

Modelamiento y Optimización

Clase 18

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Branch and Bound



Branch and Bound

Idea principal: consideremos un problema entero

$$\begin{aligned} \min \quad & c^T x \\ \text{s.a.} \quad & Ax \leq b \\ & x \in \mathbb{Z}^n \end{aligned}$$

y supongamos que la solución de la **relajación lineal** \hat{x} cumple, por ejemplo, $\hat{x}_1 = 1.3$. Podemos crear los siguientes problemas auxiliares:

$$\begin{aligned} \min \quad & c^T x \\ \text{s.a.} \quad & Ax \leq b \\ & x_1 \leq 1 \end{aligned}$$

$$\begin{aligned} \min \quad & c^T x \\ \text{s.a.} \quad & Ax \leq b \\ & x_1 \geq 2 \end{aligned}$$

Acá no hemos perdido soluciones enteras, pero si descartamos a \hat{x} . Esto es una **ramificación o branching**.

¿Qué cota obtenemos acá?

Lo peor de las ramas nos da una cota.

Ej:

Relajación

5.8

7

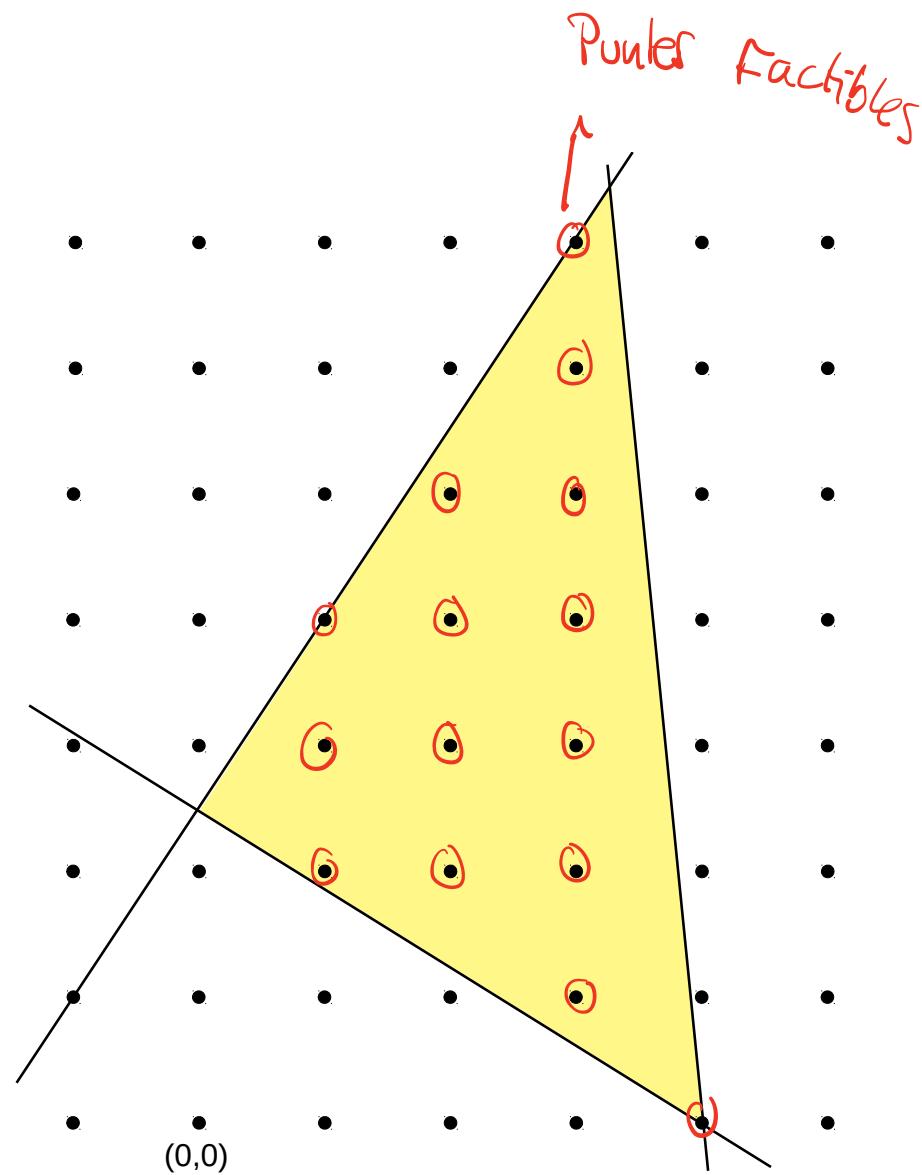
Branch
6.2

fcfm

cota nueva

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$$(PE) \min 7x + y$$
$$-3x + 2y \leq 5$$
$$5x + 8y \geq 20$$
$$20x + y \leq 800$$
$$x, y \in \mathbb{Z}$$



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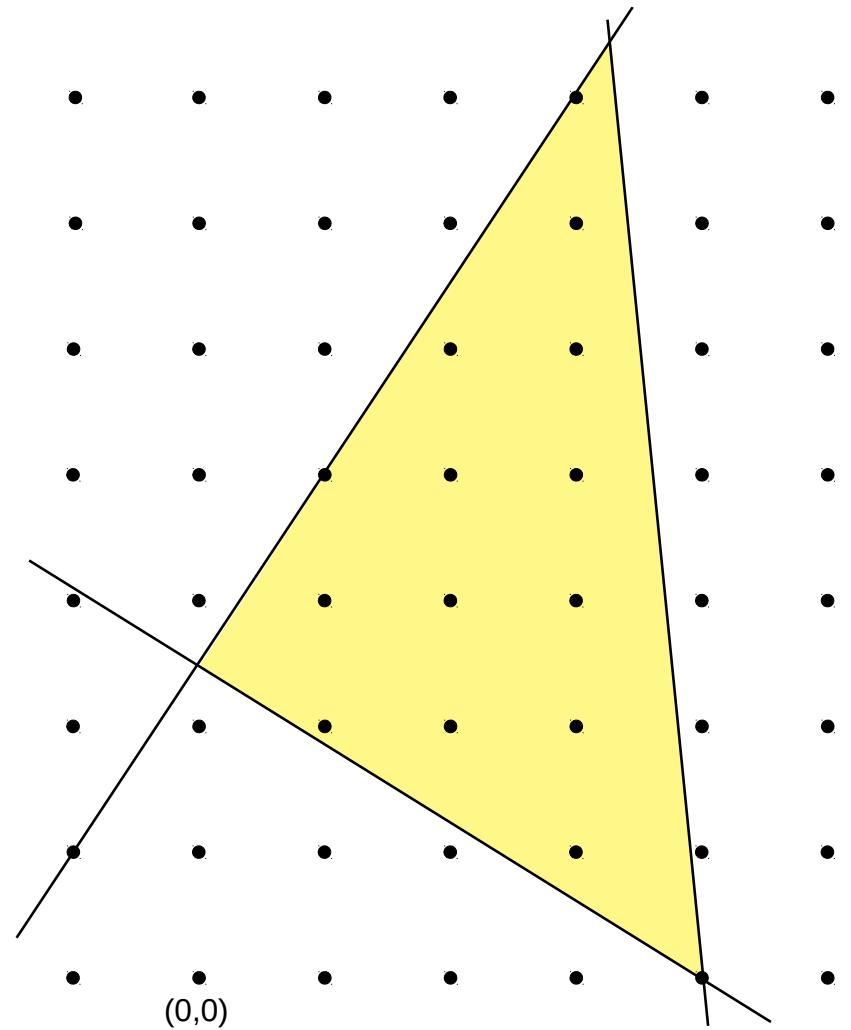
$$(P_1) \min 7x + y$$

