LIBDEPIXELIZE ALGORITHMIC DOCUMENTATION

Vinícius dos Santos Oliveira

Monday 31st March, 2014

Contents

Chapter 1

Preface

This document is a project to centralize the knowledge required to implement the Kopf-Lischinski vectorizer algorithm. It was created as part of the GSoC 2013's libdepixelize project.

Another goal of this document is to ease the comprehension of libdepixelize's source code. That said, documentation of techniques used in libdepixelize will be favored over other techniques. Keep in mind that this document is **not** documentation for the libdepixelize source code and may be useful independently from it, but libdepixelize maintainers will greatly benefit from this document.

The initial content of this document was adapted from posts published during the development of libdepixelize on Vinipsmaker labs, the original libdepixelize author's blog. The adaptation includes:

- Replacing several of the media assets by higher quality files.
- Restructure the order of the posts to make the documentation more didatic.
- Review the text and update it to refer to the libdepixelize's current techniques.

All images in this document, unless explicitly stated otherwise, were created using Inkscape and its awesome alignment tools.

This document is dedicated to the Inkscape community.

Chapter 2

Introduction

Playing video games is a part of the life of several people already and in the beginning of this new kind of entertainment, a lot of love was invested in the creation of beautiful images where the pixels were carefully put together.

Time went by on and the age of "pixel art" was momentarily... forgotten... replaced by more demanding art after more and more powerful devices started to appear.

Some people were never able to forget pixel art and continued to play ancient games just for the sake of nostalgia. The emulation community was born with the intent to revive the classic moments and it produced some awesome tools over the time.

In 2011, a new technology excited the emulation forums. This new technology was an algorithm created by Johannes Kopf and Dani Lischinski that could create resolution independent vector graphics representations out of pixel art images.

And then you came here and have this document that will surely be your lovable guide in the challenging quest of implementing the wonderful Kopf-Lischinski algorithm. What are you waiting for? Go on and proceed to the next page.

2.1 Requirements

Understanding Bézier curves is a must for this document and if the reader is not very familiar with these curves, *vinipsmaker* suggests you to read the excellent Pomax's *A Primer on Bezier Curves*. If you think Pomax's text is too advanced, you can start with Math ∩ Programming blog.

Chapter 3

Splines extraction on Kopf-Lischinski algorithm

Well, one of the phases in the algorithm is splines extraction. This phase creates a pipeline (output from one step is the input of the next) like the following:

1. It takes the color similarity graph
2. It generates a generalized Voronoi diagram
3. It groups the voronoi cells to identify visible edges
4. It generates the splines

Part 0 explains a fast method to compute the Voronoi diagram exploring special properties of the input graph. Part 1 explains a fast method to add polygons together exploring special properties of the generated Voronoi diagram and the data layout used in libdepixelize. Part 2 explains how to generate the Bézier curves. T-junction section discusses how to handle T-junction nodes.

The names of the sections are based on the titles of the original posts where they came from.

3.1 Part 0

Originally posted at `http://vinipsmaker.wordpress.com/2013/07/22/splines-extraction-on-kopf-lisc` on Monday 22$^{\text{nd}}$ July, 2013.

3.1.1 Generalized Voronoi diagram generation

Well, a Voronoi diagram is a black box where you put some points (the seeds) and you get some polygons (the cells). Each polygon contains all points that are closer to its seed than any other seed. There is a good article on Wikipedia and I won't explain any further.

Kopf-Lischinski algorithm executes a bunch of operations on a graph and it uses a Voronoi diagram to extract a visual representation from this graph. The simplest form of a Voronoi diagram works with 2D points-seeds, but we have higher-dimentional Voronoi diagrams, Voronoi diagrams using different distance functions and even Voronoi diagrams using complex non-points seeds. We are interested in these Voronoi diagrams using complex non-points seeds.

The below image has a representation of the output graph of the Kopf-Lischinski algorithm and its Voronoi diagram representation. The graph nodes are represented by circles, where nodes with the same color are similar and usually are connected. The connections of the nodes are represented by green lines.

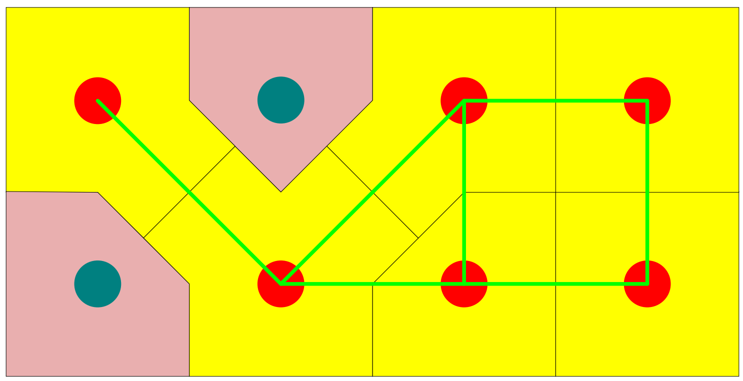


Figure 3.1: An accurate generalized Voronoi diagram

The generalized Voronoi diagram is visual — there is no special meaning to explain it in the previous image. The seeds of this diagram aren't points, they are line segments. You just need to break a green line in two and add each half to its containing cell. You can note that some polygons aren't convex.

The graph used as input has certain constraints that enable us to use some simple and fast operations instead of a full-featured and complex algorithm.

If a Voronoi cell is a polygon containing all points that are closer to its seed than any other seed, we can determine the edge of a Voronoi cell by the midpoint of two adjacent seeds. If we generate a vertex for each of its 8 directions, we will get an accurate Voronoi diagram.

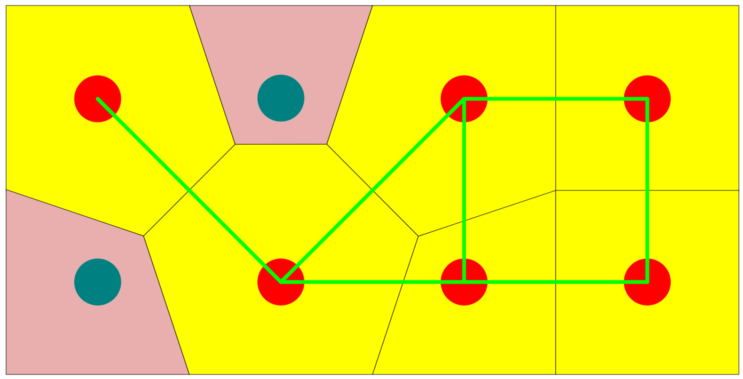


Figure 3.2: A simplified generalized Voronoi diagram

We can get a simplified Voronoi diagram by forgetting about the top, bottom, left and right vertices (if we just generate the diagonal vertices). The simplified version doesn't contain concave polygons.

The act of generating diagonal vertices is more complex than the act of generating the other vertices. We need to check if there is a connection with the other cell and, if this connection exists, generate two vertices. If the connection doesn't exist, we generate a single vertex, but its position depends on the existence of a connection between its neighbors. Look the following figure.

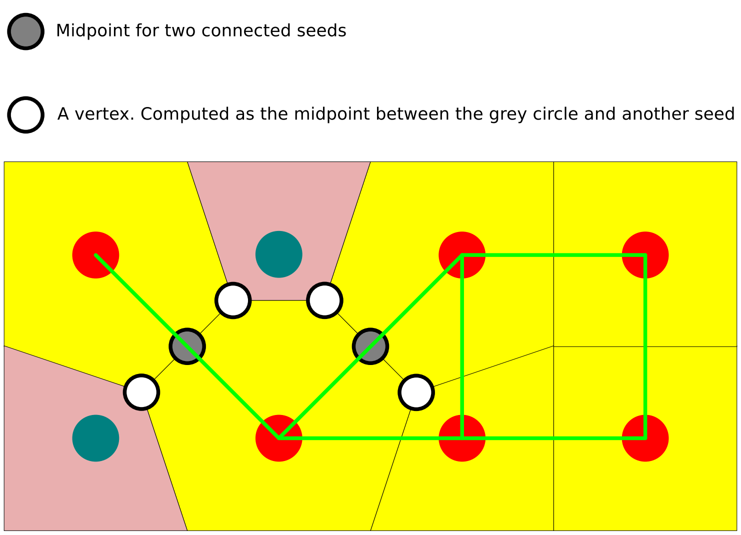


Figure 3.3: A simplified generalized Voronoi diagram (detailed)

All information we need to generate the Voronoi diagram is located within its neighbors and the only extra tool we need to generate the points is the midpoint procedure.

3.1.2 Metadata extraction

When we generate B-Splines using Kopf-Lischinski algorithm we need a way to separate points that create sharp corners and smooth points. The Kopf-Lischinski algorithm has techniques just to handle this issue. In libdepixelize, the point smoothness computation and Voronoi generation are merged in one single phase. This is the phase where we can gather lot of info about the graph and we can propagate the smoothness data to later phases.

One note about the algorithm of this “metadata extraction” section is that some points will disappear during polygon union and we don’t care if the metadata about these points is accurate, then we can simplify the algorithm exploring this property.

Unfortunately (for the implementer only), graph nodes of different colors can be grouped and if we don’t intend to discard this *extra color information* (an extension of Kopf-Lischinski on its own), a special care must be taken to handle these nodes. libdepixelize has some extensions that are documented later in this document. But for now, to ease explanation, we consider only cells of equal colors can be connected.

