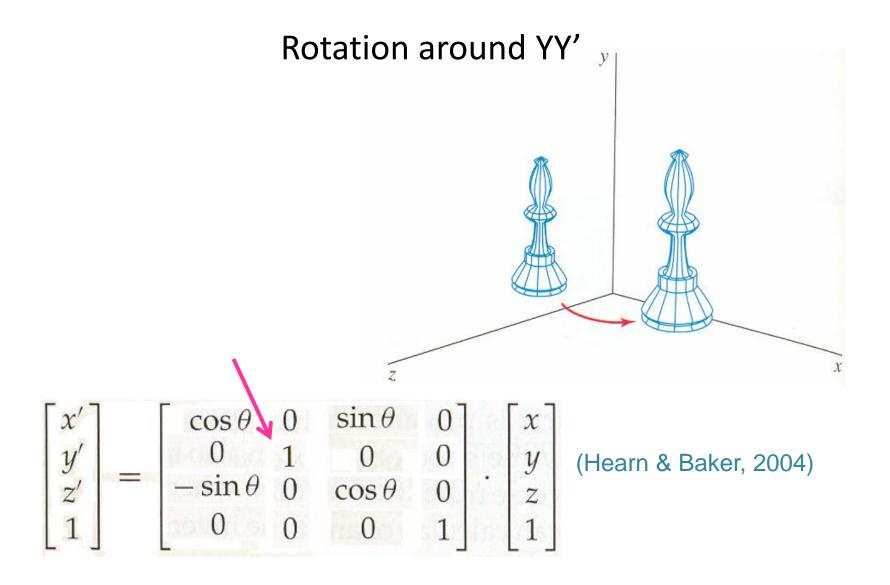
Rotation around ZZ'



$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

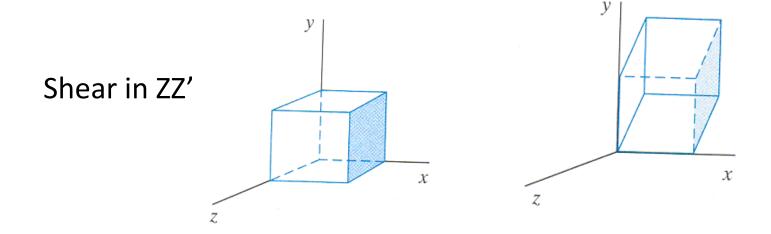
The z coordinate is maintained





Other useful 3D Transformations

- Shears
- Reflections



Transformations in three.js

Matrix4

A class representing a 4x4 matrix.

The most common use of a 4x4 matrix in 3D computer graphics is as a <u>Transformation Matrix</u>. For an introduction to transformation matrices as used in WebGL, check out <u>this tutorial</u>.

This allows a <u>Vector3</u> representing a point in 3D space to undergo transformations such as translation, rotation, shear, scale, reflection, orthogonal or perspective projection and so on, by being multiplied by the matrix. This is known as *applying* the matrix to the vector.

https://threejs.org/docs/#api/math/Matrix4

.makeTranslation (\underline{x} , \underline{y} , \underline{z})

```
\underline{x} - the amount to translate in the X axis. \underline{y} - the amount to translate in the Y axis. \underline{z} - the amount to translate in the Z axis.
```

Sets this matrix as a translation transform:

```
1, 0, 0, x,
0, 1, 0, y,
0, 0, 1, z,
0, 0, 0, 1
```

.makeScale (\underline{x} , \underline{y} , \underline{z})

```
\underline{x} - the amount to scale in the X axis.
```

 \underline{y} - the amount to scale in the Y axis.

 \underline{z} - the amount to scale in the Z axis.

Sets this matrix as scale transform:

```
x, 0, 0, 0,
0, y, 0, 0,
0, 0, z, 0,
0, 0, 0, 1
```

```
.makeRotationX ( theta )
```

<u>theta</u> — Rotation angle in radians.

Sets this matrix as a rotational transformation around the X axis by $\underline{\text{theta}}$ (θ) radians. The resulting matrix will be:

```
.makeRotationY ( theta )
```

<u>theta</u> — Rotation angle in radians.