$$\downarrow \quad -3x + 2y \leq 5$$

$$5x + 8y \geq 20$$

$$\text{relajación} \quad 20x + y \leq 800$$

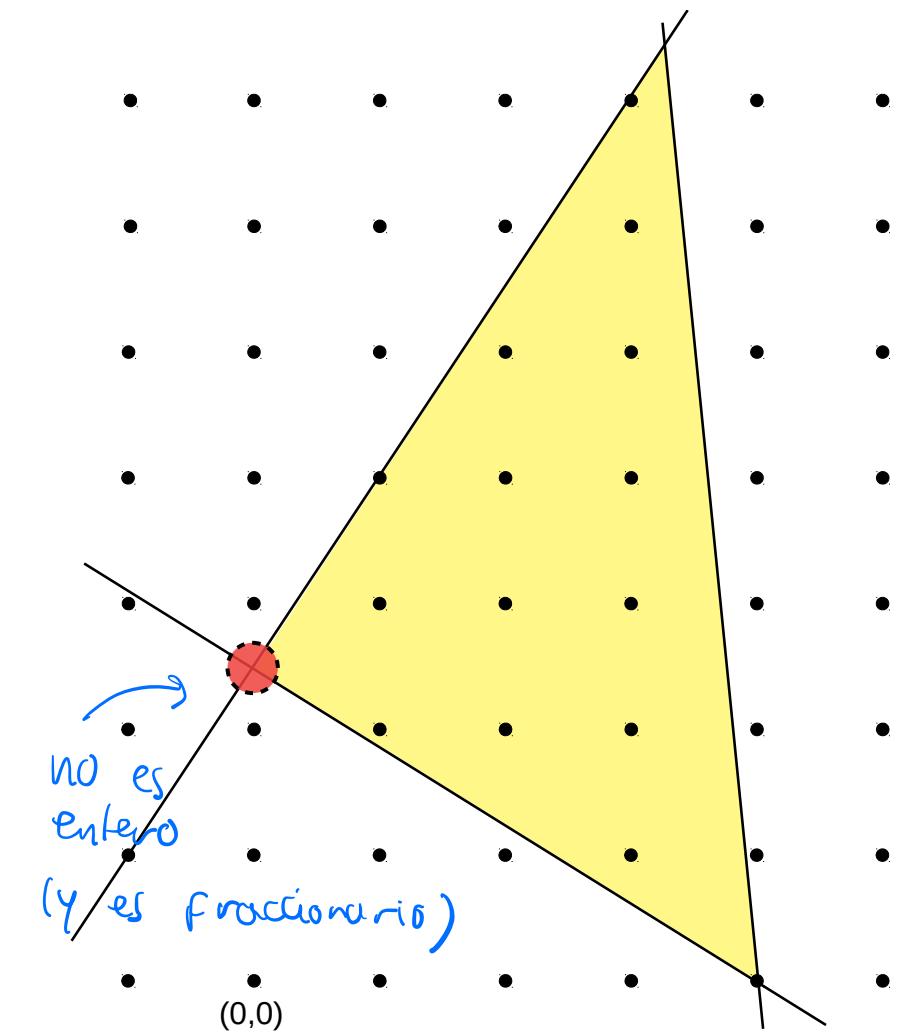
Línea 1.