The below image has some features that will be cited in this subsection to explain how the algorithm works:

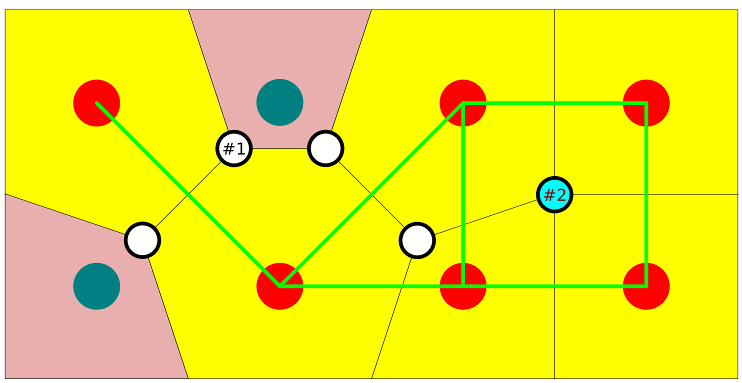


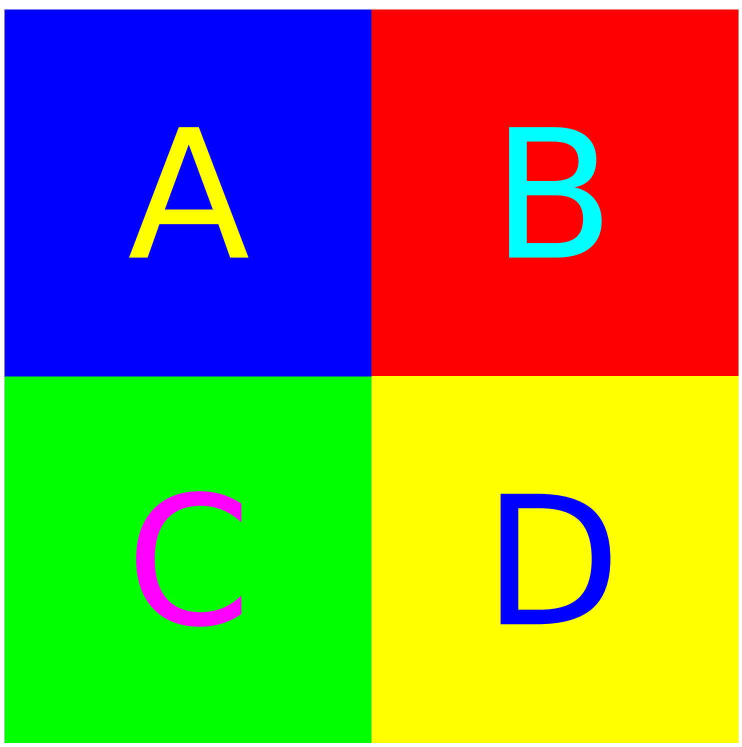
Figure 3.4: Vertex types

There are two types of vertices generated in this special Voronoi diagram:

- White type: The type of the one labelled with “#1”. Node “#1” is contained by three cells, one purple and two yellows. The two yellow cells will be merged during polygon-union. There are two possibilities for the remaining cell in this kind of node:
  - When it has the same color. Then the node will disappear during polygon-union and we don’t care about its *smoothness* attribute.
  - When it has a different color. Then we say it is a valence-2 node.

- Cyan type: The type of the one labelled with “#2”. This type of node can appear in the border of the image and isn't smooth or in the center of 4 cells and its smoothness isn't predictable in advance. If it appears in the center of four cells, then:
  - It can be in the middle of 4-connected cells and we don't care about its *smoothness* attribute, because this node will disappear.
  - It can be in the middle of a valence-2 node and will be smooth.
  - It can be a valence-3 node and things start to get complex. After the polygon union, this node will be part of three different polygon and only two of these three nodes will share the same value for the *smoothness* attribute.

With these problems explained, it's time for the algorithm! The algorithm is kind of repetitive (if we iterate the bottom-right node, compare with bottom and right cells, then do all it again, but use nodes of different directions…), but the principle is the same for all “repetitive” code, then only the important parts are documented here. If you really want to see the whole thing, see appendix A.



The above image represents the analysed data in the following code example, except for the fact that we don't know what are the colors of the cells. We are iterating on the middle point of the image and the current iterated cell is cell A. The algorithm also uses the concept of *shading edge* and *contour edge* described in Kopf-Lischinski paper.

```
A; //< cell A

// current iterated node (the one in the middle of the
// image) smoothness
A.smooth;

// It's a cyan node!
assert(!A.connected(D));
assert(!B.connected(C));

if ( A.connected(B) ) {
    if ( !C.connected(D) ) {
        bool foreign_is_contour = edge(C, D).contour();

        // the number of contour edges
        switch ( edge(A, C).contour()
                 + edge(B, D).contour()
                 + foreign_is_contour ) {
        case 2:
            A.smooth = !foreign_is_contour;
            break;
        default:
            assert(("precondition failed", false));
        // the following cases are only for
        // documentation of possible values (they don't
        // change the algorithm)
        case 3: case 1: case 0:
            // {A, B} is the pair with the angle
            // closest to 180 degrees
            A.smooth = true;
        }
    } else {
        // there might be 2-color, then A.smooth = true

        // or it might be 1-color and doesn't matter,
        // because the current node will disappear
        A.smooth = true;
    }
} else if ( A.connected(C) ) {
    // very similar to the previous condition
} else if ( B.connected(D) ) {
    bool foreign_is_contour = edge(C, D).contour();
```

```
    switch ( edge(A, B).contour()
             + edge(A, C).contour()
             + foreign_is_contour ) {
    case 2:
        A.smooth = !foreign_is_contour;
        break;
    default:
        assert(("precondition␣failed", false));
    case 3: case 1: case 0:
        // {B, D} is the pair with the angle closest to
        // 180 degrees
        A.smooth = false;
    }
} else if ( C.connected(D) ) {
    // very similar to the previous condition
} else {
    // there is a 4-color pattern, where the current
    // node won't be smooth
    A.smooth = false;
}
```

3.2 Part 1

Originally posted at `http://vinipsmaker.wordpress.com/2013/07/21/splines-extraction-on-kopf-lisc` on Sunday 21st July, 2013.

3.2.1 Polygon union

Polygons can be represented by vertice points. libdepixelize stores them in clockwise order, with the first point being part of the polygon's northwest/top-left. One important feature of the generated Voronoi diagram is that all "connected" cells share a common edge.

The algorithm can be described in 4 major steps:

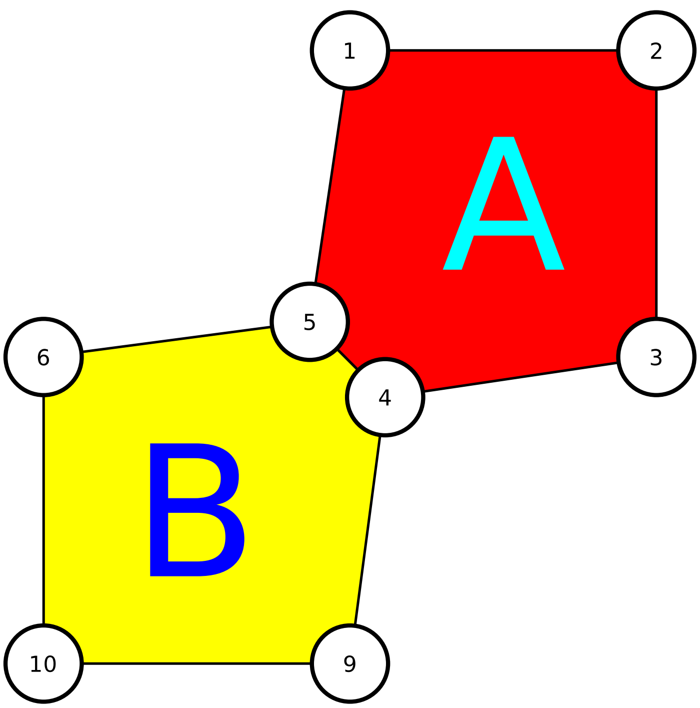
1. Find the largest common edge
2. Remove the in-between points of the common edge
3. Choose one polygon and refer to it as P (choose the smallest polygon for better performance)
   (a) Shift P such as P's head and tail are points of the common edge
4. Choose the other and refer to it as Q
   (a) Remove one of the common edge's points in Q

(b) Replace the remaining point that is part of the common edge in Q by P's points

The Voronoi cells are iterated one by one, line by line, from the left-most cell to the right-most cell. This behaviour could change in favor of a cache-oblivious algorithm.