Sets this matrix as a rotational transformation around the Y axis by $\underline{\text{theta}}$ (0) radians. The resulting matrix will be:

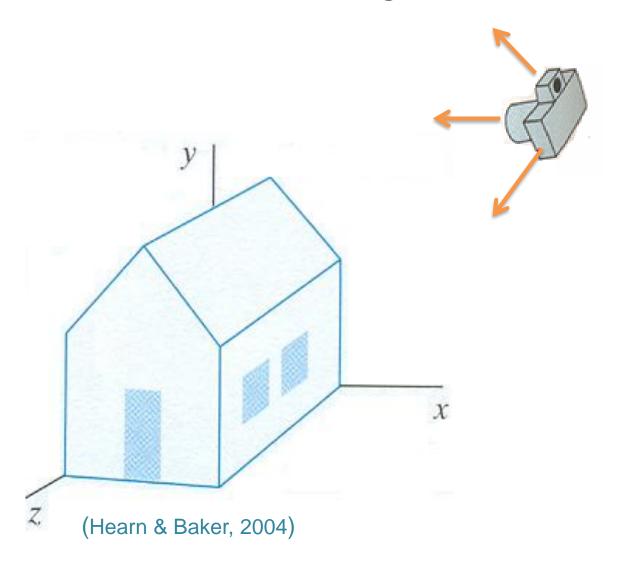
.makeRotationZ (theta)

<u>theta</u> — Rotation angle in radians.

Sets this matrix as a rotational transformation around the Z axis by $\underline{\text{theta}}$ (θ) radians. The resulting matrix will be:

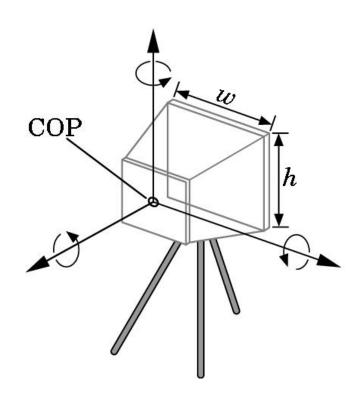
```
cos(\theta) -sin(\theta) 0 0 sin(\theta) cos(\theta) 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1
```

3D Viewing



Camera specification

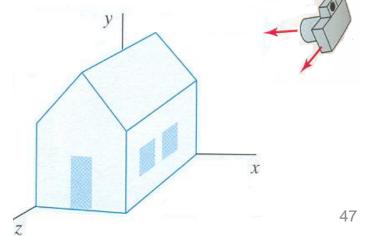
- Position and orientation
- Lens
- Image size
- Orientation of image plane



(Angel, 2012)

3D Viewing (2D representation of a 3D scene)

- Where is the observer / the camera?
 - Position ?
 - Close to the 3D scene ?
 - Far away ?
- How is the camera/observer looking at the scene ?
 - Orientation ?
- How to represent as a 2D image?
 - Projection ?



3D visualization pipeline

- Instantiate models of the scene
 - Position, orientation, size
- Establish viewing parameters
 - Camera position and orientation
- Compute illumination and shade polygons
- Perform clipping
- Project into 2D
- Rasterize

Projection (from 3D to 2D)

