Problem 1 (40%)

Question 1.1

```
count(I) :- findall(X,score(_,X,_),L), sort(L,S), length(S,I).
```

Question 1.2

One solution: `L = [peter, james, xenia]`

Question 1.3

```
print :- score(exam,X,N), N >= 50, \+ score(test,X,_), write(X), nl, fail.
print.
```

Question 1.4

```
top(T) :-
  findall(N,score(exam,_,N),L),
  sort(L,R),
  append(_,[M],R),
  findall(X,score(exam,X,M),S),
  sort(S,T).
```

Problem 2 (30%)

Question 2.1

```
X = [7,7,7,7]
Y = [7,7] ;

X = [7,7,7,7]
Y = [7,7] ;

X = [7,7,7,7]
Y = [7,7] ;

X = [7,7,7,7]
Y = [7,7] ;

No
```

Question 2.2

```
X = 1 ;

X = 2 ;

X = 3 ;

X = 7 ;

X = 8 ;

X = 9 ;

No
```

Question 2.3

```
N = 0 ;

N = 1 ;

N = 2 ;

N = -1 ;

No
```

Problem 3 (30%)

Question 3.1

$(\forall x p(x) \land \forall x q(x)) \lor \neg(\forall x(p(x) \land q(x)))$ is valid since the following tree is a closed tableau for the negated formula.

$$\neg((\forall x p(x) \land \forall x q(x)) \lor \neg(\forall x(p(x) \land q(x))))$$
$$\neg(\forall x p(x) \land \forall x q(x)), \neg\neg(\forall x(p(x) \land q(x)))$$
$$\neg(\forall x p(x) \land \forall x q(x)), \forall x(p(x) \land q(x))$$

$\overline{\text{L} \qquad \text{R}}$

$$\neg\forall x p(x), \forall x(p(x) \land q(x))$$
$$\neg p(a), \forall x(p(x) \land q(x))$$
$$\neg p(a), \forall x(p(x) \land q(x)), p(a) \land q(a)$$
$$\neg p(a), \forall x(p(x) \land q(x)), p(a), q(a)$$
$$\times$$

R

$$\neg\forall x q(x), \forall x(p(x) \land q(x))$$
$$\neg q(b), \forall x(p(x) \land q(x))$$
$$\neg q(b), \forall x(p(x) \land q(x)), p(b) \land q(b)$$
$$\neg q(b), \forall x(p(x) \land q(x)), p(b), q(b)$$
$$\times$$

Question 3.2

$(\forall x p(x) \wedge \forall x q(x)) \vee \neg(\forall x(p(x) \wedge q(x)))$ is valid since the following resolution steps produce the empty clause for the negated formula.

Skolemization:

Negated formula	$\neg((\forall x p(x) \wedge \forall x q(x)) \vee \neg(\forall x(p(x) \wedge q(x))))$
Rename bound variables	$\neg((\forall x p(x) \wedge \forall y q(y)) \vee \neg(\forall z(p(z) \wedge q(z))))$
Eliminate boolean operators	(no change)
Push negation inwards	$(\exists x \neg p(x) \vee \exists y \neg q(y)) \wedge \forall z(p(z) \wedge q(z))$
Extract quantifiers	$\exists x \exists y \forall z((\neg p(x) \vee \neg q(y)) \wedge p(z) \wedge q(z))$
Distribute matrix	(no change)
Replace existential quantifiers	$\forall z((\neg p(a) \vee \neg q(b)) \wedge p(z) \wedge q(z))$

$S_0 = \{\{\neg p(a), \neg q(b)\}, \{p(x)\}, \{q(x)\}\}$

$S_1 = S_0 \cup \{\{\neg q(b)\}\}$ since $p(a)$ and $p(x)$ have most general unifier $x = a$.

$\square$ is obtained since $q(b)$ and $q(x)$ have most general unifier $x = b$.