Lecture Notes: Introduction to Astrophysics and Cosmology

Based on lectures by **Dr. Shmuel Bialy** in 2025-26
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These notes were typed in the 2026 winter semester *Introduction to Astrophysics and Cosmology* course taught by **Dr. Shmuel Bialy** at the Technion.

The lectures were given in Hebrew and were live-translated by me to English. The document is provided as is and likely contains many errors.

If you find any mistakes or typos please let me know at `danielbreger@campus.technion.ac.il`.

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Lecture 1.

Thu, October 30, 2025

1 introduction

Since astrophysics is a very wide subject we won't be able to dive deep on each subject. The course is introductory so we'll be skipping some things like full solutions to integrals and the such.

1.1 So what is astrophysics?

Astrophysics and Cosmology could be broken down into:

Physics of stars

Solar systems planets, comets...

ISM Inter-Stellar Medium, Inter-Galactic Medium

Cosmology development, the big bang, dark energy/matter

Exotic objects black holes, neutron stars, white dwarfs...

Relativity gravitational waves, gravitational lensing

Galaxies types, development, active galactic nuclei, quasars

Explosions novas, supernovas, gamma ray bursts

and the methods used to research those things include

Observations
- **Telescopes** IR, radio, xray, gamma rays...
- **Gravitational waves**
- **Neutrinos**
- **Other particles (via probes)**

Labs

Computer simulations hydrodynamics, N-body simulations...

Analytical theory

We'll focus mostly on the analytical theory, but we'll mention the others too.

Lecture 2.

Mon, November 3rd, 2025

1.2 Regular (Baryonic) Matter

In our normal day to day life we meet protons, neutrons, and electrons. There are also other particles we come across, such as neutrinos, muons, taus, etc. Another particle we meet often is the photon, and some others we won't get into.

When the universe came into being in the big bang quarks and photons were created. those quarks then combined into protons and neutrons. protons are basically hydrogen nuclei, so hydrogen was created in the big bang. For a short period of time after the big bang there were conditions for nuclear fusion across the early universe, so about 10% of the matter created from the big bang ended up being helium, and in terms of mass it was about 25% (Because Helium is heavier than hydrogen). Elements heavier than helium weren't created at any significant quantities in the big bang. If we plot this on a graph:

The portion of mass of hydrogen of the total mass is denoted by X, hydrogen by Y, and metals, which are all the elements heavier than helium and are denoted by $Z = \frac{M_{metals}}{M}$. For our sun:

$$X_\odot = 0.7381 \tag{2.1}$$
$$Y_\odot = 0.2485 \tag{2.2}$$
$$Z_\odot = 0.0134 \tag{2.3}$$

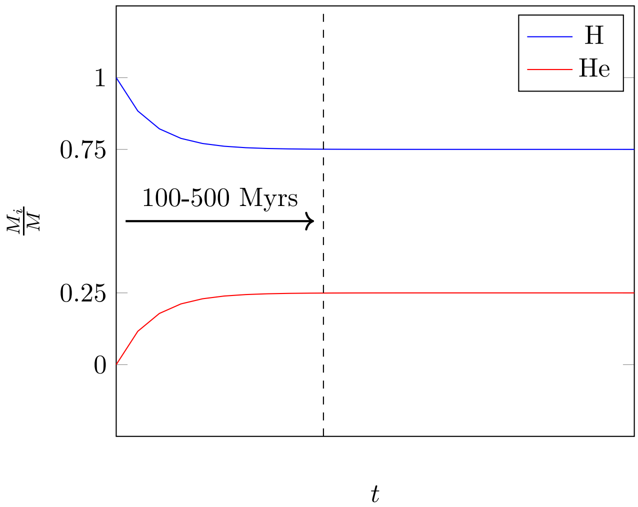

Figure 2.1: The portion of mass of the universe by element in the universe's early times.

The above was a discussion for gas and plasma. There is another kind of matter in space called dust. Dust is similar to the dust we know. It's particles about a micrometer in diameter made of water, carbon, silica, etc. Why do we mention this? In terms of the mass of the metals in the universe, about half of it is dust. Moreover, dust is a significant factor in observations of the universe, either in interfering in our ability to see things or in creating conditions that allow us to infer information. Dust also aids the creation of molecules by speeding up processes which are very slow in gasses.