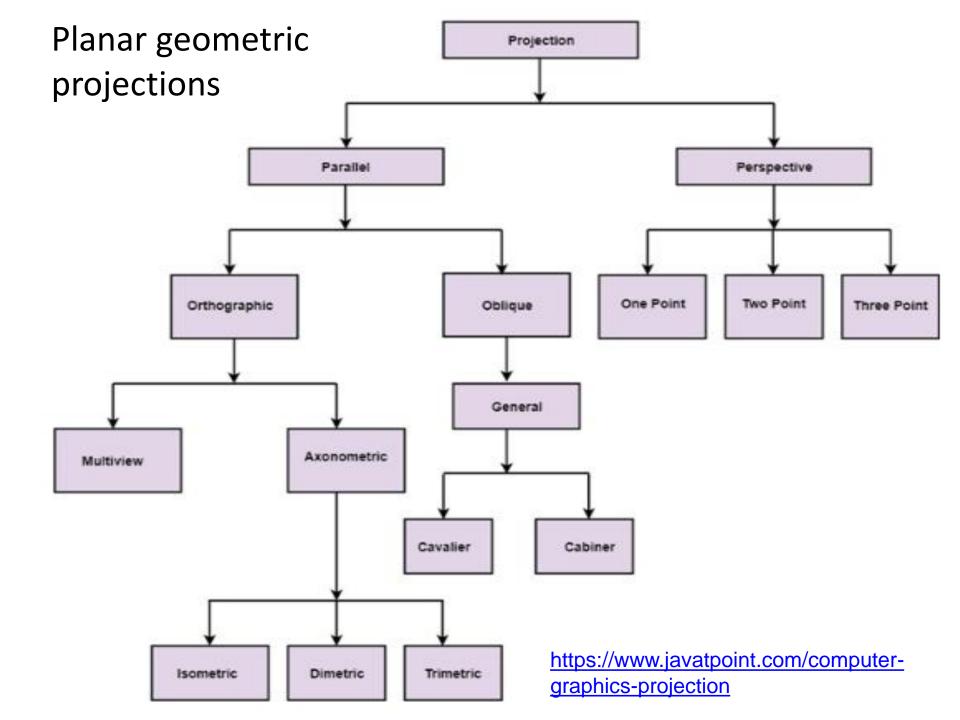
2D representations of a 3D scene may be done in different ways (in CG geometric planar projections are used: straight projectors/planar viewing plane)



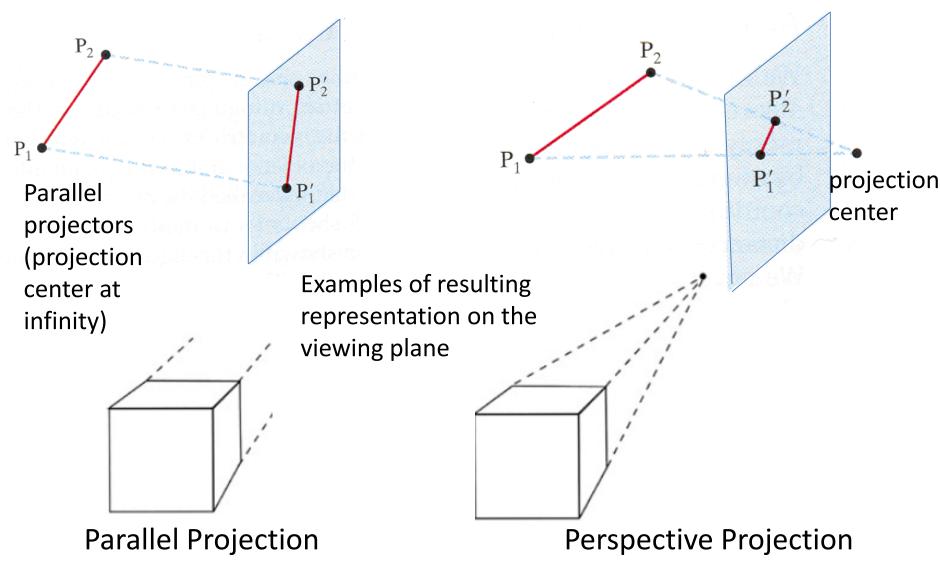


Parallel Projection (allows measures)

Perspective Projection (more realistic images)

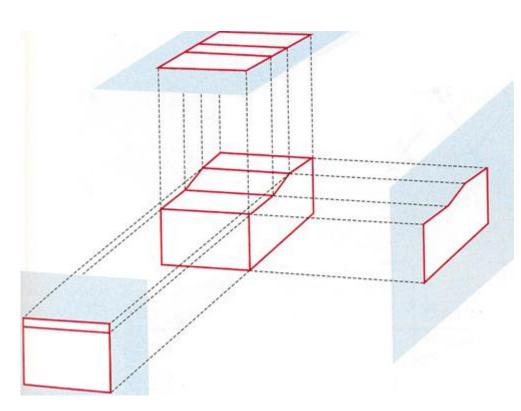


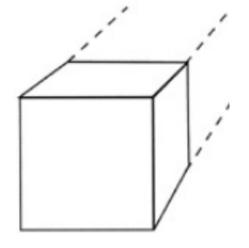
Projections



(Hearn & Baker, 2004)

