Problem 1 (40%)

Question 1.1

```
extra(A,B) :-
  w(_,_,X,A),
  w(_,_,X,B),
  w(_,_,X,C),
  C \= A,
  C \= B,
  write(X), write(' '), write(C), nl,
  fail.
extra(_,_).
```

Question 1.2

```
multi(S) :-
  findall(X,(w(_,_,X,_), findall(A,w(_,_,X,A),[_,_|_])),L),
  sort(L,S).
```

Question 1.3

```
clist(S) :-
  findall(A,w(_,_,_,A),L),
  sort(L,S).
```

Problem 2 (25%)

Question 2.1

$\forall x p(x,a,x) \to \forall x \exists y p(x,a,y)$ is valid since the following tree is a closed tableau for the negated formula.

$$\neg(\forall x p(x,a,x) \to \forall x \exists y p(x,a,y))$$
$$\forall x p(x,a,x), \neg\forall x \exists y p(x,a,y)$$
$$\forall x p(x,a,x), \neg\exists y p(b,a,y)$$
$$p(a,a,a), p(b,a,b), \neg p(b,a,a), \neg p(b,a,b), \forall x p(x,a,x), \neg\exists y p(b,a,y)$$
$$\times$$

Question 2.2

$\forall x p(x,a,x) \to \forall x \exists y p(x,a,y)$ is valid since the following resolution steps produce the empty clause for the negated formula.

Skolemization:

Negated formula	$\neg(\forall x p(x,a,x) \to \forall x \exists y p(x,a,y))$
Rename bound variables	$\neg(\forall x p(x,a,x) \to \forall z \exists y p(z,a,y))$
Eliminate boolean operators	$\neg(\neg\forall x p(x,a,x) \lor \forall z \exists y p(z,a,y))$
Push negation inwards	$\forall x p(x,a,x) \land \exists z \forall y \neg p(z,a,y)$
Extract quantifiers	$\forall x \exists z \forall y (p(x,a,x) \land \neg p(z,a,y))$
Distribute matrix	(no change)
Replace existential quantifiers	$\forall x \forall y (p(x,a,x) \land \neg p(f(x),a,y))$

$S_0 = \{\{p(x,a,x)\}, \{\neg p(f(x),a,y)\}\}$

$\square$ is obtained since $p(x,a,x)$ and $p(f(x'),a,y)$ have most general unifier $x = f(x'), y = f(x')$.

Problem 3 (35%)

Question 3.1

```
...

X = 1 ;

X = 1 ;

No
```

Question 3.2

```
X = [[1]]
Y = [[1, 2]]
Z = [1] ;

X = [[1], [1, 2]]
Y = []
Z = [1] ;

X = [[1], [1, 2]]
Y = []
Z = [1, 2] ;

No
```

Question 3.3

```
L = [1, 1, 4, 4, 2, 2, 5, 5, 3, 3] ;

No
```