DTU Course 02156 Logical Systems and Logic Programming

Mandatory Assignment 4 — Deadline Sunday 14/11 23:59
MUST BE SOLVED INDIVIDUALLY

You are only allowed to get help from the teacher and the teaching assistants.

You may use the notes and definitions on the first page of the sample exams.

You are allowed to use your computer and there is no 2 hours time limit.

For the whole assignment you must submit exactly 2 files on DTU Learn:

1. A single PDF file with extension `.pdf` with the report.

2. A single Prolog file with extension `.pl` with the programs.

Absolutely no ZIP files, no text processing documents or any other file formats.

The report should not contain any program listings (just refer to the Prolog file).

The programs should load in SWI-Prolog without any errors or warnings.

The programs must be properly documented with comments.

If possible use only the ISO Prolog features of SWI-Prolog covered in the course.

All programs must be tested and the tests must be included in the report.

In particular:

- Test a few normal cases.

- Test the special cases.

- Test with variables only (if the instantiation pattern allows variables).

Show the Prolog queries and the corresponding answers — and keep explanations short.

Problem 1 (30%)

Consider the following Prolog program serving as a database of students in a course and their scores in a test and in the exam (a score is an integer between 0 and 100):

```
score(test,xenia,50).
score(test,alice,99).
score(test,bruce,22).
score(test,carol,77).
score(test,dorit,50).
score(test,erica,22).
score(exam,peter,42).
score(exam,alice,11).
score(exam,bruce,88).
score(exam,carol,33).
score(exam,dorit,50).
score(exam,erica,66).
score(exam,james,77).
```

For example, `xenia` scored 50 in the test but did not participate in the exam, and `alice` scored 99 in the test but only 11 in the exam.

Question 1.1

Write a deterministic Prolog program `students(?List)` that succeeds if and only if `List` is the sorted list of students in the database. Duplicates are merged. A sample query:

```
?- students(S).

S = [alice, bruce, carol, dorit, erica, james, peter, xenia]

Yes
```

Question 1.2

The ministry pays 1000 dollars for each student who scored more than 40 in the exam (the score in the test is not relevant).

Write a deterministic Prolog program `money(?Integer)` that calculates the total payment for the students in the database. The result is a non-negative integer. A sample query:

```
?- money(M).

M = 5000

Yes
```

Problem 2 (30%)

The details of the following answers must be provided and explained (in particular it is not sufficient to just list the result from a Prolog program).

Question 2.1

Consider the following formula: $(p \wedge q) \rightarrow (q \wedge p)$

Use refutation and the resolution procedure. State whether this shows that the formula is valid or not.

Question 2.2

Consider the following formula: $(\forall x \exists y (p(x) \wedge \neg p(y))) \rightarrow \neg q(a)$

Use refutation and the systematic construction of a semantic tableau. State whether this shows that the formula is valid or not.

Problem 3 (40%)

Use the online SeCaV Unshortener in order to prove the following formulas:

$\forall x p(x) \rightarrow p(a)$

$p(a, a) \rightarrow \exists x \exists y p(x, y)$

$(\forall x p(x) \wedge \forall x q(x)) \rightarrow \forall x (p(x) \wedge q(x))$

Insert the SeCaV Unshortener lines in the Prolog file as a comment.

Do not include the proofs in Isabelle.

Sample lines for the formula $p \rightarrow p$:

```
/*

Imp p p

AlphaImp
  Neg p
  p
Ext
  p
  Neg p
Basic

*/
```