McGill University

Project

Something About Recurrent Neural Networks

Author:
Daniel Galeano

Supervisor:
Ioannis Psaromiligkos

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List of Abbreviations

FNN	**F**eedforward **N**eural **N**etwork
RNN	**R**ecurrent **N**eural **N**etwork
BPTT	**B**ack **P**ropagation **T**hrough **T**time
MSE	**M**ean **S**quare **E**rror

List of Symbols

Chapter 1

Background Theory

1.1 Artificial Neural Networks

Artificial neural networks are parallel distributed processing systems that are composed by neurons, their most elementary building block. Neurons are processing units that generate an output according to an activation function $\sigma(x)$, which is usually a sigmoid function like in equations 1.1 and 1.2.

Logistic Regression:

$$\sigma(h) = \begin{cases} 1, h \geq 0 \\ 0, h < 0 \end{cases} \tag{1.1}$$

Hyperbolic tangent function:

$$\sigma(h) = tanh(h) \tag{1.2}$$

Neurons are interconnected by links, which have strengths that are coded by weights. A zero weight for example would indicate that absence of connection between two neurons. The link weights can also be tuned based on experience, an important property that makes neural networks adaptable and trainable.
Figure 1.1 shows a mathematical model of a neuron based on [Ke-Lin Du, 2006], with n input signals $x_1, x_2, ..., x_n$ and an output signal y. This model also shows the weights of each of the input links of this neuron as $w_1, w_2, ..., w_n$, a bias term w_0 and an activation function $\sigma(.)$. In this model x_0 is equals to 1 to include the bias term within the same vector form.
The output of this neuron is the aggregation of the input signals, shifted by the bias term and processed by the activation function; as shown in equation 1.3.

$$y = \sigma(input + bias) = \sigma(W^T X) = \sigma(\sum_{i=0}^{n} w_i x_i) \tag{1.3}$$

1.1.1 Types of Neural Networks

The arrangement of synaptic links between neurons determines different types of neural networks. If the links are acyclic for example, the neural network is considered a FNN; while the presence of cyclic links indicates a RNN. The understanding FNNs is

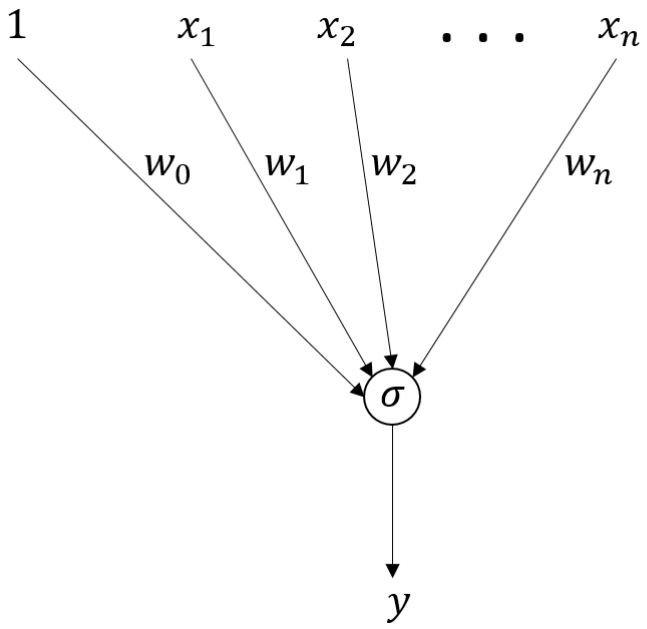

FIGURE 1.1: Mathematical Model of a Neuron.

essential to the understanding of RNNs, so a more detailed description of training and characteristics of FNNs is given in Appendix A.

1.2 Recurrent Neural Networks

Neural Networks with cyclic links are considered RNNs.

[TODO: Introduce general idea of RNNs]

1.3 Training RNNs

1.3.1 Backpropagation Through Time

BPTT is an adaption of the backpropagation method used in FNNs.

1.4 Challenges in RNNs

It has been theoretically proven that RNNs can be universal approximators of dynamic processes [KI and Y, 1993], which represents a powerful potential for modeling natural systems more accurately compared to existing methods such as FNNs. However, practical applications of RNNs have been stagnated due to theoretical and implementation challenges encountered in the this model. This section specifies some of these challenges.

1.4.1 Computational Cost

[TODO: Expand!]

1.4.2 Supervised Training

[TODO: Expand!]

Appendix A

Feedforward Neural Networks

[TODO: Introduce FFNs for discrete signals]

A.1 Terminology

Figure A.1 presents the terminology in a FNN with three input signals, a hidden layer with two neurons, and one output signal.

