# Problem 1 (50%)

## Question 1.1

```
...

L = [2, 3, 4, 5]
X = 4
Y = 5 ;

L = [3, 4, 5]
X = 3
Y = 4 ;

L = [3, 4, 5]
X = 4
Y = 5 ;

L = [4, 5]
X = 4
Y = 5 ;

No
```

## Question 1.2

```
X = 2
L = [2]
N = 1 ;

X = 3
L = [3]
N = 1 ;

No
```

## Question 1.3

```
N = 2 ;

N = 2 ;

N = -1 ;

N = 1 ;

N = 1 ;

No
```

## Question 1.4

```
expel(1,[H|T],H,T).
expel(N,[H|T],X,[H|U]) :- N > 1, N1 is N-1, expel(N1,T,X,U).
```

# Problem 2 (25%)

## Question 2.1

$\forall x(\neg p(a,b,x) \rightarrow \neg(\neg\exists x p(x,b,c) \wedge \forall x p(a,x,c)))$ is valid since the following tree is a closed tableau for the negated formula.

$$\neg(\forall x(\neg p(a,b,x) \rightarrow \neg(\neg\exists x p(x,b,c) \wedge \forall x p(a,x,c))))$$
$$\neg(\neg p(a,b,d) \rightarrow \neg(\neg\exists x p(x,b,c) \wedge \forall x p(a,x,c)))$$
$$\neg p(a,b,d), \neg(\neg(\neg\exists x p(x,b,c) \wedge \forall x p(a,x,c)))$$
$$\neg p(a,b,d), \neg\exists x p(x,b,c) \wedge \forall x p(a,x,c)$$
$$\neg p(a,b,d), \neg\exists x p(x,b,c), \forall x p(a,x,c)$$
$$\neg p(a,b,d), \neg p(a,b,c), p(a,a,c), \neg p(b,b,c), p(a,b,c), \neg p(c,b,c), p(a,c,c), \neg p(d,b,c), p(a,d,c), \neg\exists x p(x,b,c), \forall x p(a,x,c)$$
$$\times$$

## Question 2.2

$\forall x(\neg p(a,b,x) \rightarrow \neg(\neg\exists x p(x,b,c) \wedge \forall x p(a,x,c)))$ is valid since the following resolution steps produce the empty clause for the negated formula.

Skolemization:

| | |
|---|---|
| Negated formula | $\neg(\forall x(\neg p(a,b,x) \rightarrow \neg(\neg\exists x p(x,b,c) \wedge \forall x p(a,x,c)))$ |
| Rename bound variables | $\neg(\forall x(\neg p(a,b,x) \rightarrow \neg(\neg\exists y p(y,b,c) \wedge \forall z p(a,z,c)))$ |
| Eliminate boolean operators | $\neg(\forall x(\neg\neg p(a,b,x) \vee \neg(\neg\exists y p(y,b,c) \wedge \forall z p(a,z,c)))$ |
| Push negation inwards | $\exists x(\neg p(a,b,x) \wedge \forall y \neg p(y,b,c) \wedge \forall z p(a,z,c))$ |
| Extract quantifiers | $\exists x \forall y \forall z(\neg p(a,b,x) \wedge \neg p(y,b,c) \wedge p(a,z,c))$ |
| Distribute matrix | (no change) |
| Replace existential quantifiers | $\forall y \forall z(\neg p(a,b,d) \wedge \neg p(y,b,c) \wedge p(a,z,c))$ |

$S_0 = \{\{\neg p(a,b,d)\}, \{\neg p(y,b,c)\}, \{p(a,z,c)\}\}$

$\square$ is obtained since $p(y,b,c)$ and $p(a,z,c)$ have most general unifier $y = a, z = b$.

# Problem 3 (25%)

## Question 3.1

```
collect(S) :-
  findall(X,entry(_,X),L),
  sort(L,S).
```

## Question 3.2

```
details(L) :-
  entry(I,X),
  \+ member(X,L),
  write(I), write(' '), write(X), nl,
  fail.
details(_).
```