Image Analysis

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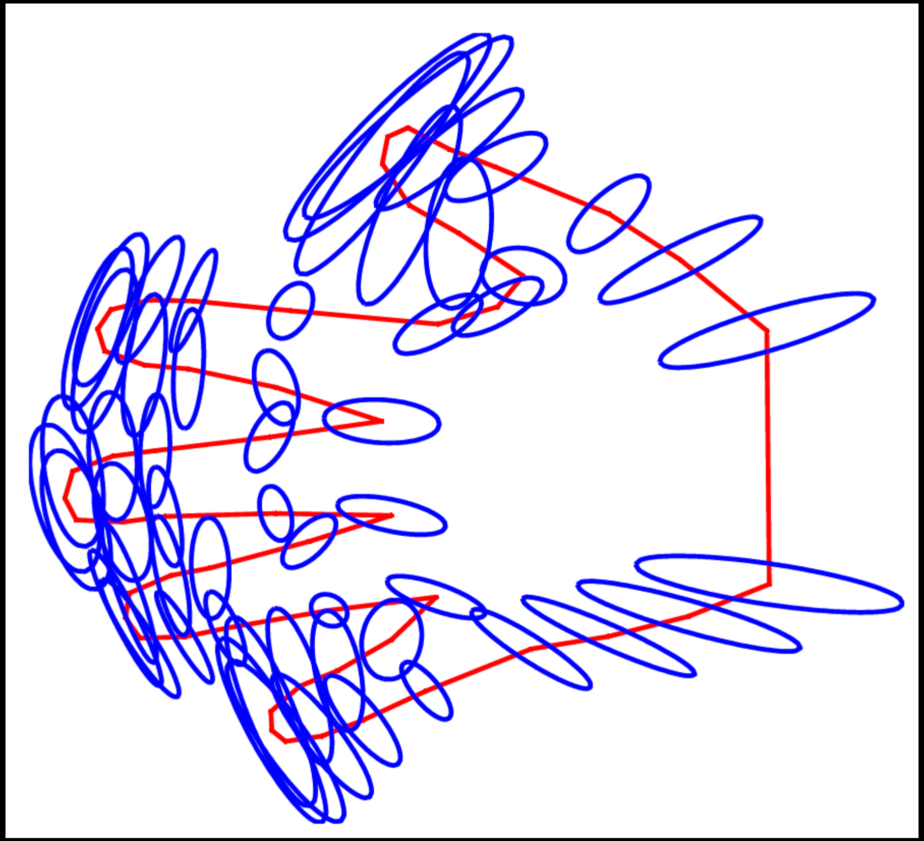
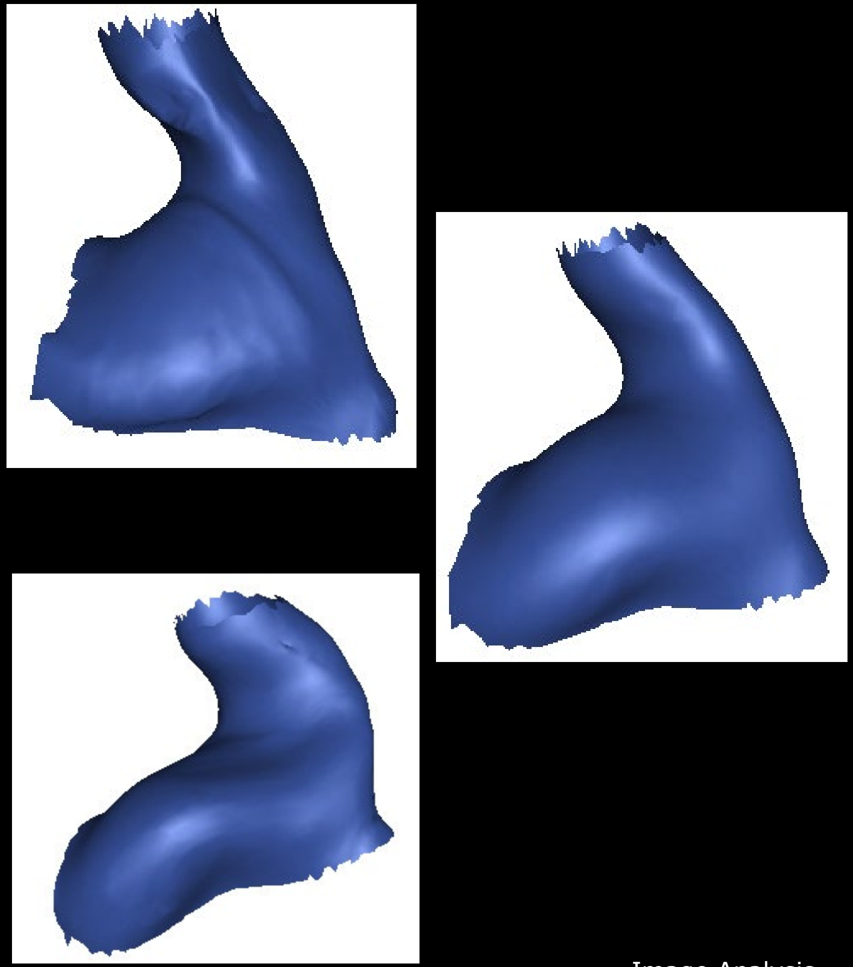
Lecture 11 – Statistical models of shape and appearance





Today's Learning Objectives

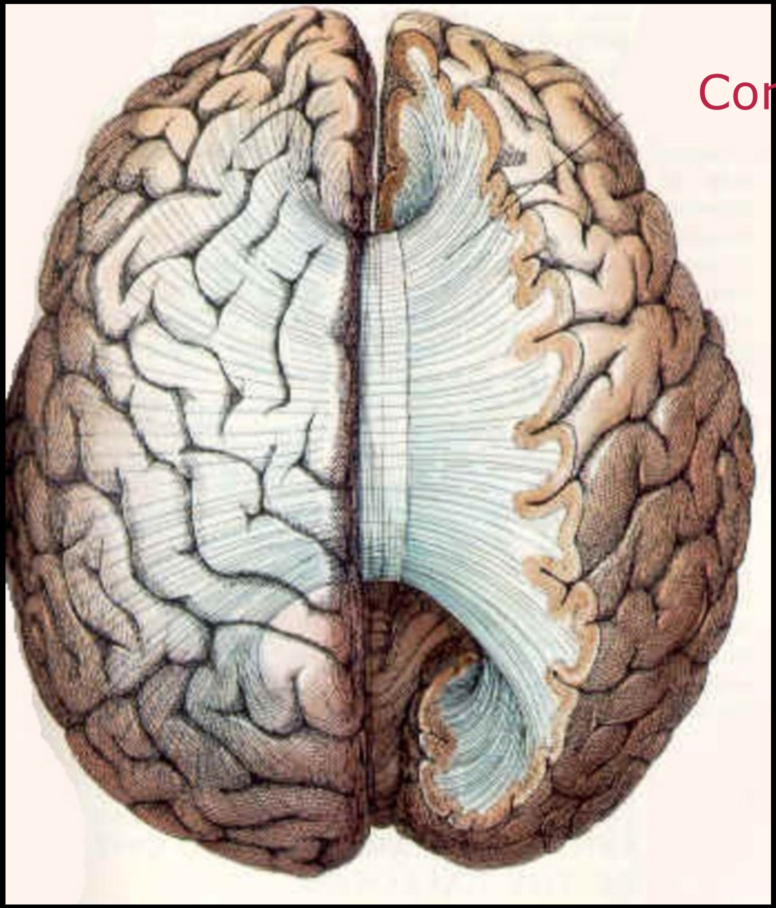
- Describe the concept of shape models
- Define the shape of an object using landmarks
- Describe point correspondence
- Describe and use the vector representation of a shape
- Describe how a shape can be seen as a point in high-dimensional space
- Explain how shapes can be aligned
- Describe how principal component analysis can be used to model shape variation
- Explain the similarity between Eigenfaces and shape and appearance models

A typical scenario

- Doctor X believes that he can "see" on a hand X-ray if the patient is in risk of arthritis!
- Specifically Doctor X is sure that the *shape of the joints* is an estimator for arthritis!

Can we verify that?

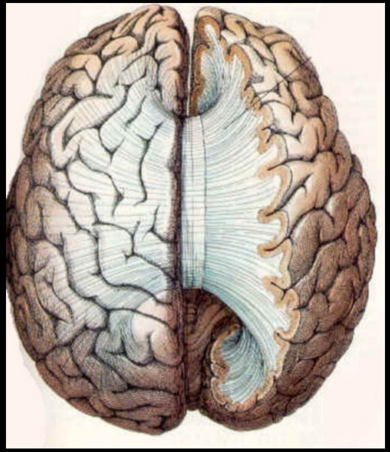
Scenario II



Corpus Callosum

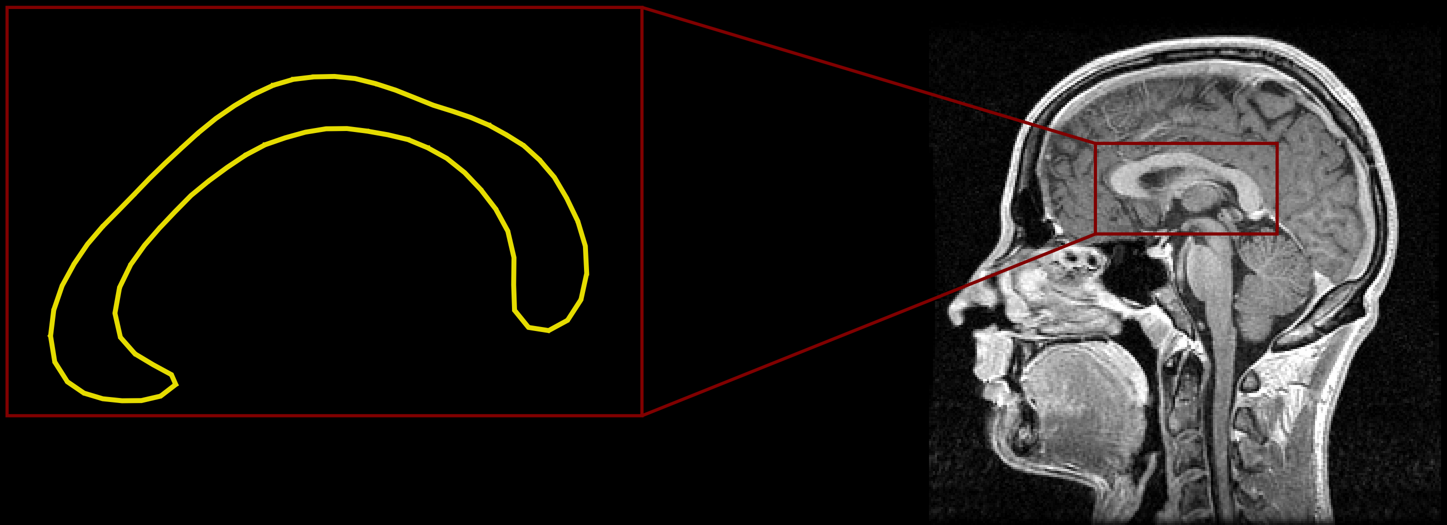
- MR images have been captured of a large group of people
- Cognitive abilities measured as well
- Is there a correlation between *how the brain looks* and how we behave?
- Does the shape of corpus callosum tell us something?

Scenario II



Corpus Callosum

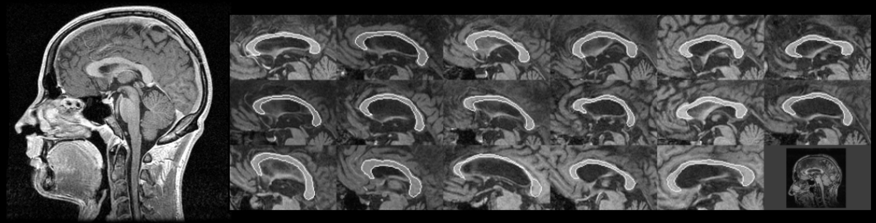
- We can get the MR slice with the corpus callosum from all the patients



Scenario III

- An experienced hearing aid fitter has seen a lot of ears!
- Some hearing aid users are very difficult to fit. Why?
- Large variation in the shape of ears
- Ear canals change shape when people chews
- Is it possible to learn about the shape and use it?

