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$$\begin{cases} y'' + 5xy' + 3y = 0 \\ y(0) = 0 \\ y'(0) = -3 \end{cases}$$

Нека $y = \sum_{0}^{\infty} a_n x^n = a_0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + \dots$

$$\Rightarrow \quad y' = \left(\sum_{0}^{\infty} a_n x^{n}\right)' = \sum_{1}^{\infty} n \cdot a_n \cdot x^{n-1}$$

$$y'' = (y')' = \left(\sum_{1}^{\infty} n \cdot a_n x^{n-1}\right)' = \sum_{2}^{\infty} n(n-1) a_n x^{n-2}$$

$$y(0) = 0 \Rightarrow a_0 + a_1 \cdot x^1 + \dots = a_0 + \dots 0 = 0 \Rightarrow \boxed{a_0 = 0}$$

$$y'(0) = -3 \Rightarrow a_1 + \underbrace{2a_2 \cdot x + \dots}_{0} = -3 \Rightarrow \boxed{a_1 = -3}$$

$$y'' + 5xy' + 3y = 0$$

$$\sum_{2}^{\infty} n(n-1) a_n x^{n-2=k} + 5x \sum_{1}^{\infty} n \cdot a_n x^{n-1} + 3 \sum_{0}^{\infty} a_n x^n = 0$$

$$\sum_{k=0}^{\infty} (k+2)(k+1) a_{k+2} x^k + \sum_{1}^{\infty} 5n \cdot a_n \cdot x^{n} + \sum_{0}^{\infty} 3a_n \cdot x^n = 0$$

$$\sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n + \sum_{1}^{\infty} 5n \cdot a_n \cdot x^{n} + \sum_{0}^{\infty} 3a_n x^n = 0$$

$$\Updownarrow$$

$$(n+2)(n+1) a_{n+2} + 5n \cdot a_n + 3a_n = 0$$

$$(n+2)(n+1)a_{n+2}+(5n+3)a_n=0$$

$$\boxed{a_{n+2}=-\frac{5n+3}{(n+2)(n+1)}\cdot a_n \qquad a_0=0 \quad a_1=-3}$$

„n" в знаменател е от по-голяме порядък, отколкото в числителя $\Rightarrow$ $\boxed{R=\infty}$

$$a_0=0 \quad a_2=-\frac{3}{2\cdot1}\cdot 0 \quad a_4=-\frac{13\,(-3)}{4\cdot3\cdot2\cdot1}\cdot 0\ldots$$

$$a_1=-3 \quad a_3=-\frac{8\,(-3)}{3\cdot2} \quad a_5=-\frac{18\,(-8)\,(-3)}{5\cdot4\cdot3\cdot2\cdot1}\ \ldots$$

алтернативно:

$$a_{n+2}=-\frac{5n+3}{(n+2)(n+1)}\,\frac{5(n-2)+3}{n\,(n-2)+1}\cdot a_{n-2}=-\frac{(5n+3)(5n-7)(5n-17)\ldots}{(n+2)!}$$

Тук отново $(n+2)!$ расте по-бързо от $(5n+3)$ през 10 факториел $\Rightarrow R=\infty$.

$$a_{2n+1}=\frac{(-1)^n(\ldots)28\cdot18\cdot8}{(2n+1)!}\overset{a_1}{\overset{\parallel}{(-3)}}=\frac{(-1)^n\,8\cdot18\cdot28\cdot\ldots\cdot(5n+3)(-3)}{(2n+1)!}$$

$$a_{2n}=\frac{(-1)^n\cdot3\cdot13\cdot23\ldots(5n+3)\cdot0}{(2n)!}=0$$

$R=\infty$

$$y=\sum_0^\infty a_nx^n=\sum_0^\infty a_{2n+1}x^{2n+1}+\sum_0^\infty \underbrace{a_{2n}}_{0}x^{2n}=\boxed{\sum_0^\infty a_{2n+1}x^{2n+1};\ R=\infty}$$

Доказателство, че $R=\infty$

Нека редицата има радиус на сходимост R.

Нека $r_n = \dfrac{a_{n+1}}{a_n}$

$r_{n+1} = \dfrac{a_{n+2}}{a_{n+1}}$

$$R = \lim_{n\to\infty} \frac{a_n}{a_{n+1}} = \lim_{n\to\infty} \frac{a_{n+1}}{a_{n+2}} \Rightarrow$$

$$R = \frac{1}{r_n} = \frac{1}{r_{n+1}} \Rightarrow$$

$$\frac{1}{R^2} = \lim_{n\to\infty} (r_n r_{n+1}) = \lim_{n\to\infty} \frac{5n+3}{(n+2)(n+1)} = 0 \Rightarrow R=\infty$$