Problem 1 (40%)

Question 1.1

```
test(N) :- score(test,_,N), !.
test(N) :- score(exam,_,N), !.
```

Question 1.2

```
print :- score(test,X,M), score(exam,X,N), M < N, write(X), nl, fail.
print.
```

Question 1.3

```
scores([],[]).
scores([H|T],L) :- score(exam,H,N), !, L = [N|U], scores(T,U).
scores([_|T],[0|U]) :- scores(T,U).

bottom(L,X) :- scores(L,R), sort(R,[X|_]).
```

Problem 2 (30%)

Question 2.1

```
X = 4
Y = 2 ;

X = 4
Y = 3 ;

X = 4
Y = 4 ;

X = 5
Y = 2 ;

X = 5
Y = 3 ;

X = 5
Y = 4 ;

X = 5
Y = 5 ;

No
```

Question 2.2

```
Z = [1, 2, 3, [], 4, [], 6, 5, [], []] ;

No
```

Question 2.3

```
N = -1 ;

N = -1 ;

N = 2 ;

No
```

Problem 3 (30%)

Question 3.1

$\forall x \forall y (p(x,y) \wedge \neg q(y,x)) \rightarrow \forall x p(x,x)$ is valid since the following tree is a closed tableau for the negated formula.

$$\neg(\forall x \forall y (p(x,y) \wedge \neg q(y,x)) \rightarrow \forall x p(x,x))$$
$$\forall x \forall y (p(x,y) \wedge \neg q(y,x)), \neg \forall x p(x,x)$$
$$\neg p(a,a), \forall x \forall y (p(x,y) \wedge \neg q(y,x))$$
$$\forall y (p(a,y) \wedge \neg q(y,a)), \neg p(a,a), \forall x \forall y (p(x,y) \wedge \neg q(y,x))$$
$$p(a,a) \wedge \neg q(a,a), \neg p(a,a), \forall x \forall y (p(x,y) \wedge \neg q(y,x)), \forall y (p(a,y) \wedge \neg q(y,a)$$
$$p(a,a), \neg q(a,a), \neg p(a,a), \forall x \forall y (p(x,y) \wedge \neg q(y,x)), \forall y (p(a,y) \wedge \neg q(y,a))$$
$$\times$$

Question 3.2

$\forall x \forall y (p(x,y) \wedge \neg q(y,x)) \rightarrow \forall x p(x,x)$ is valid since the following resolution steps produce the empty clause for the negated formula.

Skolemization:

Negated formula	$\neg(\forall x \forall y (p(x,y) \wedge \neg q(y,x)) \rightarrow \forall x p(x,x))$
Rename bound variables	$\neg(\forall x \forall y (p(x,y) \wedge \neg q(y,x)) \rightarrow \forall z p(z,z))$
Eliminate boolean operators	$\neg(\neg \forall x \forall y (p(x,y) \wedge \neg q(y,x)) \vee \forall z p(z,z))$
Push negation inwards	$\forall x \forall y (p(x,y) \wedge \neg q(y,x)) \wedge \exists z \neg p(z,z)$
Extract quantifiers	$\forall x \forall y \exists z ((p(x,y) \wedge \neg q(y,x)) \wedge \neg p(z,z))$
Distribute matrix	(no change)
Replace existential quantifiers	$\forall x \forall y ((p(x,y) \wedge \neg q(y,x)) \wedge \neg p(f(x,y), f(x,y)))$

$S_0 = \{\{p(x,y)\}, \{\neg q(y,x)\}, \{\neg p(f(x,y), f(x,y))\}\}$

$\square$ is obtained since $p(x,y)$ and $p(f(x',y'), f(x',y'))$ have most general unifier $x = f(x',y')$, $y = f(x',y')$.