Problem 1 (40%)

Question 1.1

```
count(I) :-
  findall(X,entry(_,X),L),
  sort(L,S),
  length(S,I).
```

Question 1.2

```
details(L) :-
  entry(I,X),
  member(X,L),
  write(I), write(' '), write(X), nl,
  fail.
details(_).
```

Question 1.3

```
collect(S) :-
  findall(I,entry(I,_),L),
  sort(L,S).
```

Question 1.4

```
multi(S) :-
  findall(X,(entry(_,X), findall(I,entry(I,X),[_,_|_])),L),
  sort(L,S).
```

Problem 2 (30%)

Question 2.1

$(\forall x \exists y (p(x) \land \neg p(y))) \to \neg q(a)$ is valid since the following tree is a closed tableau for the negated formula.

$\neg((\forall x \exists y (p(x) \land \neg p(y))) \to \neg q(a))$

$\forall x \exists y (p(x) \land \neg p(y)), \neg\neg q(a)$

$\forall x \exists y (p(x) \land \neg p(y)), q(a)$

$\exists y (p(a) \land \neg p(y)), \forall x \exists y (p(x) \land \neg p(y)), q(a)$

$p(a) \land \neg p(b), \forall x \exists y (p(x) \land \neg p(y)), q(a)$

$p(a), \neg p(b), \forall x \exists y (p(x) \land \neg p(y)), q(a)$

$p(a), \neg p(b), \exists y (p(a) \land \neg p(y)), \exists y (p(b) \land \neg p(y)), \forall x \exists y (p(x) \land \neg p(y)), q(a)$

$p(a), \neg p(b), \exists y (p(a) \land \neg p(y)), p(b) \land \neg p(c), \forall x \exists y (p(x) \land \neg p(y)), q(a)$

$p(a), \neg p(b), \exists y (p(a) \land \neg p(y)), p(b), \neg p(c), \forall x \exists y (p(x) \land \neg p(y)), q(a)$

$\times$

Question 2.2

$(\forall x \exists y (p(x) \land \neg p(y))) \to \neg q(a)$ is valid since the following resolution steps produce the empty clause for the negated formula.

Skolemization:

Negated formula	$\neg((\forall x \exists y (p(x) \land \neg p(y))) \to \neg q(a))$
Rename bound variables	(no change)
Eliminate boolean operators	$\neg(\neg(\forall x \exists y (p(x) \land \neg p(y))) \lor \neg q(a))$
Push negation inwards	$\forall x \exists y (p(x) \land \neg p(y)) \land q(a)$
Extract quantifiers	$\forall x \exists y (p(x) \land \neg p(y) \land q(a))$
Distribute matrix	(no change)
Replace existential quantifiers	$\forall x (p(x) \land \neg p(f(x)) \land q(a))$

$S_0 = \{\{p(x)\}, \{\neg p(f(x))\}, \{q(a)\}\}$

$\square$ is obtained since $p(x)$ and $p(f(x'))$ have most general unifier $x = f(x')$.

Problem 3 (30%)

Question 3.1

```
...

L = [3,4,5,6]
X = 5
Y = 6 ;

L = [4,5,6]
X = 4
Y = 5 ;

L = [4,5,6]
X = 5
Y = 6 ;

L = [5,6]
X = 5
Y = 6 ;

No
```

Question 3.2

```
X = []
Y = [1,2,[1],[1,2]] ;

X = [1,2,[1]]
Y = [[1,2]] ;

X = [1,2,[1],[1,2]] ;
Y = []

No
```

Question 3.3

```
N = 2 ;

N = -1 ;

N = 3 ;

N = 2 ;

N = -1 ;

No
```