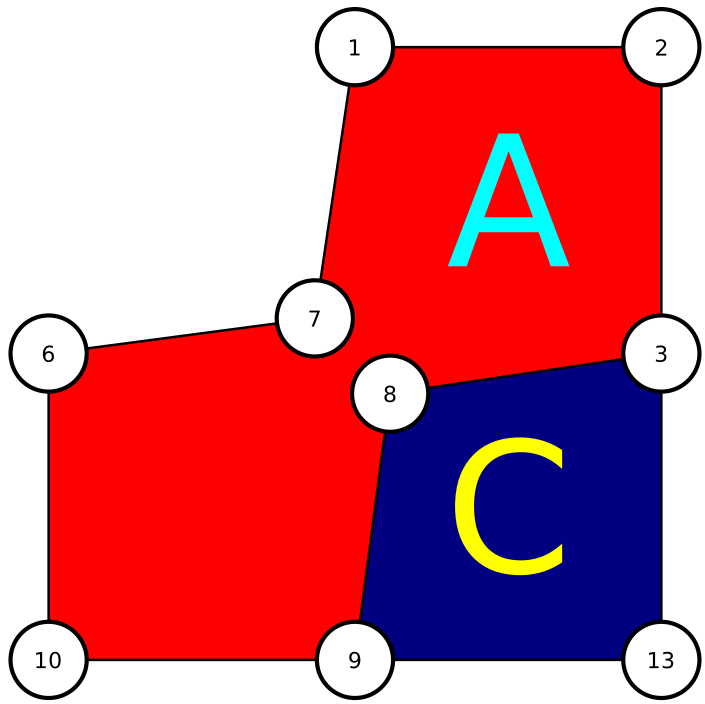
We check if (1) we already have a group of the current color, (2) the existing group has a common edge with current Voronoi cell and, then, (3) we add the current Voronoi cell to the existing polygon. Beware that the Voronoi cells from the input are always convex polygons, but the existing polygon (that groups the Voronoi cells) might be concave.

Let's see an example of the algorithm. In the example, we are iterating in the second line and we found an existing grouping polygon with a matching color (the two entities are connected), then we must add them togheter. The image below represents the situation we are interested in:

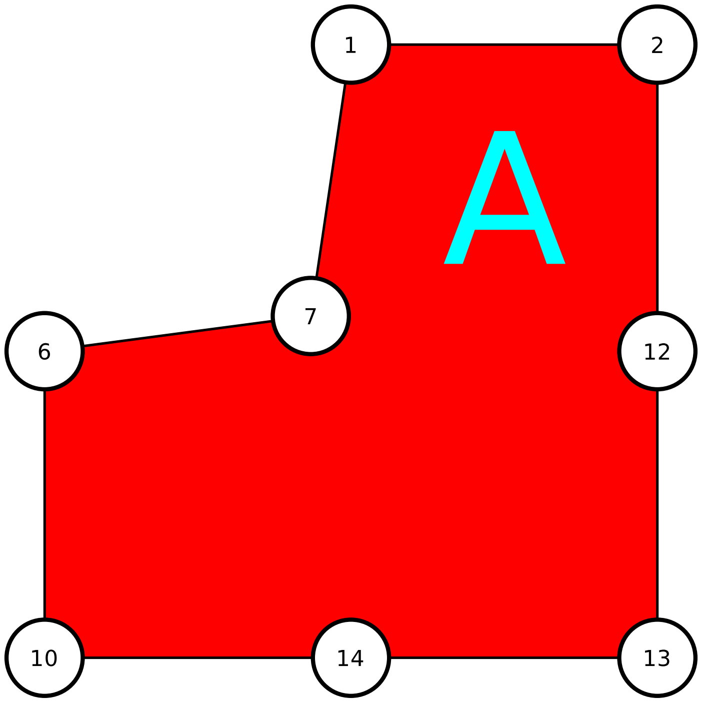


Polygon A is represented by the list of points [1, 2, 3, **4**, **5**] (common edge's points in bold). Polygon B is represented by the list of points [6, **7**, **8**, 9, 10] (common edge's points in bold). Points 4 and 8 are equal and points 5 and 7 are equal. Polygon A is the grouping polygon while polygon B is the current Voronoi cell. We shift B's list to [**8**, 9, 10, 6, **7**] and use it to get the final polygon [1, 2, 3, 8, 9, 10, 6, 7].

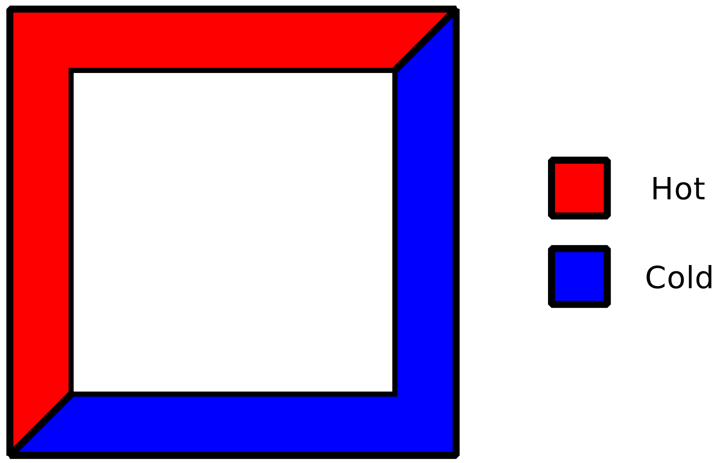
Let's do it one more time:



Polygon A is the grouping polygon, represented by [1, 2, **3**, **8**, **9**, 10, 6, 7]. Polygon C is the current Voronoi cell and is represented by [**11**, **12**, 13, **14**]. This time the largest common edge have 3 points and point 8/11 is the in-between point, then we remove it. The resulting A polygon is [1, 2, **3**, **9**, 10, 6, 7] and the resulting C polygon is [**12**, 13, **14**]. When we replace the common edge interval in A by C, we get the final polygon, [1, 2, **12**, 13, **14**, 10, 6, 7]. The final polygon is show in the image below:



One last note that might be useful in later steps is the access pattern used by the algorithm. When we add a voronoi cell to the grouping polygon, there is a hot area and a cold area. The cold area is the area where there will never be a common edge. These areas always are concetrated in the same places, like exemplified by the following image:



3.2.2 Splines generation

The previous step look simple, but there may be additional info that we may want to store for each node. This (polygon-union) is the last step where we still can gather locality info without executing a deep lookup.

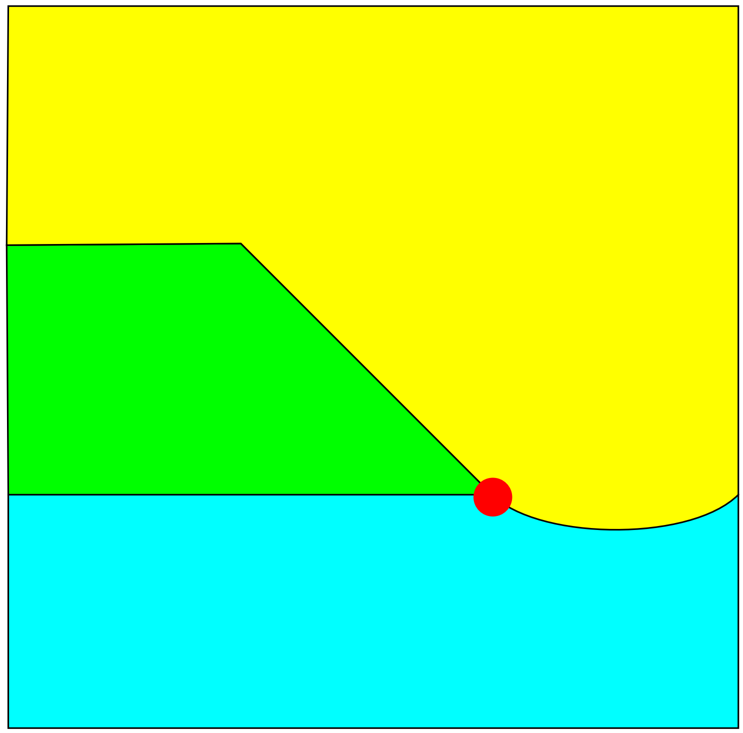


Figure 3.5: Valence-3 node (in red)

Let's refer to nodes where at most two different grouping polygons share a point as valence-2 nodes. There are valence-2 nodes, valence-3 nodes and valence-4 nodes. Valence-2 nodes are always smooth and valence-4 nodes never are smooth, but valence-3 nodes vary.

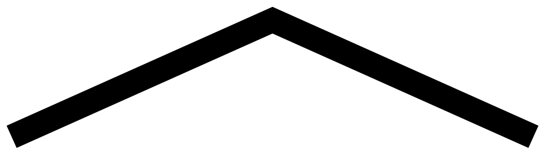


Figure 3.6: Non-smooth node

Most of the points are shared by nodes of different polygons and when we have three valence-3 nodes, exactly only one of them will be smooth. We apply Kopf-Lischinski algorithm heuristics to determine which one will be and store this info for later usage.

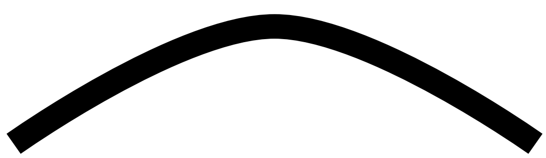


Figure 3.7: Smooth node

The complicated part about the splines extraction on Kopf-Lischinski algorithm is the overlapping between these last steps (group the Voronoi cells to identify visible edges and generate the splines).

3.2.3 A bit of performance

So, Kopf-Lischinski algorithm resembles a compiler. You have several data representation types and each type offers different operations. You explore the operations each type offers and convert it to another representation until you reach the final result. In several of the type representations used by the Kopf-Lischinski algorithm, you have a matrix and you access each element and its neighbours.