Shape Analysis

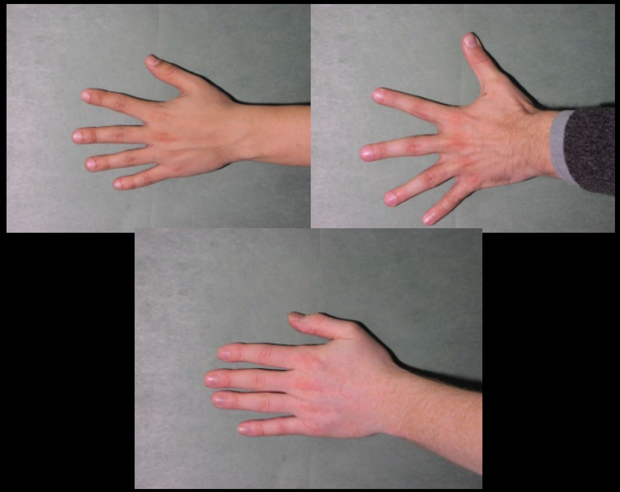


600 MR scans and behavioural data

- What can we learn from shape?
- What can we use it for?
- How do we do it?

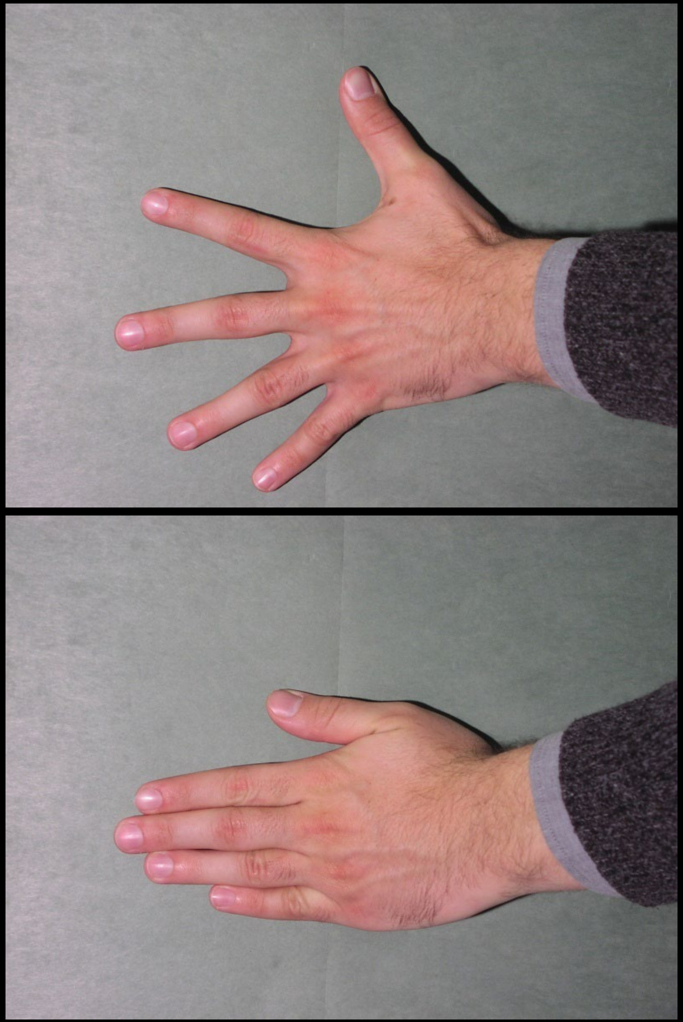
1000 historical X-rays

A boxful of something that look like ear canals



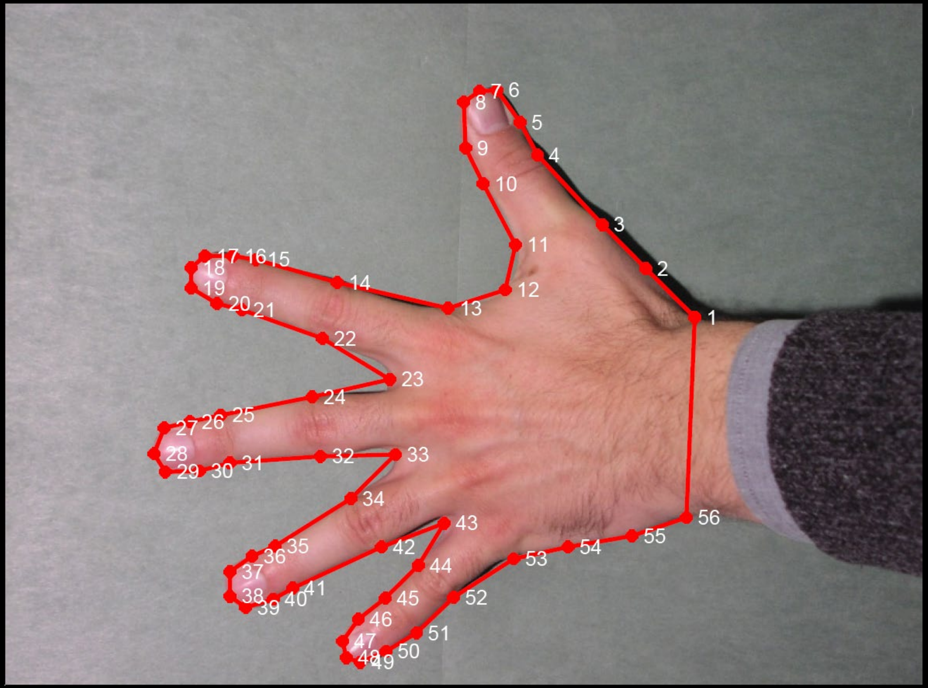
A set of hand photographs

What is shape?



- How do we define the *shape* of this hand?
- What is the shape difference between the two hands?

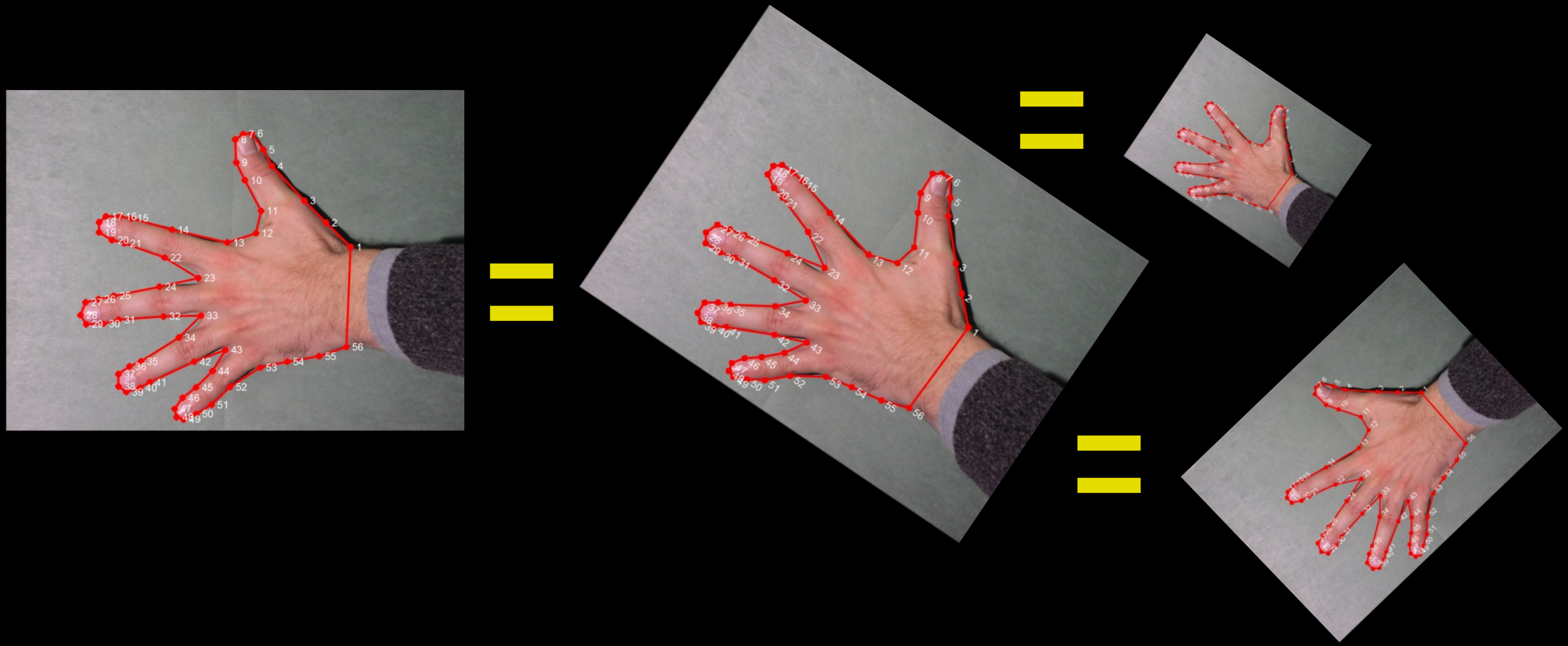
Shape definition



- Shape is defined using landmarks
  - Placed by an expert
- In this case the outer contour of the hand
- Just one of many ways!

Shape definition

Shape is all geometrical information that remains when location, scale, and rotational effects are removed



Shape definition

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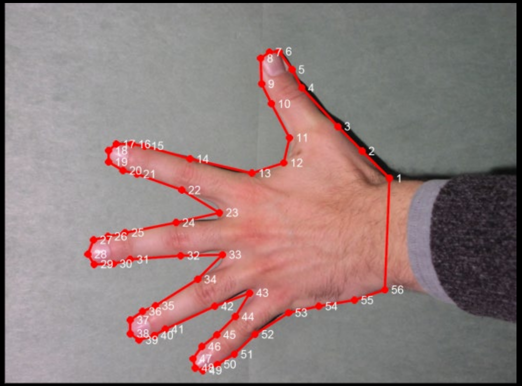
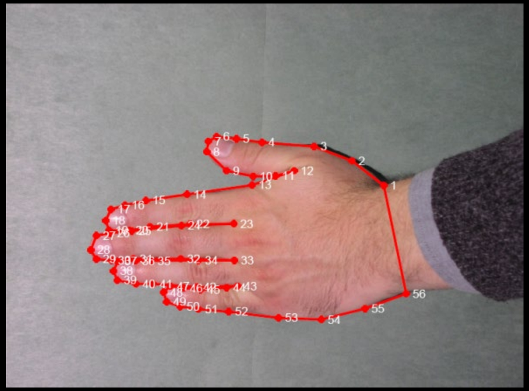
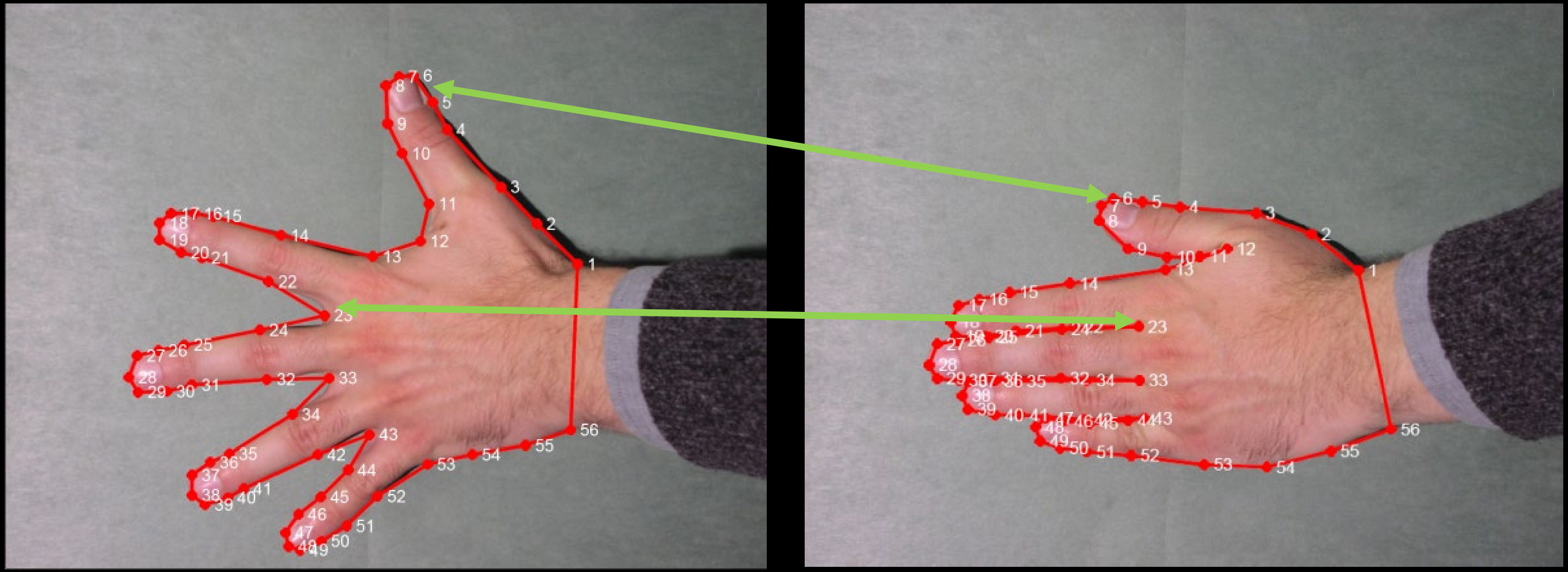


