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Trabalho III - Cálculo II
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1) $\int \frac{6x+5}{x^2+6x+9}dx$

a)

b)

c)

2)

a) $\int e^x(xe^{x^2-x}+1)dx = \int xe^{x^2-x+x}+e^x dx = \int xe^{x^2}dx + \int e^x dx$

$\int xe^{x^2}dx \qquad \int e^x dx = e^x + c$

$u = x^2$
$du = 2xdx$
$\frac{du}{2} = xdx$

$\int e^u \frac{du}{2} + e^x + c = \frac{e^u}{2} + e^x + c = \boxed{\frac{e^{x^2}}{2} + e^x + c}$

b) $\int \frac{1}{\sqrt{x-1}} + \frac{1}{-x-1}dx = \int \frac{1}{\sqrt{x-1}}dx + \int \frac{1}{-x-1}dx$

$u = x-1 \qquad w = -x-1$
$du = dx \qquad dw = -1dx$

$\int \frac{du}{u^{1/2}} + \int \frac{-dw}{w} = \frac{-2u^{-3/2}}{3} - \ln|w| + c = \boxed{\frac{-2(x-1)^{-3/2}}{3} - \ln|-x-1| + c}$

c) $\int \frac{2x(x^2-1)}{x+1}dx = \int \frac{2x(x-1)\cancel{(x+1)}}{\cancel{x+1}}dx = \int 2x(x-1)dx = \int(2x^2-2x)dx$

$\int 2x^2dx - \int 2xdx = \frac{2x^3}{3} - \frac{\cancel{2}x^2}{\cancel{2}} + c = \boxed{\frac{2x^3}{3} - x^2 + c}$

d) $\int \frac{2x+1}{2x-1}dx$

e) $\int \frac{x-1}{x^2-2x+1}dx = \int \frac{\cancel{x-1}}{(x-1)\cancel{(x-1)}}dx = \int \frac{1}{x-1}$

$u = x-1 \qquad du = dx$

$\int \frac{1}{u}du = ln(u) + c = \boxed{ln|x-1| + c}$

f) $\int \cos(x)e^{sen(x)} + e^{-x+1}dx = \int \cos(x)e^{sen(x)}dx + \int e^{-x+1}dx$

$u = sen(x) \qquad w = -x+1$

$du = \cos(x) \qquad dw = -dx$

$\int e^u du - \int e^w dw = e^u - e^w + c = \boxed{e^{sen(x)} - e^{x-1} + c}$

g) $\int sen(x+5)dx = \int sen(u)du = -\cos(u) + c = -\cos(x+5) + c$

$u = x+5 \qquad du = dx$

h) $\int \tan(x-1)dx = \int \tan(u) = \int \frac{sen(u)}{\cos(u)}du = \int \frac{-dw}{w}$

$u = x-1 \qquad w = \cos(u)$

$du = dx \qquad dw = -sen(u)du$

$ln(w) + c = -ln(cos(u)) + c = ln|sec(u)| + c = \boxed{ln|sec(x-1)| + c}$

i) $\int (2x-1)^{100}dx = \int u^{100}\frac{du}{2} = \int u^{100}\frac{du}{2} = \frac{1}{2}\frac{u^{101}}{101} + c$

$u = 2x-1 \qquad du = 2dx$

$\frac{1}{2}\frac{(2x-1)^{101}}{101} + c = \boxed{\frac{(2x-1)^{101}}{202} + c}$

j) $\int x^2 ln(2+x^3)dx = \int ln(u)\frac{du}{2} = \frac{1}{2}\int ln(u)du = u(ln(u)-1) + c$

$u = x^3 + 2 \qquad \boxed{= x^3 + 2(ln|x^3+2| - 1) + c}$

$du = 2x^2 dx$

$\frac{du}{2} = x^2 dx$

k) $\int x^2 sen(x) - xcos(x)dx = \int x^2 sen(x) - \int xcos(x)dx$

$\underline{\int x^2 sen(x)dx} = -x^2 cos(x) - \int -cos(x)2xdx =$

$u = x^2 \qquad dv = senx$
$du = -2xdx \quad v = -cos(x)$

$= -x^2cos(x) + \int cos(x)2xdx = -x^2cos(x) + (2xsen(x) - \int 2sen(x)dx) =$
$-x^2cos(x) + 2xsen(x) - 2(-cos(x)) = \underline{-x^2cos(x) + 2xsen(x) + 2cos(x)}$