Parallel Projections



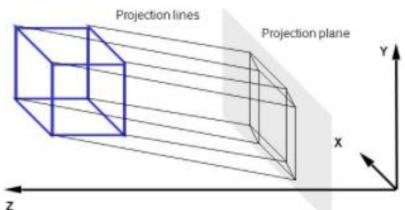


Orthographic /
Axonometric projection

Oblique projection



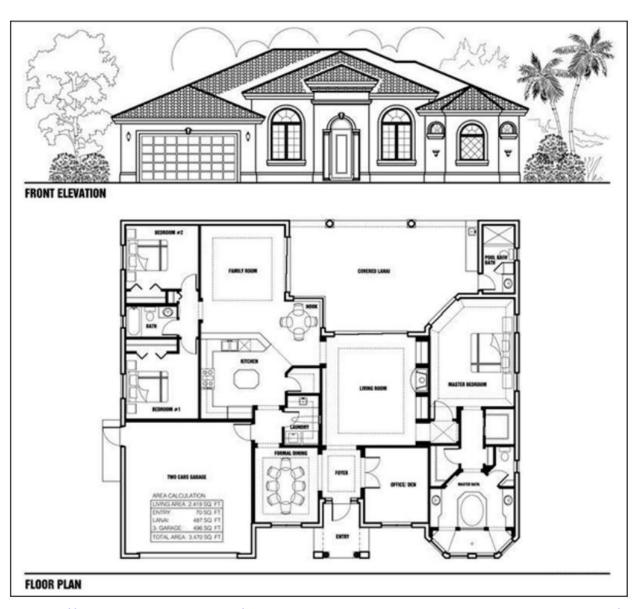
(Hearn & Baker, 2004)



Parallel projections are widely used in architecture

- Front elevation
- Floor plan

Allowing distance measures

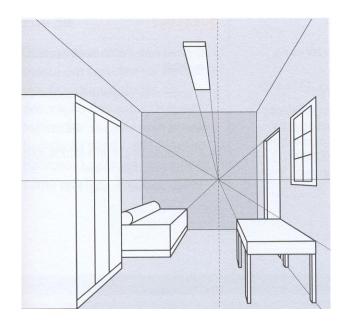


https://www.cadpro.com/easy-home-building-floor-plan-software/

Perspective Projections

Foreshortening indicates a perspective projection /





Approximate representation as a scene is seen by the eye

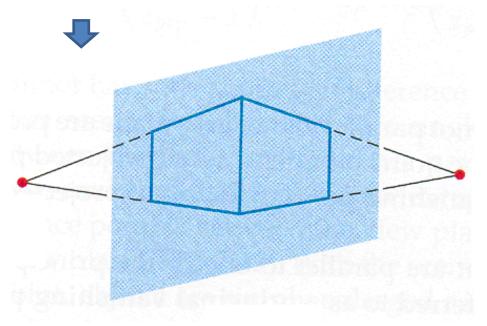
Object's dimensions along the line of sight appear shorter than its dimensions across the line of sight https://en.wikipedia.org/wiki/Perspective_(graphical)