$\neq$



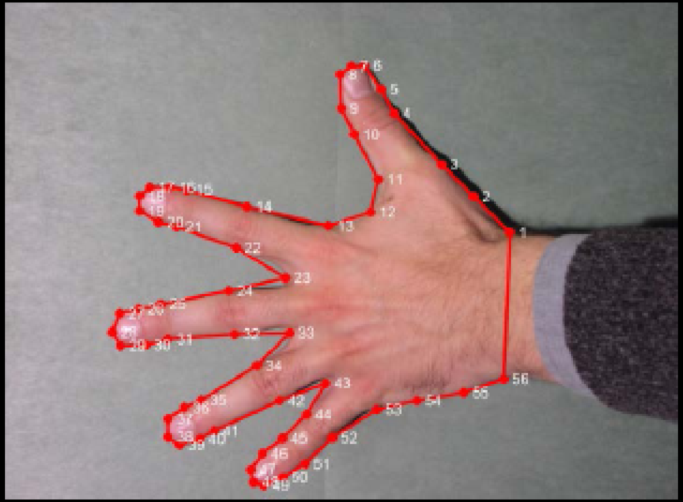
Landmarks and point correspondence

Landmarks are placed on the same place on all shapes in the training set



Shape as a vector

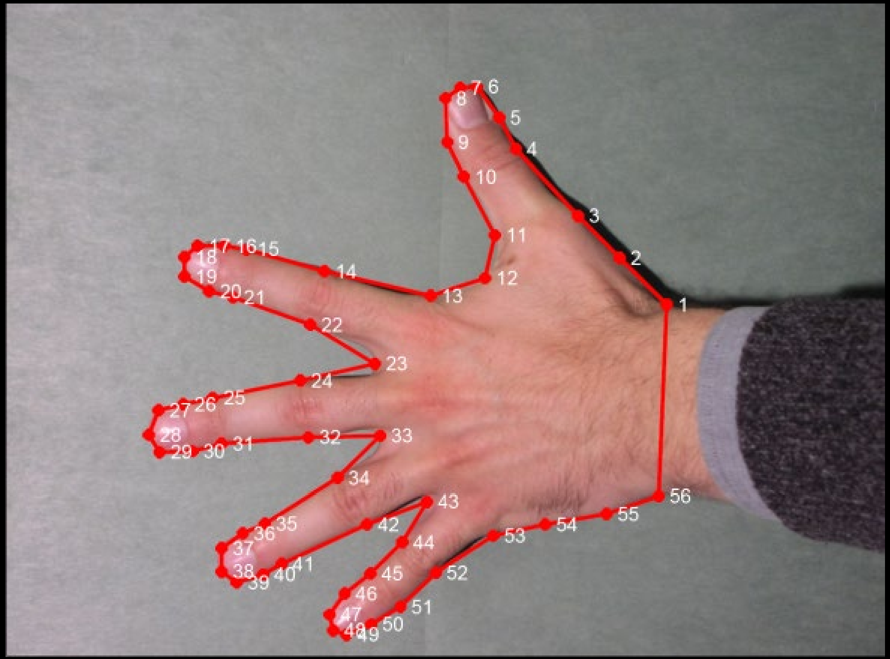
$$1 : (x_1, y_1)$$
$$2 : (x_2, y_2)$$
$$\vdots$$
$$N : (x_n, y_n)$$



- The shape is represented as an array of (x,y) coordinates
- Trick number one!
  All coordinates are put into one vector!
- n=56 points
  - Vector with 112 elements!

$$\mathbf{x} = [x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n]^T$$

Shapes in high-dimensional space

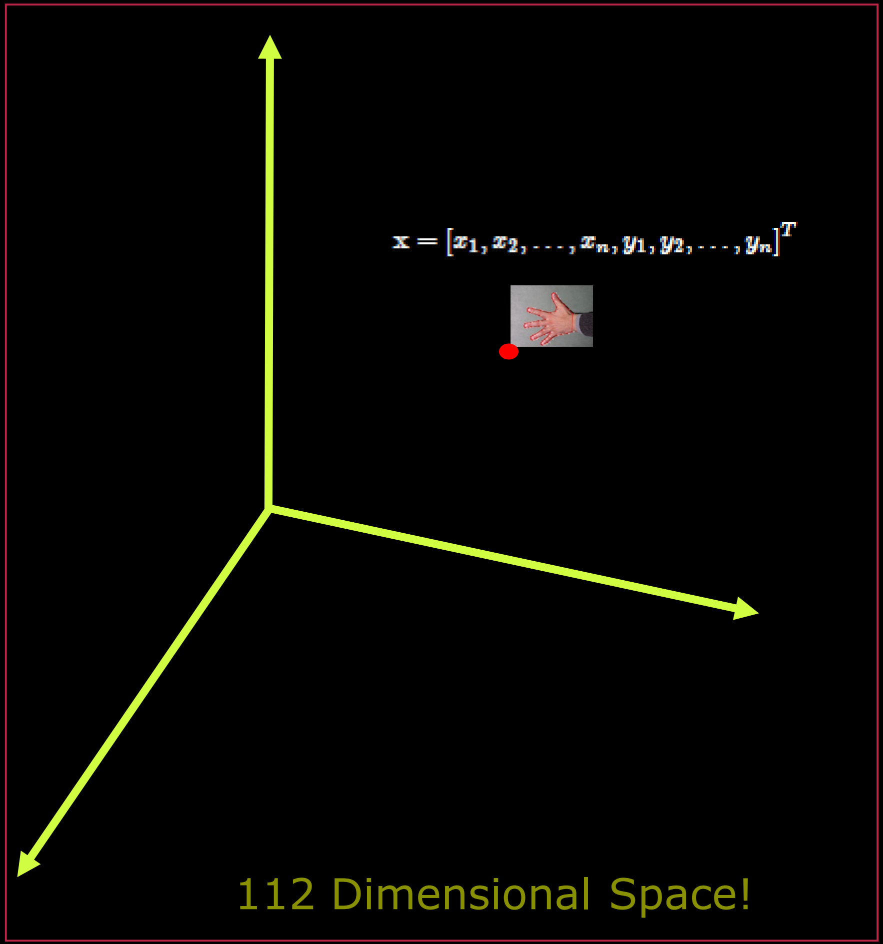


$$\mathbf{x} = [x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n]^T$$

- One hand is now described using one vector
- A vector can also be seen as a point in space!

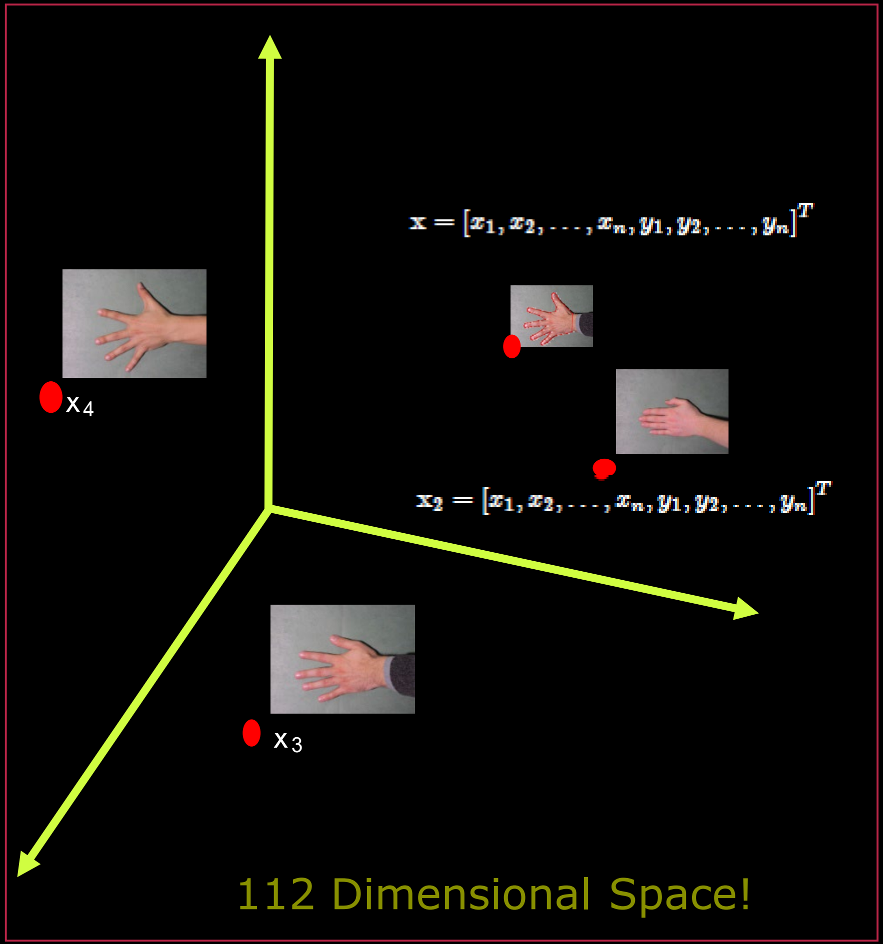
Trick number two!

Coordinates in space



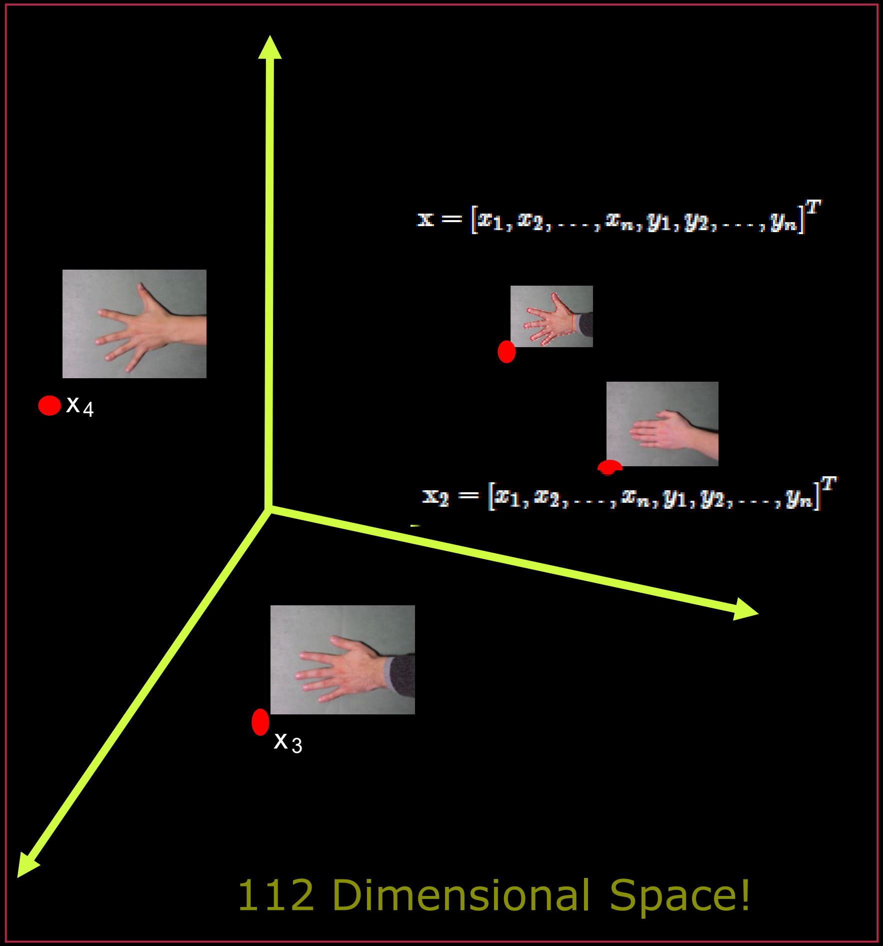
- On hand is now described using one vector
- A vector can also be seen as a coordinate in space!
- Not 2D space, not 3D space, not 4D space…
- 112 Dimensional Space!
- A hand has a position in this space!