An important definition for us will be the "average mass" which we'll use in many formulas. Assume we have a box filled with hydrogen and some helium and some metals, all with the relative quantities that are in the sun. What would the average mass per particle be? approximately a proton mass. And if the gas in the box is ionized? 0.5 proton masses. This is because we doubled the number of particles by freeing the electrons, which have approximately 0 mass relatively to the protons. This was a back of the napkin approximation just for sanity check and to have an idea of what we're expecting. The exact calculations is:

$$\overline{m} = \frac{\sum_j N_j m_j}{\sum_j N_j} \tag{2.4}$$

where j is each element type, N is their number, and m their mass. Rewriting in terms of X,Y,Z:

$$\frac{1}{\overline{m}} = \frac{\sum N_j}{\sum N_j m_j} \tag{2.5}$$
$$= \frac{M_j}{M_{tot}} \cdot \sum \frac{N_j}{M_j} \tag{2.6}$$
$$= \frac{1}{m_H}\left(X + \frac{1}{4}Y + \frac{1}{A_Z}Z\right) \tag{2.7}$$

where

$$M_j = \begin{cases} N_H m_H & H \\ N_{He} \cdot 4m_H & He \\ N_Z A_Z & else \end{cases} \tag{2.8}$$

Sometimes the following definition is used:

$$\mu = \frac{\overline{m}}{m_H} \tag{2.9}$$

and if we plug in the numbers for the sun we indeed find that it's very close to 1. We won't do this explicitly because there's a correction for the formula for a non-neutral gas, which our sun is not. For a completely ionized gas, i.e all the electrons are free, we just need to change N_j:

$$N_j = \begin{cases} 1 \to 2 & H \\ 1 \to 3 & He \\ 1 \to \left\langle \frac{A_Z}{2} \right\rangle & Else \end{cases} \tag{2.10}$$

Which gives us, for a completely ionized gas:

$$\mu = \left[2X + \frac{3}{4}Y + \frac{1}{2}Z\right]^{-1} \tag{2.11}$$

and if you insert the numbers for our sun we find

$$\langle A \rangle = 15 - 16 \tag{2.12}$$
$$\mu_{\odot} \simeq 0.6 \tag{2.13}$$

Lecture 3.

Thu, November 6th, 2025

1.3 The meaning of the symbol $\simeq$

The symbol $\simeq$ usually implies a good understanding of the physics behind the calculations. It says there's a good understanding of what can be considered negligible and what can't.

The luminosity of the sun follows

$$L_\odot = \dot{E}_{\text{nuc}} + |\dot{E}_G| \tag{3.1}$$

Observationally we can measure the luminosity and find

$$L_\odot = 3.8 \times 10^{33}\,\mathrm{erg\,s^{-1}} \tag{3.2}$$

and we can say that

$$L_\odot \simeq \dot{E}_{\text{nuc}} \tag{3.3}$$

Let's look at the total energy the Sun will output in its entire life E_{tot}.

$$\begin{aligned} E_{tot} &= L_\odot \cdot t_\odot && (3.4)\\ &\simeq (3.8 \times 10^{33}\,\mathrm{erg\,s^{-1}})(4.6\,\mathrm{Gyr}) && (3.5)\\ &= 5.6 \times 10^{50}\,\mathrm{erg} && (3.6) \end{aligned}$$

Let's look at what contributes more to the energy of the sun, the gravitational or the nuclear energy.

$$\begin{aligned} |E_G| &= \int \frac{GM(r)}{r}\,\mathrm{d}m && (3.7)\\ &= \int 4\pi\rho GM(r) r\,\mathrm{d}r && (3.8)\\ (\text{assuming } \rho = const.) &= \frac{3}{5} G \frac{M_\odot^2}{R_\odot} && (3.9)\\ &= 2 \times 10^{48}\,\mathrm{erg} && (3.10) \end{aligned}$$

To estimate the nuclear energy the sun produces we can look at the nuclear fusion process in which $4H \to He$ which gives a mass difference of

$$\begin{aligned} \Delta m &= M(4H) - M(He) && (3.11)\\ \to \Delta E &= \Delta m c^2 && (3.12) \end{aligned}$$

defining

$$\eta = \frac{\Delta E}{M_{He} c^2} = 0.7\% \tag{3.13}$$

then the nuclear energy produced in the sun is

$$\eta \cdot M_\odot c^2 \tag{3.14}$$

and using this we can find the age of the sun

$$t_{\odot} = \frac{E_{nuc}}{L_{\odot}} \approx 100\,\mathrm{Gyr} \tag{3.15}$$

but since the sun hasn't burned through all the hydrogen it has access to, we can replace

$$M_{\odot} \to M_{core} \approx 5\,\% M_{\odot} \tag{3.16}$$

which would give us a better estimate.

