

TECHNICAL UNIVERSITY OF DENMARK (DTU)

Written Sample-Exam-5, 2021

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Course: Logical Systems and Logic Programming

Course number: 02156

Exam duration: 2 hours

Aids allowed: All written works of reference

Weighting: Stated for each problem

The following basic predicates can be used when writing Prolog programs:

```
member(H, [H|_]).  
member(H, [_|T]) :- member(H,T).  
  
append([], U, U).  
append([H|T], U, [H|V]) :- append(T, U, V).
```

Here `member(?Elem, ?List)` succeeds if and only if `Elem` can be unified with one of the members of `List` and `append(?List1, ?List2, ?List3)` succeeds if and only if `List3` unifies with the concatenation of `List1` and `List2`.

Standard predicates like `is`, `fail`, `write`, `nl` and `findall` can also be used.

In the following a Prolog program is said to be deterministic if and only if it does not succeed more than once.

Assume available a deterministic predicate `sort(+List, ?Sorted)` that can be used to sort a list. Duplicates are merged as shown in the following example:

```
?- sort([3,1,4,1,2], S).
```

```
S = [1, 2, 3, 4]
```

Yes

Assume also available a predicate `length(+List, ?Integer)` that can be used to calculate the number of elements in a list.

Problem 1 (40%)

Consider the following Prolog program serving as a database of students in a course and their scores in a test and in the exam (a score is an integer between 0 and 100):

```
score(test,xenia,50).
score(test,alice,99).
score(test,bruce,22).
score(test,carol,77).
score(test,dorit,50).
score(test,erica,22).
score(exam,peter,42).
score(exam,alice,11).
score(exam,bruce,88).
score(exam,carol,33).
score(exam,dorit,50).
score(exam,erica,66).
score(exam,james,77).
```

For example, `xenia` scored 50 in the test but did not participate in the exam, and `alice` scored 99 in the test but only 11 in the exam.

Question 1.1

Write a deterministic Prolog program `test(+Integer)` that succeeds if and only if `Integer` is a score of a student who participated in the test or in the exam (or both).

Sample queries:

```
?- test(11).
```

Yes

```
?- test(12).
```

No

```
?- test(77).
```

Yes

Question 1.2

Write a deterministic Prolog program `print` that prints the students who scored more in the exam than in the test. A sample query:

```
?- print.  
bruce  
erica
```

Yes

Question 1.3

Write a deterministic Prolog program `bottom(+List,?Integer)` where `Integer` is the lowest score in the exam for the students provided in `List` (the students who did not participate in the exam are given the score 0).

Sample queries:

```
?- bottom([nigel,bruce,carol],X), bottom([alice,bruce,carol],Y).
```

```
X = 0  
Y = 11
```

Yes

```
?- bottom([bruce,carol],X), bottom([bruce],Y).
```

```
X = 33  
Y = 88
```

Yes

```
?- bottom([],X).
```

No

Hint: It is perhaps necessary to write additional programs.

Problem 2 (30%)

In the following a semicolon (;) is used to separate the solutions to a query. This corresponds to the common use of the semicolon in an interactive Prolog session.

Question 2.1

State the remaining solutions to the following query:

```
?- member(X,[1,2,3,4,5]), member(Y,[2,3,4,5]), X >= Y.
```

```
X = 2
```

```
Y = 2 ;
```

```
X = 3
```

```
Y = 2 ;
```

```
X = 3
```

```
Y = 3 ;
```

```
...
```

Question 2.2

Consider the following Prolog program:

```
p(A,[A]) :- member(A,[0,1,2,3,4,5,6,7,8,9,[]]).
```

```
p([H|T],Z) :- p(H,X), p(T,Y), append(X,Y,Z).
```

State the solutions to the following query:

```
?- p([1,[2,3,[],4],6,[5]],Z).
```

Question 2.3

Consider the following Prolog programs:

```
main(N) :- append(L,_,[a,b]), fun(L,N).
```

```
fun(L,N) :- length(L,N), N > 1, !.  
fun(_, -1).
```

State the solutions to the following query:

```
?- main(N).
```

Problem 3 (30%)

Consider the following formula: $\forall x \forall y (p(x, y) \wedge \neg q(y, x)) \rightarrow \forall x p(x, x)$

Question 3.1

Use refutation and the systematic construction of a semantic tableau. State whether this shows that the formula is valid or not.

Question 3.2

Use refutation, skolemization and the general resolution procedure. State whether this shows that the formula is valid or not.