MAT1856/APM466 Assignment 1

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Fundamental Questions - 25 points

1.
   (a) Governments issue bonds because this does not increase the supply of money, if they were to print more money this would (as a result of supply-demand dynamics) cause inflation as a result of the devaluing of existing money.

   (b) In the case where an economy is expected to stagnate or grow slowly over the long term, the long term part of the yield curve might flatten since investors will not expect to get much more return in the long term then they do now.

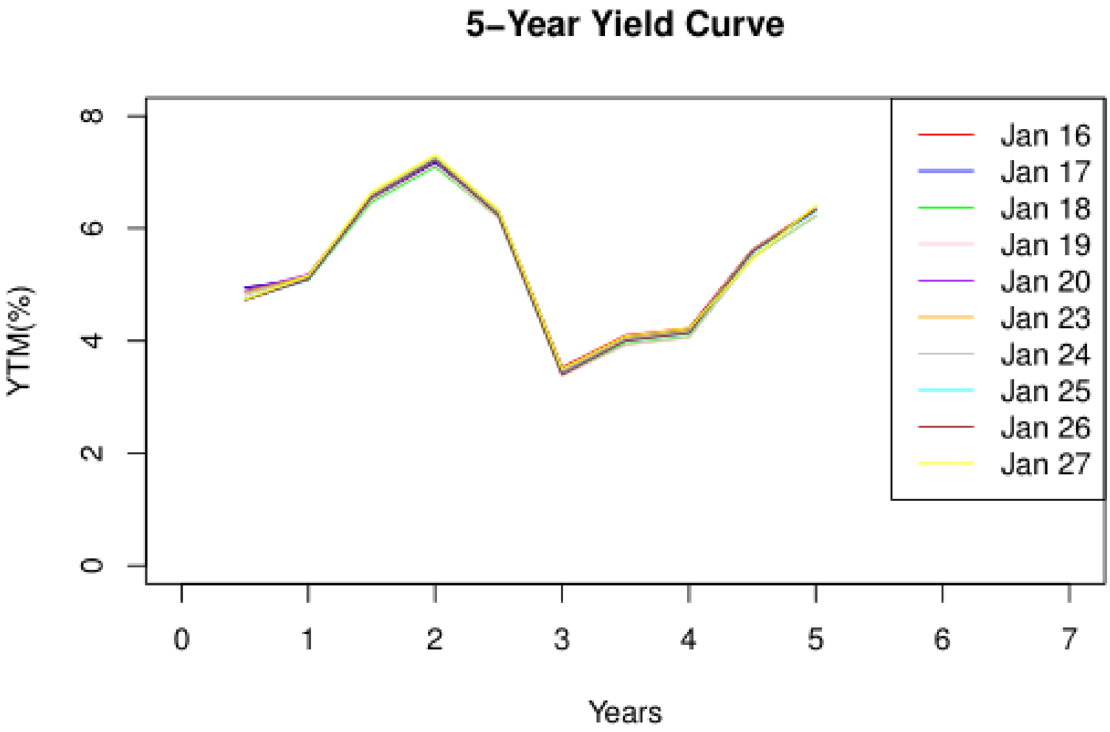
   (c) Quantitative easing is when a central bank purchases financial assets to inject liquidity into the financial system. It was employed during the COVID-19 pandemic to prevent a liquidity shortage and help the economy through the COVID-19 lockdowns.

2. Will use the bonds CAN 0.25 Aug 1 2023, CAN 0.75 Feb 1 2024, CAN 2.75 Aug 2024, CAN 3.75 Feb 1 2025, CAN 3 Oct 2025, CAN 0.25 Mar 1 2026, CAN 1 Sept 2026, CAN 1.25 Mar 2026, CAN 2.75 Sept 2026, and CAN 3.5 Mar 1 2028. The reason for this selection of bonds is that when constructing a yield curve one should ensure the quality of the bond is comparable, thus the most recently issued bond in each 6 month period was taken. For bonds issued too long ago, they would skew the results since the credit situation of the organization (the government of Canada) would be different. This also had the side effect of making bonds in similar maturity dates (6-12 months) having similar coupon amounts and refraining from using bonds which were issued a very long time ago.

