Домашно № 1

Даниел Иванов Халачев, ФН: 62547, курс II

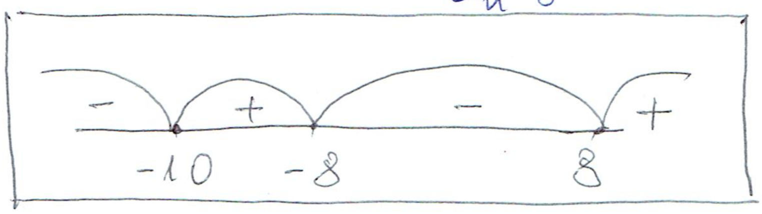
$$f(x) = \frac{2x^2+10x+80}{x-8};\quad x \neq 8$$

Нека $\underset{n\to\infty}{a_n \longrightarrow l} \Rightarrow \frac{2l^2+10l+80}{l-8} = l \Leftrightarrow 2l^2+10l+80-l^2+8l=0$

$\Leftrightarrow l^2+18l+80=0 \Leftrightarrow \underbrace{l=-10 \vee l=-8}_{\text{потенциални граници}}$

Изсл. $\boxed{a_{n+1}-a_n} = \frac{2a_n^2+10a_n+80}{a_n-8} - a_n = \frac{a_n^2+18a_n+80}{a_n-8} =$

$= \frac{(a_n+8)(a_n+10)}{a_n-8}$

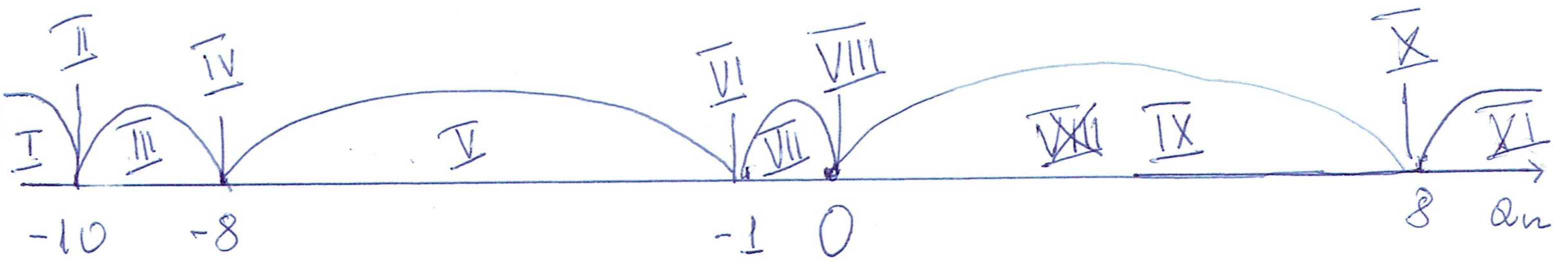


$\boxed{a_{n+1}-(-10)} = \frac{2a_n^2+10a_n+80}{a_n-8} + 10 = \frac{2a_n^2+20a_n}{a_n-8} =$

$= \frac{2a_n(a_n+10)}{a_n-8}$

$\boxed{a_{n+1}-(-8)} = \frac{2a_n^2+10a_n+80}{a_n-8} + 8 = \frac{2a_n^2+18a_n+16}{a_n-8} =$

$= \frac{2(a_n+1)(a_n+8)}{a_n-8}$



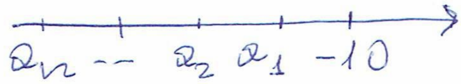
I сл. $a_1 = \lambda \in (-\infty, -10)$

$a_{n+1} - a_n < 0$

$a_{n+1} - (-10) < 0 \iff a_{n+1} < -10$

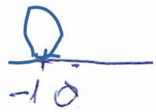
$\Rightarrow \lambda \in (-\infty, -10) \to (-\infty, -10)$

$a_{n+1} < a_n \ \forall n$



$\Rightarrow a_n \to -\infty$

II сл. $a_1 = \lambda = -10 \Rightarrow a_2 = -10 \Rightarrow a_3 = -10 \dots$



III сл. $a_1 = \lambda \in (-10, -8)$

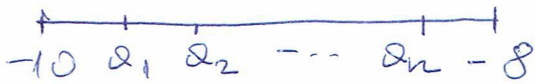
$a_{n+1} - a_n > 0$

$a_{n+1} - (-10) > 0 \iff a_{n+1} > -10$

$a_{n+1} - (-8) < 0 \iff a_{n+1} < -8$

$\lambda \in (-10, -8) \longrightarrow (-10, -8)$

$a_{n+1} > a_n$

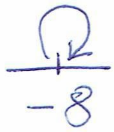


$a_n \nearrow$, $a_n < -8$ – моном. и огр.