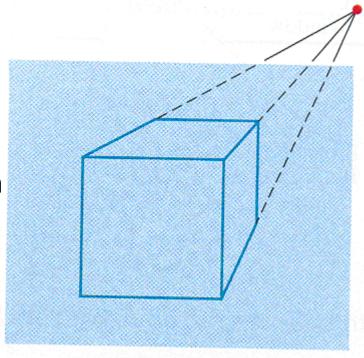
Perspective Projections

One vanishing point perspective projection

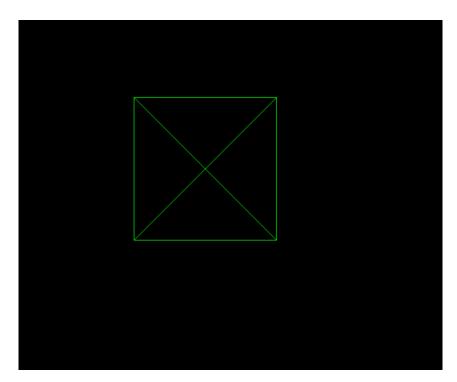


Two vanishing points perspective projection

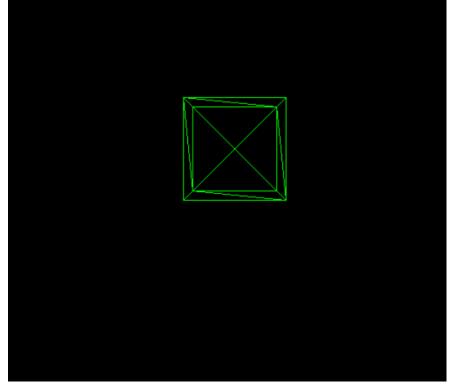




(Hearn & Baker, 2004)



Orthographic *vs* perspective camera in Three.js



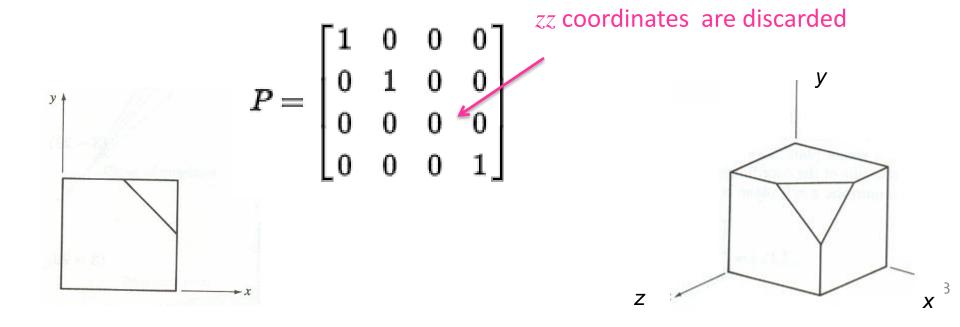
How to represent/apply projections?

- Projection matrices
- Homogeneous coordinates
- Concatenation through matrix multiplication
- Don't worry!
- Graphics APIs implement usual projections!

How to apply Projections?

Also by matrix multiplication

Example: Matrix of the orthographic projection on the *xy* plane in homogeneous coordinates:



Example of a more complex projection matrix

Compute the matrix to obtain a view allowing to make measures on the triangle face of the object.

The vertices of the face are:

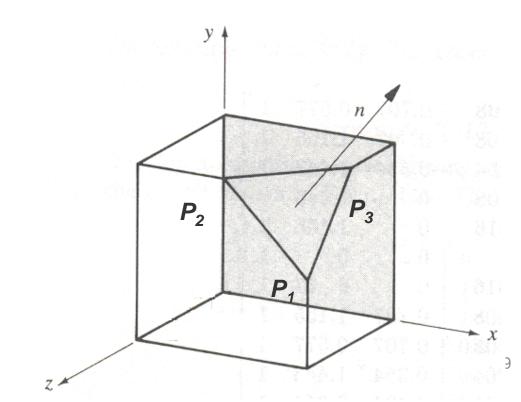
$$P_1 = (1, 0.5, 1)$$

$$P_2 = (0.5, 1, 1)$$

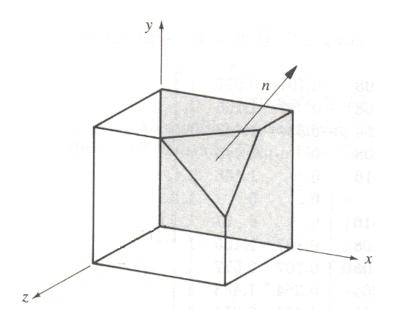
$$P_3 = (1, 1, 0.5)$$

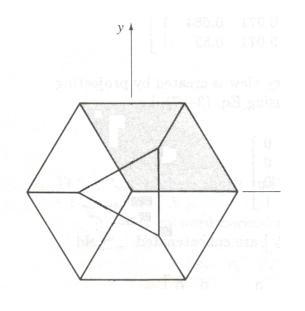
Hint:

- What is the type of projection that allows to make measures?
- Find the rotations needed ...