Hands in Space



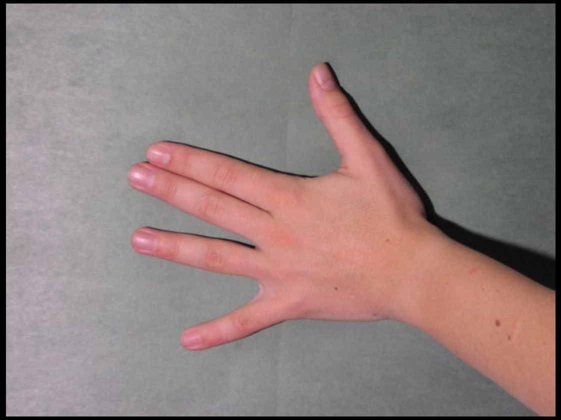
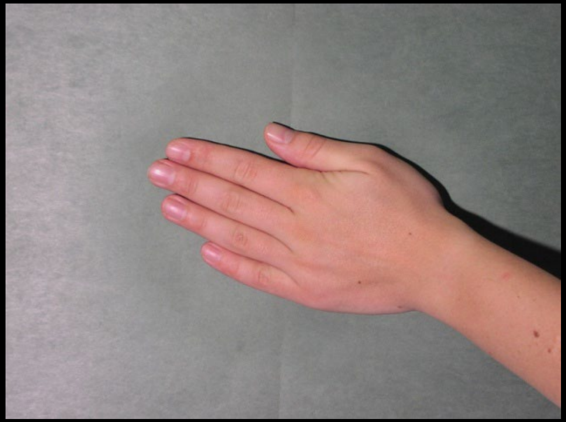
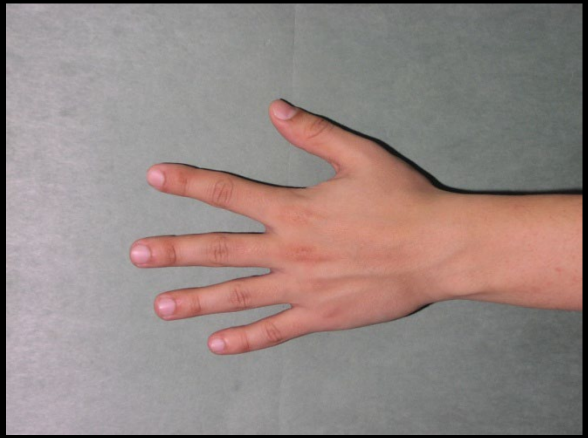
- A hand has a position in space!
- Another hand appears
  - in the same space
  - different position = different shape
- All hands have a place in this space!

Shape Analysis



- Shape analysis
  - Similar shapes are placed on “planes” in the shape-space
  - Also called a manifold

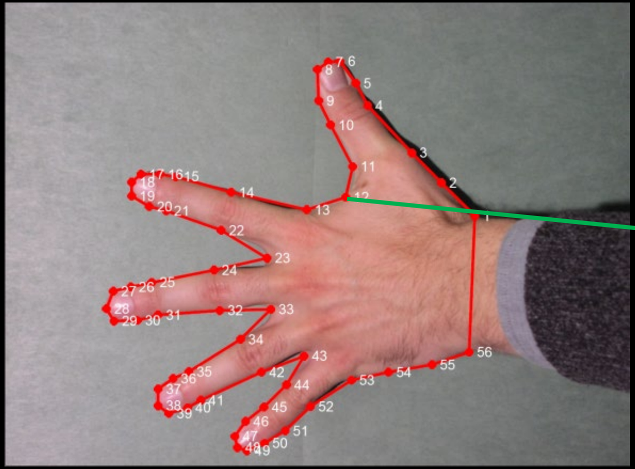
Shape alignment

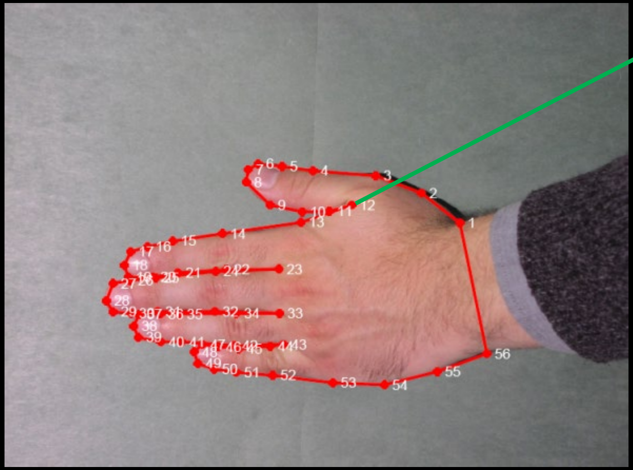
  

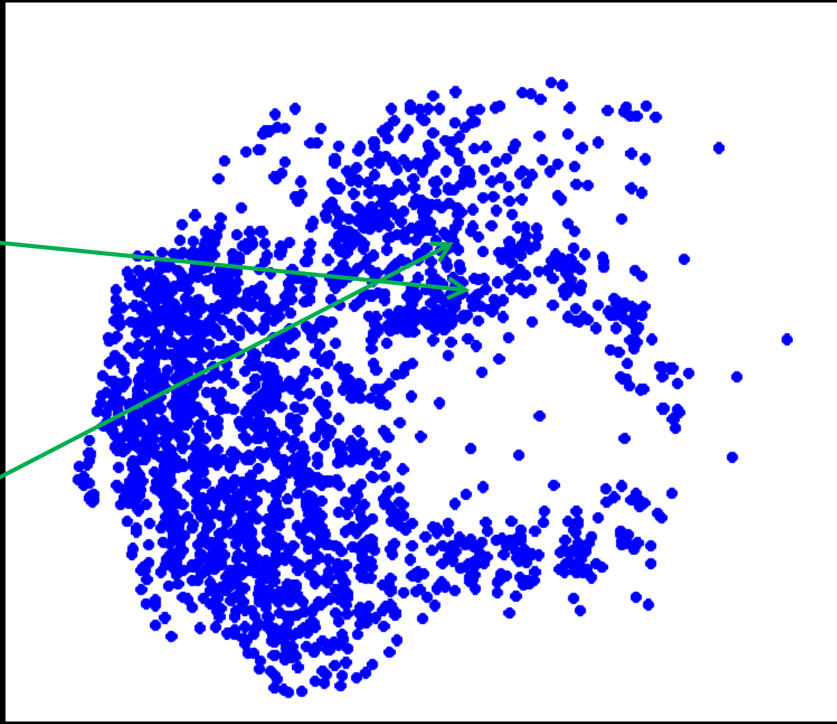
- 40 training images of hands
- 56 landmarks on each
- Placed in random location (translation+rotation)

Shape alignment

Before alignment



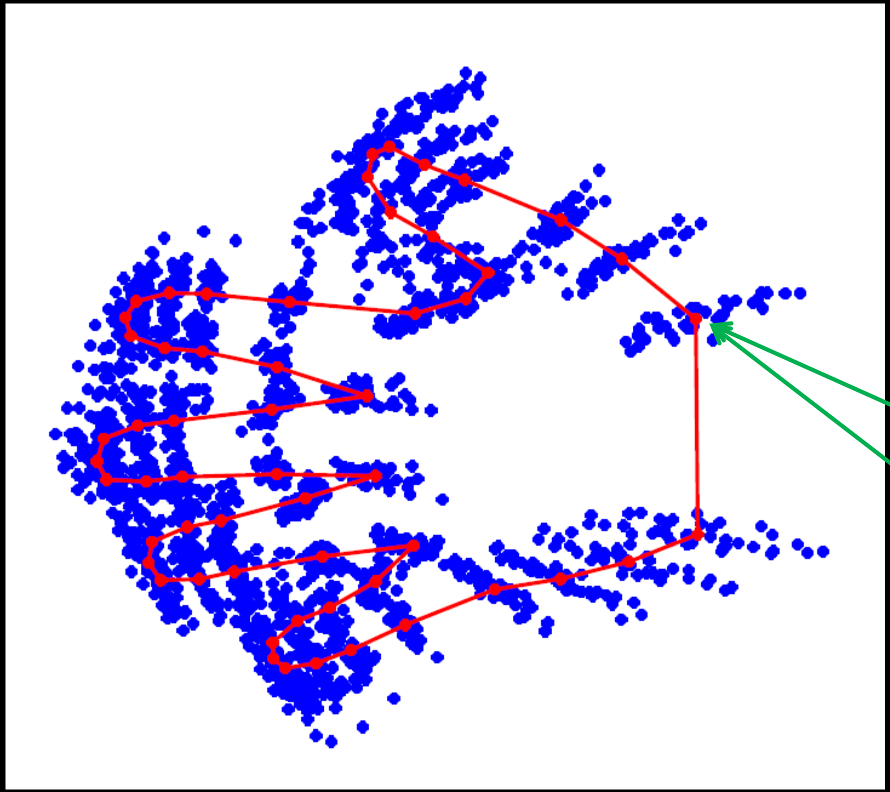




Landmarks from all hands

Needs alignment!

What is alignment?



Average shape

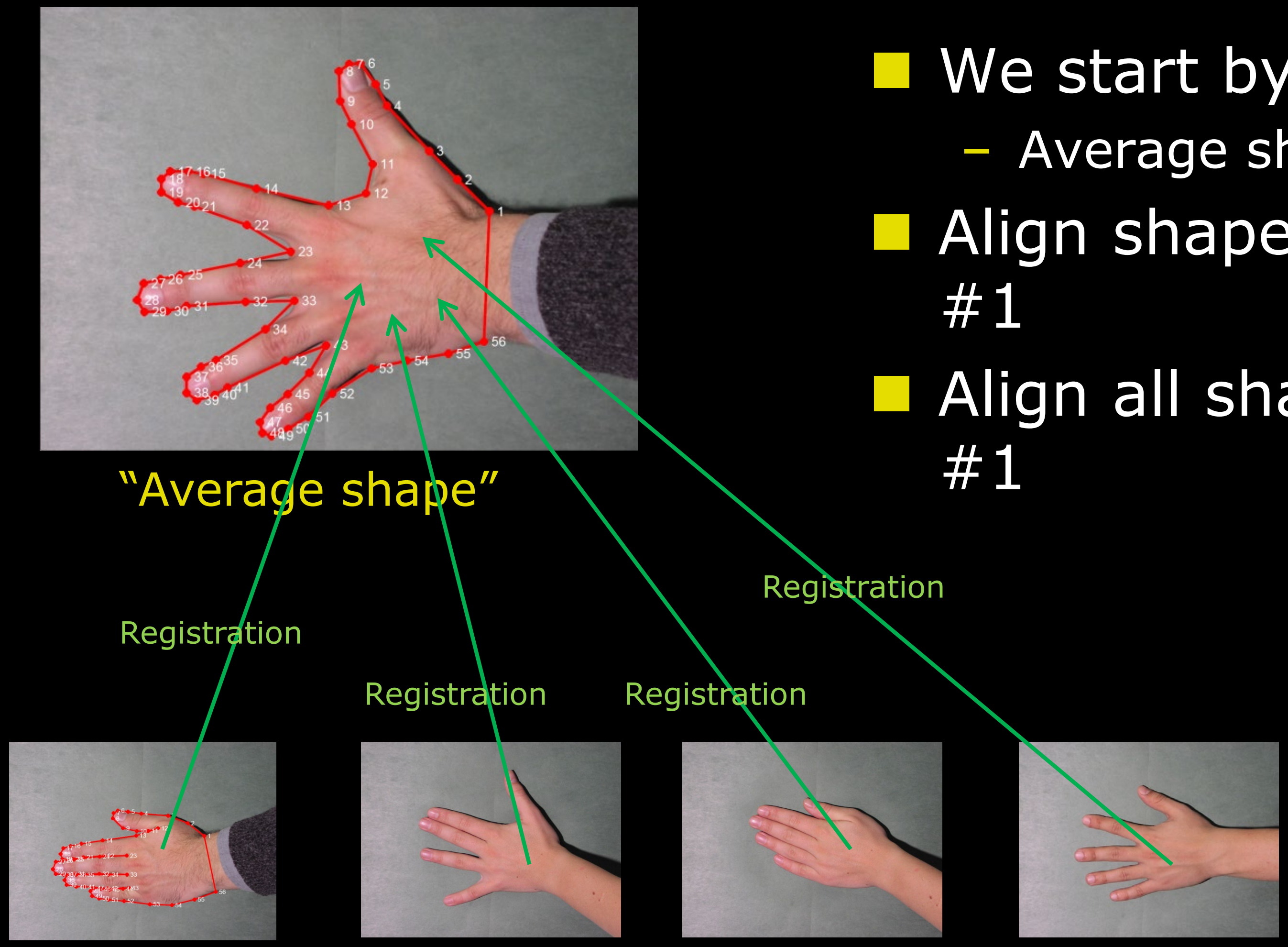
- Group wise registration
  - Not one-to-one
  - All to the average shape

$$\bar{\mathbf{x}} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{x}_i$$

$$\bar{\mathbf{x}} = [\bar{x_1}, \bar{x_2}, \ldots, \bar{x_n}, \bar{y_1}, \bar{y_2}, \ldots, \bar{y_n}]^T$$

But hey! We do not have an average shape?

