02156 Exercises-11

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Exercise 1

Unify the following pairs of atomic formulas, if possible.

$$
\begin{array}{ll}
p(a, x, f(g(y))) & p(y, f(z), f(z)) \\
p(x, g(f(a)), f(x)) & p(f(a), y, y) \\
p(x, g(f(a)), f(x)) & p(f(y), z, y) \\
p(a, x, f(g(y))) & p(z, h(z, u), f(u))
\end{array}
$$

Check the result using the file `qed.pl` available on CampusNet (Program files folder in the top folder) — but only when you have first *manually* found a most general unifier.

Examples:

```
?- test_unify(p(g(Y),f(X,h(X),Y)),p(X,f(g(Z),W,Z))).

Unify p(g(x1),f(x2,h(x2),x1))
 and  p(x2,f(g(x3),x4,x3))

x1 = x3
x2 = g(x3)
x4 = h(g(x3))

Yes

?- test_unify(p(g(Y),f(X,h(X),Y)),p(X,ff(g(Z),W,Z))).

Unify p(g(x1),f(x2,h(x2),x1))
 and  p(x2,ff(g(x3),x4,x3))

Failed (different functors) in
f(g(x1),h(g(x1)),x1) = ff(g(x3),x4,x3)

Yes

?- test_unify(p(f(Y,Z,g(Y,Z,X))),p(X)).

Unify p(f(x1,x2,g(x1,x2,x3)))
 and  p(x3)

Failed (occurs check) in
x3 = f(x1,x2,g(x1,x2,x3))

Yes

?- test_unify(p(f(a,Z),g(X)),p(f(a,Y),g(g(Z)))).

Unify p(f(a,x1),g(x2))
 and  p(f(a,x3),g(g(x1)))

x1 = x3
x2 = g(x3)

Yes
```

Exercise 2

Consider the following formulas:

$$p(a) \qquad \exists x p(x) \qquad \forall x p(x) \qquad \forall x p(x) \rightarrow \exists x p(x) \qquad \exists x p(x) \rightarrow \forall x p(x) \qquad \exists x((\neg p(a) \rightarrow (q \wedge \neg q)) \rightarrow p(x))$$

$$\forall x(p(x) \rightarrow q(x)) \rightarrow (\forall x p(x) \rightarrow \exists x q(x)) \qquad \forall x(p(x) \rightarrow q(x)) \rightarrow (\exists x p(x) \rightarrow \exists x q(x))$$

Use refutation, skolemization and the general resolution procedure and state whether this shows that the formulas are valid or not.

Check the result using the file `qed.pl` available on CampusNet (Program files folder in the top folder) — but only when you have first *manually* worked on the formulas.

```
?- qed(~ (all(X, p(f(X))) & all(X, ~ p(X)))).

~(Ax1p(f(x1)) & Ax1~p(x1))

Ax1Ax2(p(f(x1)) & ~p(x2))

[p(f(x1))][~p(x2)]

Resolve [p(f(x1))]
  and   [~p(x2)]

x2 = f(x1)

[][p(f(x1))][~p(x2)]

Yes
```

Exercise 3

Consider the following formula: $\neg\forall x\forall y((p(x) \vee p(y)) \wedge (\neg p(x) \vee \neg p(y)))$

First use refutation and the systematic construction of a semantic tableau. State whether this shows that the formula is valid or not.

Then use refutation, skolemization and the general resolution procedure. Again state whether this shows that the formula is valid or not.

Finally check the result using the file `qed.pl` available on CampusNet (Program files folder in the top folder) and provide a brief explanation of the result.

Exercise 4

Write a program `ins(+List1,?List2)` that succeeds if and only if `List2` is a permutation of `List1` and the elements in `List2` are ordered by the standard order. Use the Insertionsort algorithm, that is, insert in turn the elements in an initially empty list that is kept sorted (the program cannot be called `is` since this is a built-in predicate). Duplicates must not be merged. You should use a program `insert` to do the insertions, for example:

```
?- insert(b,[a,c],L).

L = [a, b, c]

Yes
```

Make sure that the following query fails: `insert(b,[a],[b,a])`.

State the result of the following query without running it first.

```
?- A = 1, C = _, ins([D,C,B,A],L), B = 2, D = 4.
```