Trabalho 4 - Calculo II

1 - a) $\int \frac{2x^3}{\sqrt{1+x^2}}dx = \int \frac{x^2}{\sqrt{u}}du = \int \frac{u-1}{\sqrt{u}}du = \int \frac{u}{\sqrt{u}}du - \int \frac{1}{\sqrt{u}}du$

$u = 1+x^2 \Rightarrow x^2 = u-1$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$= \frac{2u^{3/2}}{3} - 2\sqrt{u} + C$

$= \frac{2(1+x^2)^{3/2}}{3} - 2\sqrt{1+x^2} + C$

b) $\int \frac{8}{2+x^2}dx = \int \frac{8(\sqrt{2}\sec^2\theta)\,d\theta}{2+(\sqrt{2}\,tg\theta)^2} = 8\int \frac{\sqrt{2}\sec^2\theta}{2+(}$

$x = \sqrt{2}\,tg\theta$

$dx = \sqrt{2}\sec^2\theta\,d\theta$

c) $\int \frac{2x^2}{\sqrt{9-x^2}}dx = \int \frac{2(3\,sen\theta)^2}{\sqrt{9-(3\,sen\theta)^2}} = \int \frac{18\,sen^2\theta}{\sqrt{9(1-sen^2\theta)}}d\theta = \frac{18}{3}\int \frac{sen^2\theta\,d\theta}{\sqrt{1-sen^2\theta}}$

$x = 3\,sen\theta$

$dx = 3\cos\theta\,d\theta$

$\theta = arc\,sen\left(\frac{x}{3}\right)$

$6\int \frac{1-\cos^2\theta}{3\cos\theta}d\theta = 6\int 3\,sen^2\theta\,d\theta$

$18\int sen^2\theta\,d\theta = -9\cos(2\theta) + C$

$\int \frac{2x^2}{\sqrt{9-x^2}} = -9\cos\left(2\,arc\,sen\left(\frac{x}{3}\right)\right) + C$

d) $\int -\frac{x^3}{\sqrt{1+x^2}}dx = -\int \frac{u\sqrt{u}}{\sqrt{u+1}}\frac{du}{2\sqrt{u}} = -\int \frac{u}{\sqrt{u+1}}du = \int \frac{v-1}{\sqrt{v}}d$

$u = x^2 \Rightarrow x^2 = u \Rightarrow x^3 = u\sqrt{u}$ $\quad v = u+1 \Rightarrow u = v-$

$du = 2x\,dx$ $\quad dv = du$

$dx = \frac{du}{2x}$

$\int \frac{v-1}{v}dv = -\frac{2}{3}(v-2)\sqrt{v+1} =$

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$$-\frac{2}{3}(u-1)\sqrt{u+2}+c = -\frac{2}{3}(x^2-1)\sqrt{x^2+2}+c$$

$$\int \frac{-x^3}{\sqrt{1+x^2}}\,dx = -\frac{2}{3}(x^2-1)\sqrt{x^2+2}+c$$

e) $\int 2x\sqrt{4+x^2}\,dx = \int (4\,tg\,\theta\sqrt{4+4tg^2\theta})(2\sec^2\theta)\,d\theta =$

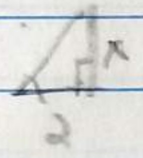
$x = 2\,Tg\,\theta$ $\quad \int 4\,tg\,\theta\sqrt{4(1+tg^2\theta)}\,(2\sec^2\theta)\,d\theta =$

$dx = 2\sec^2\theta$

$Tg^2\theta = 1-\sec^2\theta$ $\quad 2\int 4\,tg\,\theta\sqrt{(\sec^2\theta)}\,(2\sec^2\theta)\,d\theta =$

$Tg\,\theta = \frac{x}{2}$

$16\int (Tg\,\theta\sec\theta)(\sec^2\theta)\,d\theta = 16\int u^2\,du = \frac{u^3}{3}+c$



$u = \sec\theta$ $\quad = \frac{16\sec^3\theta}{3}+c$

$du = Tg\,\theta$

f) $\int \sqrt{(x+1)(x-1)}\,dx = \int \sqrt{x^2-1}\,dx = \int \sqrt{\sec^2 u - 1}\,(\sec u\,tg\,u)\,du =$