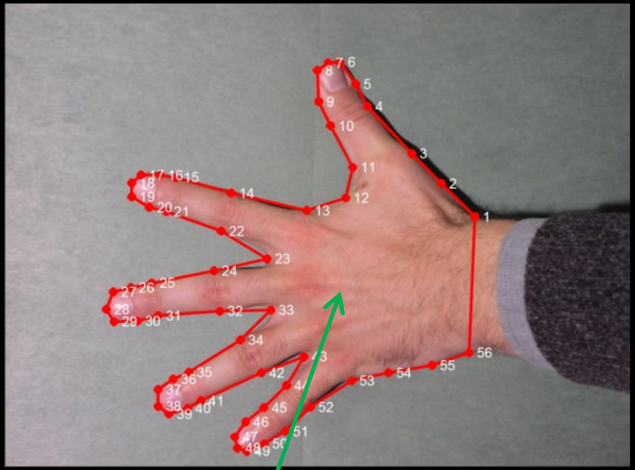
Procrustes Analysis (alignment)



Shape #2

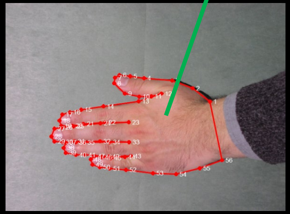
- We start by defining
  - Average shape = Shape #1
- Align shape #2 to shape #1
- Align all shapes to shape #1

Landmark based registration



“Average shape”

Registration

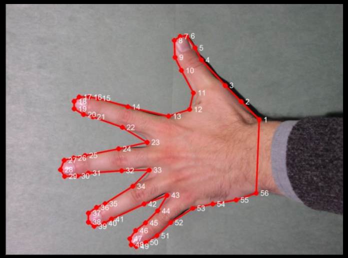


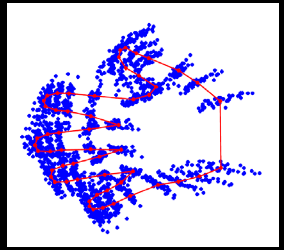
Shape #2

- Shape #2 is transformed to fit the average shape
  - Translation
  - Rotation
  - Scaling
  - = Similarity Transform
- Result
  - Shape #2 is placed *on top of* the average shape

Procrustes Analysis

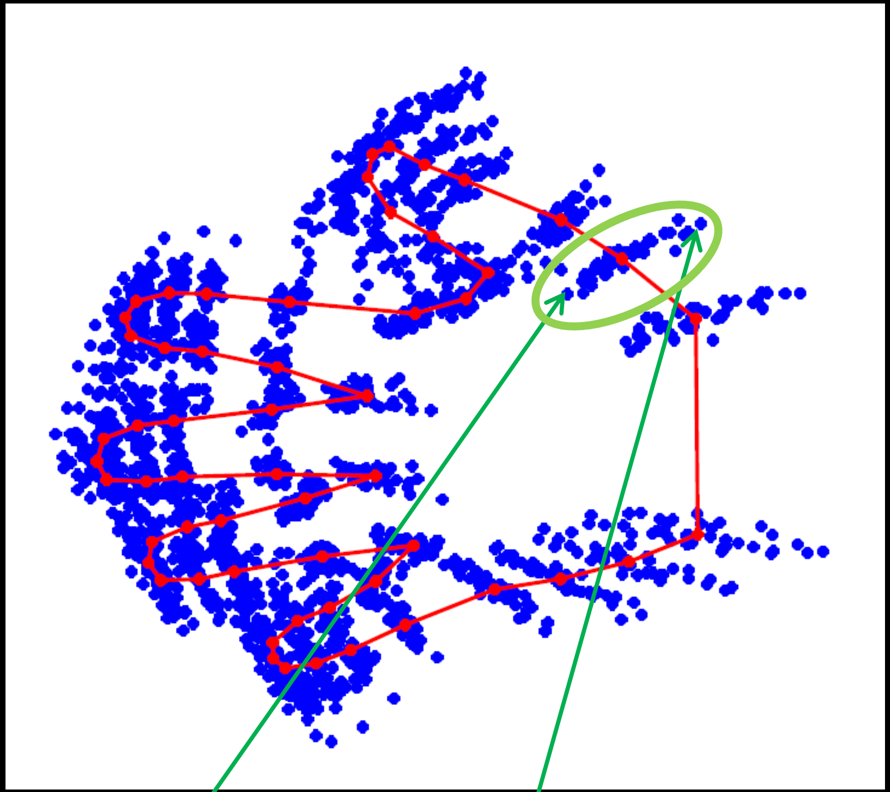
1. Average shape is set to shape #1
2. Register all shapes to the average shape
   - Landmark based registration
3. Recompute the average shape
4. If average shape changed return to step 2.





$$\bar{\mathbf{x}} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{x}_i$$

Aligned shapes – what now

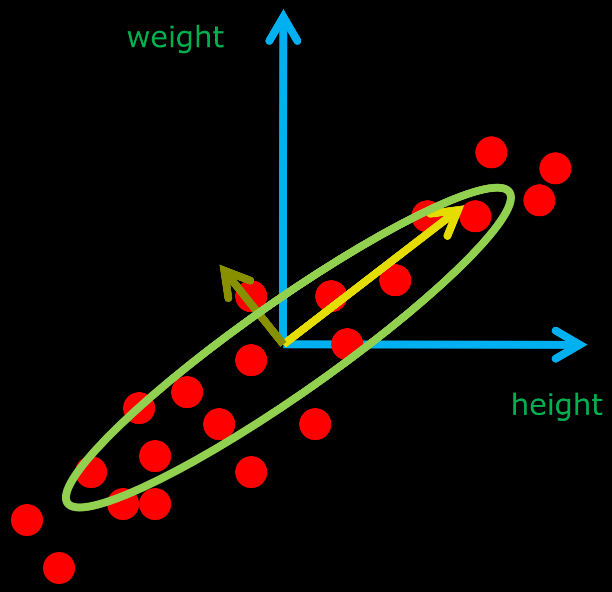


Shape #16

Shape #27

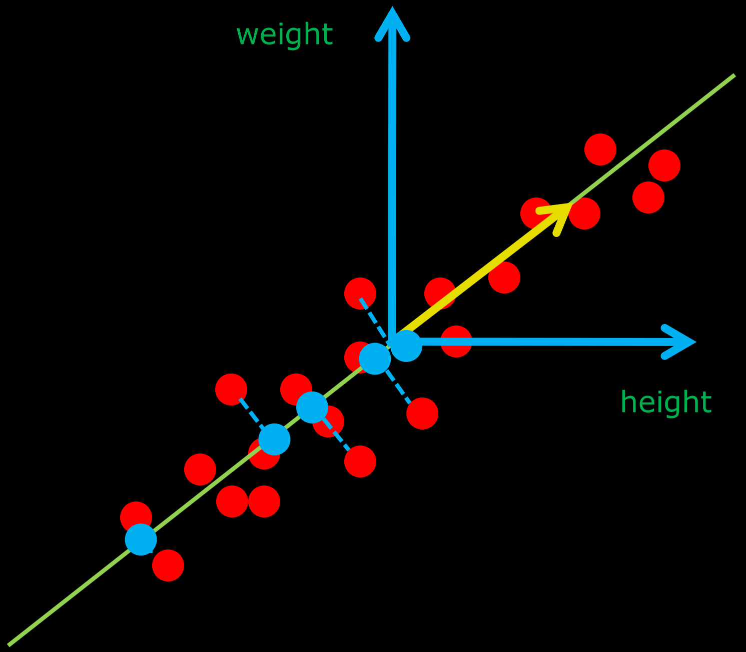
- Individual landmark variation
  - Over the training set
- What shape is the variation?

Principal Component Analysis (PCA)



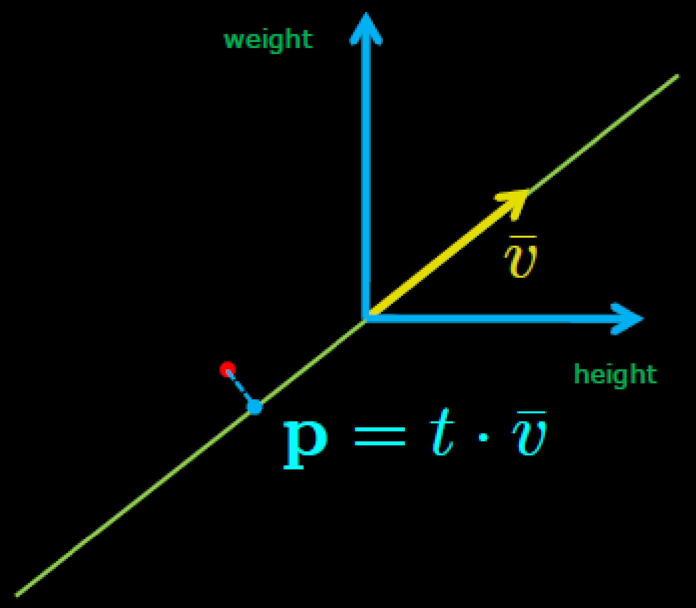
- PCA
  - Main axis in data
  - Eigenvectors
  - Eigenvalues
- Size of Eigenvalues describe explained variance

Principal Component Analysis (PCA)



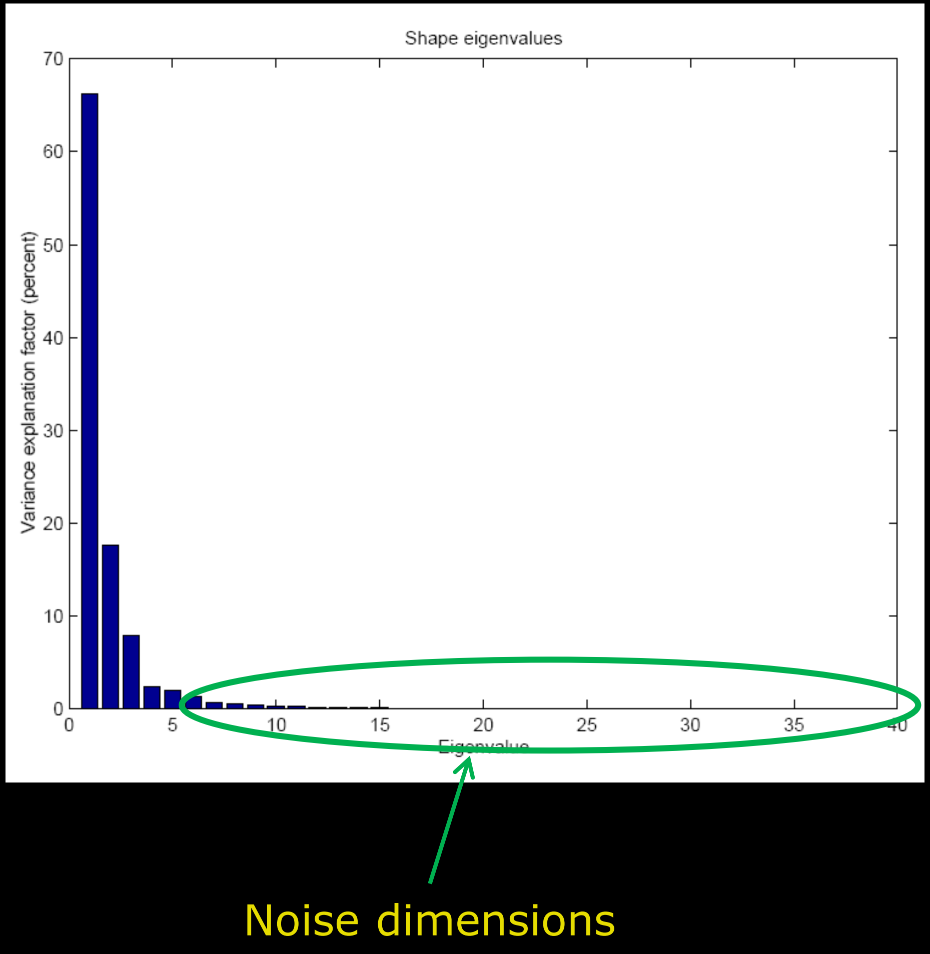
- We throw away the *noise dimensions*
- Points projected to the line

Principal Component Analysis (PCA)