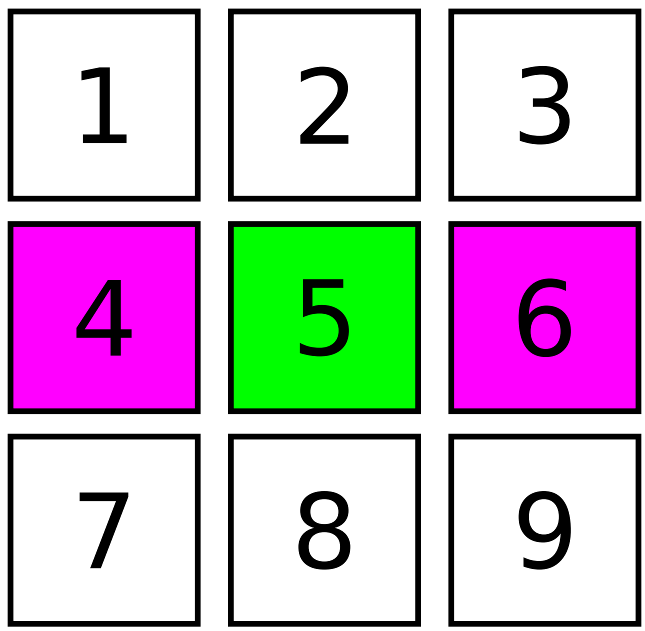


Figure 3.8: Access pattern representation

The particular implementation for libdepixelize stores and access the elements linearly like [1, 2, 3, 4, 5, 6, 7, 8, 9]. It could make good use of processor cache, but ain't that easy. Suppose we are iterating on element 5, then we need to access all its neighbours, but only neighbours 4 and 6 may be in cache, especially in large images. This is the first problem in cache usage of the implementation, but we cannot remove this usage pattern, because it's part of the algorithm and there is a data dependency among the elements. However, we can change the memory layout to favor the usage pattern. This technique is called cache-oblivious algorithm and future versions of libdepixelize might implement it.

Another problem with the current data access pattern is that each element store a complex object that may point to other regions of memory and add a level of indirection that can contribute to cache miss. One idea that can increase the cache hit is the one behind the Small String Optimization. This change would highly increase data locality and fits well in Kopf-Lischinski algorithm, because the complex objects stored by each element in every phase tends to have a small maximum number of subelements.

3.3 Part 2

Originally posted at `http://vinipsmaker.wordpress.com/2013/08/13/splines-extraction-on-kopf-lisc` on Tuesday 13th August, 2013.

3.3.1 Once upon a time... a polygon with holes appeared

The polygon-union described in part 1 works correctly and there are no better answers than the correct answer, but these polygons were meant to be B-Splines and what have been done till now is insufficient.

Consider the following image.

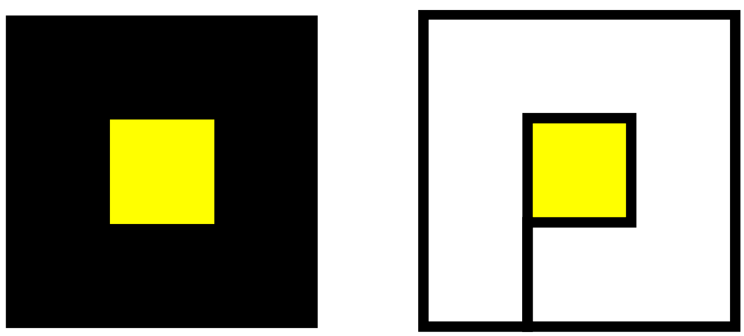


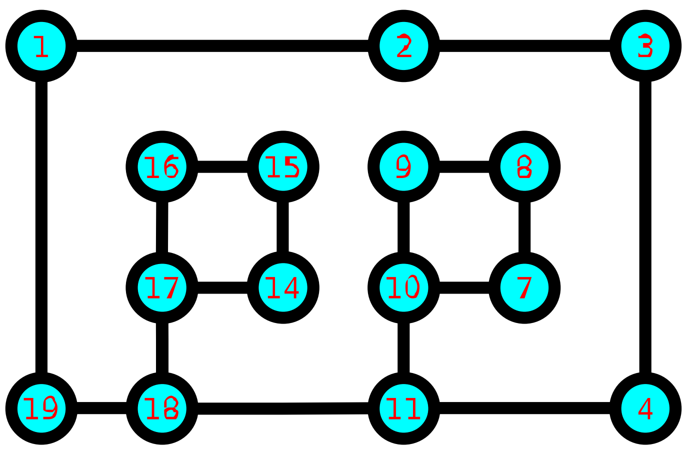
Figure 3.9: Normal polygon (left) and bordered polygon (right)

The left polygon is the polygon what you see before and after the polygon-union. They are visually indistinguishable. The right polygon has an outline to allow you understand its internal representation. You can understand how the polygon-union generates a polygon with a "hole" seeing the right polygon.

If the polygon-union don't generate visually identifiable pixels, then there shouldn't exist any problem, but when we convert the polygon to a B-Spline, some areas of the image won't be filled. The polygons won't fit correctly, like show in the image below.

Figure 3.10: After B-Spline conversion

The solution is to create some holes. With the representation of polygons used in libdepixelize, this task is very simple and the key operation to accomplish this goal is to find the "edges that the polygon share with itself". The following paragraphs explain this task.



The above image has 2 pairs of hidden points: pair <5, 6>, that is equal to pair <11, 10>, and pair <12, 13>, that is equal to pair <18, 17>.

The algorithm outline is:

1. Iterate over the points and, for each point, try to find another point that is equal to it. For each point
   (a) Get the internal sequence. To gather the internal sequence, just iterate over more nodes to get the "largest common edge".
   (b) Make sure the "largest common edge" has a size greater than 1 (e.g. is not only a common vertex, but a common edge). Use this sequence to construct the internal "hole" (or holes, if more than one) polygon and then remove it from the original polygon.

Beware that *use a sequence to construct the internal "hole" polygon* is another algorithm (and ain't trivial). It goes like this (let's name it *fill_holes*):

1. Create an empty sequence of nodes. Let's name it *hole*.
2. Let's name the beginning of our input sequence *region_begin*.
3. Iterate over the nodes of sequence and for each node:
   (a) Find another node equal to this one and if there is such node:
      i. Let's name our new finding *aux*.
      ii. Append a copy of the range from *region_begin* until the current node (do **not** confuse the current node with *aux*) into *hole*
      iii. We can see a common vertex: Current node and *aux*. Find the largest common edge and there will be references to two more nodes. Call *fill_holes* with this new range.
      iv. Forget about the range between current node and *aux*. We won't see these points in the next iteration and *region_begin* will refer to *aux* from now on.
4. Append the range from *region_begin* until the end of input sequence into *hole*.
5. *hole* is ready. Do something with it (e.g. store it).

The *fill_holes* procedure is complicated and you should check libdepixelize's source code (*HomogeneousSplines::_fill_holes*) to kill any ambiguity or doubt.

- Remark #1: libdepixelize do the steps of the outmost algorithm (the one removing nodes from the sequence) in backward order, to avoid too much moves of elements in the internal vector.
- Remark #2: The polygon is represented in clockwise order, but the holes will be represented in counter-clockwise order, but there is no reason to normalize. You can safely ignore this (de)normalization feature.
- Remark #3: The reason behind the special procedure to make sure the common sequence is an edge and not a vertex come from the fact that the code to fix the position of T-junction nodes creates a pattern where the polygon shares a point with itself.

3.3.2 And then, the polygon was meant to be a B-Spline

It's a chapter about splines extraction and it's fair to end with this step.

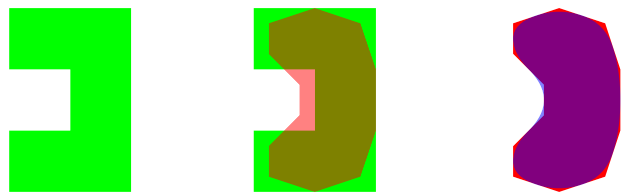


Figure 3.11: It's evolution, baby

The algorithm is very simple. The points you get through the polygon-union algorithm in part 1 will be the control points of quadratic Bézier curves and the interpolating/ending points of each Bézier curve will be the middle points between two adjacent control points. This is the special case where all points are smooth. The image below can help you get the idea.

Figure 3.12: Locations of the B-Spline points

If you have non-smooth points, then all you need to do is interpret them as interpolating points, instead of control points. You'll have three interpolating points that are part of two straight lines. There is a bit more of theory to understand why the generated B-Spline is smooth.

3.4 T-junctions

In this section the problem of how to adjust the endpoint of a curve to properly lie on the other curve will be discussed.