$Tg\,u = \frac{\operatorname{sen} u}{\cos u} = \frac{\frac{\sqrt{1-x^2}}{x}}{1/x}$ $\quad x = \sec u$ $\quad \sec = \frac{1}{\cos u}$ $\quad \cos u =$

$dx = \sec u\,tg\,u\,du$

$= \int Tg\,u\,(\sec u\,tg\,u)\,du = \int Tg^2 u\sec u\,du = \int (\sec^2 u - 1)\sec u\,du$

$= \int \sec^3 u - \sec u\,du = \int \sec^3 u\,du - \int \sec u\,du =$

$= \int \sec^2 u\sec u\,du - \int \sec u\,du = \frac{\sec u\,tg\,u + \ln|\sec u + tg\,u|}{2} -$

$\ln|\sec u\,tg\,u| + c =$

g) $\int \frac{1}{x\sqrt{x^2-1}}\,dx = \int \frac{1}{\sec\theta\sqrt{\sec^2\theta-1}} \sec\theta \, tg\theta \, d\theta$

$x = \sec\theta$

$dx = \sec\theta \, tg\theta \, d\theta$

$= \int \frac{1}{\cancel{\sec\theta \, tg\theta}} \cancel{\sec\theta \, tg\theta} \, d\theta$

$= \int d\theta = \theta + c = \arccos\left(\frac{1}{x}\right) + c$

$x = \frac{1}{\cos\theta}$

$\cos\theta = \frac{1}{x}$

h) $\int \frac{1}{x\sqrt{x^2-4}}\,dx = \int \frac{1}{2\sec\theta\sqrt{4\sec^2\theta-4}} 2\sec\theta \, tg\theta \, d\theta =$

$x = 2\sec\theta$

$dx = 2\sec\theta \, tg\theta \, d\theta$ $\quad = \int \frac{1}{2\sec\theta\sqrt{4(\sec^2\theta-1)}} \cancel{2}\sec\theta \, tg\theta \, d\theta =$

$x = \frac{2}{\cos\theta} \to \cos\theta = \frac{2}{x}$ $\quad = \frac{1}{2}\int \frac{1}{\sec\theta|tg\theta|} \sec\theta \, tg\theta \, d\theta = \frac{1}{2}\int d\theta = \frac{1}{2}\theta + c$

$0 < \theta < \frac{\pi}{2}$ $\quad = \frac{1}{2}\arccos\left(\frac{2}{x}\right) + c$

i) $\int \frac{x}{x\sqrt{x^2-4}}\,dx = \int \frac{2\sec\theta}{2\sec\theta\sqrt{4(\sec^2\theta-1)}} 2\sec\theta \, tg\theta \, d\theta = \int \sec\theta \, d\theta, \ 0 < \theta < \frac{\pi}{4}$

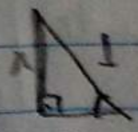
$x = 2\sec\theta$ $\quad = \ln|T_g\theta + \sec\theta| + c = \ln|\sqrt{x^2-4} + x|$

$dx = 2\sec\theta tg\theta d\theta$

j) $\int \frac{\sqrt{1-x^2}}{x^2}\,dx = \int \frac{\sqrt{1-sen^2\theta}}{sen^2\theta} \cos\theta \, d\theta = \int \frac{\sqrt{\cos^2\theta}}{sen^2\theta} . \cos\theta \, d\theta$

$x = sen\,\theta$ $\quad = \int \frac{\cos^2\theta}{sen^2\theta} d\theta, \ -\frac{\pi}{2} < \theta < \pi/2$