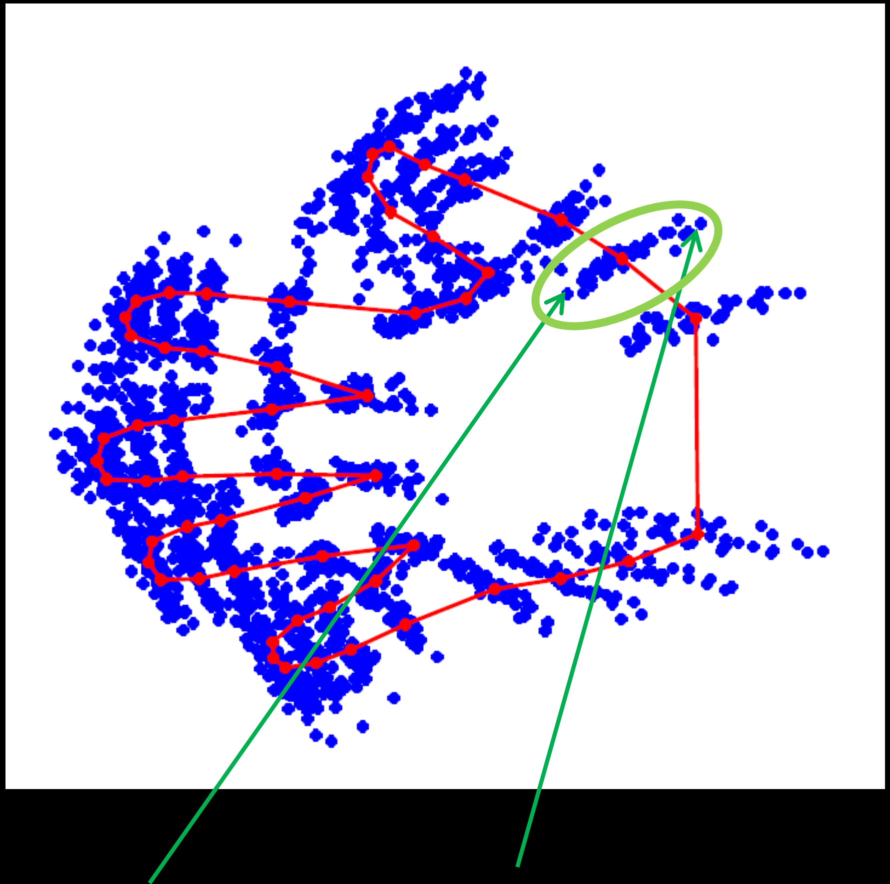
- We throw away the *noise dimensions*
- Points projected to the line
- A point can now be described by one parameter *t*
- We have reduced the number of dimensions

How many dimensions should we keep?



- Plot the Eigenvalues
- Explains how *important* each dimension is
- Cut away noise dimensions

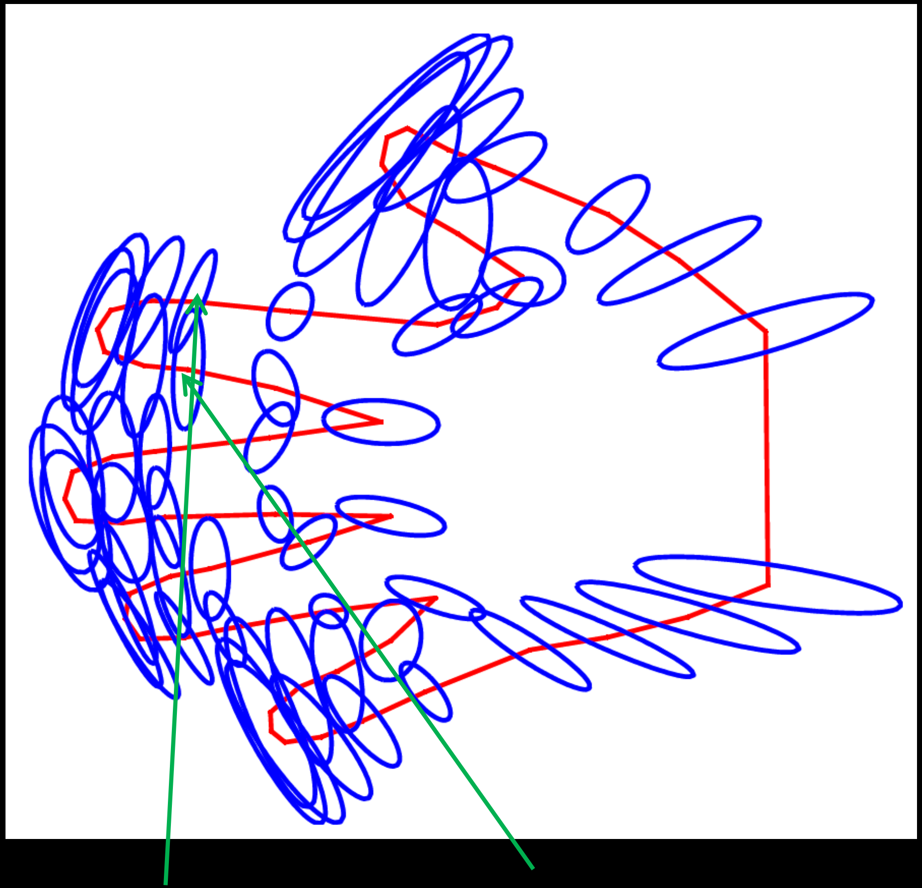
Aligned shapes – what now



Shape #16 Shape #27

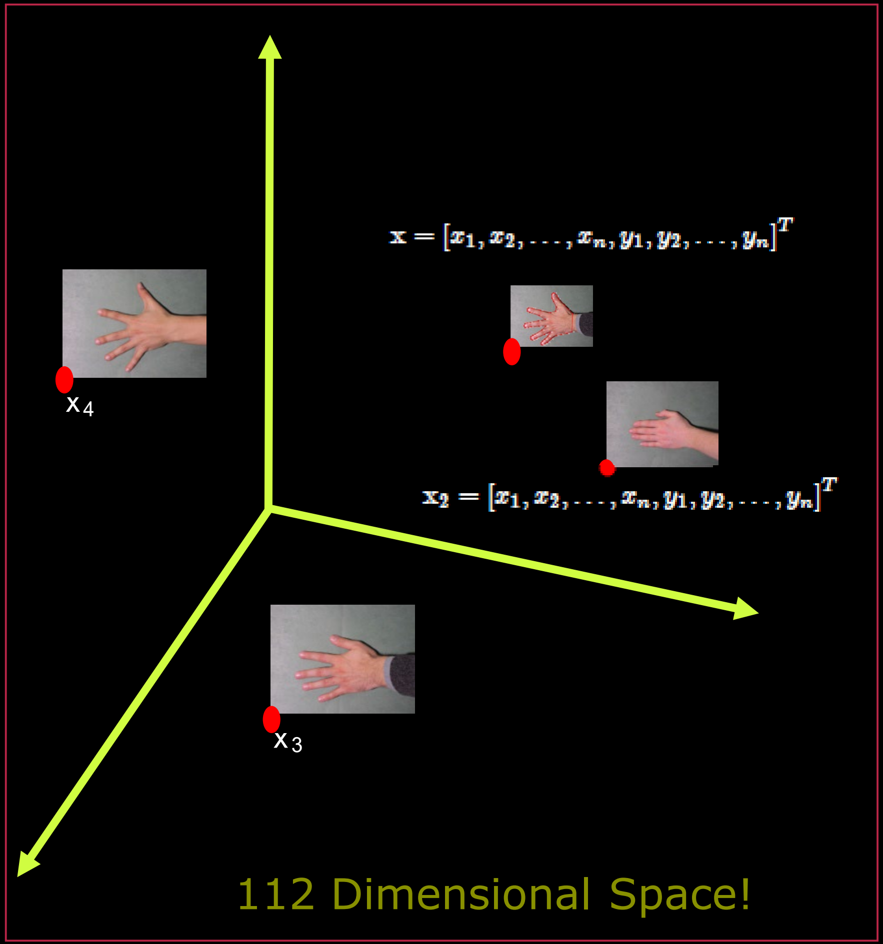
- Individual landmark variation
  - Over the training set
- What shape is the variation?

PCA Analysis



- PCA analysis on individual landmarks
- Describes the major direction of variation
- Landmarks are correlated!
- The movement over the shape is connected
- Return to shape space

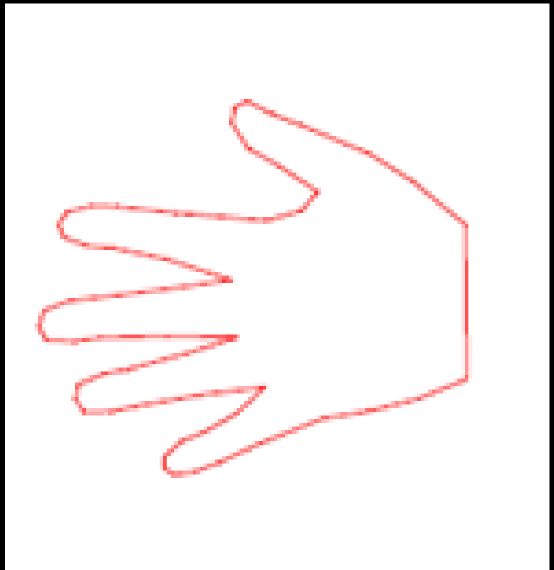
PCA in shape space



- Instead of doing PCA on 2D points we do it on 112D points
- Examine if our 40 *shapes is lying on a plane* in 112D space
- We find the directions that spans the maximum variation in shape space

Start by computing the shape average

$$\bar{\mathbf{x}} = \frac{1}{s}\sum_{i=1}^{s}\mathbf{x}_i$$

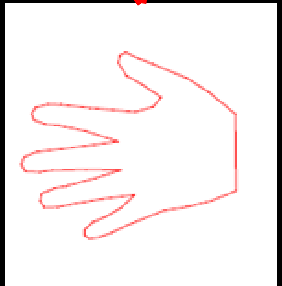


Since we do this on the aligned shapes –
this is the Procrustes average

Do the eigenvector analysis

$$\mathbf{S} = \frac{1}{s-1}\sum_{i=1}^{s}(\mathbf{x}_i - \bar{\mathbf{x}})(\mathbf{x}_i - \bar{\mathbf{x}})^T$$

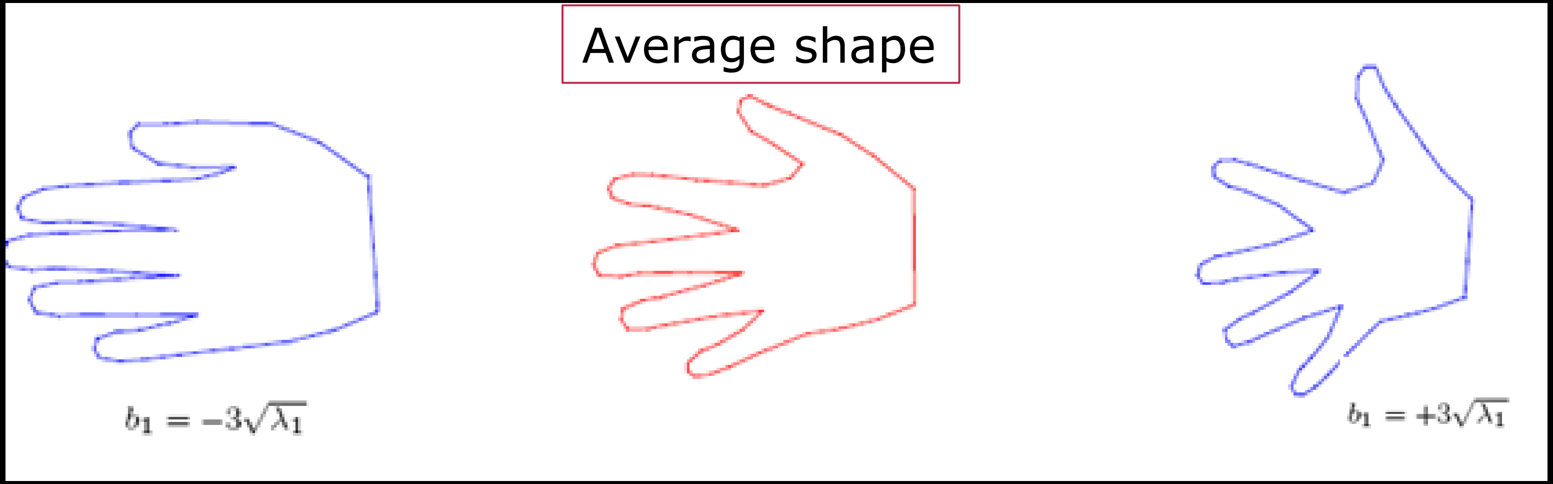
- Computing the covariance of the shape data