2 The Structure of Stars

We'll use the Sun as an example, then we'll hit equations.

2.1 The Sun

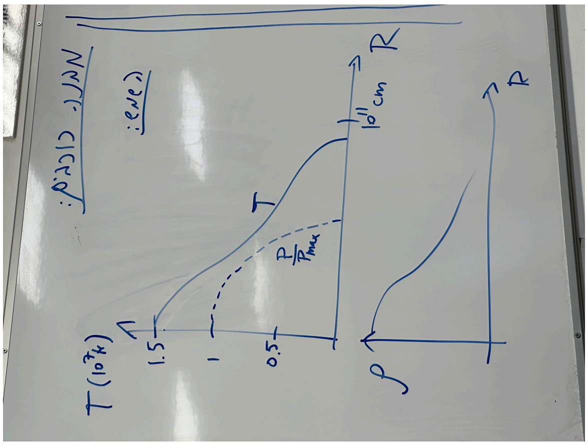

Figure 3.1: The temperature in the sun as a function of the distance from it core, the relative pressure compared to the core, and the density.

Our goal in the coming weeks will be to derive differential equations to figure out how the different profiles behave. As for some numbers:

$$R_{\odot} = 7\times 10^{10}\,\mathrm{cm} \qquad T_C = 1.5\times 10^{7}\,\mathrm{K} \qquad \rho_C = 150\,\mathrm{g\,cm^{-3}} \tag{3.17}$$
$$\overline{\rho} = 1.4\,\mathrm{g\,cm^{-3}} \qquad P_C = 10^{11}\,\mathrm{Atm} \qquad M_{\odot} = 2\times 10^{33}\,\mathrm{g} \tag{3.18}$$
$$L_{\odot} = 3.8\times 10^{33} erg/s \tag{3.19}$$

2.2 Star Structure Equations

The equations describing the structure of stars are:

$$\frac{dP}{dr} = -G\frac{M_r\rho}{r^2} \tag{3.20}$$
$$\frac{dM_r}{dr} = 4\pi r^2\rho \tag{3.21}$$
$$\frac{dL_r}{dr} = 4\pi r^2\rho\epsilon \tag{3.22}$$
$$\frac{dT}{dr} = -\frac{3\kappa\rho}{4acT^3}\frac{L_r}{4\pi r^2} \tag{3.23}$$

The first equation is called the hydrostatic equation and is equivalent to conservation of momentum, the second is the continuum equation and is equivalent to conservation of mass, the third is the energy production equation and is equivalent to conservation of energy, and the final one is the radiation diffusion equation.

2.3 The Hydrostatic Equation

The equation can be derived from an equality of forces. We look at a shell at radius r with mass $\mathrm{d}m$ and ask which forces act on it. On one hand there's gravitation pulling it inwards and on the other side there's a force resulting from a difference in pressure.

$$
\begin{aligned}
|\mathrm{d}F_g| &= \frac{GM_r\,\mathrm{d}m}{r^2} &(3.24)\\
&= \frac{GM_r}{r^2}4\pi r^2\rho\,\mathrm{d}r &(3.25)\\
|\mathrm{d}F_P| &= A\cdot(P(r)-P(r+\mathrm{d}r)) &(3.26)\\
&= 4\pi r^2\left(-\frac{dP}{dr}\,\mathrm{d}r\right) &(3.27)
\end{aligned}
$$

equating the two forces we can find

$$\frac{dP}{dr} = -\rho\frac{GM_r}{r^2} \tag{3.28}$$

which is indeed the hydrostatic equation.

Applying The Equation To Earth

As a back of the envelope calculation, we can use the equation to find the gravitational acceleration of earth. Noticing that

$$\frac{GM_r}{r^2} = g \tag{3.29}$$

we can rewrite the hydrostatic equation as

$$\frac{dP}{dr} = -\rho g \tag{3.30}$$

We can also find the speed of sound

$$c_s = \sqrt{\gamma\frac{dP}{d\rho}} \tag{3.31}$$

which means

$$\frac{dP}{d\rho} \approx c_s^2 \tag{3.32}$$

so

$$
\begin{aligned}
g &\sim \frac{dP}{d\rho}\frac{1}{h} &(3.33)\\
&= \frac{(340\,\mathrm{m\,s^{-1}})^2}{10\,\mathrm{km}} &(3.34)\\
&= 10\,\mathrm{m/s^2} &(3.35)
\end{aligned}
$$

Lecture 4.