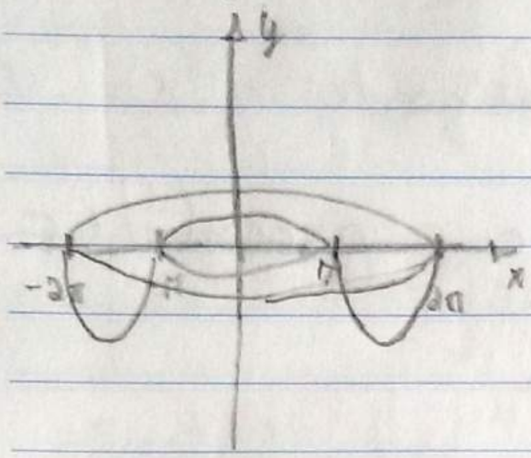
$dx = \cos\theta \, d\theta$



$z = \sqrt{1-x^2}$ $\quad = \int \cot g^2\theta \, d\theta = \int (\csc^2\theta - 1) d\theta = \int \csc^2\theta \, d\theta - \int d\theta$

$= -\cot\theta - \theta + c = -\frac{\sqrt{1-x^2}}{x} - arcsen(x) + C$

2- a) $y = sen(x)$ $\quad x \in [\pi, 2\pi]$



$$\int_\pi^{2\pi} \pi(sen\,x)^2\,dx = \pi \int_\pi^{2\pi} sen^2 x\,dx = \pi \int_\pi^{2\pi} \frac{1-\cos(2x)}{2}$$

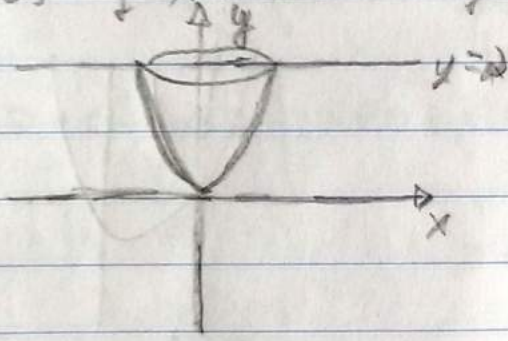
$$= \left[\pi\left(x - \frac{sen(2x)}{2}\right)\right]_\pi^{2\pi}$$

$$= \pi\left(2\pi - \frac{sen(4\pi)}{2}\right) - \pi\left(\pi - \frac{sen(2\pi)}{2}\right)$$

$$= \frac{2\pi^2 - \pi\,sen(4\pi)}{2} - \frac{\pi^2 - \pi\,sen(2\pi)}{2}$$

$$= \pi^2 - \frac{\pi^2}{2} = \frac{\pi^2}{2}$$

b) $y = x^2 + 2x$ $\quad y=0$ e $y=2$



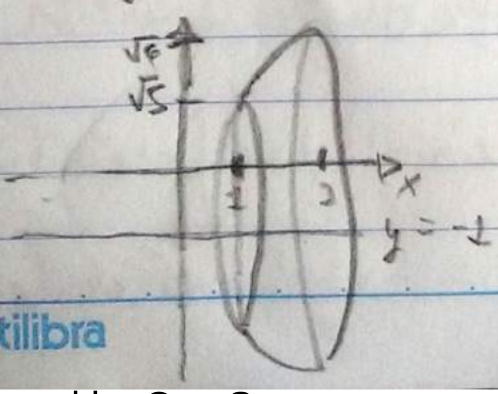
$$2\pi\int_0^2 x(x^2+2x)\,dx = 2\pi\int x^3+2x^2\,dx$$

$$= \left[\frac{\pi x^4}{2} + \frac{4x^3\pi}{3}\right]_0^2$$

$$= \left(8\pi + \frac{32\pi}{3}\right) = \frac{56\pi}{3}$$

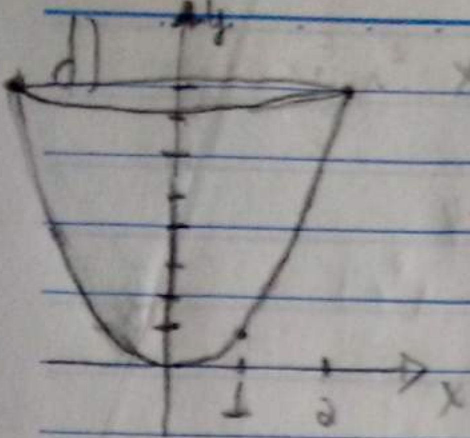
c) $y = \sqrt{x+4}$ $\quad x \in [1,2]$ $\quad$ reta $y=-1$