3. Given several stochastic processes, the eigenvalues (in descending order) indicate which variables have the greatest influence on the variability of the stochastic curve. If the only variables to predict with are the different points on the stochastic, then it indicates which points have the greatest impact on variability. The eigenvectors indicate the direction of the change, whether it be increasing or decreasing.

Empirical Questions - 75 points

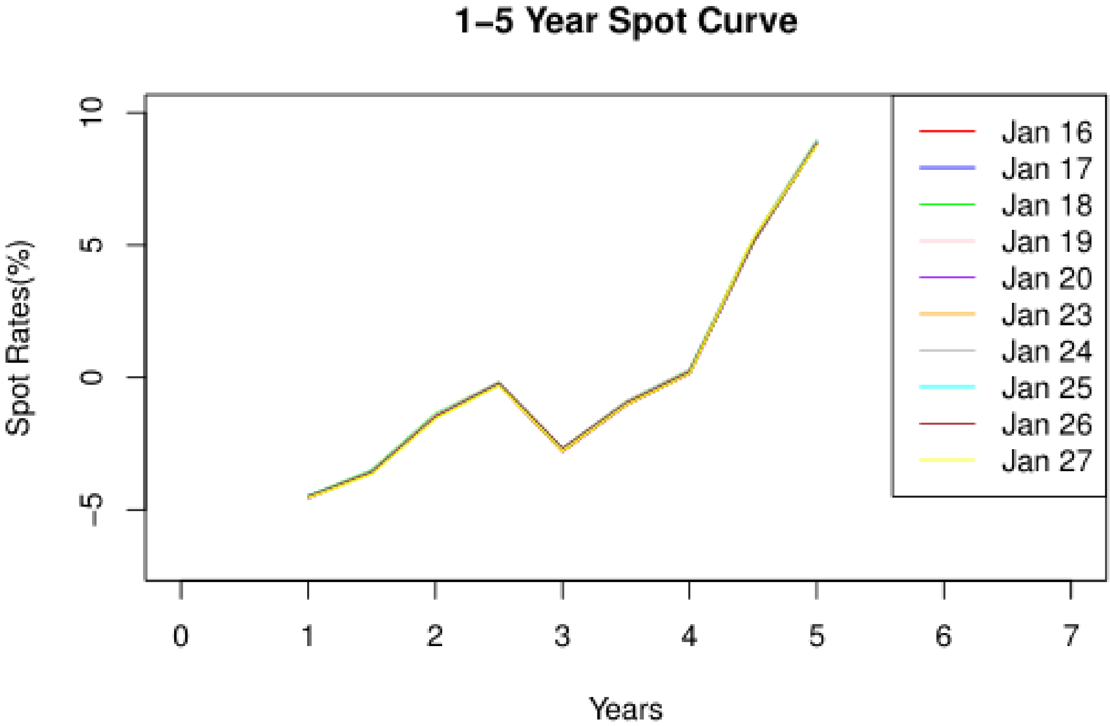
4.
   (a) In this case linear interpolation was used. This was done in order to most directly communicate biannual changes in the yield curve, rather then introduce some noise via smoothing, which can also be misleading (since it may give the impression more data points are used). The same reasoning is used for linear interpolations use in the upcoming spot and forward curves.



As one can see from the curve it has a sort of inverted shape. This indicates a market expectation for interest rates to drop (from a recession) and thus the yield curve does not have its traditional shape (increasing).

(b) Pseudo code:

Repeat what follows (for loop) for each bond 1 year from Feb 1 2023:
Set months to be the number of months between feb 1 2023 and the maturity date.
Set total=0
for i from 1 to months/6:
    if i not months/6:
        total=total+coupon/(2*(1+yield curve rate at month 6*i/100))
    if i is months/6 (maturity date):
        total=total+(coupon/2+100)/(1+yield curve rate at month 6*i/100))
total-100 is the desired spot rate value for the bonds since they have notional value 100, the spot curve will be formed via interpolation of the spot rates, repeated for each day.



As one can see from the spot rates, it is roughly increasing but starts negative. This indicates the existence of bonds on the market which don't have return along the lines of their notional value, given the current yield curve. This is in line with market prices, as they are in some cases significantly below 100.