Mon, November 10th, 2025

2.4 The State Equation

The gas in stars is to a good approximation an ideal gas so we can use the state equation of an ideal gas

$$P = Nk_BT \tag{4.1}$$

we can multiply and divide by the average mass of each particle

$$P = \frac{n\overline{m}}{\overline{m}}k_BT = \frac{\rho}{m_p\mu}k_BT \tag{4.2}$$

A quick aside on why the state equation is true

This is taken from Feynman's lectures on physics volume 2 chapter 39-2. Assume we have a wall of area A and particles with density n, each having velocity v_x and they impact the wall head on. As we know $F = \frac{dp}{dt}$.

The distance the particles travel over a period of time of δt is $\delta x = v_x\delta t$. So the rate at which particles hit over the same time period is $\delta N = \frac{1}{2}nAv\delta t$. Each particles transfers a momentum of $2p_1 = 2mv$ therefore the total momentum transferred is

$$\delta p = mnAv_x^2\delta t \tag{4.3}$$

therefore the force is

$$F = \frac{dp}{dt} = mnAv_x^2 \tag{4.4}$$

and the pressure is

$$P = \frac{F}{A} = mnv_x^2 \tag{4.5}$$

and if the velocity isn't a constant but rather there's a distribution of velocities then

$$P = mn\langle v_x^2\rangle \tag{4.6}$$

when in 3 dimensions

$$\langle v^2\rangle = \langle v_x^2\rangle + \langle v_y^2\rangle + \langle v_z^2\rangle = 3\langle v_x^2\rangle \tag{4.7}$$

So the pressure is

$$P = \frac{1}{3}mn\langle v^2\rangle \tag{4.8}$$

If we define temperature as follows

$$\frac{1}{2}m\langle v^2\rangle = \frac{3}{2}k_BT \tag{4.9}$$

and then insert this into our equation for pressure we get

$$P = nk_BT \tag{4.10}$$

Thermal energy is

$$U = \frac{1}{2}m\langle v^2\rangle N \tag{4.11}$$

so its density is

$$u = \frac{1}{2}m\langle v^2 c\rangle n \tag{4.12}$$

therefore we can insert this into the expression for pressure we have and find

$$P = \frac{2}{3}u \tag{4.13}$$

This is correct for mono-atomic gas such as hydrogen. In the more complicated case we can define the adiabatic index

$$\gamma_{ad} = \frac{C_p}{C_v} = 1 + \frac{2}{f} \tag{4.14}$$

where f is the number of degrees of freedom, and then we can find that

$$P = (\gamma_{ad} - 1)u \tag{4.15}$$

2.5 Generalizing To Radiation Pressure

The distribution of the velocities of the massive particles is the Boltzmann distribution. On the other hand, the distribution of the momenta of the photons is the Planck distribution:

$$\frac{dn}{d\nu} = n_\nu = \frac{8\pi\nu^2}{c^3}\frac{1}{e^{\frac{h\nu}{k_B T}} - 1} \tag{4.16}$$

and

$$u_\nu = n_\nu h\nu = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{\frac{h\nu}{k_B T}} - 1} \tag{4.17}$$

To get the pressure, rather than doing an entirely new development, we can say our previous development of the pressure using a wall and particles was good it just needs some adjustments. We had:

$$P = \frac{1}{3}mn\langle v^2\rangle = \langle\frac{1}{3}nv \cdot mv\rangle = \langle\frac{1}{3}nv \cdot p\rangle \tag{4.18}$$

inserting the momentum and velocity of photons we find

$$\begin{aligned} P &= \langle\frac{1}{3}nc\frac{h\nu}{c} && (4.19)\\ &= \langle\frac{1}{3}nh\nu\rangle && (4.20)\\ &= \frac{1}{3}\langle u\rangle && (4.21)\\ &= \frac{1}{3}u && (4.22) \end{aligned}$$