$y = \sqrt{5}$ e $y = \sqrt{6}$ $\qquad V = \frac{11\pi}{2}$



$$\pi\int_1^2 x+4\,dx = \left[\pi\left(\frac{x^2}{2}+4x\right)\right]_1^2 = \left[\frac{x^2\pi}{2} + 4\pi x\right]_1^2$$

$$(2\pi + 8\pi) - \left(\frac{\pi}{2} + 4\pi\right) = 10\pi - \frac{9\pi}{2} = \frac{11\pi}{2}$$

d) 

$x \in [0, 2]$ $\quad y = x^2$

$$2\pi \int_0^2 x^4\,dx = \left[\frac{2\pi x^5}{5}\right]_0^2 = \frac{64\pi}{5}$$

3- $V = \frac{4}{3}\pi R^3$

Usando Disco:

Para uma semi circunferência $x^2+y^2=2^2$ e girando em torno de x temos $\int_{-2}^{2} \pi\left(\sqrt{4-x^2}\right)^2 dx = \int_{-2}^{2} \pi(4-x^2)\,dx$

Então:

$$\pi\left[4x - \frac{x^3}{3}\right]_{-2}^{2} = \left(8\pi - \frac{2^3\pi}{3}\right) - \left(-8\pi + \frac{2\pi^3}{3}\right) = \left(\frac{24\pi - 2^3\pi}{3}\right) - \left(\frac{-24\pi + 2\pi^3}{3}\right)$$

$$= \pi\left[\left(\frac{3r^3 - r^3}{3}\right) - \left(\frac{-3r^3 + r^3}{3}\right)\right] = \frac{\pi}{3}\left(2r^3 + 2r^3\right)$$

$$V = \frac{4}{3}\pi r^3$$

Usando cascas cilíndricas:

Para uma semi circunferência $x^2+y^2=2^2$, girando em torno de y temos $\int_{-2}^{2} 2\pi\left(\sqrt{2^2-x^2}\right)x\,dx = 2\pi\int_{-2}^{2} x\sqrt{4-x^2}\,dx =$

$\operatorname{sen}\theta = \frac{x}{2}$

$x = 2\operatorname{sen}\theta$ $\qquad 2\pi \int_{-2}^{2} 2\operatorname{sen}\theta\sqrt{4-4\operatorname{sen}^2\theta}\;2\cos\theta\,d\theta =$

$dx = 2\cos\theta\,d\theta$

$$= 2\pi \int_{-2}^{2} 2\operatorname{sen}\theta \cdot 2\cos\theta \sqrt{4(1-\operatorname{sen}^2\theta)}\,d\theta =$$

$$= 4\pi \int_{-2}^{2} 2\operatorname{sen}\theta\, 2\cos\theta \cdot \cos^2\theta\,d\theta = 16\pi \int_{-2}^{2} \operatorname{sen}\theta\cos^2\theta\,d\theta = 16\pi\int_{-2}^{2} u^2(-du)$$

$u = \cos\theta$

$du = -\operatorname{sen}\theta\,d\theta$

$$= -16\pi \int_{-2}^{2} u^2\,du = -16\pi\left[\frac{\cos^3\theta}{3}\right]_{-2}^{2} =$$

$$-8\pi\left[\frac{\cos^3(\ldots)}{3}\right]_{-2}^{2}$$

4- $y = x^4 + 5$ e $y = x^2 + 5$

a) $x^4 + 5 = x^2 + 5$ eixo $y = -4$

$x^4 - x^2 = 0$ $x^4 + 9 = x^2 + 9$

$x = 0$ ou $x = \pm 1$

$\pi \int_0^1 (x^2+9)^2 dx - \pi \int_0^1 (x^4+9)^2 dx = \pi \int_0^1 x^4 + 18x^2 + 81\, dx - \pi \int_0^1 x^8 + 18x^4 + 81\, dx$

$\left[\frac{\pi x^5}{5} + 6\pi x^3 + 81\pi x\right]_0^1 - \left[\frac{\pi x^9}{9} + \frac{18\pi x^5}{5} + 81\pi x\right]_0^1 =$