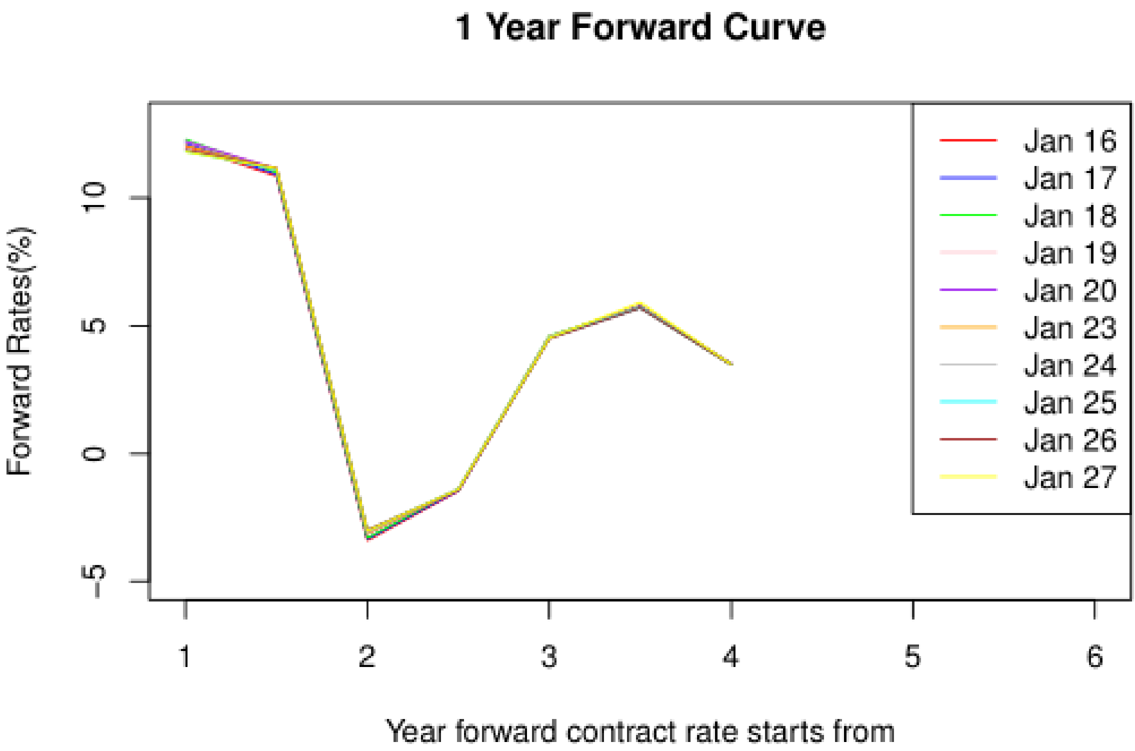
(c) Pseudo code:

Repeat what follows (for loop) for each bond with maturity 2-5 years from Feb 1 2023:
Set months_early to be the number of months between the Feb 1 2023 and the maturity date.

Let spot_late be the spot rate at maturity and spot_early be the spot rate 1 year at Feb 1 2024

forward=[(1+spot_late)^(6*(months_early-12)]/[(1+spot_early)^(6*months_early)]

forward is the desired forward rate, forward curve will be formed via linear interpolation of the forward rates.



In this curve we see the large changes in the spot rate reflected in the forward curve.

