# DTU Course 02156 Logical Systems and Logic Programming (2021)

Week	Date	Main Topics (Prolog Programming in All Lessons)
35 #01	31/8	Course Prerequisites & Tutorial on Logical Systems and Logic Programming
36 #02	7/9	Chapter 1 - Introduction (Prolog Note)
37 #03	14/9	Chapter 2 - Propositional Logic: Formulas, Models, Tableaux
38 #04	21/9	Chapter 3 - Propositional Logic: Deductive Systems
39 #05	28/9	"Isabelle" - Propositional Logic: Sequent Calculus Verifier (SeCaV)
40 #06	5/10	Chapter 4 - Propositional Logic: Resolution
41 #07	12/10	Chapter 7 - First-Order Logic: Formulas, Models, Tableaux
42		(Autumn Vacation)
43 #08	26/10	Chapter 8 - First-Order Logic: Deductive Systems
44 #09	2/11	"Isabelle" - First-Order Logic: Sequent Calculus Verifier (SeCaV)
45 #10	9/11	Chapter 9 - First-Order Logic: Terms and Normal Forms
46 #11	16/11	Chapter 10 - First-Order Logic: Resolution
47 #12	23/11	Chapter 11 - First-Order Logic: Logic Programming
48 #13	30/11	Chapter 12 - First-Order Logic: Undecidability and Model Theory & Course Evaluation

Responsible: Associate Professor Jørgen Villadsen <jovi@dtu.dk>

# **Assignments & Exam**

# MUST BE SOLVED INDIVIDUALLY

Assignment-1 Deadline Sunday 26/9 (Available Wednesday 15/9)

Assignment-2 Deadline Sunday 10/10 (Available Wednesday 29/9)

Assignment-3 Deadline Sunday 31/10 (Available Wednesday 13/10)

Assignment-4 Deadline Sunday 14/11 (Available Wednesday 3/11)

Assignment-5 Deadline Thursday 2/12 (Available Wednesday 17/11)

Written Exam Tuesday 14/12 (2 Hours / No Computer / All Notes Allowed)

The mandatory assignments and the written exam are evaluated as a whole – even if you do well in the mandatory assignments then you still must do decent in the written exam in order to pass the course!

A TEACHER MUST IMMEDIATELY REPORT ANY SUSPICION OF CHEATING TO THE STUDY ADMINISTRATION FOR FURTHER ACTIONS

# Agenda — Week #9

The Hardest Logic Puzzle Ever

SeCaV

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A solution to "The Hardest Logic Puzzle Ever" was provided by Boolos in *The Harvard Review of Philosophy* (1996).

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Many possibilities: FOL tableaux, deductive systems, resolution, ...

#### De Bruijn Index

The arrows indicate the references of variables:

Here is the same formula using de Bruijn indices instead:

$$g(0) \longrightarrow (\forall. \ p(0) \longrightarrow (\exists. \ q(0) \longrightarrow (p(1) \lor q(2))))$$
 free variable  $free \ variable$ 

Quantifiers such as  $\exists x$  are replaced by  $\exists$  as the variable referencing is implicit. A variable 0 is bound by the innermost quantifier, a variable 1 inside two quantifiers is bound by the outermost quantifier, and so on. A variable references a free variable when its index exceeds or equals the number of quantifiers it is bound by.



# **Sequent Calculus Verifier**

SeCaV formalizes first-order logic with constants and functions

SeCaV verifies one-sided sequent calculus proofs

SeCaV uses the Isabelle proof assistant

SeCaV is a tool for teaching logic

Jørgen Villadsen

Asta Halkjær From

Alexander Birch Jensen

**Anders Schlichtkrull**