$= \frac{\pi}{5} + 6\pi + 81\pi - \left(\frac{\pi}{9} + \frac{18\pi}{5} + 81\pi\right) = \frac{112\pi}{45}$

b) $2\pi \int_0^1 x(x^4+5)\, dx - \int_0^1 2\pi x(x^2+5)\, dx =$

$2\pi \int_0^1 x^5 + 5x\, dx - 2\pi \int_0^1 x^3 + 5x\, dx = \left[\frac{\pi x^6}{3} + 5\pi x^2\right]_0^1 - \left[\frac{\pi x^4}{2} + 5x\pi\right]_0^1$

$\frac{\pi}{3} + 5\pi - \frac{\pi}{2} + 5\pi = \frac{16\pi}{3} - \frac{11\pi}{2}$

$V = \frac{16\pi}{3} - \frac{11\pi}{2} = -\frac{\pi}{6}$

c) $\pi \int_0^1 (x^2+5)^2 dx - \pi \int_0^1 (x^4+5)^2 dx = \left[\frac{\pi x^5}{5} + \frac{10x^3\pi}{3} + 25\pi x\right]_0^1$

$- \left[\frac{\pi x^9}{9} + 2\pi x^5 + 25\pi x\right]_0^1$

$$\frac{\pi}{5} + \frac{10\pi}{3} + 25\pi - \left(\frac{\pi}{9} + 2\pi + 25\pi\right) = \frac{64\pi}{45}$$

5 - a) $y = -8x$ e $y = x^2 + 16$

$$-8x = x^2 + 16$$

$$x^2 + 8x + 16 = 0$$

$$\Delta = 64 - 64 = 0$$

$$x = \frac{-8 \pm 0}{2} \quad x = -4$$

$$\int_{-4}^{0} x^2 + 16\,dx - \int_{-4}^{0} -8x\,dx = \left[\frac{x^3}{3} + 16x\right]_{-4}^{0} - \left[-4x^2\right]_{-4}^{0}$$

$$= \frac{4^3}{3} + 64 + 64 = 128 + \frac{64}{3} = \frac{384 + 64}{3} = \frac{448}{3} =$$

b) $y = x - 3$ e $y = x^2 - 3$

$$x^2 - 3 = x - 3$$

$$x^2 - x = 0 \quad \to x(x-1) = 0$$

$$x = 0 \text{ ou } x = 1$$

$$\int_{0}^{1} x - 3\,dx - \int_{0}^{1} x^2 - 3\,dx = \left[\frac{x^2}{2} - 3x\right]_{0}^{1} - \left[\frac{x^3}{3} - 3x\right]_{0}^{1}$$

$$= \frac{1}{2} - 3 - \left(\frac{1}{3} - 3\right) = -\frac{5}{2} + \frac{8}{3} = \frac{-15 + 16}{6} = \frac{1}{6}$$

c) $y_1 = x$ e $y_2 = e^{\frac{2x+4}{14}}$, com $x \in [4, 6]$

p/ $x = 5$

$y_2 = e$

$$\int_{4}^{6} e^{2x+4}\,dx - \int_{4}^{6} x\,dx = \left[2e^{2x+4}\right]_{4}^{6} - \left[\frac{x^2}{2}\right]_{4}^{6} = \left[2e^{16} - 2e^{12}\right] - \left[\frac{36}{2} - \frac{16}{2}\right]$$

$u = 2x + 4$

$$= 2e^{16} - 2e^{12} - 10$$

$du = 2\,dx$

d) $y = x^3+2$ e $y = x^4+2$

$$x^3+2 = x^4+2$$
$$x^3 - x^4 = 0$$
$$x^3(1-x) = 0$$
$$x = 0 \text{ ou } x = 1$$

$$\int_0^1 x^3+2\,dx - \int_0^1 x^4+2\,dx = \left[\frac{x^4}{4}+2x\right]_0^1 - \left[\frac{x^5}{5}+2x\right]_0^1 = \frac{1}{4}+2-\left(\frac{1}{5}+2\right)$$
$$= \frac{9}{4} - \frac{11}{5} = \frac{45-44}{20} = \frac{1}{20}$$

