Problem 1 (50%)

Question 1.1

```
L = [1, 2, 3]
X = 1 ;

L = [1, 2, 3]
X = 2 ;

L = [1, 2, 3]
X = 3 ;

No
```

Question 1.2

```
L = []
R = [] ;

L = [1]
R = [1] ;

L = [1, 2]
R = [2, 1] ;

L = [1, 2, 3]
R = [3, 2, 1] ;

No
```

Question 1.3

```
cutoff([],[]).
cutoff([H|_],[]) :- H < 0.
cutoff([H|T],[H|U]) :- H >= 0, cutoff(T,U).

cutoff2([],[]).
cutoff2([H|_],U) :- H < 0, !, U = [].
cutoff2([H|T],[H|U]) :- cutoff2(T,U).
```

Problem 2 (25%)

Question 2.1

```
fighter(X) :- beat(X,[_|_]), beat(_,L), member(X,L), !.
```

Question 2.2

```
count :- beat(X,L), length(L,N), write(N), write(' '), write(X), nl, fail.
count.
```

Problem 3 (25%)

Question 3.1

Original formula: $\exists y \forall x (p(y) \rightarrow p(x))$

Since the following tree is a closed tableau for the negated formula, the original formula is valid.

$$
\begin{array}{l}
\neg \exists y \forall x (p(y) \rightarrow p(x)) \\
\neg \forall x (p(a) \rightarrow p(x)), \neg \exists y \forall x (p(y) \rightarrow p(x)) \\
\neg (p(a) \rightarrow p(b)), \neg \exists y \forall x (p(y) \rightarrow p(x)) \\
p(a), \neg p(b), \neg \exists y \forall x (p(y) \rightarrow p(x)) \\
p(a), \neg p(b), \neg \forall x (p(a) \rightarrow p(x)), \neg \forall x (p(b) \rightarrow p(x)), \neg \exists y \forall x (p(y) \rightarrow p(x)) \\
p(a), \neg p(b), \neg (p(a) \rightarrow p(c)), \neg \forall x (p(b) \rightarrow p(x)), \neg \exists y \forall x (p(y) \rightarrow p(x)) \\
p(a), \neg p(b), \neg p(c), \neg \forall x (p(b) \rightarrow p(x)), \neg \exists y \forall x (p(y) \rightarrow p(x)) \\
p(a), \neg p(b), \neg p(c), \neg (p(b) \rightarrow p(d)), \neg \exists y \forall x (p(y) \rightarrow p(x)) \\
p(a), \neg p(b), \neg p(c), p(b), \neg p(d), \neg \exists y \forall x (p(y) \rightarrow p(x)) \\
\times
\end{array}
$$

Question 3.2

Original formula: $\exists y p(y) \rightarrow \forall x p(x)$

Since the following tree is an open tableau for the negated formula, the original formula is not valid.

$$
\begin{array}{l}
\neg (\exists y p(y) \rightarrow \forall x p(x)) \\
\exists y p(y), \neg \forall x p(x) \\
p(a), \neg \forall x p(x) \\
p(a), \neg p(b) \\
\odot
\end{array}
$$

Hence $\exists y \forall x (p(y) \rightarrow p(x))$ and $\exists y p(y) \rightarrow \forall x p(x)$ are not logically equivalent.