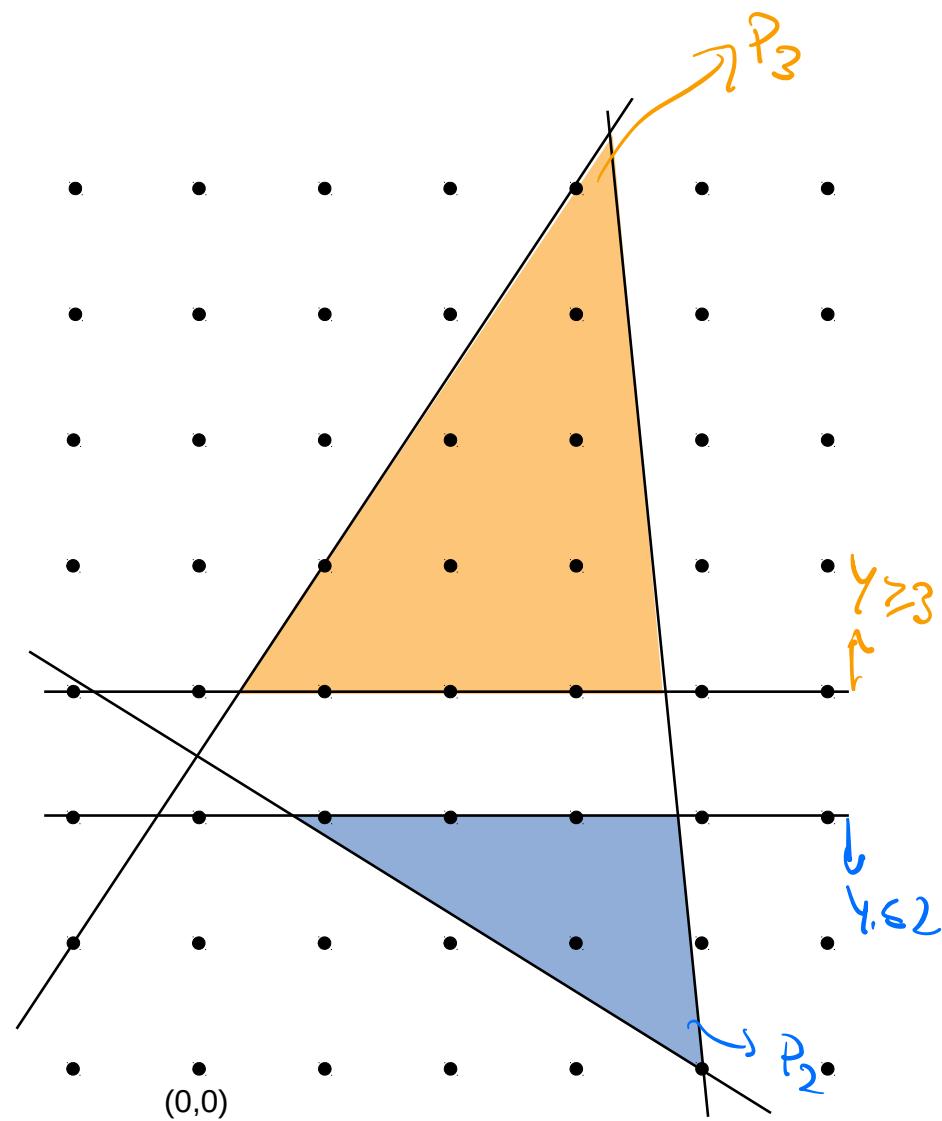
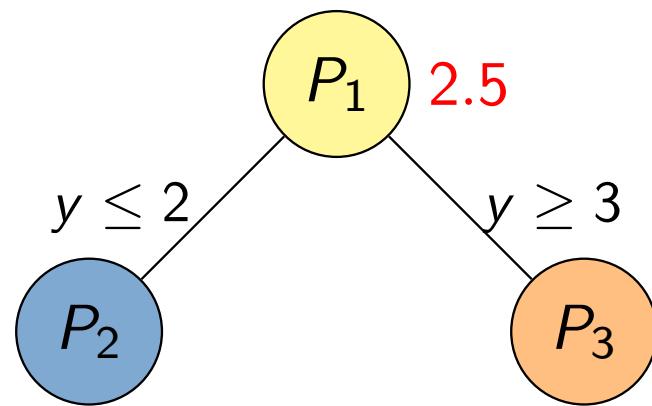
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P_1 2.5

