Даниел Иванов Халачев, № 62547, „си"

$$A = \int_0^{\frac{1}{8}} e^{-3x^2} dx$$

$$e^x = \sum_0^{\infty} \frac{x^k}{k!} = x + \frac{x^2}{2} + \dots \Rightarrow e^{-3x^2} = \sum_0^{\infty} \frac{(-3x^2)^k}{k!} = \sum_0^{\infty} \frac{(-1)^k 3^k x^{2k}}{k!} \Rightarrow$$

$$A = \int_0^{\frac{1}{8}} e^{-3x^2} dx = \int_0^{\frac{1}{8}} \sum_{k=0}^{\infty} \frac{(-1)^k 3^k x^{2k}}{k!} dx = \sum_0^{\infty} \frac{(-1)^k 3^k}{k!} \int_0^{\frac{1}{8}} x^{2k} dx =$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k 3^k}{k!} \left(\frac{x^{2k+1}}{2k+1} \right) \Bigg|_0^{\frac{1}{8}} = \sum_0^{\infty} \frac{(-1)^k 3^k}{k!} \cdot \frac{1}{8^{2k+1}(2k+1)} =$$

$$= \sum_0^{\infty} \frac{(-1)^k 3^k}{k!\, 8^{2k+1} (2k+1)} = \sum_0^{\infty} (-1)^k a_k; \quad a_k = \frac{3^k}{k!\, 8^{2k+1}(2k+1)}$$

A - знакопроменлив ред

$a_k \searrow 0$ - монотонно клони към 0 $\Big\} \circledast$

$$A = \sum_0^{\infty} \frac{(-1)^k 3^k}{k!\, 8^{2k+1}(2k+1)} = \sum_0^{n} \frac{(-1)^k 3^k}{k!\, 8^{2k+1}(2k+1)} + \underbrace{\sum_{n+1}^{\infty} \frac{(-1)^k 3^k}{k!\, 8^{2k+1}(2k+1)}}_{R - \text{ грешка.}}$$

По условие трябва $|R| < 10^{-4} \Rightarrow$

$$|R| = \left| \sum_{n+1}^{\infty} \frac{(-1)^k 3^k}{k!\, 8^{2k+1}(2k+1)} \right| = \sum_{n+1}^{\infty} \frac{3^k}{k!\, 8^{2k+1}(2k+1)} \leq |a_{n+1}|, \text{ защото}$$

$$a_{n+1} - a_{n+2} + a_{n+3} - \dots = a_{n+1} - \underbrace{(a_{n-2} - a_{n-3})}_{>0} + \underbrace{(a_{n-4} - a_{n-5})}_{>0} + \dots < a_{n+1}$$

$$R < \frac{3^{n+1}}{8^{2(n+1)+1}(2(n+1)+1)(n+1)!} = \frac{3^{n+1}}{8^{2n+3}(2n+3)(n+1)!}$$

$n = ?\ R < 10^{-4}$

$$\rightarrow n=1 \Rightarrow \frac{3^2}{8^5 \cdot 5 \cdot 2} = \frac{9}{10 \cdot 8^5} < 10^{-4}$$

$$?\ n=0 \Longrightarrow \frac{3}{8^3 \cdot 3} > 10^{-4} \Rightarrow \boxed{n=1}$$

$$A \approx \sum_{k=0}^{1} \frac{(-1)^k 3^k}{8^{2k+1}(2k+1) \cdot k!} = \frac{\overset{8^2}{1}}{8} - \frac{\cancel{3}}{8^3 \cdot \cancel{3}} = \frac{64-1}{8^3} = \boxed{\frac{63}{8^3}} =$$

$$\approx \boxed{0{,}123046}$$