

# DTU Course 02156 Logical Systems and Logic Programming (2021)

Week	Date	Main Topics (Prolog Programming in All Lessons)
35 #01	31/8	Course Prerequisites & Tutorial on Logical Systems and Logic Programming
36 #02	7/9	Chapter 1 - Introduction (Prolog Note)
37 #03	14/9	Chapter 2 - Propositional Logic: Formulas, Models, Tableaux
38 #04	21/9	Chapter 3 - Propositional Logic: Deductive Systems
39 #05	28/9	"Isabelle" - Propositional Logic: Sequent Calculus Verifier (SeCaV)
40 #06	5/10	Chapter 4 - Propositional Logic: Resolution
41 #07	12/10	Chapter 7 - First-Order Logic: Formulas, Models, Tableaux
42		(Autumn Vacation)
43 #08	26/10	Chapter 8 - First-Order Logic: Deductive Systems
44 #09	2/11	"Isabelle" - First-Order Logic: Sequent Calculus Verifier (SeCaV)
45 #10	9/11	Chapter 9 - First-Order Logic: Terms and Normal Forms
46 #11	16/11	Chapter 10 - First-Order Logic: Resolution
47 #12	23/11	Chapter 11 - First-Order Logic: Logic Programming
48 #13	30/11	Chapter 12 - First-Order Logic: Undecidability and Model Theory & Course Evaluation

**Responsible: Associate Professor Jørgen Villadsen <jovi@dtu.dk>**

## **Assignments & Exam**

**MUST BE SOLVED INDIVIDUALLY**

**Assignment-1 Deadline Sunday 26/9 (Available Wednesday 15/9)**

**Assignment-2 Deadline Sunday 10/10 (Available Wednesday 29/9)**

**Assignment-3 Deadline Sunday 31/10 (Available Wednesday 13/10)**

**Assignment-4 Deadline Sunday 14/11 (Available Wednesday 3/11)**

**Assignment-5 Deadline Thursday 2/12 (Available Wednesday 17/11)**

**Written Exam Tuesday 14/12 (2 Hours / No Computer / All Notes Allowed)**

**The mandatory assignments and the written exam are evaluated as a whole – even if you do well in the mandatory assignments then you still must do decent in the written exam in order to pass the course!**

**A TEACHER MUST IMMEDIATELY REPORT ANY SUSPICION OF CHEATING TO THE STUDY ADMINISTRATION FOR FURTHER ACTIONS**

Propositional Logic: Truth Tables

Clavius's Law  $(\neg A \rightarrow A) \rightarrow A$

Tautology

If there are no true sentences then there are true sentence

(for instant the sentence: there are no true sentences)

So there are true sentences

Ludwig Wittgenstein

1889 – 1951

[https://en.wikipedia.org/wiki/Ludwig\\_Wittgenstein](https://en.wikipedia.org/wiki/Ludwig_Wittgenstein)

Christopher Clavius

1538 – 1612

[https://en.wikipedia.org/wiki/Christopher\\_Clavius](https://en.wikipedia.org/wiki/Christopher_Clavius)

Clavius's Law = Consequentia Mirabilis (Latin: "admirable consequence")

[https://en.wikipedia.org/wiki/Consequentia\\_mirabilis](https://en.wikipedia.org/wiki/Consequentia_mirabilis)

## Agenda — Week #2

About the course

Prolog note (pages 1-8)

Tracer — More Next Week

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Elementary logics, including propositional and first-order logics

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Proof systems, deductive systems and/or refutation systems

Problem solving techniques, like the backtracking algorithm

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A Prolog system like SWI-Prolog displays a prompt `?-` and waits for a query, for example (the dot `.` marks the end of the query):

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?- halt.
```

The predicate `halt` terminates the Prolog system immediately and is therefore rarely used.



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The predicate `halt` terminates the Prolog system immediately and is therefore rarely used.

Except for special queries with a predicate like `halt` and queries that loop forever it is characteristic of queries that they either succeed or fail.

## Succeed

Here is a query that succeeds:

```
?- write('Hello World'), nl.  
Hello World
```

Yes

Recent versions of the SWI-Prolog system uses true instead of Yes and false instead of No (a dot . is also added at the end).

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The program writes the greeting and a new line.

The word Yes means that the query succeeds.

Whether queries succeed or fail have nothing to do with errors or exceptions.

The , (comma) means “and” (logical conjunction) and ; (semicolon) means “or” (logical disjunction to be used later).

## Fail

Here is a query that fails:

```
?- write('Hello World'), nl, fail.  
Hello World
```

No

## Fail

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```

No

The word No means that the query fails.  
Observe that the greeting is written before the failure.

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```
?- write('Hello World'), nl, fail.  
Hello World
```

No

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Observe that the greeting is written before the failure.

The following query fails before the greeting is written:

```
?- fail, write('Hello World'), nl.
```

No

The predicate `fail` always fails, but is nevertheless quite useful :-)

## Succeed More Than Once

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```

The predicate `true` succeeds once.

The use of the predicates `true` and `repeat` is not recommended for novices.

## Variables

A variable should always start with an uppercase letter (A . . . Z) and the remaining letters in variables can be lowercase letters and/or numbers.

X    H    T    Args    ArgsRest    V123

An exception is the special anonymous variable `_` (underscore) that simply is a new variable for each occurrence.

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There is no type system in Prolog, hence a variable can hold any kind of data.

## Constants

Although there is no type system in Prolog there are different kinds of constants, in particular atoms like `a` and numbers like `2` (decimal numbers are also possible).

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Constants are terms (so are variables), and compound terms can easily be formed:

```
f(1,2,3)    tree(tree(nil,nil),nil)    (1,2)    [a,b,c]
```

Here `f` and `tree` are called functors (atoms are also functors).

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```

Here `f` and `tree` are called functors (atoms are also functors).

Again no types are given or inferred, and there are no restrictions on elements of a list:

```
[1,a,g([]),nil]
```



# Resolution

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Such definite clauses are also called facts and program clauses and a query is called a goal clause.

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The answer to a query is a set of substitutions:

?- length([a,b,c],N).

N = 3

Yes

The answer means that the query succeeds with 3 substituted for N.

## Equality

The infix predicate = is simply defined as follows:

$X = X.$

It is completely symmetric and arithmetic expressions are not evaluated (but the term  $2+2$  is equal to itself).

?-  $2+2 = 2+2$ ,  $X = 2+2$ ,  $2+2 = Y$ .

$X = 2+2$

$Y = 2+2$  ;

No

In SWI-Prolog a ; (semicolon) can be entered after the answer in order to search for other answers, but there are no other way that the query can succeed in this case as indicated by the No for failure.

## Unification

Prolog uses unification which means that variables can appear anywhere in terms:

```
?- tree(tree(nil,A),B) = tree(C,tree(nil,nil)).
```

```
B = tree(nil, nil)
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C = tree(nil, A)
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Yes

Here no ; (semicolon) was entered after the first and only answer.

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Variables like A are called logical variables.

## Lists

Special notation for lists with head and tail:

?-  $L = [X,Y,Z]$  ,  $[H|T] = L$  ,  $L = [a,b,c]$  ,  $[A,B|R] = L$  .

$L = [a, b, c]$

$X = a$

$Y = b$

$Z = c$

$H = a$

$T = [b, c]$

$A = a$

$B = b$

$R = [c]$  ;

No

Note that A and B are the first two elements in the list.

## Arithmetic

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The following special infix predicates evaluate A and B

$A < B$	$A > B$	Less than / Greater than
$A \leq B$	$A \geq B$	Equal or less than / Greater than or equal
$A == B$	$A \neq B$	Equal / Not equal
$X \text{ is } A$		Unification with X

For example (here  $//$  is integer division):

?-  $X \text{ is } (2+3)//2.$

$X = 2 ;$

No

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Hence the second clause is really:  $p :- (q, r) ; (s, t).$

The use of “or” is not recommended for novices and can always be avoided by additional clauses and/or predicates.

## A Sample Program — To Be Revised Later

Here is a program that writes the elements of a list in reverse order:

```
main(Args) :-  
    Args = []  
    ;  
    Args = [Arg|ArgsRest],  
    main(ArgsRest), write(Arg), nl.
```

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main(Args) :-  
    Args = []  
    ;  
    Args = [Arg|ArgsRest],  
    main(ArgsRest), write(Arg), nl.
```

Predicate `main` has no reserved role in ISO Prolog.

```
?- main([a,b,c]).
```

c

b

a

Yes

## Conventions

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This makes good sense since due to the lack of a type system the instantiation patterns are just conventions, whereas predicates with different arities are always distinguished.