5. Via computation of covariance in R:

```
## [1] "The covariance matrix of log-returns of yield is"
##                [,1]          [,2]          [,3]          [,4]          [,5]
##  [1,]  3.281226e-05  5.011890e-05  2.472506e-05 -1.601466e-05 -2.558352e-05
##  [2,]  5.011890e-05  1.009801e-04  3.956955e-05 -4.221713e-05 -3.292984e-05
##  [3,]  2.472506e-05  3.956955e-05  2.540824e-05 -1.753198e-05 -1.662883e-05
##  [4,] -1.601466e-05 -4.221713e-05 -1.753198e-05  2.333128e-05  6.654736e-06
##  [5,] -2.558352e-05 -3.292984e-05 -1.662883e-05  6.654736e-06  2.222728e-05
##  [6,] -4.431920e-06  2.181275e-06  5.887221e-07 -6.256028e-06  6.891191e-06
##  [7,]  1.785633e-05  1.886370e-05  2.516973e-05 -1.054183e-05 -1.214556e-05
##  [8,]  9.617591e-06  4.351088e-06  7.991896e-06  1.747442e-06 -9.707090e-06
##  [9,] -3.744349e-05 -5.556812e-05 -3.636054e-05  2.241833e-05  2.699707e-05
##                [,6]          [,7]          [,8]          [,9]
##  [1,] -4.431920e-06  1.785633e-05  9.617591e-06 -3.744349e-05
##  [2,]  2.181275e-06  1.886370e-05  4.351088e-06 -5.556812e-05
##  [3,]  5.887221e-07  2.516973e-05  7.991896e-06 -3.636054e-05
##  [4,] -6.256028e-06 -1.054183e-05  1.747442e-06  2.241833e-05
##  [5,]  6.891191e-06 -1.214556e-05 -9.707090e-06  2.699707e-05
##  [6,]  6.928692e-06 -1.970982e-06 -5.805172e-06  3.331256e-06
##  [7,] -1.970982e-06  3.999854e-05  1.416550e-05 -3.935495e-05
##  [8,] -5.805172e-06  1.416550e-05  9.012875e-06 -1.530154e-05
##  [9,]  3.331256e-06 -3.935495e-05 -1.530154e-05  5.522568e-05
```

```
## [1] "The covariance matrix of forward rates is"
##                [,1]          [,2]          [,3]          [,4]          [,5]
##  [1,]  1.572513e-04  1.047343e-04  2.139340e-04  7.592542e-05 -9.460416e-05
##  [2,]  1.047343e-04  1.234887e-04  1.406102e-04  1.859833e-05 -5.116734e-05
##  [3,]  2.139340e-04  1.406102e-04  3.998354e-04  1.118074e-04 -7.575552e-05
##  [4,]  7.592542e-05  1.859833e-05  1.118074e-04  5.618447e-05 -4.909269e-05
##  [5,] -9.460416e-05 -5.116734e-05 -7.575552e-05 -4.909269e-05  8.571671e-05
##  [6,]  4.065151e-05  3.828515e-05  1.544346e-04  1.972927e-05  2.686171e-05
##  [7,]  8.375589e-05  3.169551e-05  1.458326e-04  5.688457e-05 -4.045975e-05
##  [8,]  6.241668e-05  2.298147e-05  5.954111e-05  3.942686e-05 -5.442056e-05
##  [9,] -9.268318e-05 -5.318960e-05 -1.057307e-04 -4.842587e-05  6.778147e-05
##                [,6]          [,7]          [,8]          [,9]
##  [1,]  4.065151e-05  8.375589e-05  6.241668e-05 -9.268318e-05
##  [2,]  3.828515e-05  3.169551e-05  2.298147e-05 -5.318960e-05
##  [3,]  1.544346e-04  1.458326e-04  5.954111e-05 -1.057307e-04
##  [4,]  1.972927e-05  5.688457e-05  3.942686e-05 -4.842587e-05
##  [5,]  2.686171e-05 -4.045975e-05 -5.442056e-05  6.778147e-05
##  [6,]  1.039475e-04  4.504432e-05 -1.156420e-05 -3.267058e-06
##  [7,]  4.504432e-05  6.427420e-05  3.414780e-05 -4.729487e-05
##  [8,] -1.156420e-05  3.414780e-05  3.743524e-05 -4.468779e-05
##  [9,] -3.267058e-06 -4.729487e-05 -4.468779e-05  5.991008e-05
```

