

Lecture Notes: Introduction to Astrophysics and Cosmology

Based on lectures by **Dr. Shmuel Bialy** in 2025-26
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These notes were typed in the 2026 winter semester *Introduction to Astrophysics and Cosmology* course taught by **Dr. Shmuel Bialy** at the Technion.

The lectures were given in Hebrew and were live-translated by me to English. The document is provided as is and likely contains many errors.

If you find any mistakes or typos please let me know at danielbreger@campus.technion.ac.il.

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Thu, October 30, 2025

1 introduction

Since astrophysics is a very wide subject we won't be able to dive deep on each subject. The course is introductory so we'll be skipping some things like full solutions to integrals and the such.

1.1 So what is astrophysics?

Astrophysics and Cosmology could be broken down into:

Physics of stars

Solar systems planets, comets...

ISM Inter-Stellar Medium, Inter-Galactic Medium

Cosmology development, the big bang, dark energy/matter

Exotic objects black holes, neutron stars, white dwarfs...

Relativity gravitational waves, gravitational lensing

Galaxies types, development, active galactic nuclei, quasars

Explosions novae, supernovae, gamma ray bursts

and the methods used to research those things include

Observations Telescopes IR, radio, xray, gamma rays...

Gravitational waves

Neutrinos

Other particles (via probes)

Labs

Computer simulations hydrodynamics, N-body simulations...

Analytical theory

We'll focus mostly on the analytical theory, but we'll mention the others too.

Mon, November 3rd, 2025

1.2 Regular (Baryonic) Matter

In our normal day to day life we meet protons, neutrons, and electrons. There are also other particles we come across, such as neutrinos, muons, taus, etc. Another particle we meet often is the photon, and some others we won't get into.

When the universe came into being in the big bang quarks and photons were created. those quarks then combined into protons and neutrons. protons are basically hydrogen nuclei, so hydrogen was created in the big bang. For a short period of time after the big bang there were conditions for nuclear fusion across the early universe, so about 10% of the matter created from the big bang ended up being helium, and in terms of mass it was about 25% (Because Helium is heavier than hydrogen). Elements heavier than helium weren't created at any significant quantities in the big bang. If we plot this on a graph:

The portion of mass of hydrogen of the total mass is denoted by X , hydrogen by Y , and metals, which are all the elements heavier than helium and are denoted by $Z = \frac{M_{\text{metals}}}{M}$. For our sun:

$$X_{\odot} = 0.7381 \quad (2.1)$$

$$Y_{\odot} = 0.2485 \quad (2.2)$$

$$Z_{\odot} = 0.0134 \quad (2.3)$$

The above was a discussion for gas and plasma. There is another kind of matter in space called dust. Dust is similar to the dust we know. It's particles about a micrometer in diameter made of water, carbon, silica, etc. Why do we mention this? In terms of the mass of the metals in the universe, about half of it is dust. Moreover, dust is a significant factor in observations of the universe, either in interfering in our ability to see things or in creating conditions that allow us to infer information. Dust also aids the creation of molecules by speeding up processes which are very slow in gasses.

An important definition for us will be the "average mass" which we'll use in many formulas. Assume we have a box filled with hydrogen and some helium and some metals, all with the relative quantities that are in the sun. What would the average mass per particle be? approximately a proton mass. And if the gas in the box is ionized? 0.5 proton masses. This is because we doubled the number of particles by freeing the electrons, which have approximately 0 mass relatively to the protons. This was a back of the napkin approximation just for sanity check and to have an idea of what we're expecting. The exact calculations is:

$$\bar{m} = \frac{\sum_j N_j m_j}{\sum_j N_j} \quad (2.4)$$

where j is each element type, N is their number, and m their mass. Rewriting in terms of X, Y, Z :