Average shape

Shape number i in the training set

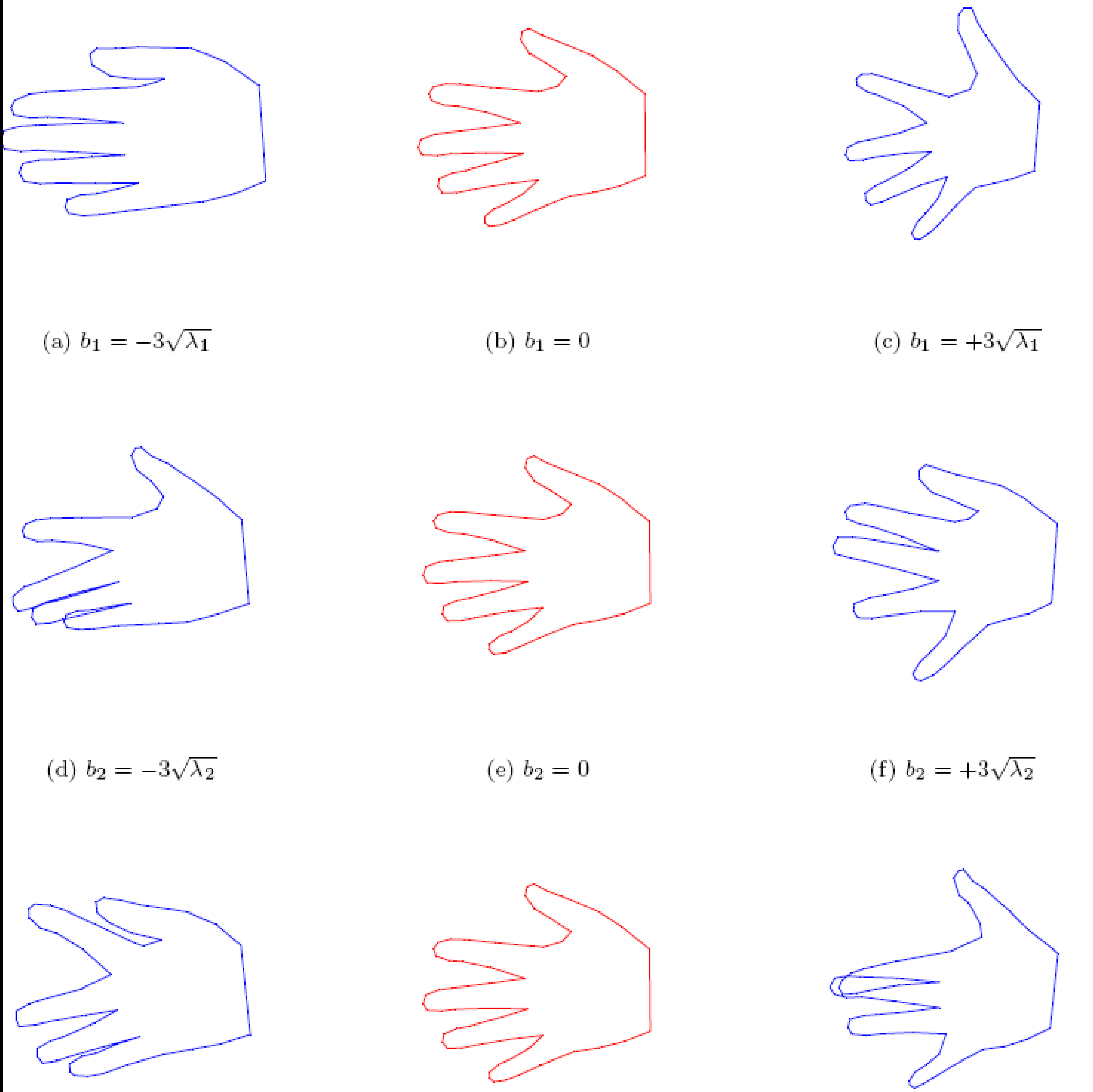
Visualizing variation



Visualizing the first principal component

$$\mathbf{x} \approx \bar{\mathbf{x}} + \Phi \mathbf{b}$$

Φ contains the t eigenvectors



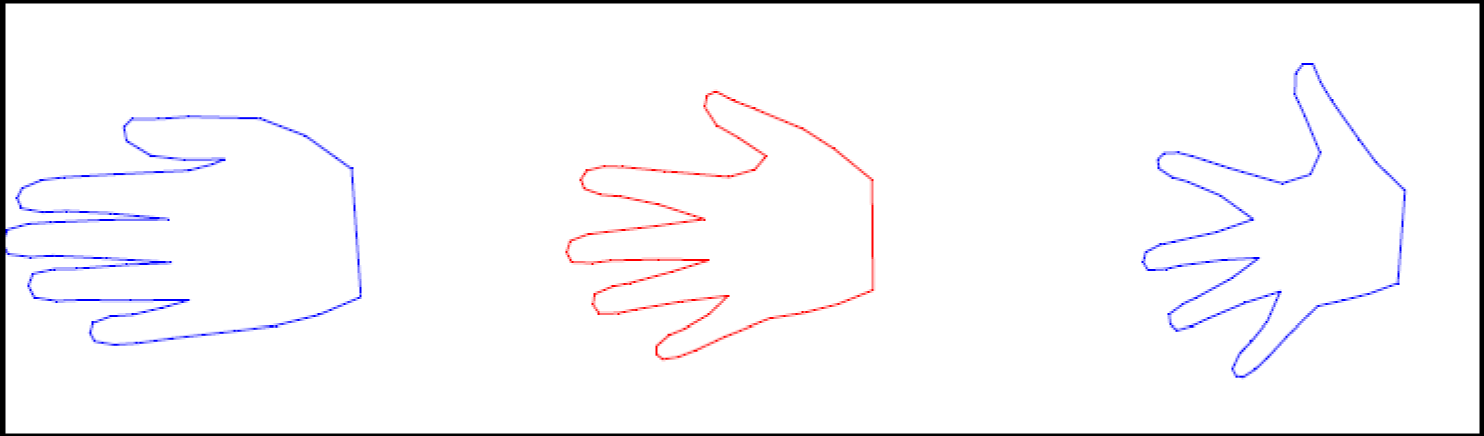
(a) $b_1 = -3\sqrt{\lambda_1}$ (b) $b_1 = 0$ (c) $b_1 = +3\sqrt{\lambda_1}$

(d) $b_2 = -3\sqrt{\lambda_2}$ (e) $b_2 = 0$ (f) $b_2 = +3\sqrt{\lambda_2}$

(g) $b_3 = -3\sqrt{\lambda_3}$ (h) $b_3 = 0$ (i) $b_3 = +3\sqrt{\lambda_3}$

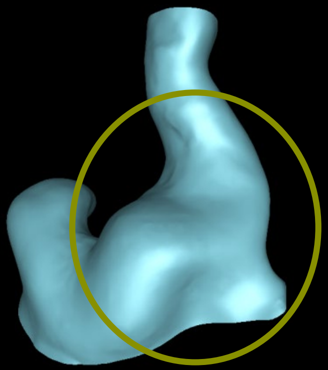
Results of Shape Analysis

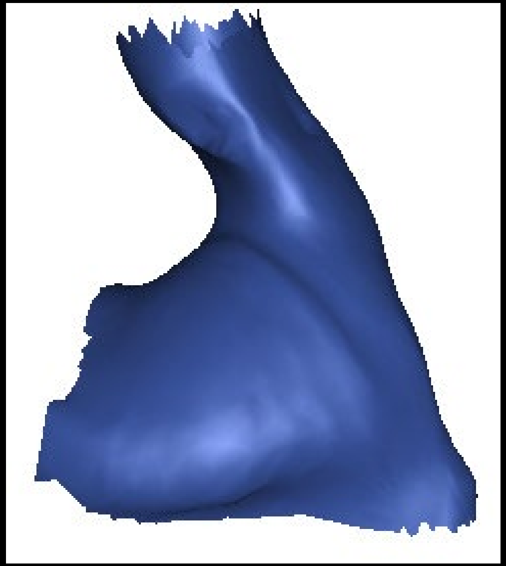
- Visualisation of the major variation of the shape over a population



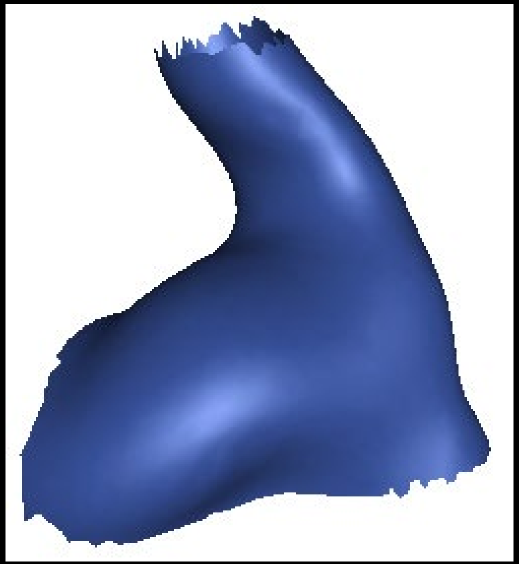
Hearing Aid Design

- Main variation of the shape of the ear canal
- Found using principal component analysis
- First mode of variation
- 7 modes explain 95% of the total variation

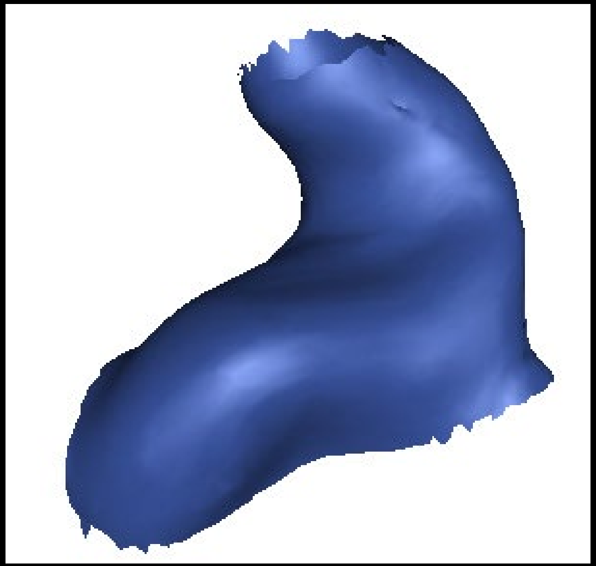




Average-1. mode



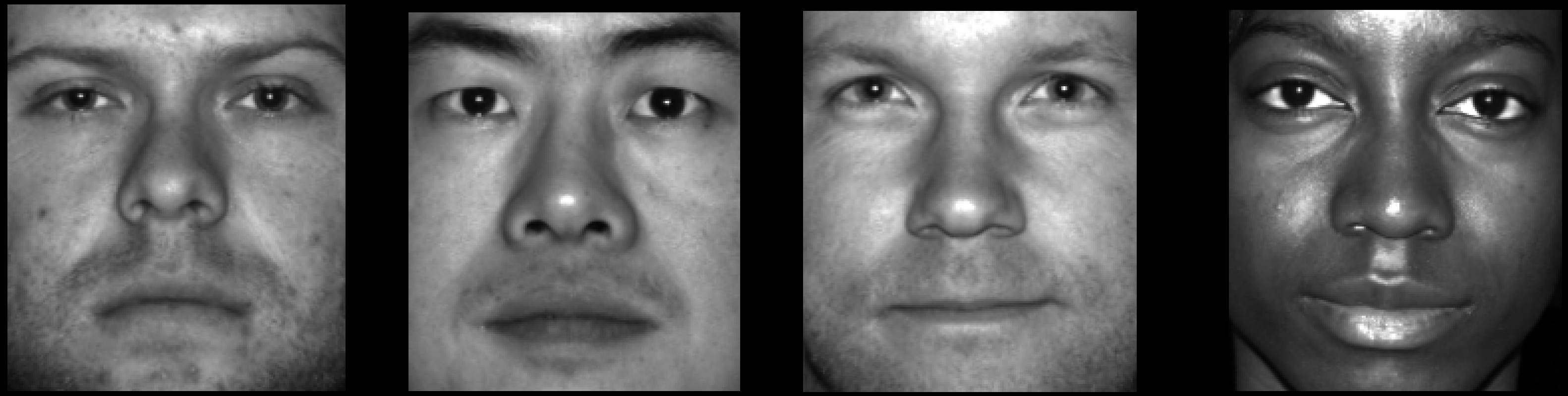
Average



Average+1. mode

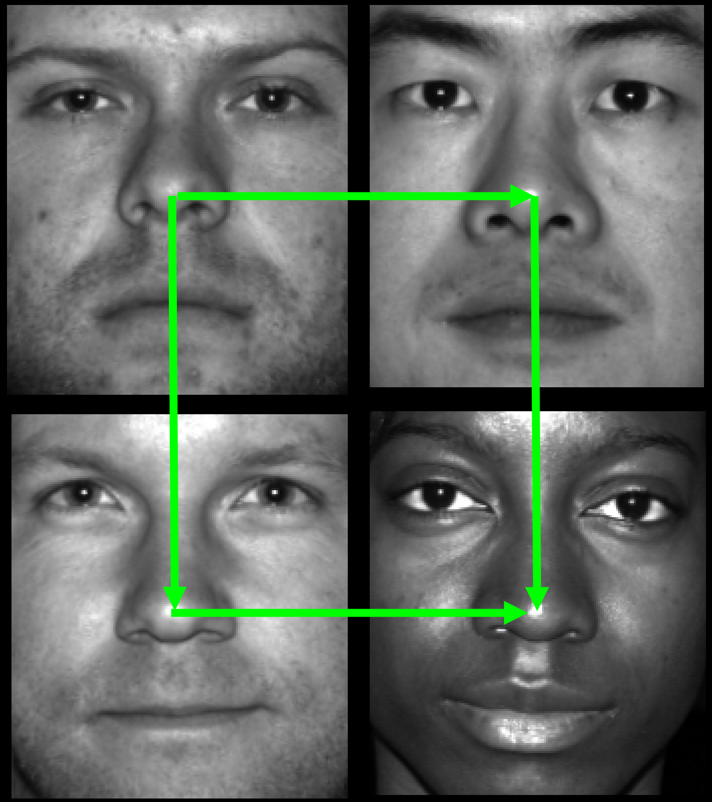
Modelling shape and appearance

- A model that can both model the shape of an object and the appearance (the texture)
- Texture: The pattern of intensities (or colors) across an image patch



