Problem 1 (40%)

Question 1.1

Two solutions: `X = peter` and `X = james`

Question 1.2

```
print :- score(exam,X,_), score(test,X,N), N >= 50, write(X), nl, fail.
print.
```

Question 1.3

```
check :- findall(_,score(test,_,_),N), findall(_,score(exam,_,_),N).
```

Problem 2 (25%)

Question 2.1

$\exists x(p(x) \to \forall x p(x))$ is valid since the following tree is a closed tableau for the negated formula.

$$\neg\exists x(p(x) \to \forall x p(x))$$
$$\neg(p(a) \to \forall x p(x)), \neg\exists x(p(x) \to \forall x p(x))$$
$$p(a), \neg\forall x p(x), \neg\exists x(p(x) \to \forall x p(x))$$
$$p(a), \neg p(b), \neg\exists x(p(x) \to \forall x p(x))$$
$$p(a), \neg p(b), \neg(p(a) \to \forall x p(x)), \neg(p(b) \to \forall x p(x)), \neg\exists x(p(x) \to \forall x p(x))$$
$$p(a), \neg p(b), \neg\forall x p(x), \neg(p(b) \to \forall x p(x)), \neg\exists x(p(x) \to \forall x p(x))$$
$$p(a), \neg p(b), \neg\forall x p(x), p(b), \neg\exists x(p(x) \to \forall x p(x))$$
$$\times$$

Question 2.2

$\exists x(p(x) \to \forall x p(x))$ is valid since the following resolution steps produce the empty clause for the negated formula.

Skolemization:

Negated formula	$\neg\exists x(p(x) \to \forall x p(x))$
Rename bound variables	$\neg\exists x(p(x) \to \forall y p(y))$
Eliminate boolean operators	$\neg\exists x(\neg p(x) \lor \forall y p(y))$
Push negation inwards	$\forall x(p(x) \land \exists y \neg p(y))$
Extract quantifiers	$\forall x \exists y(p(x) \land \neg p(y))$
Distribute matrix	(no change)
Replace existential quantifiers	$\forall x(p(x) \land \neg p(f(x)))$

$S_0 = \{\{p(x)\}, \{\neg p(f(x))\}\}$

$\square$ is obtained since $p(x)$ and $p(f(x'))$ have most general unifier $x = f(x')$.

Problem 3 (35%)

Question 3.1

```
...

X = [3,2]
Y = 2 ;

X = [3,2,1]
Y = 3 ;

No
```

Question 3.2

```
X = [1]
Y = [2,3,[1],[1,2],[1,2,3]] ;

X = [1,2]
Y = [3,[1],[1,2],[1,2,3]] ;

X = [1,2,3]
Y = [[1],[1,2],[1,2,3]] ;

No
```

Question 3.3

```
L = [10, 40, 20, 50, 30]
R = [10, 20, 30, 40, 50] ;

No
```