Per Kopf-Lischinski, a T-junction node will be the node that is common for three adjacent splines. For one of the splines, this will be a smooth node. Using the the shading/contour edges heuristic with an unspecified input image, we get the following output:

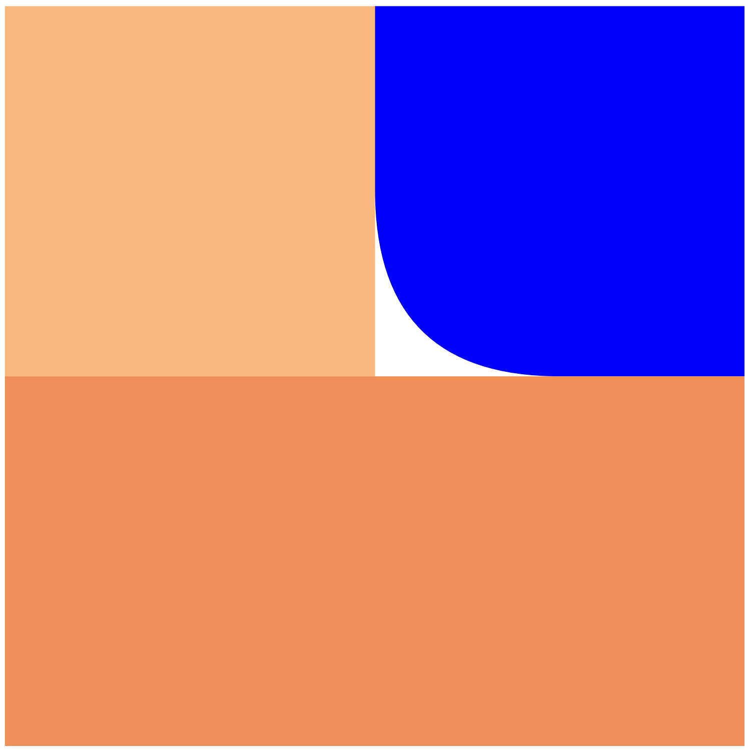
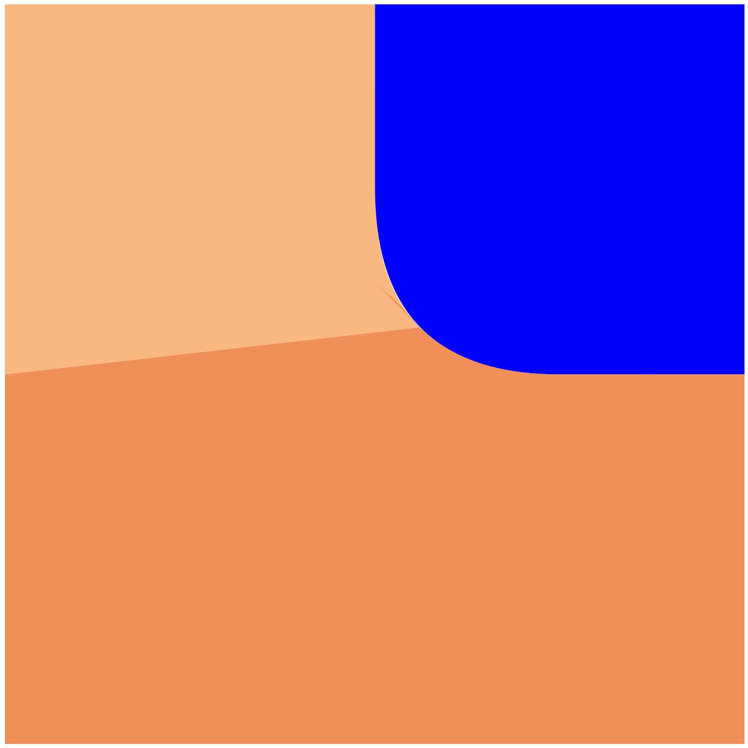


Figure 3.13: The non-blue splines share a shading edge

Some splines got visually disconnected and created an undesired result. To fix this issue we adjust the endpoint of the non-blue curves to properly lie on the blue curve. The end result will be the following image:



Kopf-Lischinski paper don't document extensively how to "adjust the endpoint of a curve to properly lie on the other curve" and you'll need to verify that the algorithm documented here is correct based on the sample images provided in the paper.

You may had learnt how to generate the splines in part 2. And you know that if you change the position of a node, the midpoint between this node and adjacent nodes will change. With the midpoint being changed, the parts that previously were fitting together will visually disconnect or overlap. Given that, we cannot simply adjust some node position without screwing up parts of the curve that were previously correct using the poor and simple data representation from the other steps.

We don't want to change the representation too, because this is exactly the type of representation expected by the optimizer in the next steps. We can, however, insert extra nodes without screwing up the previously correct parts of the curve and if we take care to make some of them collinear, then they won't interfere with the visual representation we got.

Combining the previous idea with the idea of subdivision from De Casteljau's algorithm it's possible to adjust the "endpoint" of the non-blue curves to properly lie on the blue curve. It's just a matter of finding the correct positions.

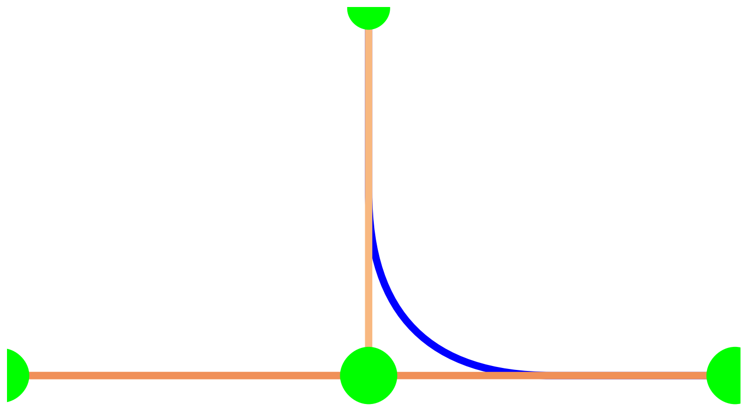


Figure 3.14: **Splines** with control points highlighted with green circles

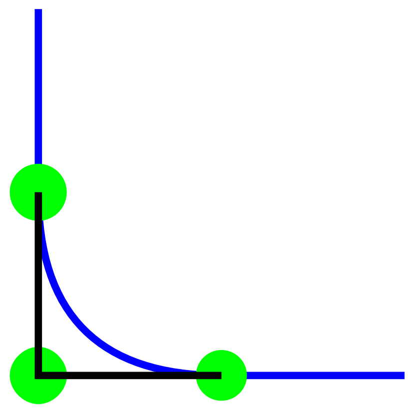


Figure 3.15: Blue **Bézier curve** with its points highlighted with green circles

After we subdivide a Bézier curve into two, there will be a new Bézier curve for each other curve whose “endpoint” should properly lie on this curve. As such, we only focus on one curve at each time. Now that we have the Bézier curve, we need to compute the position of the new splines nodes exploring the *midpoint* feature. This process is represented in the next picture.

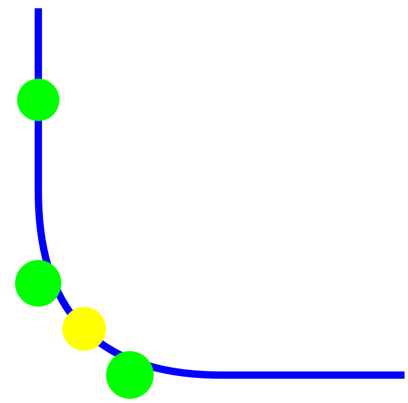


Figure 3.16: **Spline** with its control points highlighted with green circles. Yellow point is the point where the "endpoint" of the other curves should be placed.

The final result will have 5 points instead of one for a "vertex":

- The initial point to avoid screwing parts of the curve that previously were correct. Note that this point won't be visible, because it will be collinear with its neighbours points.
- The new three points to properly adjust the "endpoint" of this curve. Note that extreme points (the first and the third) won't be visible, because they are collinear with its neighbour points.
- The endpoint of the **Bézier curve**, then the two curves that are adjusted won't overlap. Note that this point is **not** smooth.

You may want to store this *visibility* metadata for easily discard these extra points when you convert the splines to Bézier curves in later steps.

This is the technique to "adjust the endpoint of a curve to properly lie on the other curve", but this problem is not so easy, because (1) you'll have to implement code to handle each node in the three splines and (2) the position of nodes will also depend on the color similarity with neighbour splines. The problem #2 is exemplified in the image below (when compared to previous examples):

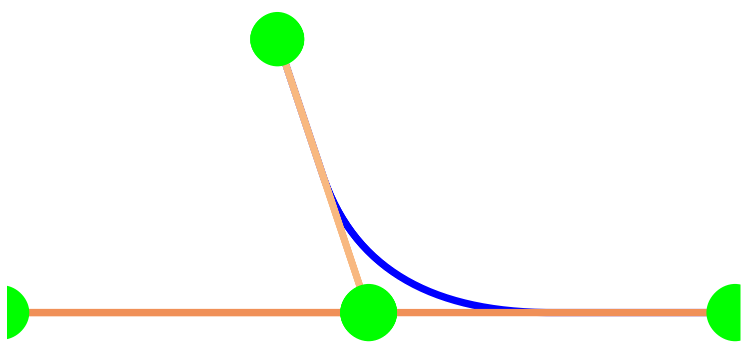


Figure 3.17: **Splines** with its control points highlighted with green circles.

Usually you will have to handle four different possibilities for **each** of the five nodes. This means four large conditional branches. Nothing pleasant, but you can reduce the code if you find the *amount* that each condition contributes to the final result.

You can find these conditions playing with experiments and boolean algebra or, using a more elaborate approach, solving a simple linear system. This text won't document all possible constants (at least not for this initial release) that you can find, but you can check libdepixelize source code if you are in such a hurry. This text will, however, give an example of how to solve a linear system to find such constants.

These conditions are always related to connectivy among local nodes and will be translated in expressions like "is node X connected to its topleft node?".

The constants $\frac{3}{16}$, $\frac{1}{16}$ and others happen a lot in this problem. If you know a cool name for these constants, ping *vinipsmaker* and he will be happy to use your suggestion to name these constants.

3.4.1 Solving the linear system

Suppose that you have conditions A and B to determine the position of the nodes, then you have four possibilities/branches to determine the position of the nodes.