Resulting matrix:

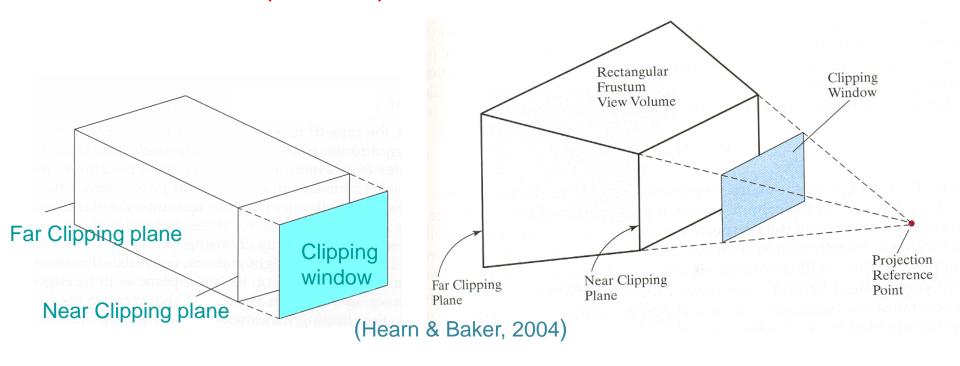




$$\mathbf{M}_{\text{final}} = \begin{bmatrix} 2/\sqrt{6} & -1/\sqrt{6} & -1/\sqrt{6} & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

How to limit what is observed and represented?