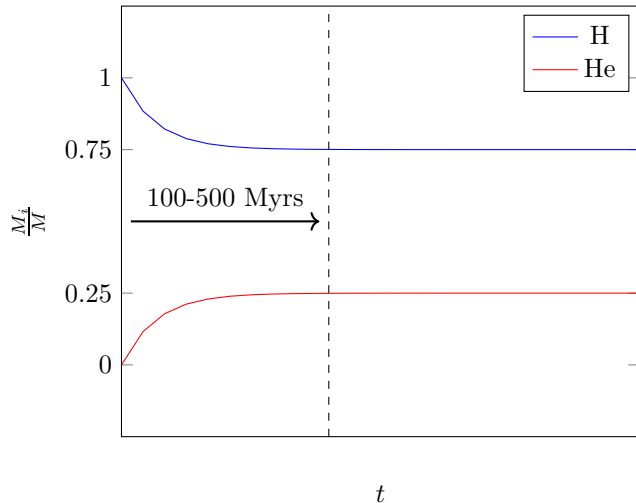


Figure 2.1: The portion of mass of the universe by element in the universe's early times.

$$\frac{1}{\bar{m}} = \frac{\sum N_j M_j}{\sum N_j m_j M_j} \quad (2.5)$$

$$= \frac{M_j}{M_{tot}} \cdot \sum \frac{N_j}{M_j} \quad (2.6)$$

$$= \frac{1}{m_H} \left(X + \frac{1}{4}Y + \frac{1}{A_Z}Z \right) \quad (2.7)$$

where

$$M_j = \begin{cases} N_H m_H & H \\ N_{He} \cdot 4m_H & He \\ N_Z A_Z & else \end{cases} \quad (2.8)$$

Sometimes the following definition is used:

$$\mu = \frac{\bar{m}}{m_H} \quad (2.9)$$

and if we plug in the numbers for the sun we indeed find that it's very close to 1. We won't do this explicitly because there's a correction for the formula for a non-neutral gas, which our sun is not. For a completely ionized gas, i.e all the electrons are free, we just need to change N_j :

$$N_j = \begin{cases} 1 \rightarrow 2 & H \\ 1 \rightarrow 3 & He \\ 1 \rightarrow \langle \frac{A_Z}{2} \rangle & Else \end{cases} \quad (2.10)$$

Which gives us, for a completely ionized gas:

$$\mu = \left[2X + \frac{3}{4}Y + \frac{1}{2}Z \right]^{-1} \quad (2.11)$$

and if you insert the numbers for our sun we find

$$\langle A \rangle = 15 - 16 \quad (2.12)$$

$$\mu_{\odot} \simeq 0.6 \quad (2.13)$$

Thu, November 6th, 2025

1.3 The meaning of the symbol \simeq

The symbol \simeq usually implies a good understanding of the physics behind the calculations. It says there's a good understanding of what can be considered negligible and what can't.

The luminosity of the sun follows

$$L_{\odot} = \dot{E}_{\text{nuc}} + |\dot{E}_G| \quad (3.1)$$

Observationally we can measure the luminosity and find

$$L_{\odot} = 3.8 \times 10^{33} \text{ erg s}^{-1} \quad (3.2)$$

and we can say that

$$L_{\odot} \simeq \dot{E}_{\text{nuc}} \quad (3.3)$$

Let's look at the total energy the Sun will output in its entire life E_{tot} .

$$E_{\text{tot}} = L_{\odot} \cdot t_{\odot} \quad (3.4)$$

$$\simeq (3.8 \times 10^{33} \text{ erg/s})(4.6 \text{ Gyr}) \quad (3.5)$$

$$= 5.6 \times 10^{50} \text{ erg} \quad (3.6)$$

Let's look at what contributes more to the energy of the sun, the gravitational or the nuclear energy.

$$|E_G| = \int \frac{GM(r)}{r} dm \quad (3.7)$$

$$= \int 4\pi\rho GM(r)r dr \quad (3.8)$$

$$(\text{assuming } \rho = \text{const.}) = \frac{3}{5}G \frac{M_{\odot}^2}{R_{\odot}} \quad (3.9)$$

$$= 2 \times 10^{48} \text{ erg} \quad (3.10)$$

To estimate the nuclear energy the sun produces we can look at the nuclear fusion process in which $4H \rightarrow He$ which gives a mass difference of

$$\Delta m = M(4H) - M(He) \quad (3.11)$$

$$\rightarrow \Delta E = \Delta mc^2 \quad (3.12)$$

$$(3.13)$$

defining

$$\eta = \frac{\Delta E}{M_{He}c^2} = 0.7\% \quad (3.14)$$

then the nuclear energy produced in the sun is

$$\eta \cdot M_{\odot}c^2 \quad (3.15)$$

and using this we can find the age of the sun

$$t_{\odot} = \frac{E_{nuc}}{L_{\odot}} \approx 100 Gyr \quad (3.16)$$

but since the sun hasn't burned through all the hydrogen it has access to, we can replace

$$M_{\odot} \rightarrow M_{core} \approx 5\% M_{\odot} \quad (3.17)$$

which would give us a better estimate.