Suppose that, for the first node, you find the following positions for the x-axis given the input conditions:

$\boldsymbol{A}$	$\boldsymbol{B}$	position
true	*true*	0.0625
true	*false*	0.09375
false	*true*	0.09375
false	*false*	0.125

Now you can convert the previous table to a linear system replacing *false* by 0, *true* by 1 and adding a column completely filled with ones for the base value. For the previous table, this procedure will generate the following augmented matrix:

$$\left(\begin{array}{ccc|r} 1 & 1 & 1 & 0.0625 \\ 1 & 0 & 1 & 0.09375 \\ 0 & 1 & 1 & 0.09375 \\ 0 & 0 & 1 & 0.125 \end{array}\right)$$

After solve the system, you'll get the values $-\frac{1}{32}$, $-\frac{1}{32}$ and $\frac{1}{8}$ for χ_1, χ_2 and χ_3, respectively. χ_3 is the base value, then the equation for the x position of the first node will be:

$$\texttt{node.pos.x} = \frac{1}{8} - A\frac{1}{32} - B\frac{1}{32}$$

Because χ_1 and χ_2 values are equal in this particular case, we can reduce the formula:

$$\texttt{node.pos.x} = \frac{1}{8} - (A + B)\frac{1}{32}$$

That's it! Have fun finding the constants for each x and y for each node for each "adjust the endpoint of a curve to properly lie on the other curve" procedure.

Chapter 4

The *extra color information* patch

When you apply the polygon-union algorithm from part 1, you face a challenging question: What should we do if we find polygons that have similar colors and these colors are different? Kopf-Lischinski paper don't explore this issue at all and the results from their supplementary material show random results (sometimes the color information is simply lost and sometimes they are preserved and smoothed by the rendering technique chosen).

libdepixelize's approach is to preserve as much color information as possible and this approach led to some extensions to the algorithm (with more to come). The first rule is:

$$\textbf{Don't discard color information\ldots ever!} \tag{4.1}$$

The effect of the rule 4.1 is that only polygons of the same color should be united together.

But if I don't discard color information, the splines will become visually disconnected. What should I do? First, let's review the node types with the following image:

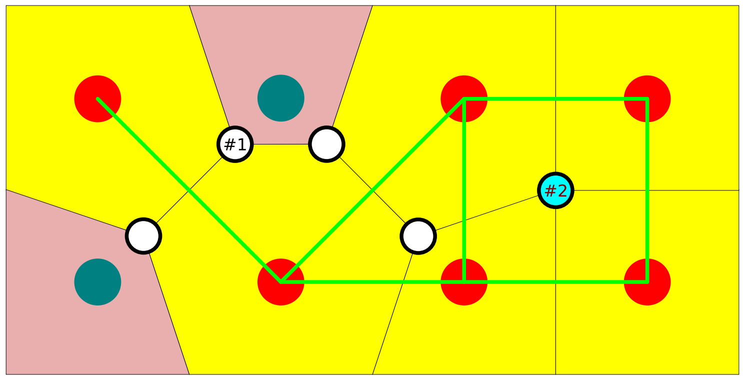


Figure 4.1: Vertex types

There are special heuristics about what to do for each node type and these heuristics are organized in some sections below.

Another interesting figure to introduce is the *boof* character. *Boof* has a color palette and pixel patterns that help to explain the heuristics responsible for extra color information. The Voronoi diagram generated by boof input is shown below.

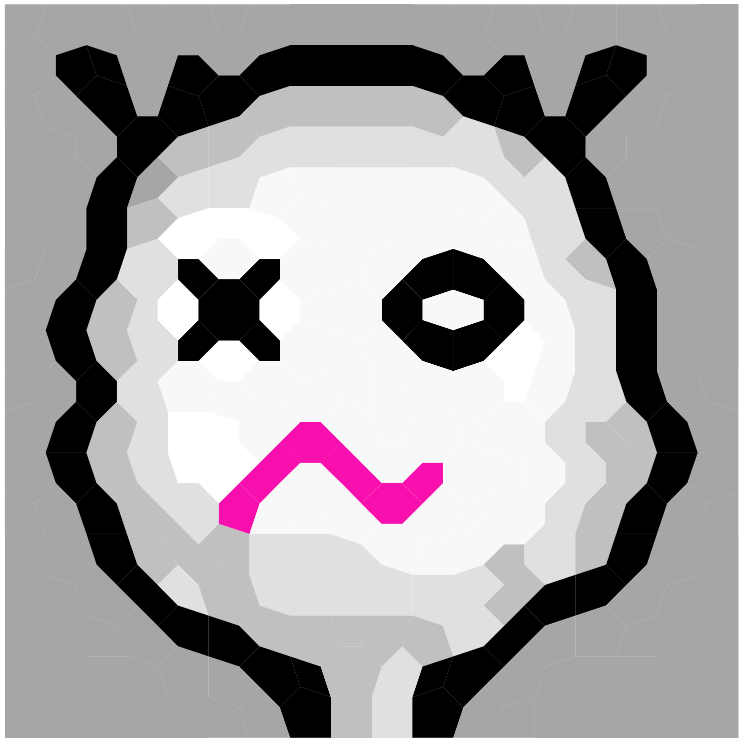


Figure 4.2: *Boof*, kindly created by Jabiertxo Arraiza Cenoz and “licensed” under public domain (CC0).

4.1 Cyan (#2) nodes

If you find a node for which the surrounding cells have similar colors, but not equal, then you check if the colors of both sides of an edge are equal. There are four edges and the node will be smooth if the total number of “equal colors” counts two.

The following picture highlight the separating edges you need to check.

Figure 4.3: Cyan node where the interesting separating edges are highlighted in green.

The result of this heuristic for the previous picture follows:

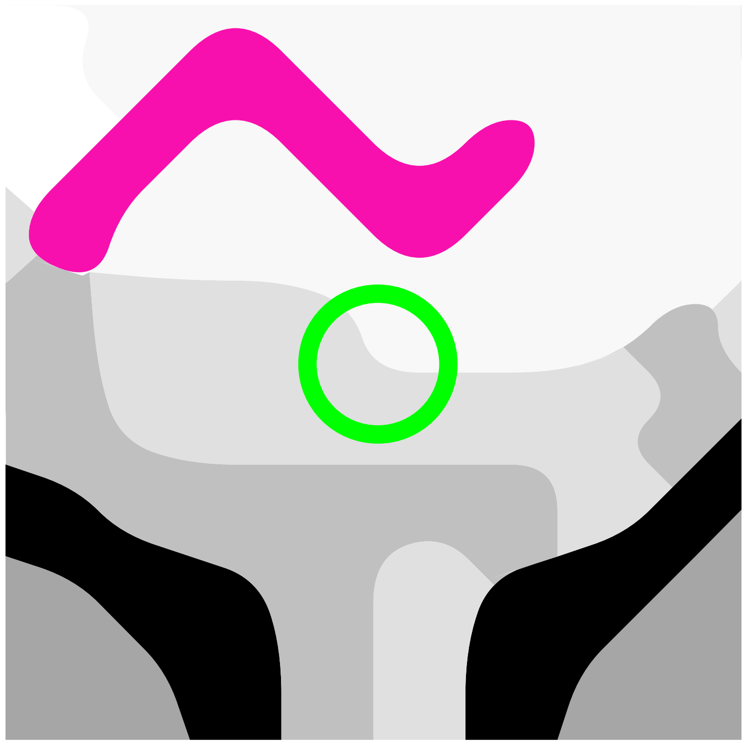


Figure 4.4: Cyan node highlighted with a green circle.

4.1.1 Why does it work?

We have a heuristic and its results are quite good, but maybe you're wondering... why does it work?

First requirement for *some* of these heuristics work is:

$$\text{The answer must be the same for all nodes sharing the same position.} \tag{4.2}$$

And our heuristic fulfill the requirement 4.2, because it uses the same local data for all four nodes sharing the same position.

If a heuristic don't fulfill the requirement 4.2, then it should have extra effects such as "adjust B-Spline" to produce good results.

The other requirement for a good heuristic is:

$$\text{The result shouldn't create distant or overlapping segments.} \tag{4.3}$$

And this heuristic fulfill the requirement 4.3 too, because it only creates smooth nodes when three cells are going to be grouped. If three cells of the four will be grouped, then the result will be two large polygons. When the region affected by the smooth node is shared by two polygons only, the two will perfectly complete each other.

4.2 White (#1) nodes

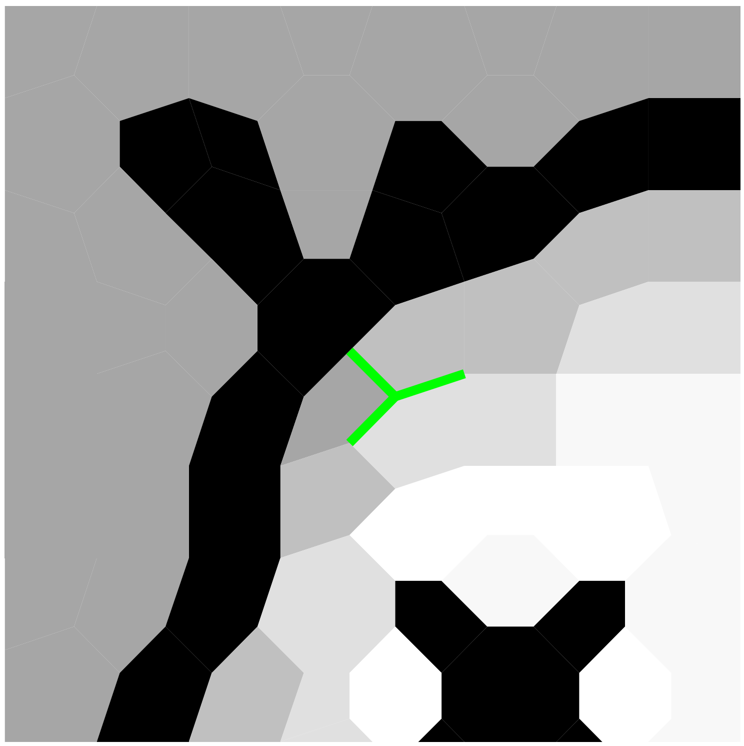


Figure 4.5: White node with the borders/edges highlighted in green.