The arity can also be used for functors: `tree/2`, `nil/0`, etc.

## A Sample Program — Revised

Same program in the recommended style:

```
% Prolog file main.pl
```

```
/*  
main(+List)
```

```
Writes the elements of List in reverse order.  
*/
```

```
main([]).  
main([Arg|ArgsRest]) :-  
    main(ArgsRest),  
    write(Arg),  
    nl.
```

## Length

Consider a predicate `length` that can be used to calculate the number of elements in a list.

```
length(+List,?Integer)
```

For example:

```
?- length([a,b,c],N).
```

```
N = 3 ;
```

```
No
```

SWI-Prolog has a flexible built-in `length` predicate:

```
length(?List,?Integer)
```

## Length Problems

The following program works for the simple `length(+List,?Integer)` instantiation pattern:

```
length([],0).  
length([_|T],N1) :- length(T,N), N1 is N+1.
```

However it loops forever for the flexible `length(?List,?Integer)` instantiation pattern:

```
?- length(X,-1).
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The built-in `length` predicate in SWI-Prolog uses advanced features to avoid these problems.

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The `length` predicate is not in ISO Prolog.



## Auto-Loaded Predicates

In SWI-Prolog the predicate `length(?List,?Integer)` is built-in, which means that it cannot be redefined.

In SWI-Prolog the basic predicates `member(?Elem,?List)` and `append(?List,?List,?List)` are library auto-loaded, which means that they can be redefined, but are otherwise like built-in predicates.

None of `length`, `member` and `append` are ISO Prolog predicates.

By the way, `sort(+List,?Sorted)` is a built-in ISO predicate for sorting a list (duplicates are removed):

```
?- sort([3,1,4,1,2],S).
```

```
S = [1, 2, 3, 4]
```

Yes

## Member

```
/*
```

```
member(?Elem,?List)
```

Succeeds iff Elem can be unified with one of the members of List.

```
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```

```
member(H,[H|_]).
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```
member(H,[_|T]) :- member(H,T).
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The member predicate is not in ISO Prolog.

## Append

```
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append(?List1,?List2,?List3)
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Succeeds iff List3 unifies with  
the concatenation of List1 and List2.

```
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append([],U,U).
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append([H|T],U,[H|V]) :- append(T,U,V).
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## Examples 1

?- member(b,[a,b,c]).

Yes

?- member(a,[b,c,d]).

No

?- member(X,[a,b,c]), member(X,[b,c,d]).

X = b ;

X = c ;

No



## Examples 2

?- append(X,Y,[a,b,c]).

X = []

Y = [a, b, c] ;

X = [a]

Y = [b, c] ;

X = [a, b]

Y = [c] ;

X = [a, b, c]

Y = [] ;

No

## Examples 3

?- append([a,b],Y,[a,\_,c]).

Y = [c]

Yes

?- append(X,Y,Z).

X = []

Y = Z

Yes

The predicates member/2 and append/3 can be used with any instantiation patterns.

## Tracer — More Next Week

?- trace, member(X,[a,b]).

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Call: (1) member(\_0,[a,b]) ?

Exit: (1) member(a,[a,b]) ?

X = a ;

Redo: (1) member(\_0,[a,b]) ?

Call: (2) member(\_0,[b]) ?

Exit: (2) member(b,[b]) ?

Exit: (1) member(b,[a,b]) ?

X = b ;

Redo: (2) member(\_0,[b]) ?

Call: (3) member(\_0,[]) ?

Fail: (3) member(\_0,[]) ?

Fail: (2) member(\_0,[b]) ?

Fail: (1) member(\_0,[a,b]) ?

No