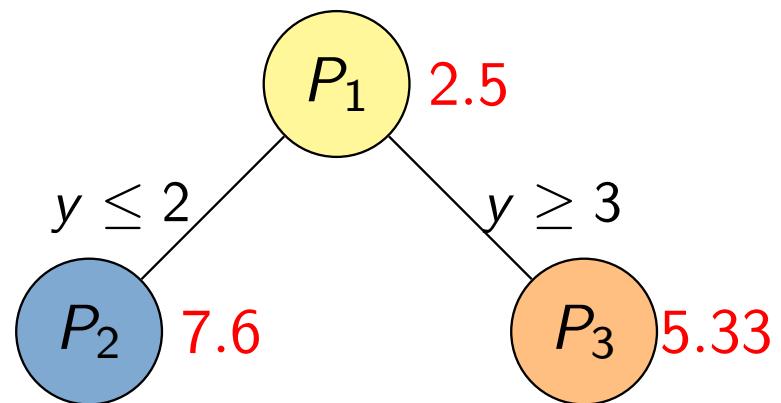
valor P.E ≥ 2.5



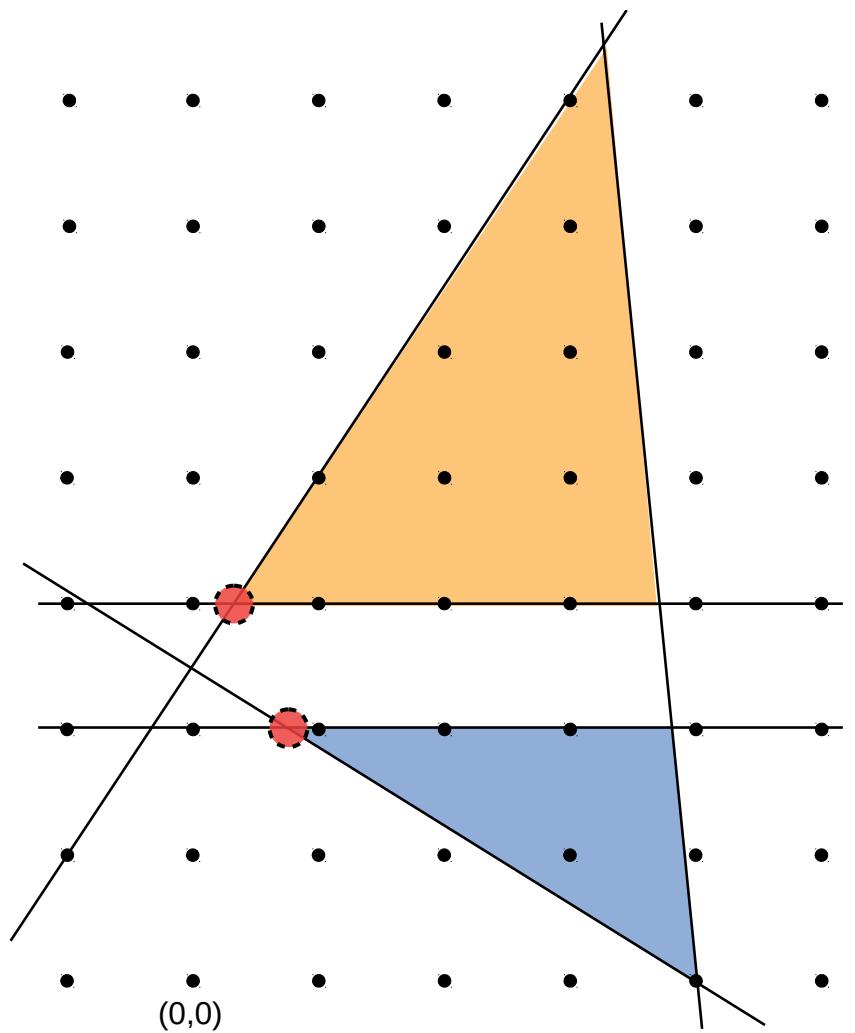
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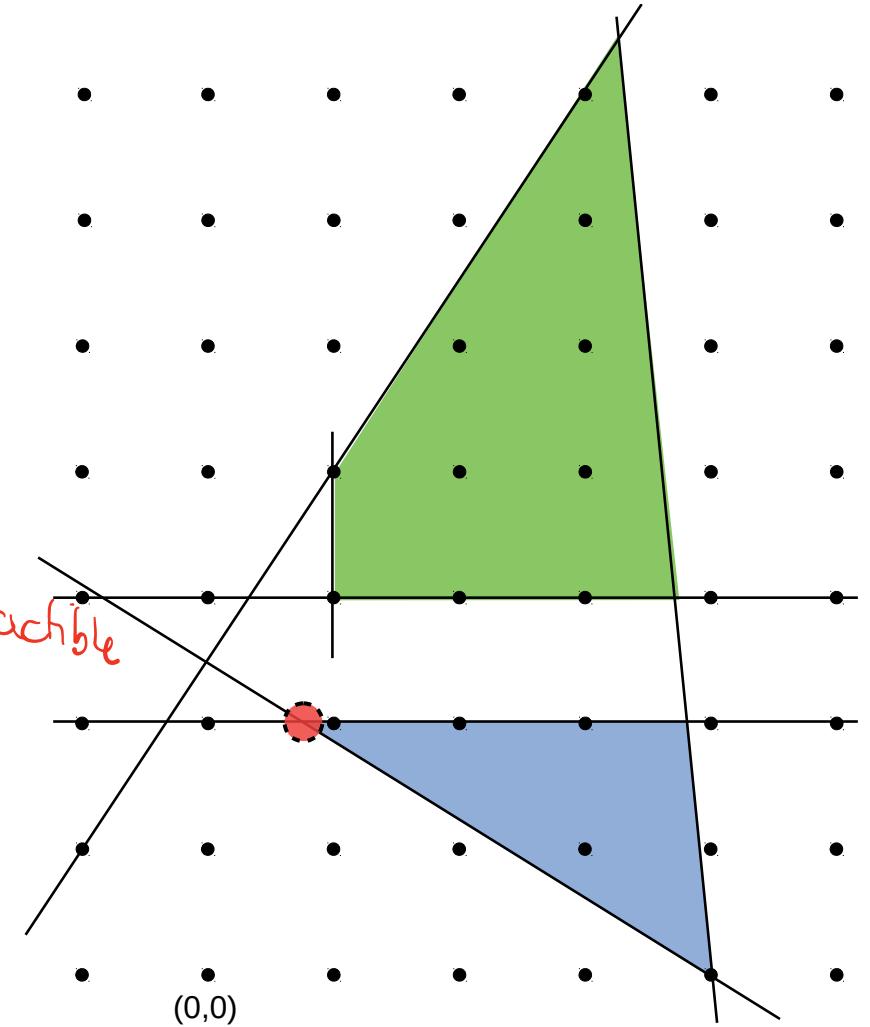
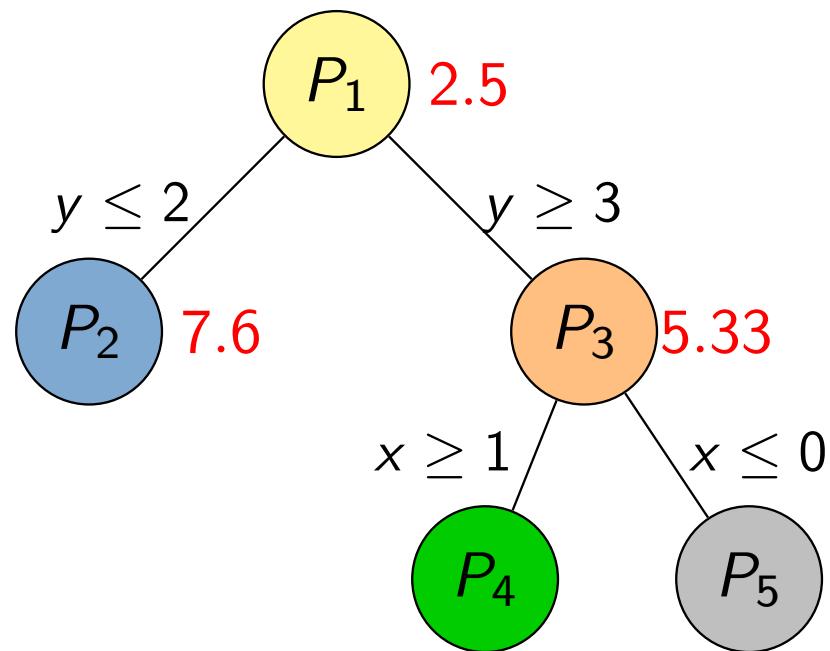
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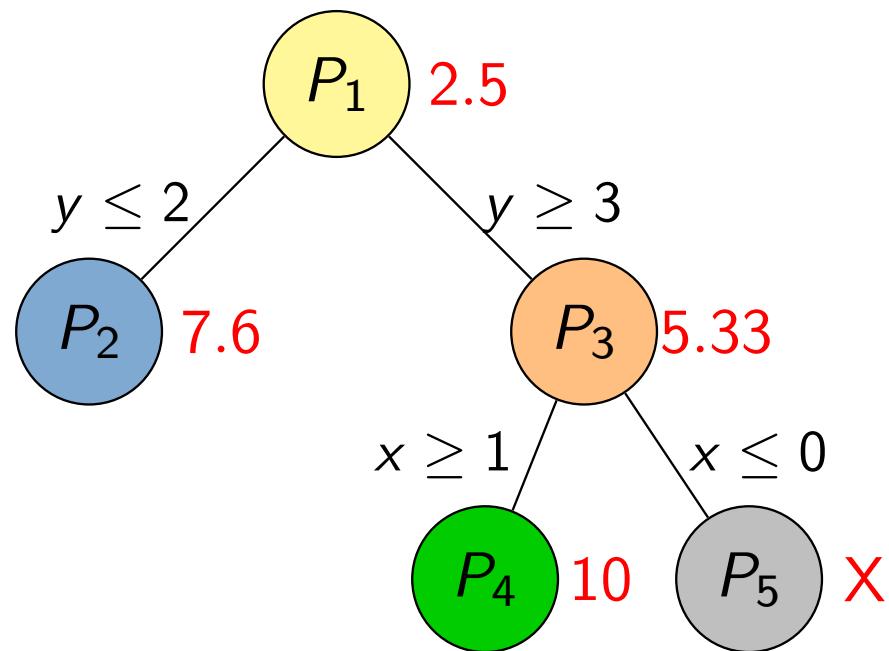
Valor P.E ≥ 5.33