- Clipping window on the projection plane
- View volume (frustum) in 3D



Examples using Three.js

three.js r87

featured projects

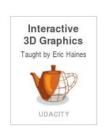
submit project

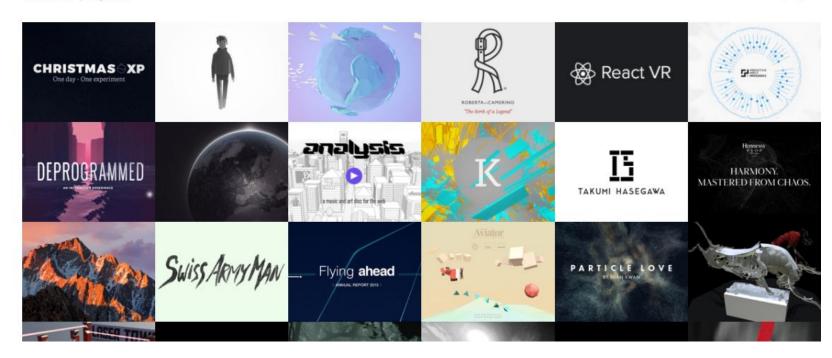
documentation examples

download

source code questions forum irc slack google+

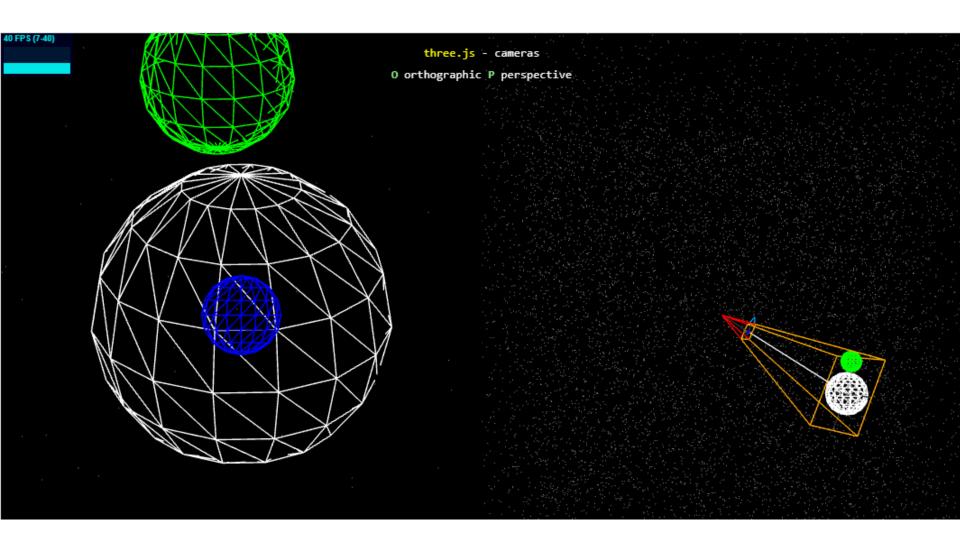
editor



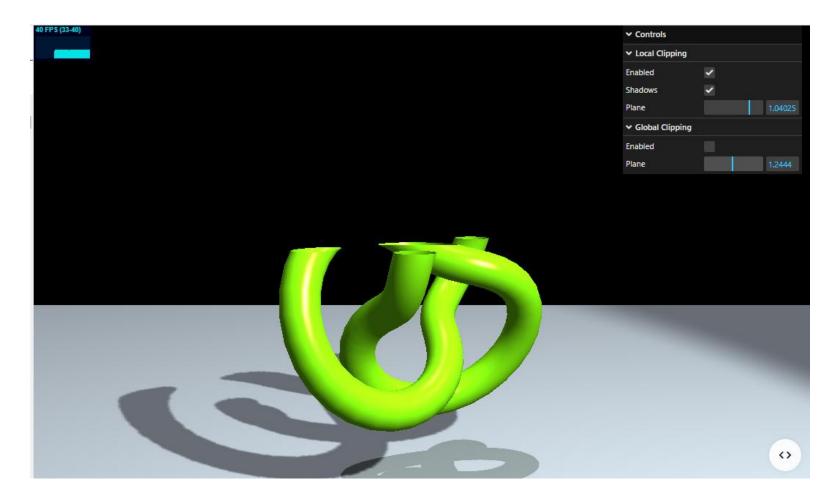


https://threejs.org/

Projections



Clipping (and shadows)



https://threejs.org/examples/#webgl_clipping

Thee.js first example

1. Defining the scene, the camera and where the scene is rendered

```
var scene = new THREE.Scene();
var camera = new THREE.PerspectiveCamera( 75,
window.innerWidth / window.innerHeight, 0.1, 1000 );
var renderer = new THREE.WebGLRenderer();
renderer.setSize( window.innerWidth, window.innerHeight );
document.body.appendChild( renderer.domElement );
```

2. Creating an object and camera position

```
var geometry = new THREE.BoxGeometry(1,1,1);
var material = new THREE.MeshBasicMaterial( {
  color: 0x00ff00 } );
var cube = new THREE.Mesh( geometry, material );
scene.add( cube );
camera.position.z = 5;
```

3. Scene rendering

```
function render() {
requestAnimationFrame(render);
renderer.render(scene, camera);
}
```

4. Scene animation

```
render();
cube.rotation.x += 0.1;
cube.rotation.y += 0.1;
```

Adding lights and shading

```
var material = new THREE.MeshPhongMaterial({
  ambient: '#006063',
  color: '#00abb1',
  specular: '#a9fcff',
  shininess: 100
  });
```

Some reference books

- S. Marschner, P. Shirley, Fundamentals of Computer Graphics, 5th ed., A K Peters/CRC Press, 2021
 Fundamentals of Computer Graphics, 5th Edition (oreilly.com)
- D. Hearn and M. P. Baker, Computer Graphics with OpenGL, 3rd Ed., Addison-Wesley, 2004
- E. Angel and D. Shreiner, *Introduction to Computer Graphics*, 6th Ed., Pearson Education, 2012
- Hughes, J., A. Van Dam, et al., Computer Graphics, Principles and Practice, 3rd Ed., Addison Wesley, 2013
 Hughes/Computer Graphics, 3/E (oreilly.com)