And if we equate this to the more general pressure equation using the adiabatic constant we find

$$\gamma_{\text{photons}} = \frac{4}{3} \tag{4.23}$$

to rewrite this in terms of temperature we remember that

$$u = \int_0^\infty u_\nu \, \mathrm{d}\nu = \int \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{\frac{h\nu}{k_B T}} - 1} \tag{4.24}$$

$$= [x = \frac{h\nu}{k_B T}] = 8\pi \frac{(kT)^4}{(hc)^3} \int_0^\infty x^3 \frac{\mathrm{d}x}{e^x - 1} \tag{4.25}$$

$$= 8\pi \frac{(kT)^4}{(hc)^3} \cdot \frac{\pi^4}{15} \tag{4.26}$$

we can define a constant

$$a = \frac{8\pi^5 k_B^4}{15(hc)^3} \tag{4.27}$$

and have

$$u = aT^4 \tag{4.28}$$

and find that the total pressure is

$$P_{tot} = nk_B T + \frac{1}{3} a T^4 \tag{4.29}$$

2.6 Nuclear Fusion

quick look at the periodic table. quick look at the standard model. the dominant nuclear fusion process is the pp-chain.

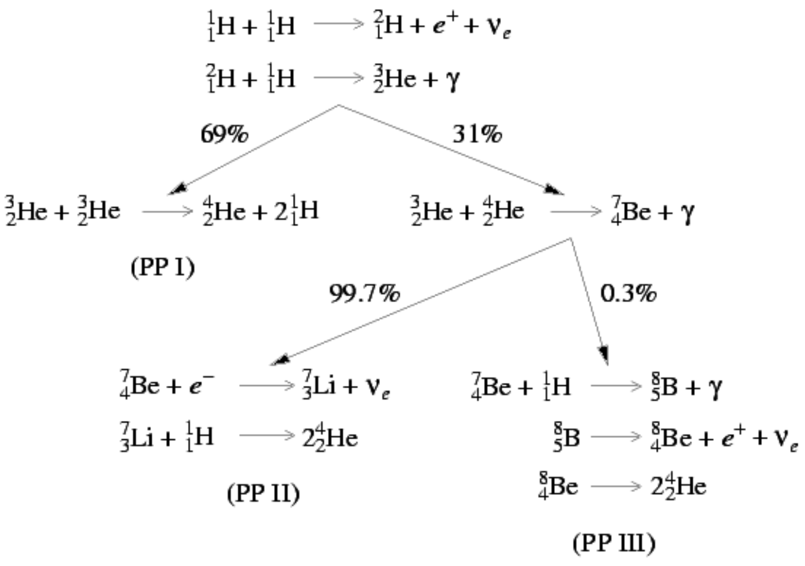

Figure 4.1: https://burro.cwru.edu/academics/Astr221/StarPhys/ppchain.html

The efficiency of the process is

$$\eta = \frac{E}{Mc^2} = \frac{26.7MeV}{4m_p c^2} = 0.7\% \tag{4.30}$$

for context of how much energy that is, we know that the sun's luminosity is $l_\odot = 4 \times 10^{33}$ erg/sec. This means that per second it produces 4×10^{26} J, which is 10^6 times the yearly consumption of all humanity.

Lecture 5.

Thu, November 13th, 2025

To find the luminosity we need to also figure out the pace of the processes, which we'll call r_{xi} whose units are $\mathrm{s}^{-1}\mathrm{cm}^{-3}$, x indicating the target particle being hit by the particle i. First though let's check if nuclear fusion can happen in the sun at all.

The electrostatic potential (in cgs) is given by

$$U = \frac{e^2}{r} = \frac{1.44\,\mathrm{MeV}}{(r/fm)} \tag{5.1}$$

where

$$e = 4.8 \times 10^{-10}\,\mathrm{esu} \qquad\qquad fm = 10^{-13}\,\mathrm{m} \tag{5.2}$$

and the temperature at the core of the sun is $T_c \simeq 1 \times 10^7\,\mathrm{K}$ and using the expression for the energy at the core

$$\frac{3}{2}k_B T = E \implies E_c \sim keV \tag{5.3}$$

This means that the thermal particles have only 0.1% of the energy required to jump the energy barrier imposed by the coulomb force's repulsion to reach the area the strong force attracts them. Seemingly if the system were purely classical, fusion could not happen. Let's consider an edge case where maybe fusion happens because a random small number of particles just happen to have a high enough energy to pass the barrier.

To do that we can calculate how many particles on average there are that have an energy with approximately a mega-electronvolt of energy. For a rough estimate we can use statistical mechanics

$$e^{-\frac{E}{k_B T}} \sim e^{-1000} = 10^{-435} \tag{5.4}$$

where we inserted the temperature at the core of the sun and the energy we want to have to breach the barrier. The number is so small that there would on average be less than one particle in the entire universe that would have that energy. For that reason we didn't calculate it accurately, since it wouldn't have mattered.