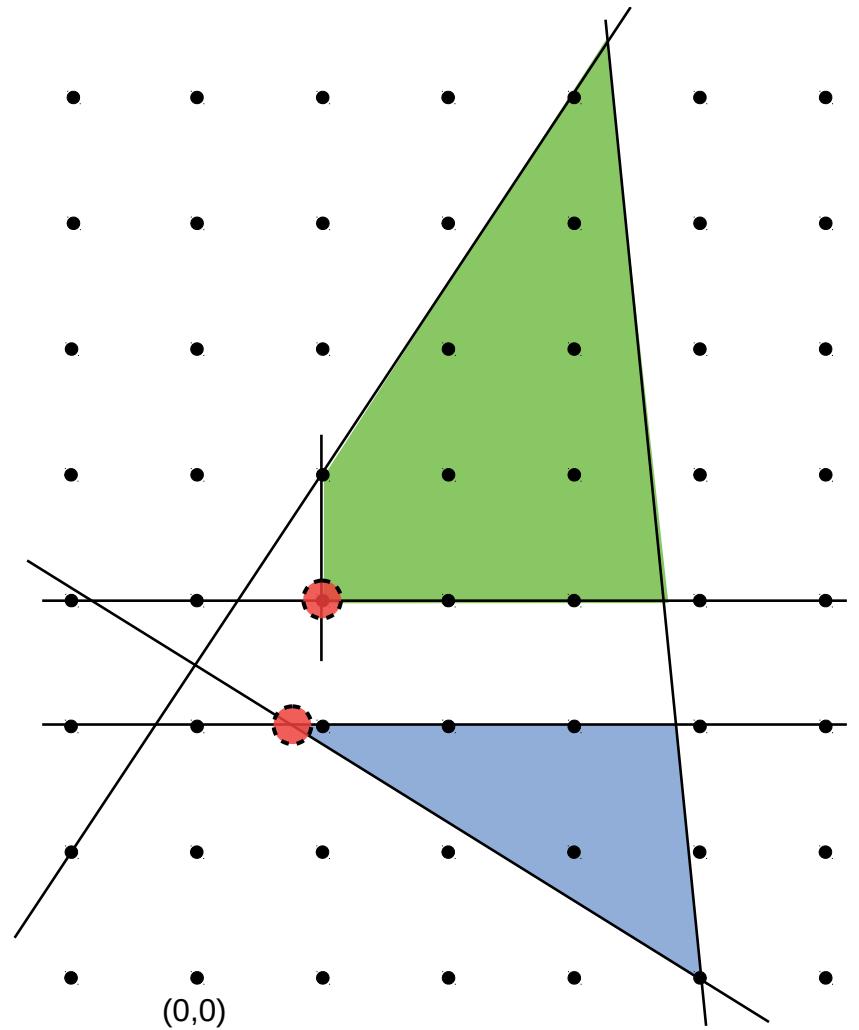
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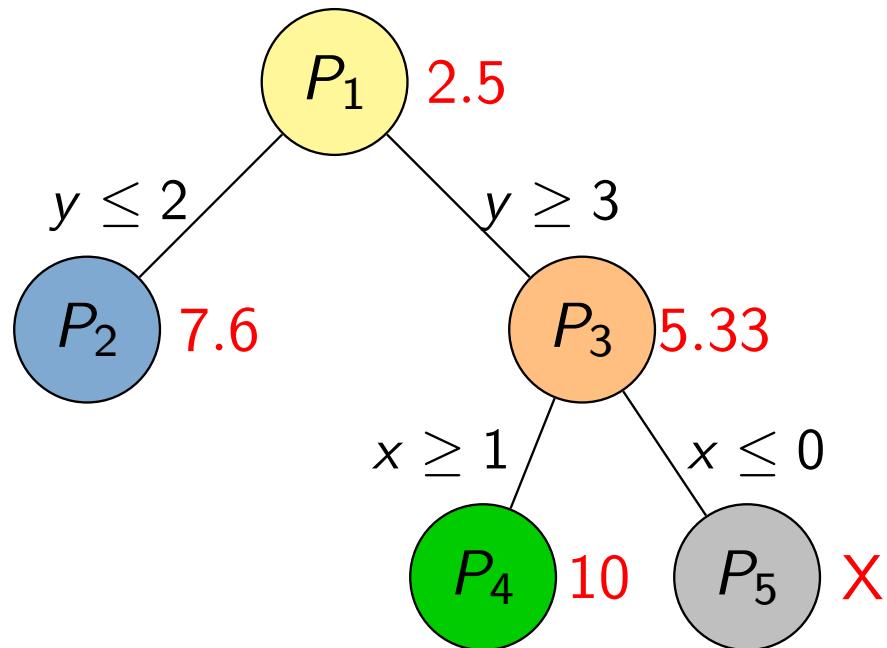
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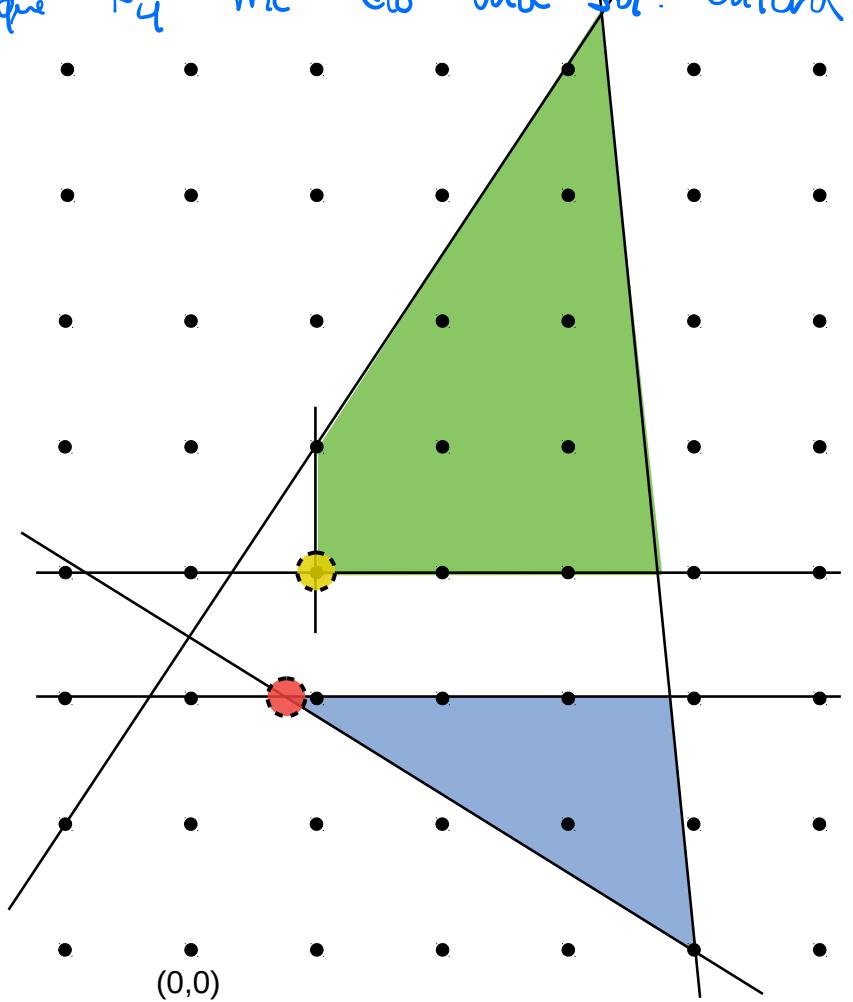
Valor P.E ≥ 7.6