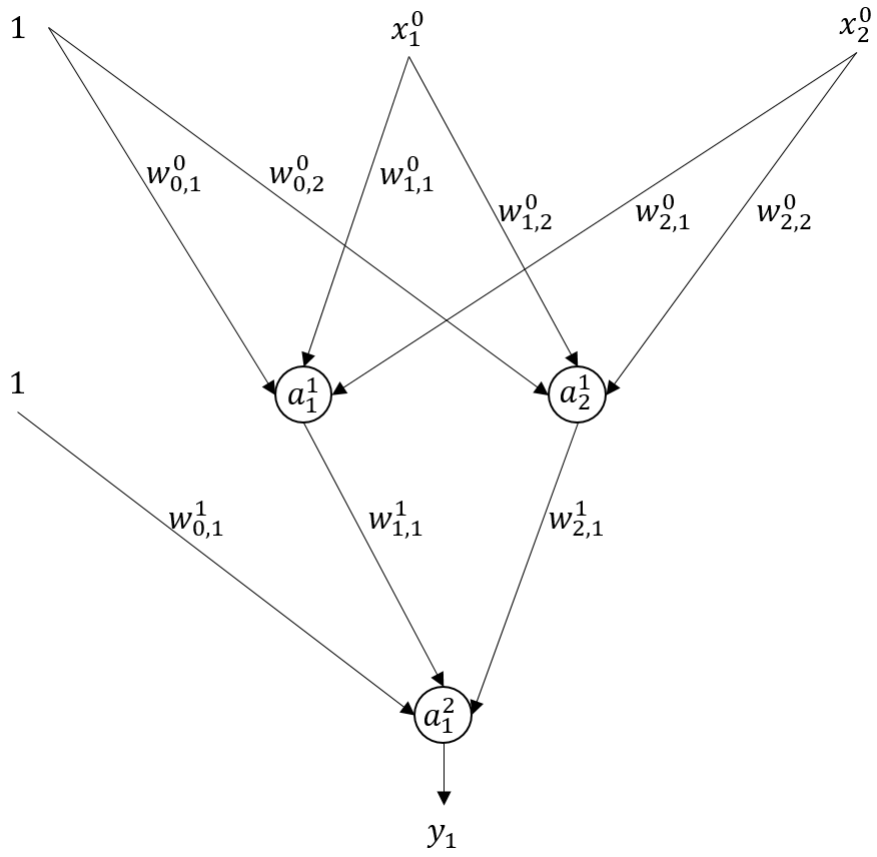

FIGURE A.1: Neural Networks Terminology

In this figure w_{ij}^m represents the weight in the synaptic link that joins neuron a_i^m with neuron a_j^{m+1}. Also m indicates the layer in the neural network, where the input layer has $m = 0$, and the output layer $m = 2$. The two input signals are given by x_1^0, x_2^0,

and the output signal by y_1. Even though it is not shown in the figure, the output of the hidden neurons a_1^1 and a_2^1 are x_1^1 and x_2^1 respectively.

A.2 Backpropagation Training in FNNs

Supervised training of FNNs can be done through the backpropagation method, which consists in propagating the output error with respect to the training data down all the internal layers in the network in order to adjust the weights while optimizing a model. The enumeration below presents the steps of a pseudo algorithm that implements backpropagation training in a FNN. Each of these steps is further explained in the remaining of this section.

Step 1 Initialize parameters in Neural Network

Step 2 Perform Forward Pass

Step 3 Compute Error

Step 4 Update link Weights

Step 5 Repeat Steps 2 to 4 until reducing Error to target value

A.2.1 Initialization and Forward Pass

The weights in a neural network are initialized randomly, typically to small values. Forward pass is the process through which a neural network is triggered with a given input signal to activate internal and output neurons.
[TODO: Expand on forward pass and initialization of parameters]

A.2.2 Error Computation

The error represents the difference between the predicted output y^* calculated in forward pass, and the real output y which is given as part of the training data. The MSE function A.1 is commonly use to measure these errors between predicted and real output for a training data set with n samples.

$$E = \sum_{i=1}^{n} E_i = \frac{1}{2} \sum_{i=1}^{n} (y_i^* - y_i)^2 \tag{A.1}$$

A.2.3 Weigths Optimization

The weights in a neural network represent the trainable parameters, and their optimization requires the calculation of the error gradient with respect to each of the weights $\frac{\partial E}{\partial w_{i,j}}$. The weigthts in each layer are adjusted through gradient steps in order to approach a minimum solution of the error function. Function A.2 shows the gradient step calculation to update a weight, where γ is the learning rate.

$$w_{i,j}^{new} = w_{i,j} - \gamma \frac{\partial E}{\partial w_{i,j}} \tag{A.2}$$

Error Gradient Computation

The computation of the gradients is performed first with respect to the weights that connect to the output layer, and then backpropagates down the neural network until reaching the weights that connect to the input layer. Equation A.3 shows a generalization of the error gradient with respect to weights $w_{i,j}^{M-1}$ of the synaptic links that connect to the output neuron a_1^M, where M is the total number of layers in a neural network.

$$\frac{\partial E_i}{\partial w_{i,j}^M} = \frac{\partial}{\partial w_{i,j}^M} \frac{1}{2}(y_i^* - y_i)^2 = (y_i^* - y_i) \frac{\partial}{\partial w_{i,j}^M} (\sigma(W_{i,j}^M X_i^{M-1}) - y_i) \tag{A.3}$$

[TODO: Keep Expanding on backpropagation training]

Appendix B

Appendix B title

Bibliography

Ke-Lin Du, M.N.S. Swamy (2006). *Neural Networks in a Softcomputing Framework*.

KI, Funahashi and Nakamura Y (1993). "Approximation of dynamical systems by continuous time recurrent neural networks". In: *Neural Networks*, 801–806.