e) $y = x+1$ e $y = x^2+1$

$$x^2+1 = x+1$$
$$x^2 - x = 0$$
$$x(x-1) = 0$$
$$x = 0 \text{ ou } x = 1$$

$$\int_0^1 x+1\,dx - \int_0^1 x^2+1\,dx = \left[\frac{x^2}{2}+x\right]_0^1 - \left[\frac{x^3}{3}+x\right]_0^1 = \frac{1}{2}+1-\left(\frac{1}{3}+1\right)$$
$$= \frac{3}{2} - \frac{4}{3} = \frac{9-8}{6} = \frac{1}{6}$$

f) $y = \cos(3x)$ e $y = \operatorname{sen}(3x)$, com $x \in [0, \pi]$

$$\int_0^\pi \operatorname{sen}(3x)\,dx - \int_0^\pi \cos(3x)\,dx = \Big[3\cos(3x)\Big]_0^\pi - \Big[-3\operatorname{sen}(3x)\Big]_0^\pi =$$
$$= 3\cos(3\pi) + 3\operatorname{sen}(3\pi)$$

5- a) $$\int_{-4}^{6} x^2 e^x\,dx = \left[x^2e^x - \int e^x 2x\,dx\right]_{-4}^{6} = \left[x^2e^x - 2\int e^x x\,dx\right]_{-4}^{6}$$

$u = x^2$ $\quad dv = e^x dx$

$du = 2x\,dx$ $\quad v = e^x$

$u = x$ $\quad dv = e^x dx$

$du = dx$ $\quad v = e^x$

$$= \left[x^2e^x - 2\left(xe^x - \int e^x dx\right)\right]_{-4}^{6} = \left[x^2e^x - 2xe^x + 2e^x\right]_{-4}^{6}$$

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$$= \left(36e^{6} - 12e^{6} + 2e^{6}\right) - \left(16e^{-4} + 8e^{-4} + 2e^{-4}\right)$$

$$= 26e^{6} - 26e^{-4}$$

b) $$\int_0^{\pi/2} \cos^3(x)\,\text{sen}(x)\,dx = \left[\int \cos^3 x\,(1 - \cos^2 x)\,dx\right]_0^{\pi/2} = \int_0^{\pi/2} \cos^3 x - \cos^5 x\,dx$$

$$\int_0^{\pi/2} \cos^3 x\,dx - \int_0^{\pi/2} \cos^5 x\,dx = \int_0^{\pi/2} (1 - \text{sen}^2 x)\cos x\,dx - \int_0^{\pi/2} (1 - \text{sen}^2 x)^2 \cos x\,dx$$

$u = \text{sen}\,x$

$du = \cos x\,dx$

$$= \left[\text{sen}\,x - \frac{\text{sen}^3 x}{3}\right]_0^{\pi/2} - \int_0^{\pi/2} (1 - u^2)^2\,du =$$

$$= \left[\text{sen}\,x - \frac{\text{sen}^3 x}{3}\right]_0^{\pi/2} - \int_0^{\pi/2} 1 + u^4 - 2u^2\,du = \left[u - \frac{2u^3}{3} + \frac{u^5}{5}\right]_0^{\pi/2}$$

$$= \left[\text{sen}\,x - \frac{\text{sen}^3 x}{3}\right]_0^{\pi/2} - \left[\text{sen}\,x - 2\frac{\text{sen}^3 x}{3} + \frac{\text{sen}^5 x}{5}\right]_0^{\pi/2}$$

$$= \text{sen}\frac{\pi}{2} - \frac{\text{sen}^3 \pi/2}{3} - \left(\text{sen}\,\pi/2 - 2\frac{\text{sen}^3 \pi/2}{3} + \frac{\text{sen}^5 \pi/2}{5}\right)$$

$$= \cancel{1} - \frac{1}{3} - \cancel{1} - \frac{2}{3} + \frac{1}{5} = \frac{5 - 10 + 3}{15} = \frac{-2}{15}$$

c) $$\int_0^{2\pi} \cos^3(x) + \text{sen}^2(x)\,dx = \int_0^{2\pi} \cos^3(x)\,dx + \int_0^{2\pi} \text{sen}^2(x)\,dx$$

$$= \left[\text{sen}\,x - \frac{\text{sen}^3 x}{3}\right]_0^{2\pi} + \int_0^{2\pi} \frac{1 - \cos(2x)}{2}\,dx =$$

$$= \left[\text{sen}\,x - \frac{\text{sen}^3 x}{3}\right]_0^{2\pi} + \left[\frac{x}{2} - \frac{\text{sen}(2x)}{4}\right]_0^{2\pi} = 0 - 0 + \pi - 0 = \pi$$

d) $\int_0^1 \ln(5x+2)\,dx = 5\int_0^1 \ln(u)\,dx = u\ln(u) - \int \frac{u}{u}\,du =$