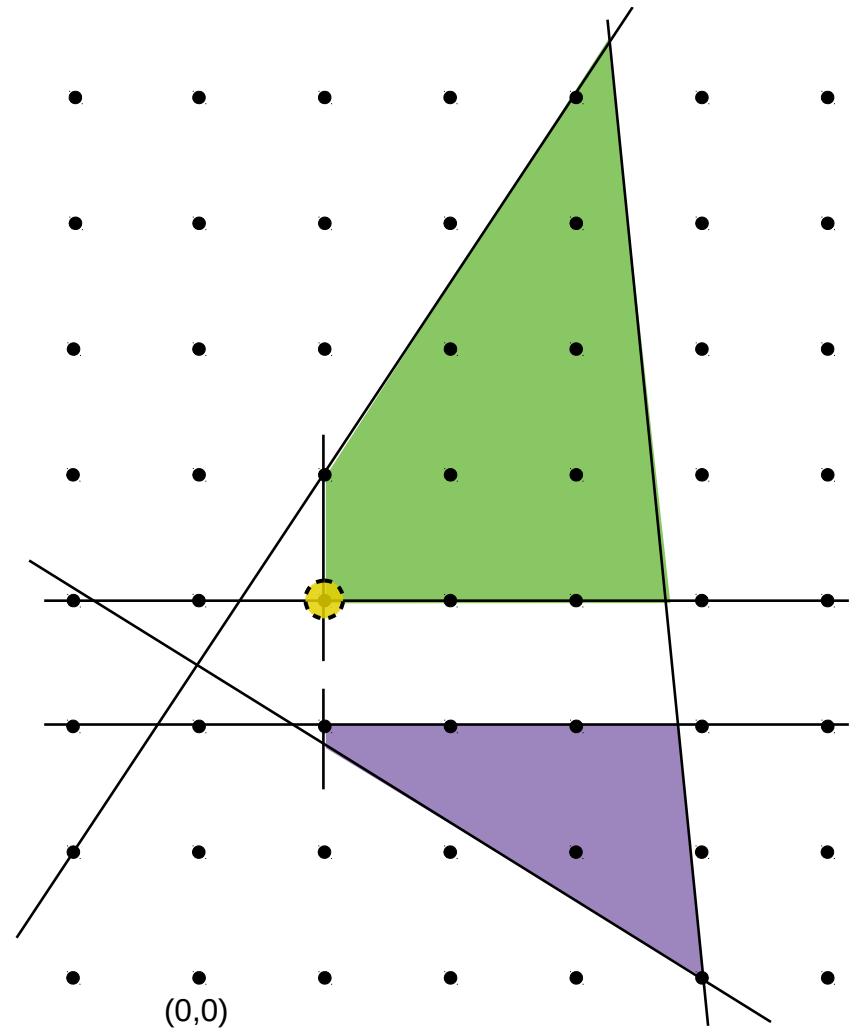
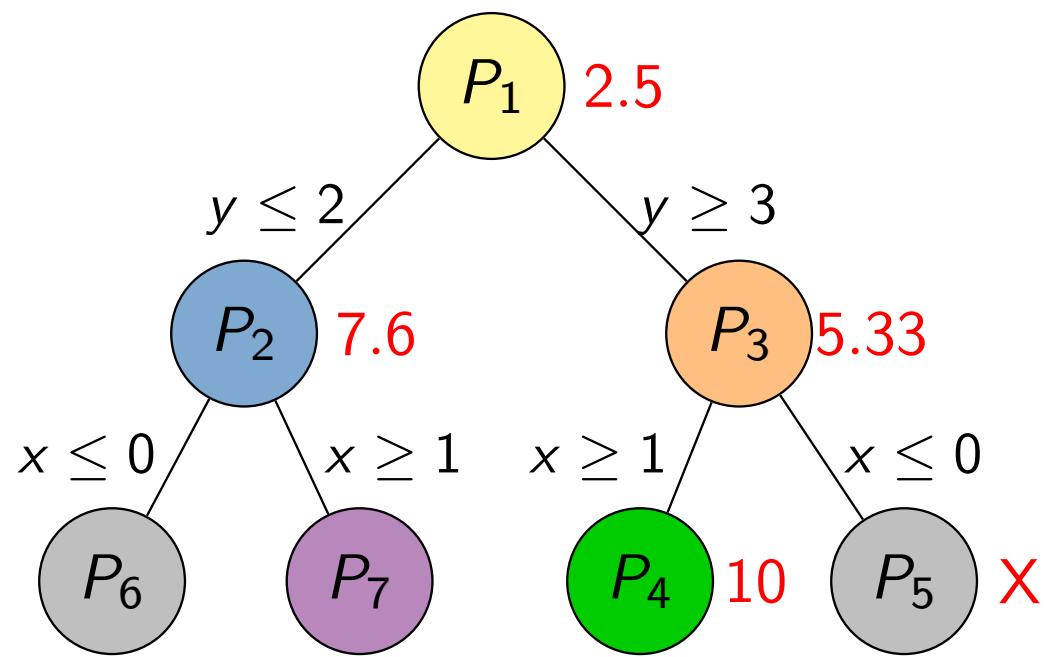
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nos dan un "gap", es decir una medida de "que tan bien estamos".
10 \geq Valor P.E \geq 7.6
Porque P_4 me dio una Sol. entera.