$w = 2x \qquad dt = cos(x)$
$dw = 2dx \quad t = sen(x)$

$\underline{\int xcos(x)dx} = xsen(x) - \int sen(x)dx = \underline{xsen(x) + cos(x)}$

$u = x \qquad dv = cos(x)$

$du = dx \quad v = senx(x)$

$\boxed{-x^2cos(x) + 2xsen(x) + 2cos(x) - (xsen(x) + cos(x))}$

l) $\int sen^2(x)cos^2(x)dx = cos^2(x)(x - sen(x)cos(x) - \int(x - sen(x)cos(x))2cos(x)sen(x)dx$

$u = cos^2(x) \qquad dv = sen^2(x)$
$du = 2cos(x)sen(x)dx \quad v = x - sen(x)cos(x)$

m) $\int \frac{x+1}{x^2-1}dx = \int \frac{\cancel{x+1}}{\cancel{(x+1)}(x-1)}dx = \int \frac{1}{x-1}dx = \int \frac{1}{u}du = ln(u) + c = \boxed{ln|x-1| + c}$

$u = x - 1 \qquad du = dx$

n) $\int \frac{sen(2x)}{cos^2(x)}dx = \int \frac{2sen(x)\cancel{cos(x)}}{\cancel{cos^2(x)}}dx = 2\int \frac{sen(x)}{cos(x)}dx = 2\int \frac{-du}{u} = -2ln(u) + c = \boxed{-2ln(cos(x)) + c}$

$sen(2x) = 2sen(x)cos(x) \qquad u = cos(x) \qquad du = -sen(x)dx$

o) $\int e^{x^3}x^2dx = \int e^u \frac{du}{2} = \frac{e^u}{2} + c = \boxed{\frac{e^{x^3}}{2} + c}$

$u = x^3 \qquad du = 2x^2dx$
$. \qquad \frac{du}{2} = x^2dx$

p) $\int \frac{3x^2+4}{\sqrt{x^3+x}} = \int(\frac{3x^2+1}{\sqrt{x^3+x}} + \frac{3}{\sqrt{x^3+x}})du = \int \frac{du}{u^{1/2}} + \int \frac{3}{u^{1/2}}du = \int u^{-1/2}du + \int 3u^{-1/2}du$

$u = x^3 + x \qquad du = 3x^2 + 1$

q) $\int -(e^{x^2} + 2x)(xe^{x^2} + 1)dx$

$u = -e^{x^2} + 2x \qquad dv = xe^{x^2} + 1$
$du = -e^{x^2}2x + 2 \quad v =$

r) $\int$

s) $\int sen(x+3)cos(x+3)dx = \int udu = \frac{u^2}{2} + c = \boxed{\frac{sen(x+3)^2}{2} + c}$

$u = sen(x+3) \qquad du = cos(x+3)dx$

t) $\int arctg(x) + \frac{1}{1+x^2}dx = \int arctg(x) + arctg(x) = \boxed{xarctg(x) - \frac{ln(x^2+1)}{2} + arctg(x) + c}$

u) $\int xe^{-x}dx = \boxed{xe^{-x} - e^{-x} + c}$

$u = x \qquad dv = e^{-x} \qquad xe^{-x} - \int e^{-x}dx$
$du = dx \qquad v = e^{-x}$

3)

a) $\int x^2e^xdx = x^2e^x - \int e^x 2xdx = x^2e^x - 2\int e^x xdx = x^2e^x - 2(xe^x - \int e^x dx) + c = \boxed{x^2e^x - 2xe^x + 2e^x + c}$

$u = x^2 \qquad dv = e^x \qquad w = x \qquad dt = e^x$
$du = 2xdx \qquad v = e^x \qquad dw = dx \qquad t = e^x$

b) $\int e^x senx(x)dx = e^x sen(x) - \int e^x cos(x)dx =$

$u = sen(x) \qquad dv = e^x \qquad w = cos(x) \qquad dt = e^x$
$du = cos(x) \qquad v = e^x \qquad dw = -sen(x) \qquad t = e^x$

$\int e^x senx(x)dx = e^x sen(x) - (e^x cos(x) - \int -e^x sen(x)dx) = e^x sen(x) - e^x cos(x) - \int e^x sen(x)dx$