For this type of node, we know in advance that two of the cells surrounding it have similar colors and we will refer to them as the *twin cells*. We will refer to the other cell as the *third cell*.

The node within the *third cell* will always be smooth.

The nodes within the *twin cells* will be smooth **if** any two of the three cells have the same color. If the node is **not** smooth, then it needs to be properly adjusted using the technique from T-junctions section to properly lie on the smooth curve affected by the *third cell*.

You can see some examples of this heuristic below:

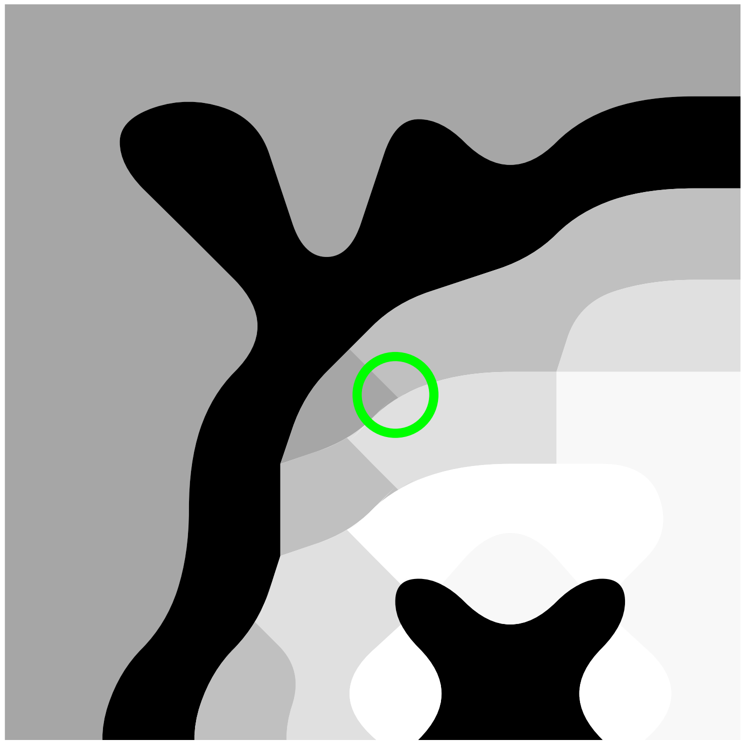


Figure 4.6: White node heuristic producing a non-smooth node.

This heuristic doesn't fulfill requirement 4.2, but it will produce a good result thanks to the special handling of non-smooth nodes using the trick from T-junctions section.

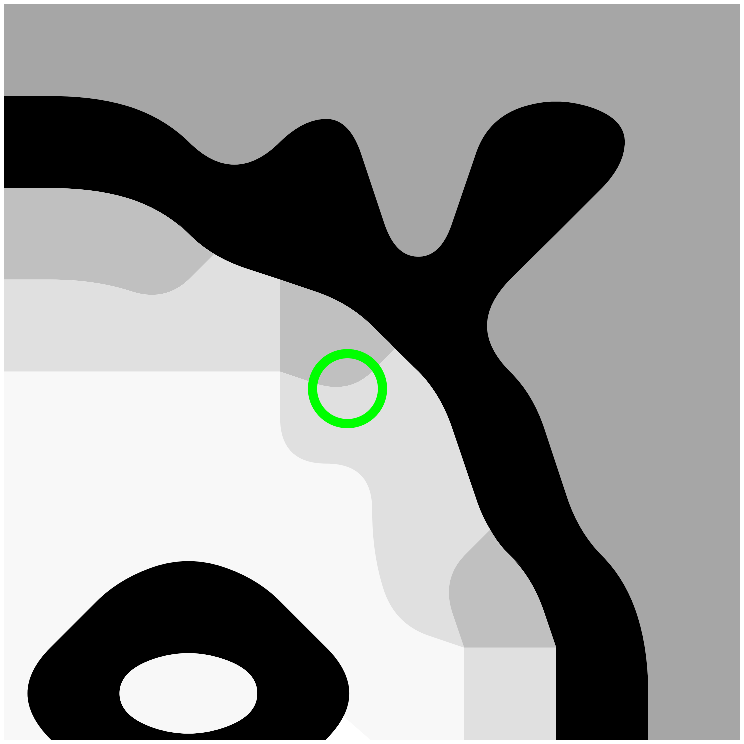


Figure 4.7: White node heuristic producing a smooth node.

4.3 The evil pattern

Thanks to the rule 4.1, we lose the transitivity property. The lack of transitivity introduces some unusual patterns that I refer to as an *evil pattern*. I identified some evil patterns.

4.3.1 The evil pattern #1

An example of the *evil pattern #1* follows:

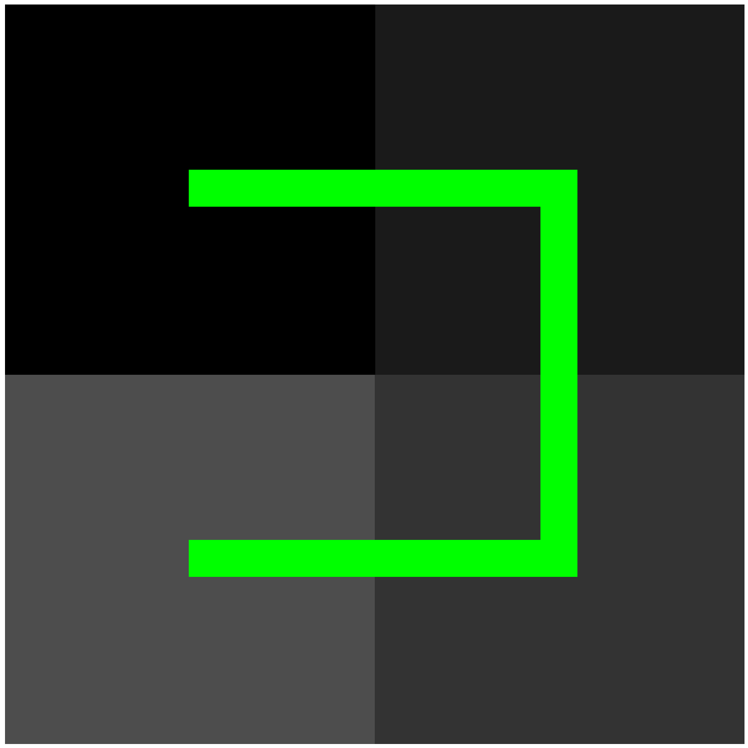


Figure 4.8: An evil pattern. A green line represents a connection (based on the similarity graph).

We handle the *evil pattern #1* using the rules for the cyan nodes. This pattern doesn't generate extra complication.

4.3.2 The evil pattern #2

An example of the *evil pattern #2* follows:

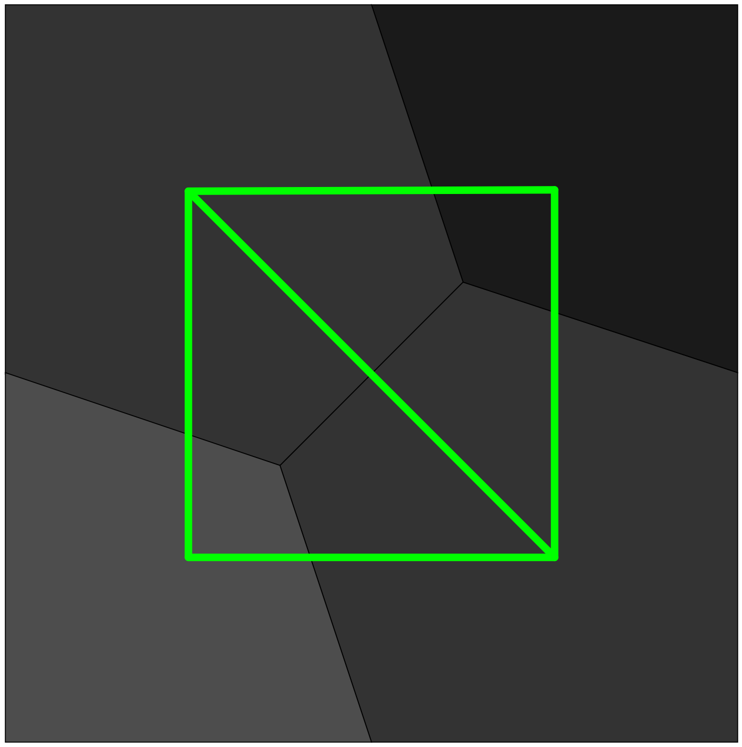


Figure 4.9: An evil pattern. A green line represents a connection (based on the similarity graph). The conversion to a Voronoi diagram based on the rules described in this document was already done in this sample.