We'll need to take into consideration the effects of quantum tunneling, but before we do that, we need to discuss the action cross-section. We'll imagine a ball x with effective surface area σ being bombarded with particles i.

The number of particles hitting the area is given by

$$\mathrm{d}N = \sigma v \delta t \cdot n_i \tag{5.5}$$

so the rate at which they hit is

$$\frac{dN}{dt} = \sigma v n_i \tag{5.6}$$

If we have an environment with many x particles then

$$r_{xi} = \sigma v n_i \cdot n_x \tag{5.7}$$

where n_x is the density of the particles x. In the more general case if we have a distribution of velocities then we have

$$v \implies v(E) \qquad\qquad n_i \implies \frac{dn_i}{dE}\,\mathrm{d}E \tag{5.8}$$

and then

$$r_{xi} = \int \sigma(E) v(E) n_x$$

with the Stefan-Boltzmann law being

$$\frac{dn_i}{dE} = \frac{2n_i}{\sqrt{\pi}} \frac{\sqrt{E}}{(k_B T)^{3/2}} e^{-\frac{E}{k_B T}} \tag{5.10}$$

our goal now is to find $\sigma(E)$. For quantum tunneling we'll write

$$\sigma = \sigma_0 P \tag{5.11}$$

where P is the chance to tunnel and σ_0 is the effective area. As for σ_0:

$$\begin{aligned} \sigma_0 &\approx \pi R^2 && (5.12) \\ &= \pi \lambda_{DB}^2 && (5.13) \\ &= \pi \left(\frac{h}{p}\right)^2 && (5.14) \\ &= \pi \frac{h^2}{2\mu E} && (5.15) \end{aligned}$$

where μ is the reduced mass, since we're working with a collision of two particles. or, in a general more accurate form

$$\sigma_0 = \frac{S}{E} \tag{5.16}$$

To find the chance to tunnel we'll refer to quantum mechanics but this isn't a course on quantum mechanics so it's just a quick overview.

The Schrdinger equation is

$$\frac{\hbar^2}{2\mu} \nabla^2 \Psi = \left(V(r) - E\right) \Psi \tag{5.17}$$

where our energy is

$$E = \frac{3}{2} k_B T = \frac{z_1 z_2 e^2}{r_1} \tag{5.18}$$

and our potential is

$$V(r) = \begin{cases} \frac{Z_A Z_B e^2}{r} & r > r_0 \\ 0 & r < r_0 \end{cases} \tag{5.19}$$

Solving for this potential is complicated. We'll approximate it using a step function.

Its height should be the average of the potential

$$\langle V(r) \rangle = \frac{\int\limits_{r_0}^{r_1} 4\pi r^2 V(r) \, \mathrm{d}r}{\int\limits_{r_0}^{r_1} 4\pi r^2 \, \mathrm{d}r} = \frac{3}{2} E \tag{5.20}$$

and a step function is a problem we know how to solve. Inside the step we have

$$\frac{\hbar^2}{2\mu} \nabla^2 \Psi = \left(\frac{3}{2} E - E\right) \Psi \tag{5.21}$$

and since we're working in spherical coordinates

$$\frac{\hbar}{2\mu r}\frac{\partial^2}{\partial r^2}(r\Psi) = \frac{1}{2}E\Psi \tag{5.22}$$

whose solution is

$$\Psi = A\frac{e^{\beta r}}{r} \qquad\qquad \beta = \frac{\sqrt{\mu E}}{\hbar} \tag{5.23}$$

and therefore the chance to find a particle in an area δr is proportional to $|\Psi|\,\delta r$ and the probability of tunneling is

$$P_{\text{tunnel}} = \frac{4\pi r_0^2\delta r\,|\Psi(r_0)|^2}{4\pi r_1^2\delta r\,|\Psi(r_1)|^2} = \frac{e^{2\beta r_0}}{e^{2\beta r_1}} = e^{2\beta(r_0-r_1)} \approx e^{-2\beta r_1} \tag{5.24}$$

inserting the parameters we found earlier we find

$$P = e^{-\frac{2\sqrt{\mu E}}{\hbar}\frac{Z_A Z_B e^2}{E}} \tag{5.25}$$

* * *