Back to lecture 3: Eigenfaces

Face data

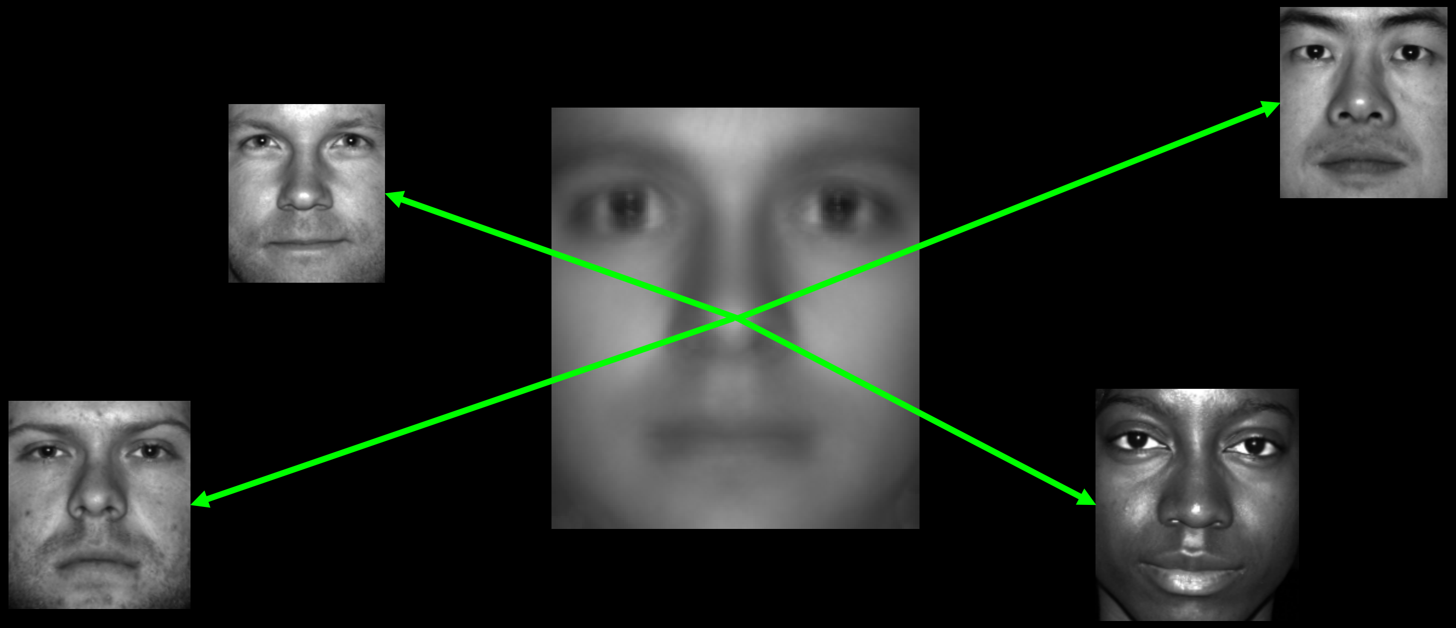


- 38 face images
  - 168 x 192 grayscale
- Aligned
  - The anatomy is placed ”in the same position in all image”
- Same illumination conditions on the images we use

The Extended Yale Face Database B

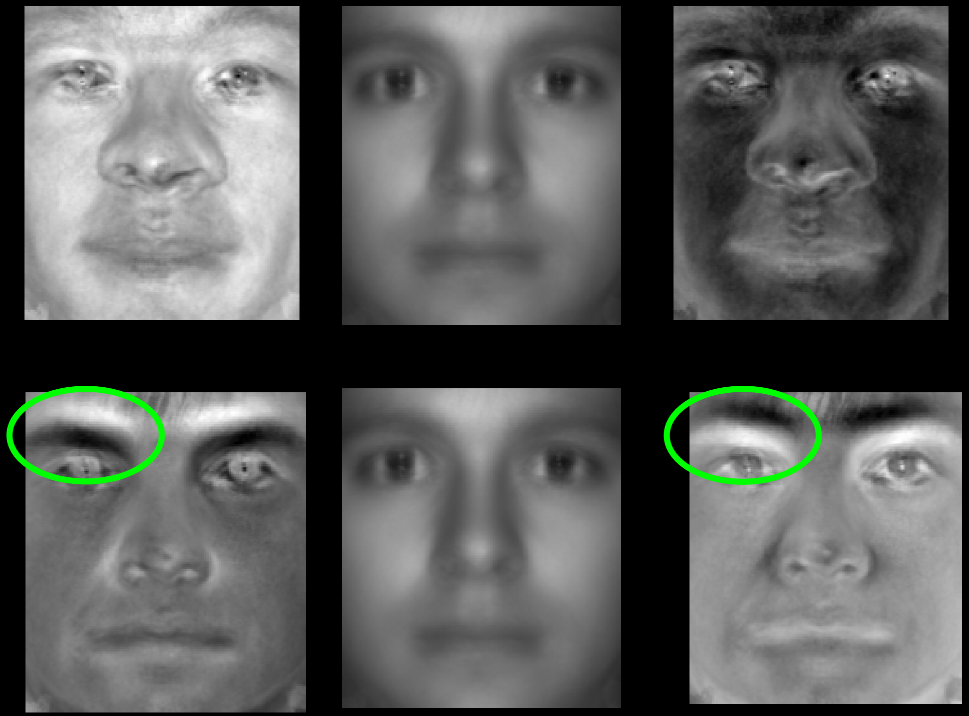
Analyzing the deviation from the mean face

- We want to do the principal component analysis on the *deviations from the average face*



Visualizing the PCA faces

Main deviations from the average face



First PC – 40% of variation

Second PC – 8% of variation

-PC

Average face

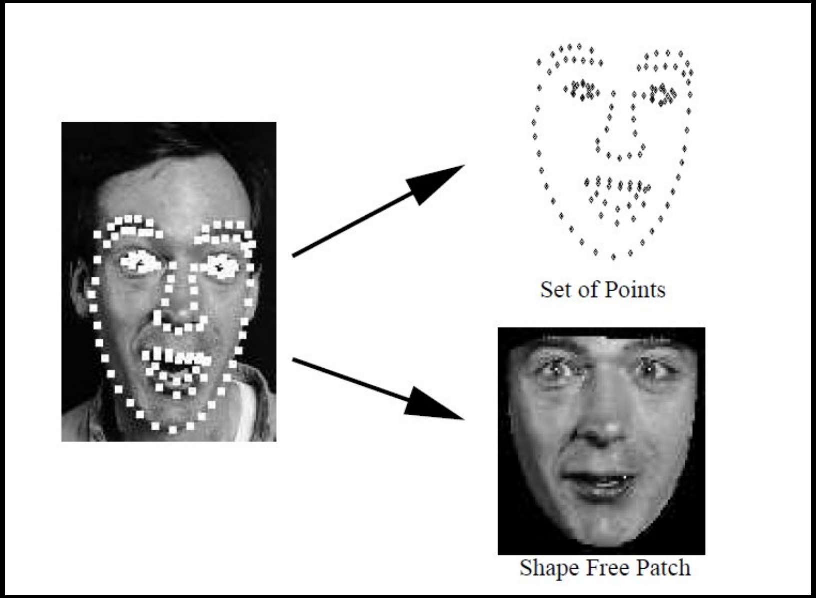
+PC

A tool to see major variations – brow lifting

Eigenfaces: Modelling texture

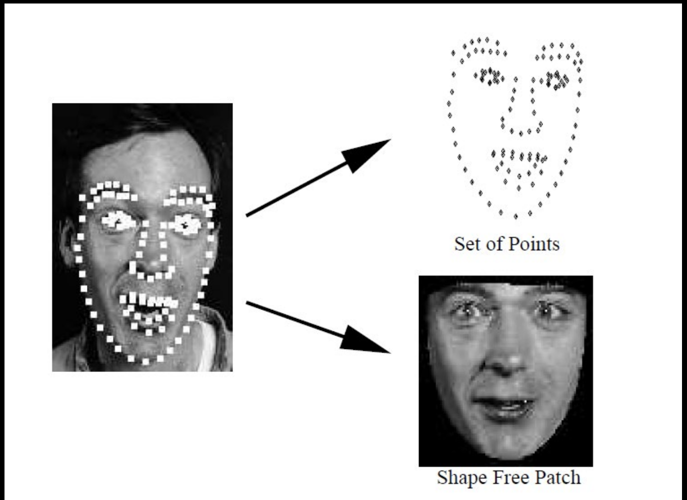
- The modelling of the pure appearance
- Without removing variation in shape
- No *decoupling* of shape and appearance

Decoupling shape and texture



- Warp each face to average shape using the landmarks
- Non-linear geometrical transformation
- Sample the texture from the warped face

Eigenfaces on warped faces



- Same PCA modelling as in the Eigenfaces approach
- Just slightly different notation

$$\mathbf{g} = \bar{\mathbf{g}} + \mathbf{P}_g \mathbf{b}_g$$

Combined shape and appearance model

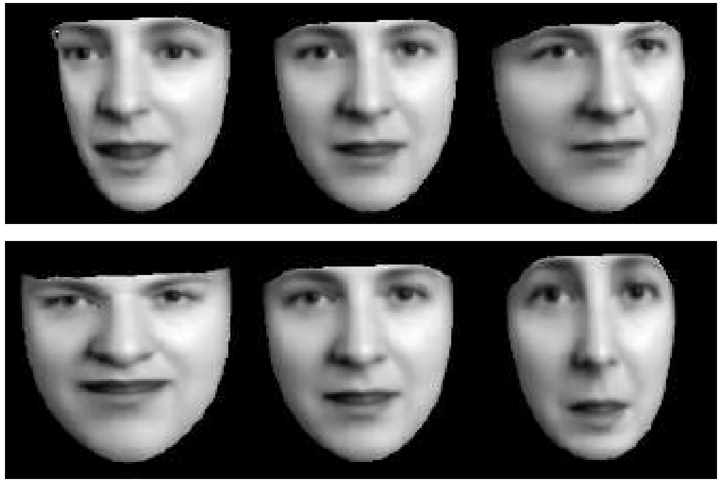


Figure 5.2: First two modes of shape variation (± 3 sd)

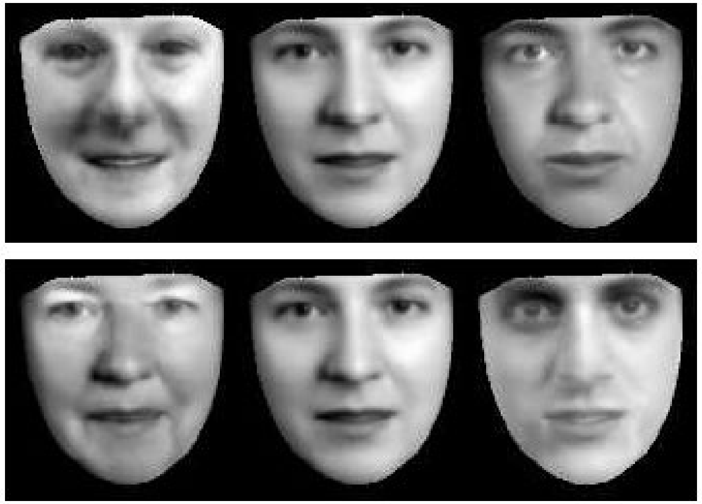


Figure 5.3: First two modes of grey-level variation (± 3 sd)

Facial Analysis

- Demo of AAM explorer