

02156 - Logical Systems and Logic Programming
Fall 2021



DTU - Technical University of Denmark

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Assignment 1

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Problem 1

Question 1.1

Truth table is constructed using semantics written in the assignment description.

A	B	$\neg A$	$A \wedge B$	$A \vee B$	$A \rightarrow B$	$A \leftrightarrow B$
T	T	F	T	T	T	T
T	F	F	F	T	F	F
T	X	F	X	T	X	X
F	T	T	F	T	T	F
F	F	T	F	F	T	T
F	X	T	F	X	T	$\neg X$
X	T	$\neg X$	X	T	T	X
X	F	$\neg X$	F	X	$\neg X$	$\neg X$
X	X	$\neg X$	X	X	T	T

The lack of semantics for X will make some cases just terminate with $\neg X$. An interesting observation from comparison is that some logical operations disregard or will work just fine without classical truth values. Like implication $X \rightarrow X$ will give True, as with implication it does not matter what value it is operating with.

Question 1.2

When p is T:

$$\neg T \wedge T = F \wedge T = \underline{\underline{F}}$$

When p is F:

$$\neg F \wedge F = T \wedge F = \underline{\underline{F}}$$

When p is X:

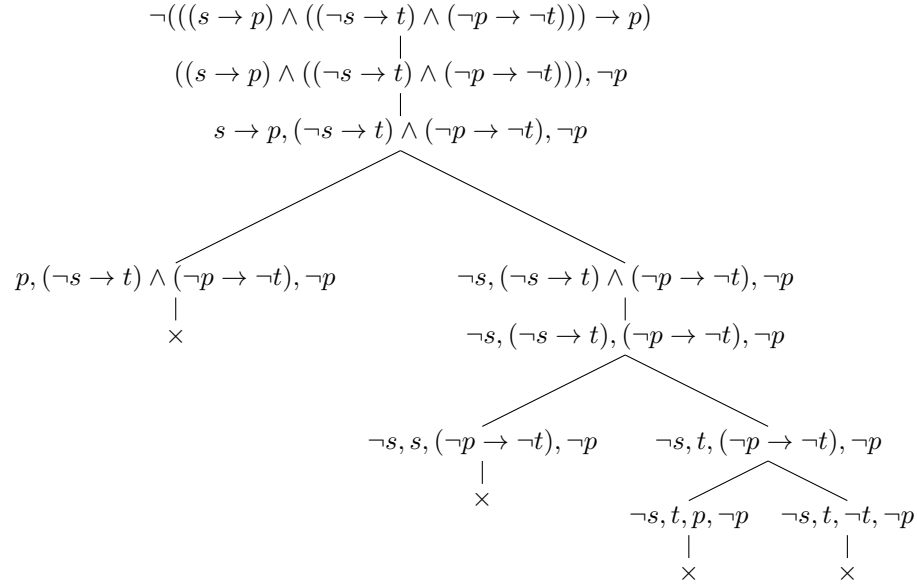
$$\neg X \wedge X = \underline{\underline{F}}$$

As there is no semantics for the negation of X, there is no better evaluation of $\neg X$. Hence we conclude the X case to be false, as the values is not equal and neither of them is T.

Problem 2

Question 2.1

Refuting the validity of the expression, we negate it and try to find counter example:



Concluding a complete closed tableau, thus we can say that the formula is valid hence we can say it's a tautology.

Question 2.2

Considering the logical equivalence, we observe that it can be used to argument for the following:

$$s \rightarrow p \equiv \neg p \rightarrow \neg s$$

and

$$\neg p \rightarrow \neg t \equiv t \rightarrow p$$

Therefor we can "swap" the parts marked with underline below to show equivalence with the formular:

$$((\underline{s \rightarrow p}) \wedge ((\neg s \rightarrow t) \wedge (\underline{\neg p \rightarrow \neg t}))) \rightarrow p$$