Problem 1 (30%)

Question 1.1

```
L = [2, 3]
X = 2 ;

L = [2, 3]
X = 3 ;

L = [3]
X = 3 ;

No
```

Question 1.2

```
L = []
R = [] ;

L = [1, 2]
R = [2] ;

L = [1, 2, 3, 4]
R = [2, 4] ;

No
```

Question 1.3

```
same_length(L,R) :- length(L,N), length(R,N).
```

```
same_length([],[]).
same_length([_|L],[_|R]) :- same_length(L,R).
```

Problem 2 (30%)

Question 2.1

Original formula: $\exists x p(a,x,b) \lor \forall x \neg p(a,x,b)$

Since the following tree is a closed tableau for the negated formula, the original formula is valid.

$\neg(\exists x p(a,x,b) \lor \forall x \neg p(a,x,b))$

$\neg\exists x p(a,x,b), \neg\forall x \neg p(a,x,b)$

$\neg\neg p(a,c,b), \neg\exists x p(a,x,b)$

$p(a,c,b), \neg\exists x p(a,x,b)$

$p(a,c,b), \neg p(a,a,b), \neg p(a,b,b), \neg p(a,c,b), \neg\exists x p(a,x,b)$

$\times$

Question 2.2

Skolemization:

Negated formula	$\neg(\exists x p(a,x,b) \lor \forall x \neg p(a,x,b))$
Rename bound variables	$\neg(\exists x p(a,x,b) \lor \forall y \neg p(a,y,b))$
Eliminate boolean operators	(no change)
Push negation inwards	$\forall x \neg p(a,x,b) \land \exists y p(a,y,b)$
Extract quantifiers	$\forall x \exists y(\neg p(a,x,b) \land p(a,y,b))$
Distribute matrix	(no change)
Replace existential quantifiers	$\forall x(\neg p(a,x,b) \land p(a,f(x),b))$

$S_0 = \{\{\neg p(a,x,b)\}, \{p(a,f(x),b)\}\}$ is the set of clauses for the negated formula.

$\square$ is obtained since $p(a,x,b)$ and $p(a,f(x'),b)$ have most general unifier $x = f(x')$.

Since the empty clause is produced for the negated formula, the original formula is valid.

Problem 3 (40%)

Question 3.1

```
count :-
  findall(Z,(edge(Z,_) ; edge(_,Z)),X),
  sort(X,Y),
  member(Z,Y),
  findall(_,edge(_,Z),L),
  findall(_,edge(Z,_),R),
  length(L,M),
  length(R,N),
  S is M-N,
  write(Z), write(' '), write(S), nl,
  fail.
count.
```

Question 3.2

```
test(A,Z,P) :- path([A],Z,P).

path([A|_],A,P) :- !, P = [A].
path(V,Z,[A|P]) :-
  V = [A|_],
  edge(A,B),
  \+ member(B,V),
  path([B|V],Z,P).
```