Problem 1 (50%)

Question 1.1

```
summarize(Size) :- findall(_,db(_,_,_),L), length(L,Size).
```

Question 1.2

```
dump :- \+ dump(1).

dump(X) :- db(X,N,A), write(N), write(' '), write(A), nl, X1 is X+1, dump(X1).
```

Question 1.3

```
check1 :- \+ (db(X,_,A), db(Y,_,A), X =\= Y).
```

Question 1.4

```
check2 :- \+ (db(X,NX,_), db(Y,NY,_), X < Y, NX < NY).
```

Question 1.5

```
check3 :- findall(X,db(X,_,_),L), sort(L,S), length(L,N), length(S,N), check(S).

check(S) :- check(S,1).

check([],_).
check([X|S],X) :- X1 is X+1, check(S,X1).
```

Problem 2 (25%)

Question 2.1

```
word(X) :- w(_,_,X,_), !.
```

Question 2.2

```
count(I) :- findall(X,w(_,_,_,X),L), sort(L,R), length(R,I).
```

Question 2.3

```
sum(I) :- findall(X,w(_,X,_,_),L), sum(L,I).

sum([],0).
sum([H|T],I) :- sum(T,J), I is H+J.
```

Problem 3 (25%)

Question 3.1

Skolemization:

Negated formula	$\neg\exists y\forall x(p(y) \rightarrow p(x))$
Rename bound variables	(no change)
Eliminate boolean operators	$\neg\exists y\forall x(\neg p(y) \vee p(x))$
Push negation inwards	$\forall y\exists x(p(y) \wedge \neg p(x))$
Extract quantifiers	(no change)
Distribute matrix	(no change)
Replace existential quantifiers	$\forall y(p(y) \wedge \neg p(f(y)))$

$S_0 = \{\{p(x)\}, \{\neg p(f(x))\}\}$ is the set of clauses for the negated formula.

$\square$ is obtained since $p(x)$ and $p(f(x'))$ have most general unifier $x = f(x')$.

Since the empty clause is produced for the negated formula, the original formula is valid.

Question 3.2

A formula in CNF must be a conjunction of disjunctions of literals and hence the three formulas p, $p \vee q$ and $(\neg p \vee \neg q) \wedge r$ are in CNF.

The atoms $p(x)$ and $q(y)$ are not unifiable since the functors are different.

The atoms $p(f(x), x)$ and $p(y, y)$ are not unifiable due to occurs-check.