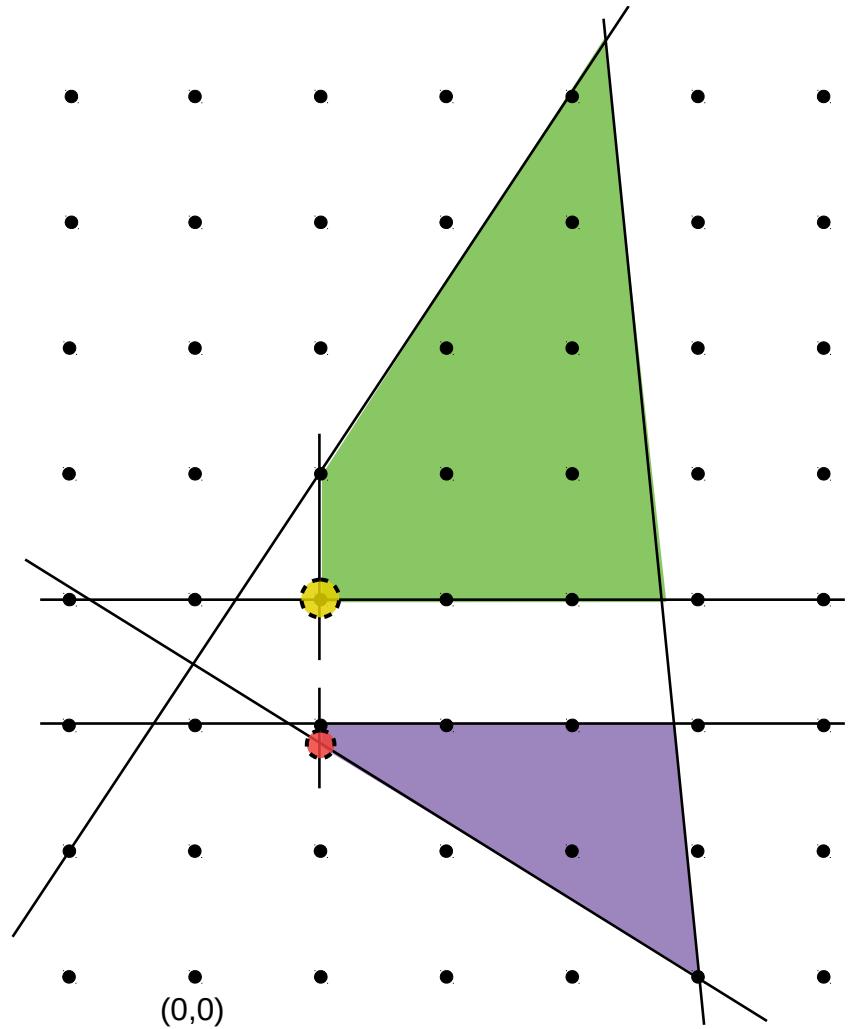
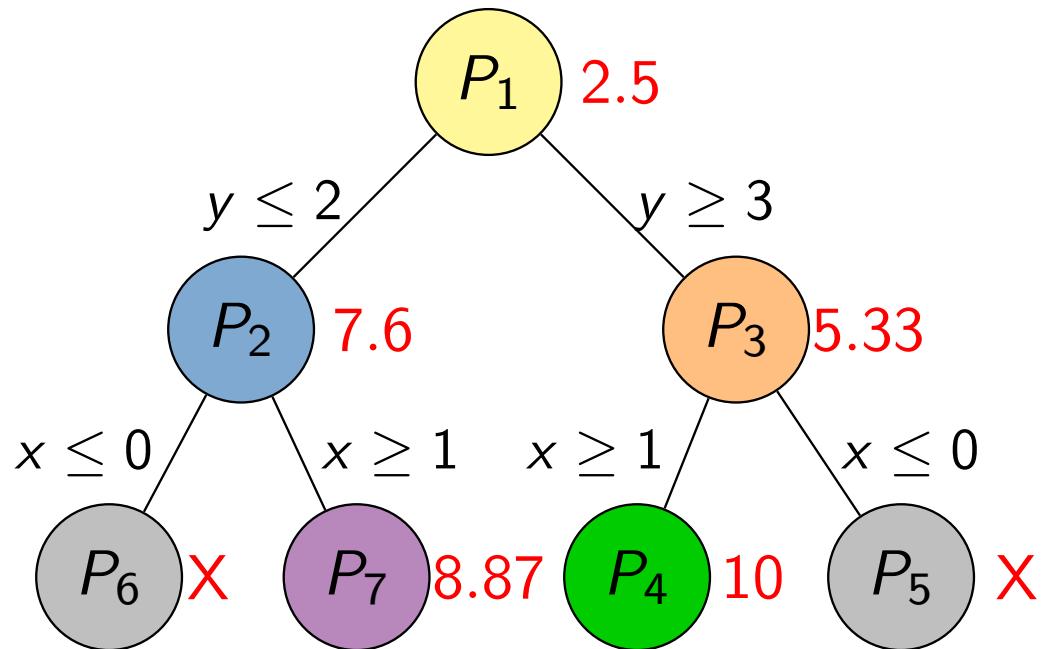