$u = 5x+2$ $\quad u=\ln(u)$ $\quad dv = du$

$du = 5\,dx$ $\quad du = \frac{1}{u}du$ $\quad v = u$

$= \left[5\left(u\ln(u) - u\right)\right]_0^1 = \left[5(5x+2)\ln(5x+2) - 5(5x+2)\right]_0^1$

$= \left[25x + 10\ln(5x+2) - 25x - 10\right]_0^1 = 35\ln(7) - 35$

e) $\int_{-2}^{4} 2 + e^{-x+4}\,dx = \int_{-2}^{4} 2\,dx - \int_{-2}^{4} e^u\,du = \left[2x\right]_{-2}^{4} - \left[e^{-x+4}\right]_{-2}^{4}$

$u = -x+4$

$du = -dx$

$= (8+4) - (1+e^6) = 13 + e^6$

f) $\int_0^{\pi} e^{x+1}\cos(x+1)\,dx = \int_0^{\pi} e^u\cos(u)\,du = e^u\cos(u) - \int e^u - \text{sen}\,u\,du$

$u = x+1$ $\quad u = \cos(u)$ $\quad dv = e^u du$

$du = dx$ $\quad du = -\text{sen}(u)du$ $\quad v = e^u$

$= \left[e^u\cos(u) - e^u - \cos(u)\right]_0^{\pi} = \left[e^{x+1}\cos(x+1) - e^{x+1} - \cos(x+1)\right]_0^{\pi}$

$= e^{\pi+1}\cos(\pi+1) - e^{\pi+1} - \cos(\pi+1)$

g) $\int_{-2\pi}^{\pi} 1 - \cos^2(x)\,dx = \int_{-2\pi}^{\pi} dx - \int_{-2\pi}^{\pi} \frac{1}{2}\left(\cos(2x) + 1\right)dx =$

$= \left[x\right]_{-2\pi}^{\pi} - \left[\frac{x}{2} + \frac{\text{sen}\,x\cos x}{2}\right]_{-2\pi}^{\pi} = (\pi + 2\pi) - \left(\frac{\pi}{2} + 0 + \pi + 0\right)$

$= \int_{-2\pi}^{\pi} 1 - \cos^2(x)\,dx = 3\pi - \frac{3\pi}{2} = \frac{3\pi}{2}$

h) $\int_{-2}^{3} x\cos(x)\,dx = x\,\text{sen}(x) - \int \text{sen}(x)\,dx = \Big[x\,\text{sen}(x) - \cos(x)\Big]_{-2}^{3}$

$u = x \quad dv = \cos(x)\,dx$

$du = dx \quad v = \text{sen}(x)$

$= \Big(3\,\text{sen}(3) - \cos(3)\Big) - \Big(-2\,\text{sen}(-2) - \cos(-2)\Big) =$

$= 3\,\text{sen}(3) - \cos(3) - 2\,\text{sen}(2) - \cos(2)$

i) $\int_{3}^{5} x - x\,e^x\,dx = \int_{3}^{5} x\,dx - \int_{3}^{5} x\,e^x\,dx =$

$u = x \quad dv = e^x\,dx$

$du = dx \quad v = e^x$

$= \left[\frac{x^2}{2}\right]_3^5 - \left[x\,e^x - \int e^x\,dx\right]_3^5 = \left(\frac{25}{2} - \frac{9}{2}\right) - \Big[x\,e^x - e^x\Big]_3^5 =$

$= \frac{16}{2} - \left(5e^5 - e^5 - 3e^3 + e^3\right) = \frac{16}{2} - 5e^5 + e^5 + 3e^3 - e^3$