6. Via computation of eigenvalues and eigenvectors in R:

```
## [1] "The eigenvectors and eigenvalues for the covariance matrix of log-returns of yield are"
## eigen() decomposition
## $values
## [1]  2.332212e-04  5.358666e-05  2.663140e-05  2.485628e-06  6.948831e-21
## [6] -2.541428e-23 -2.744331e-21 -6.974806e-21 -3.461399e-20
##
## $vectors
##              [,1]         [,2]        [,3]        [,4]        [,5]       [,6]
##  [1,] -0.35508669  0.005756684  0.34647341 -0.28892196  0.00000000  0.4742297
##  [2,] -0.60250353  0.548106827  0.02790149  0.28295812  0.19819065  0.2134876
##  [3,] -0.31363531 -0.136332951 -0.17162669 -0.52553572 -0.45538857  0.1857743
##  [4,]  0.24368990 -0.277354991  0.44332891 -0.22438427  0.52005397  0.3453106
##  [5,]  0.24848966  0.075307457 -0.52653985  0.23667899  0.04681983  0.5761486
##  [6,]  0.01269223  0.159752128 -0.41787480 -0.59270301  0.17785653 -0.1258770
##  [7,] -0.26257238 -0.608729061 -0.38621703  0.19054436  0.19104833  0.2198719
##  [8,] -0.09539272 -0.319797785  0.21663275  0.25408472 -0.53399522  0.0643249
##  [9,]  0.46173459  0.316258840  0.07228094 -0.04174582 -0.35694564  0.4189321
##             [,7]        [,8]         [,9]
##  [1,]  0.0000000  0.00000000 -0.667432979
##  [2,] -0.1034010 -0.15507601  0.368954990
##  [3,]  0.2160606  0.31410155  0.436084715
##  [4,] -0.1751042  0.04826582  0.440583685
##  [5,] -0.2619269  0.43240860 -0.097970704
##  [6,] -0.4990040 -0.38584418 -0.055165724
##  [7,]  0.2684166 -0.46699484  0.007693789
##  [8,] -0.6690192 -0.17611920  0.096164447
##  [9,]  0.2733491 -0.53852296  0.110332500
```

```
## [1] "The eigenvectors and eigenvalues for the covariance matrix of forward rates are"
## eigen() decomposition
## $values
## [1]  7.972304e-04  1.990613e-04  9.175191e-05  2.450469e-20  1.239919e-20
## [6]  1.005769e-20  8.514139e-21 -1.062094e-21 -5.134547e-20
##
## $vectors
##             [,1]        [,2]         [,3]        [,4]       [,5]         [,6]
##  [1,] -0.4249889  0.24874663  0.101336769 -0.64100627  0.0000000  0.000000000
##  [2,] -0.2776909  0.06162911  0.817086686 -0.06786473 -0.1047432  0.187605318
##  [3,] -0.6853278 -0.35184970 -0.090619009  0.47692535 -0.1809157 -0.002041677
##  [4,] -0.2156665  0.13479511 -0.410840005 -0.27849911  0.3432477  0.400063758
##  [5,]  0.2043617 -0.51315867  0.004940826 -0.46469191 -0.4104053 -0.328474288
##  [6,] -0.2066552 -0.59257785 -0.002603780 -0.18701067  0.5169658 -0.087441745
##  [7,] -0.2593067 -0.01900559 -0.339840377 -0.16431364 -0.4880985  0.089964827
##  [8,] -0.1449421  0.30081948 -0.170693789  0.02440741 -0.3114314 -0.151390510
##  [9,]  0.2310146 -0.29500024  0.020972582 -0.03129757 -0.2599301  0.811292113
##              [,7]         [,8]       [,9]
##  [1,]  0.00000000  0.000000000  0.5799581
##  [2,] -0.01458639  0.006071277 -0.4477017
##  [3,] -0.29731299  0.113979970  0.1916689
##  [4,] -0.41878914  0.177527226 -0.4518813
##  [5,] -0.37252211  0.207186259 -0.1446194
##  [6,]  0.52836082  0.079904472 -0.1035171
##  [7,]  0.33807866 -0.580347612 -0.3040955
##  [8,]  0.42331838  0.726513511 -0.1784332
##  [9,]  0.15676213  0.203763803  0.2575561
```

The first eigenvalue (which is relatively large) indicates that there is at least a significant amount of the variability in yields or forward rates explained by what the yield or forward rate is the day before.

References and GitHub Link to Code

Link to Code: https://github.com/DanielG-1/APM466-A1

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