The original Kopf-Lischinski paper describes fully connected graphs with redundant crossing connections. The approach was to remove the redundant crossing connections.

The existence of crossing connections prevents the conversion of the pixel graph to Voronoi diagrams and all of them must be erased. This behaviour is kept here.

The original Kopf-Lischinski paper considers some crossing connections as redundant, because no extra care was taken to preserve more color information, but the *evil pattern #2* requires an extra rule to produce better results. We handle these nodes by **NOT** removing the diagonal connection and handling them with the heuristics for white nodes.

There is, however, an extra step that replaces the "*remove safe and redundant crossing connections*" step described in the original Kopf-Lischinski paper. The new algorithm should detect if only one of the crossing connections glue nodes of equal colors (as opposed to similar colors) and preserve it over the other connection. If the detection fails, just fallback to the old Kopf-Lischinski technique. We'll handle the new nodes using the same rules for white nodes.

Note that this new rule is **not** a classic heuristic that will compete with the other heuristics (islands, sparse pixels, ...).

This approach is an improvement, because it'll favor a pattern where cells that share the same color will share an edge. Thus, they will be summed together in the same polygon in later processing steps, creating splines that are more smooth, natural and can be optimized more easily.

4.3.3 The evil pattern #3

An example of the *evil pattern #3* follows:

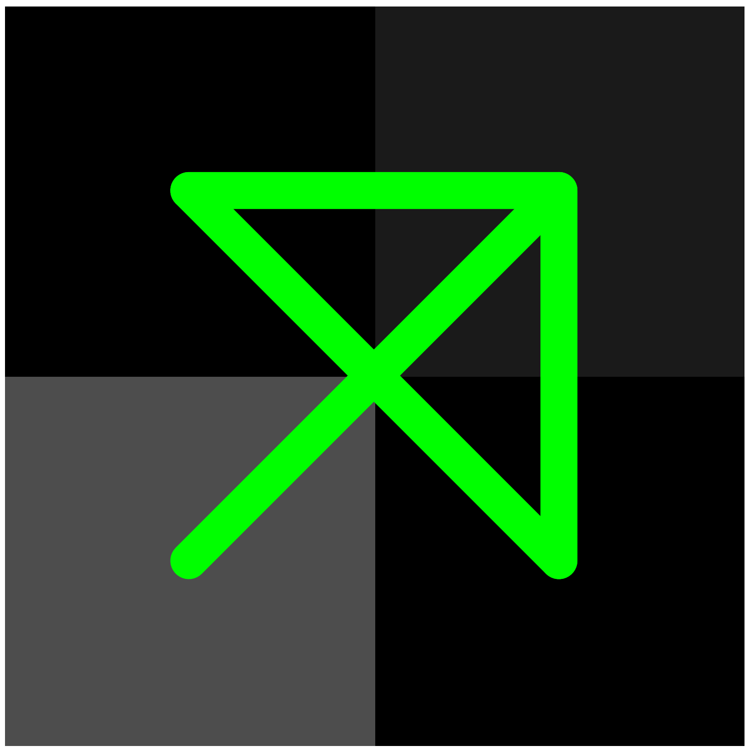


Figure 4.10: An evil pattern. A green line represents a connection (based on the similarity graph). The colors of the nodes on the main diagonal are the same.

The *evil pattern #3* is resolved by the technique developed to handle the evil pattern #2. If only one of the two connections share the **same** color, then this should be the connection preserved.

4.3.4 Other evil patterns

There are other the evil patterns (the evil pattern #3, where the colors of the main diagonal are different, for instance). I give up here.

The new rules are added as an extra step on the process. They are not treated like the Kopf-Lischinski's heuristics to resolve crossing connections. I think maybe would be possible to get some of the new rules and describe versions that are more general and can be added to the "container" that holds the old heuristics. To make this happen, I'd need to define the concept of "similar color" as a set and operations on top of the set, the notion of interval and other things and logical reasoning on top of all that. A bit of mathematical work to improve the quality a little more, but I wanna to investigate the use of La*b* colors (an old suggestion by Nathan) to replace the current concept of "similar colors". I didn't replaced the current space color until now, because YUV and La*b* behave differently and I couldn't just convert the YUV constants that define the boundary between dissimilar colors to La*b* to achieve better results. The current YUV constants were taken from HQx filter and I need to define a methodology to find new constants.

The advantage of a rule that is more general and unifies behaviour is just the beauty of a simpler rule that handles the real problem, as opposed to several branches that are consequence of the real problem. It'd abstract the nature of the problem better. It'd make the code simpler. It'd handle more branches. Being a single rule that affect more branches, it'd be easier to test and better at convincing me that the improvement is real and there will be no loss of quality in other images.

It'd be interesting to investigate the range of voting in all heuristics and try to come up with "fair" multipliers/strength.

Chapter 5

Optimizing the B-Splines

The vector arts generated by the previous steps are fine if you apply small factors of zoom, but after magnifying the image several times some staircasing artifacts become clear. To remove the staircasing artifacts, you need to optimize the B-Spline's control points (do **not** confuse the control points of the B-Spline with the control points of the Bézier curves, or you will kill continuity and the result will look awful).

Kopf-Lischinski proposes the below *energy* formula as the term to optimize:

$$E^{(i)} = E_s^{(i)} + E_p^{(i)}$$

$E_s^{(i)}$ stands for smoothness energy and $E_p^{(i)}$ stands for positional energy.

The smoothness energy formula given on the paper is:

$$E_s^{(i)} = \int\limits_{s \in r(i)} |k(s)| \mathrm{d}s$$

$k(s)$ stands for curvature at point s. The curvature formula for parametric equation is given below:

$$k = \frac{x'y'' - y'x''}{(x'^2 + y'^2)^{\frac{3}{2}}}$$

If you've read Pomax's *A Primer on Bezier Curves*, then you already demystified the $k(s)$ formula and you are ready to integrate it, but the Pomax's text is abstract enough for Bézier curves of any order, then I'll help you further giving the required formulas for quadratic Bézier curves:

$$Bezier(2, t) = (1 - t)^2 w_0 + 2(1 - t)tw_1 + t^2 w_2$$

$$Bezier'(2, t) = 2(1 - t)(w_1 - w_0) + 2t(w_2 - w_1)$$

$$Bezier''(2,t) = 2(w_2 - 2w_1 + w_0)$$

Replace the w_i term by the node position on the dimension you want to compute.

Numerical integration is easy and you can use any method you like. There is a good text about numerical integration on Math $\cap$ Programming and after you find the methods names, Wikipedia is good enough. If you want a deep understanding on this topic, *Heath's Scientific Computing* book is a good start. You should use the $\{0..1\}$ range for the integration.

The positional energy formula is given on the paper and uses the norm concept.

Appendix A

Complete smoothness code (without the extra color patch)

Originally "published" at `https://gist.github.com/vinipsmaker/6065604` on Tuesday 23rd July, 2013.

```
if ( A.connected(B) ) {
    if ( !C.connected(D) ) {
        bool foreign_is_contour = edge(C, D).contour();

        // the number of contour edges
        switch ( edge(A, C).contour()
                 + edge(B, D).contour()
                 + foreign_is_contour ) {
        case 2:
            A.smooth = !foreign_is_contour;
            break;
        default:
            assert(("precondition failed", false));
        // the following cases are only for
        // documentation of possible values (they don't
        // change the algorithm)
        case 3: case 1: case 0:
            // {A, B} is the pair with the angle
            // closest to 180 degrees
            A.smooth = true;
        }
    } else {
        // there might be 2-color, then A.smooth = true
```

```
            // or it might be 1-color and doesn't matter,
            // because the current node will disappear
            A.smooth = true;
        }
    } else if ( A.connected(C) ) {
        if ( !B.connected(D) ) {
            bool foreign_is_contour = edge(B, D).contour();

            // the number of contour edges
            switch ( edge(A, B).contour()
                     + edge(C, D).contour()
                     + foreign_is_contour ) {
            case 2:
                A.smooth = !foreign_is_contour;
                break;
            default:
                assert(("precondition␣failed", false));
            // the following cases are only for
            // documentation of possible values (they don't
            // change the algorithm)
            case 3: case 1: case 0:
                // {A, C} is the pair with the angle
                // closest to 180 degrees
                A.smooth = true;
            }
        } else {
            // there might be 2-color, then A.smooth = true

            // or it might be 1-color and doesn't matter,
            // because the current node will disappear
            A.smooth = true;
        }
    } else if ( B.connected(D) ) {
        bool foreign_is_contour = edge(C, D).contour();

        switch ( edge(A, B).contour()
                 + edge(A, C).contour()
                 + foreign_is_contour ) {
        case 2:
            A.smooth = !foreign_is_contour;
            break;
        default:
            assert(("precondition␣failed", false));
        case 3: case 1: case 0:
            // {B, D} is the pair with the angle closest to
```

```
        // 180 degrees
        A.smooth = false;
    }
} else if ( C.connected(D) ) {
    bool foreign_is_contour = edge(B, D).contour();

    switch ( edge(A, C).contour()
             + edge(A, B).contour()
             + foreign_is_contour ) {
    case 2:
        A.smooth = !foreign_is_contour;
        break;
    default:
        assert(("precondition␣failed", false));
    case 3: case 1: case 0:
        // {C, D} is the pair with the angle closest to
        // 180 degrees
        A.smooth = false;
    }
} else {
    // there is a 4-color pattern, where the current
    // node won't be smooth
    A.smooth = false;
}
```