2 The Structure of Stars

We'll use the Sun as an example, then we'll hit equations.

2.1 The Sun

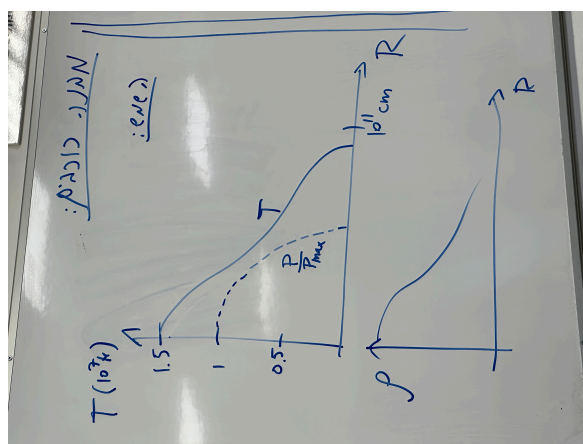


Figure 3.1: The temperature in the sun as a function of the distance from it core, the relative pressure compared to the core, and the density.

Our goal in the coming weeks will be to derive differential equations to figure out how the different profiles behave. As for some numbers:

$$R_{\odot} = 7 \times 10^{10} cm \quad T_C = 1.5 \times 10^7 \quad \rho_C = 150 gcm^{-3} \quad (3.18)$$

$$\bar{\rho} = 1.4 gcm^{-3} \quad P_C = 10^{11} Atm \quad M_{\odot} = 2 \times 10^{33} \quad (3.19)$$

$$L_{\odot} = 3.8 \times 10^{33} erg/s \quad (3.20)$$

2.2 Star Structure Equations

The equations describing the structure of stars are:

$$\frac{dP}{dr} = -G \frac{M_r \rho}{r^2} \quad (3.21)$$

$$\frac{dM_r}{dr} = 4\pi r^2 \rho \quad (3.22)$$

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \epsilon \quad (3.23)$$

$$\frac{dT}{dr} = -\frac{3\kappa \rho}{4acT^3} \frac{L_r}{4\pi r^2} \quad (3.24)$$

The first equation is called the hydrostatic equation and is equivalent to conservation of momentum, the second is the continuum equation and is equivalent to conservation of mass, the third is the energy production equation and is equivalent to conservation of energy, and the final one is the radiation diffusion equation.

2.3 The Hydrostatic Equation

The equation can be derived from an equality of forces. We look at a shell at radius r with mass dm and ask which forces act on it. On one hand there's gravitation pulling it inwards and on the other side there's a force resulting from a difference in pressure.

$$|dF_g| = \frac{GM_r dm}{r^2} \quad (3.25)$$

$$= \frac{GM_r}{r^2} 4\pi r^2 \rho dr \quad (3.26)$$

$$|dF_P| = A \cdot (P(r) - P(r + dr)) \quad (3.27)$$

$$= 4\pi r^2 \left(-\frac{dP}{dr} dr \right) \quad (3.28)$$

equating the two forces we can find

$$\frac{dP}{dr} = -\rho \frac{GM_r}{r^2} \quad (3.29)$$

which is indeed the hydrostatic equation.

Applying The Equation To Earth

As a back of the envelope calculation, we can use the equation to find the gravitational acceleration of earth. Noticing that

$$\frac{GM_r}{r^2} = g \quad (3.30)$$

we can rewrite the hydrostatic equation as

$$\frac{dP}{dr} = -\rho g \quad (3.31)$$

We can also find the speed of sound

$$c_s = \sqrt{\gamma \frac{dP}{d\rho}} \quad (3.32)$$

which means

$$\frac{dP}{d\rho} \approx c_s^2 \quad (3.33)$$

so

$$g \sim \frac{dP}{d\rho} \frac{1}{h} \quad (3.34)$$

$$= \frac{(340m/s)^2}{10km} \quad (3.35)$$

$$= 10m/s^2 \quad (3.36)$$

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