$2\int e^x senx(x)dx = e^x sen(x) - e^x cos(x) = \boxed{\frac{e^x sen(x) - e^x cos(x)}{2} + c}$

c) $\int (ln(x) + arctg(x))dx = \boxed{x(ln|x| - 1) + xarctg(x) - \frac{ln(x^2+1)}{2} + c}$

d) $\int x^2 ln(x)dx = \frac{x^3 ln(x)}{3} - \int \frac{x^3}{3x}dx = \frac{x^3 ln(x)}{3} - \frac{1}{3}\int x^2 dx = \frac{x^3 ln(x)}{3} - \frac{1}{3}.\frac{x^3}{3} = \boxed{\frac{x^3 ln(x)}{3} - \frac{x^3}{9} + c}$

$u = ln(x) \qquad dv = x^2$
$d = \frac{1}{x} \qquad v = \frac{x^3}{3}$

e) $\int x^2 cos(2x)dx = \frac{x^2 sen(2x)}{2} - \int \frac{sen(2x)2x}{2}dx = \frac{x^2 sen(2x)}{2} - (\frac{\cancel{-2}xcos(2x)}{\cancel{2}} + \int \frac{\cancel{-2}cos(2x)}{\cancel{2}}dx)$

$u = x^2 \qquad dv = cos(2x) \qquad w = 2x \qquad dv = sen(2x)$
$du = 2xdx \qquad v = \frac{sen(2x)}{2} \qquad dw = 2 \qquad v = \frac{-cos(2x)}{2}$

$= \boxed{\frac{x^2 sen(x)}{2} + \frac{xcos(2x)}{2} - \frac{sen(2x)}{2} + c}$

f) $\int e^x cos(x)dx = \int e^x sen(x) - \int sen(x)e^x dx = e^x sen(x) - (-e^x cos(x) - \int -cos(x)e^x dx) = e^x sen(x) + e^x cos(x) - \int cos(x)e^x dx$

$u = e^x \qquad dv = cos(x) \qquad w = e^x \qquad dt = cos(x)$
$du = e^x \qquad v = sen(x) \qquad dw = e^x \qquad t = -cos(x)$

$\boxed{\frac{e^x sen(x) + e^x cos(x)}{2} + c}$

g) $\int xe^{2x}dx = xe^{2x} - \int e^{2x}dx = \boxed{xe^{2x} - e^{2x} + c}$

$u = x \qquad dv = e^{2x}$
$du = dx \qquad v = e^{2x}$

4)

a) $\int \frac{1}{(x+2)(x-3)} = \frac{A}{(x+2)} + \frac{B}{(x-3)}$

$1 = A(x-3) + B(x+2))$
$x = -2 => 1 = -5A \qquad A = -1/5$
$x = 3 => 1 = 5B \qquad B = 1/5$

$-1/5 \int \frac{1}{x+2}dx + 1/5 \int \frac{1}{x+3}dx = \boxed{-1/5ln|x+2| + 1/5ln|x+3| + c}$

b) $\int \frac{x}{x^2+3x+10}dx = \int \frac{x}{(x+5)(x-2)} = \frac{A}{(x+5)} + \frac{B}{(x-2)}$

$x = A(x-2) + B(x+5)$
$A = 5/7 \qquad B = 2/7$
$5/7 \int \frac{x}{x+5}dx + 2/7 \int \frac{x}{x-2}dx = \boxed{5/7(x - 5ln|x+5|) + 2/7(x + 2ln|x-2|) + c}$

c) $\int \frac{2+4x}{(x+8)(x-3)}dx = \int \frac{2}{(x+8)(x-3)} + \frac{4x}{(x+8)(x-3)}dx$

d) $\int \frac{1}{(x-6)^2(x-1)}dx = \frac{Ax+B}{(x-6)^2} + \frac{C}{(x-1)}$

$1 = Ax + B(x-1) + C(x-6)^2$
$x = 1 => 1 = 25C$
$x = 6 => 1 = 6A + 5B \qquad A = 4/30 \qquad B + C = 0 \quad B = -C$

$\int \frac{4/30x - 1/25}{(x-6)^2)} + \frac{1/25}{x-1}dx = \boxed{1/25(\frac{-5}{x-6} - ln|x-6 + ln|x-1|) + c}$

e) $\int \frac{1}{x^2(x-2)} = \frac{Ax+B}{x^2} + \frac{C}{x-2}$

$Ax + B(x-2) + Cx^2 = 1$
$C = 0 \qquad x = 2 => 1 = 2A \quad A = 1/2 \qquad A = -B$

$\int \frac{1/2x - 1/2}{x^2}dx = 1/2 \int \frac{x-1}{x^2} = \boxed{\frac{xln(2-x) - xln(x) + 2}{4x} + c}$