$\Rightarrow a_n$ – сходяща

$\lim\limits_{n\to\infty} a_n = -8$

IV сл. $a_1 = \lambda = -8 \Rightarrow a_2 = -8 \Rightarrow$



V сл. $a_1 = \lambda \in (-8, -1)$

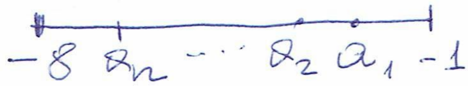
$a_{n+1} - a_n < 0$

$a_{n+1} - (-8) > 0 \iff a_{n+1} > -8$

$\lambda \in (-8, -1) \longrightarrow (-8, -1)$

$a_{n+1} < a_n < -1$

$a_{n+1} > -8$ $\Rightarrow$



$a_n \searrow$, $a_n > -8 \Rightarrow$

мон. нам. и огран. $\Rightarrow$

$\lim\limits_{n\to\infty} a_n = -8$

VI сл. $a_1 = \lambda = -1 \Rightarrow a_2 = -8$

VII сл. $a_1 = \lambda \in (-1; 0)$

$a_{n+1} - a_n < 0$

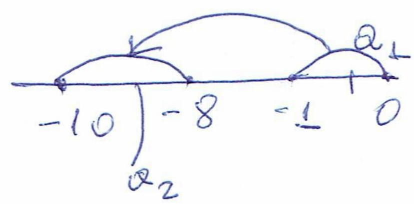
$a_{n+1} - (-8) < 0 \iff a_{n+1} < -8$

$a_{n+1} - (-10) > 0 \iff a_{n+1} > -10$

$a_{n+1} < a_n \iff a_2 < a_1$

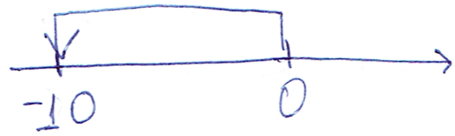
$a_2 < -8$

$a_2 > -10$



$\Rightarrow \lambda \in (-1; 0) \longrightarrow (-10, -8)$

VIII сл. $a_1 = x = 0 \Rightarrow a_2 = -10$

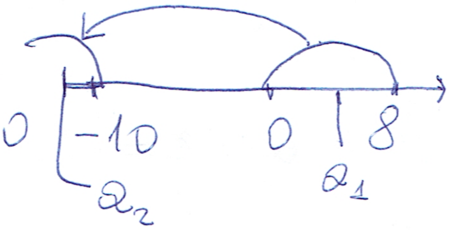


IX сл. $a_1 = x \in (0; 8)$

$a_{n+1} - a_n < 0$

$a_{n+1} - (-10) < 0 \Leftrightarrow a_{n+1} < -10$

$\left\{ \begin{array}{l} a_2 < a_1 \\ a_2 < -10 \end{array} \right.$

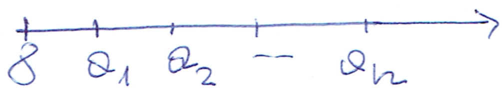


$\Rightarrow x \in (0, 8) \longrightarrow (-\infty, -10)$

X сл. $a_1 = x = 8 \Rightarrow \overset{\circ}{x}$ $\nexists a_2, \dots, a_n$

XI сл. $a_1 = x \in (8; +\infty)$

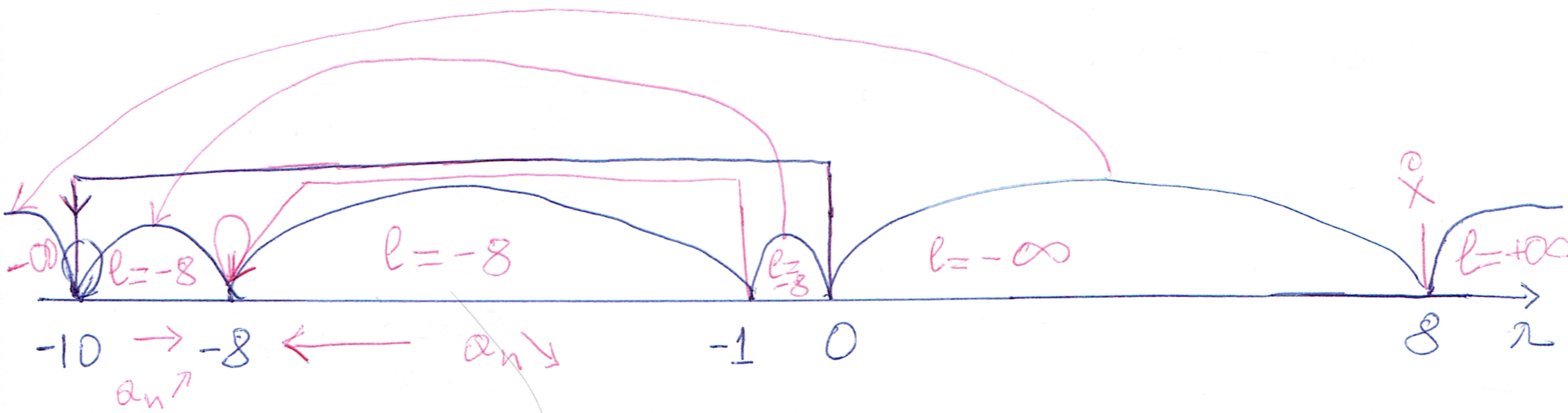
$\left. \begin{array}{l} a_{n+1} - a_n > 0 \\ a_{n+1} - 8 > 0 \Leftrightarrow a_{n+1} > 8 \end{array} \right\} \Rightarrow a_{n+1} > a_n > 8$



$\left. \begin{array}{l} a_n \nearrow \text{ монот. раст.} \\ a_n > 8 - \text{огран. отдолу} \end{array} \right\} \Rightarrow$

$\lim a_n = +\infty$

$a_n \to \infty$



Отговор:

$$\lim_{n\to\infty} a_n = l = \begin{cases} -\infty; & x \in (-\infty, -10) \cup (0; 8) \\ -10; & x = \{-10, 0\} \\ -8; & x \in (-10; 0) \cup \{-8\} \\ +\infty; & x \in (8; +\infty) \end{cases}$$