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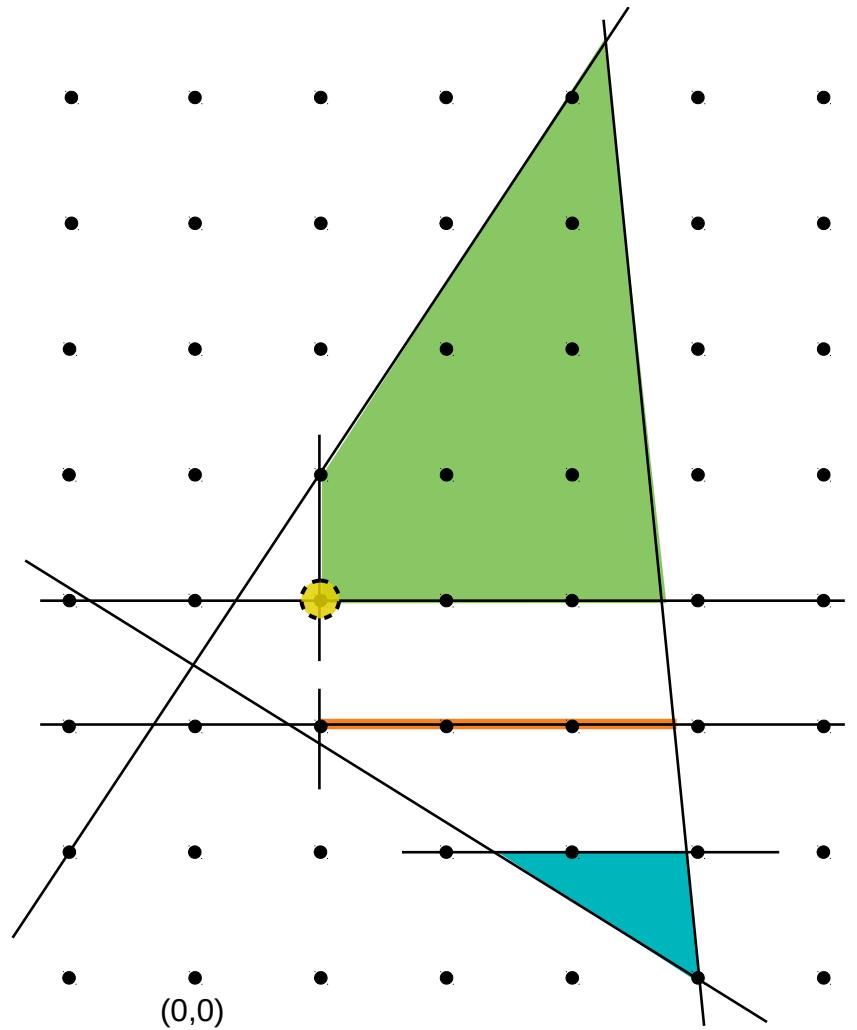
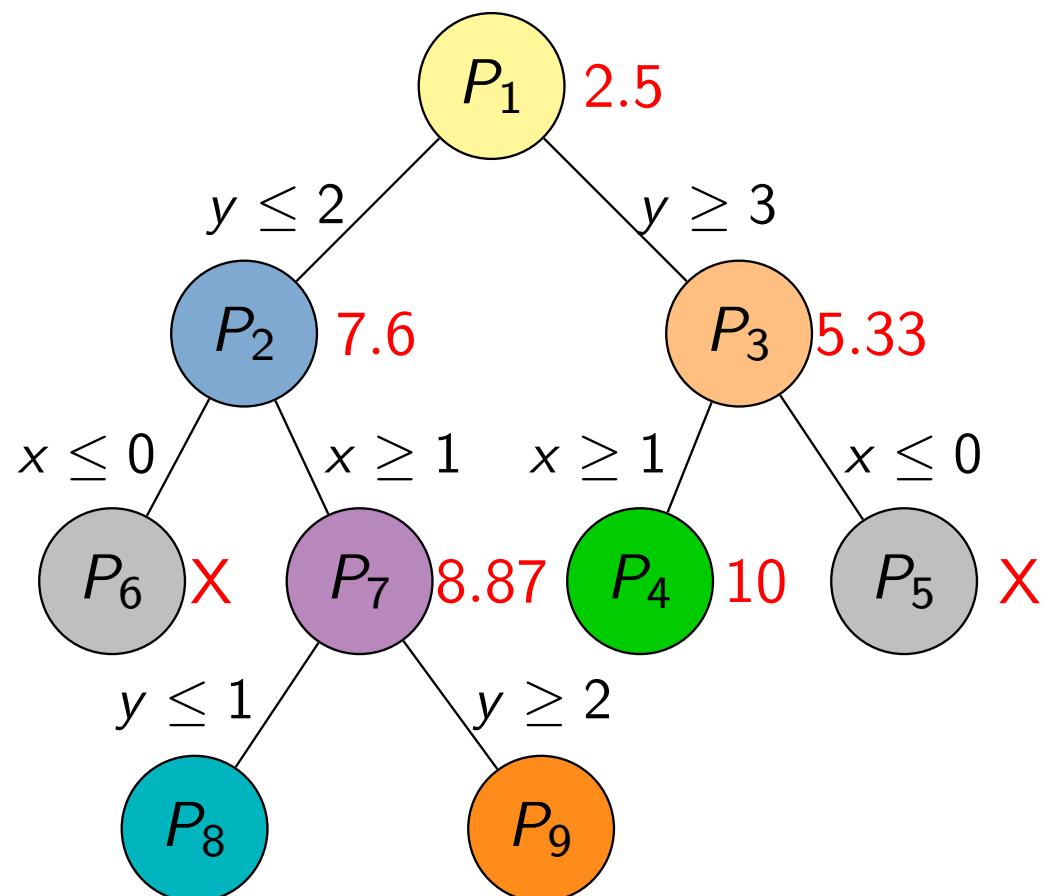


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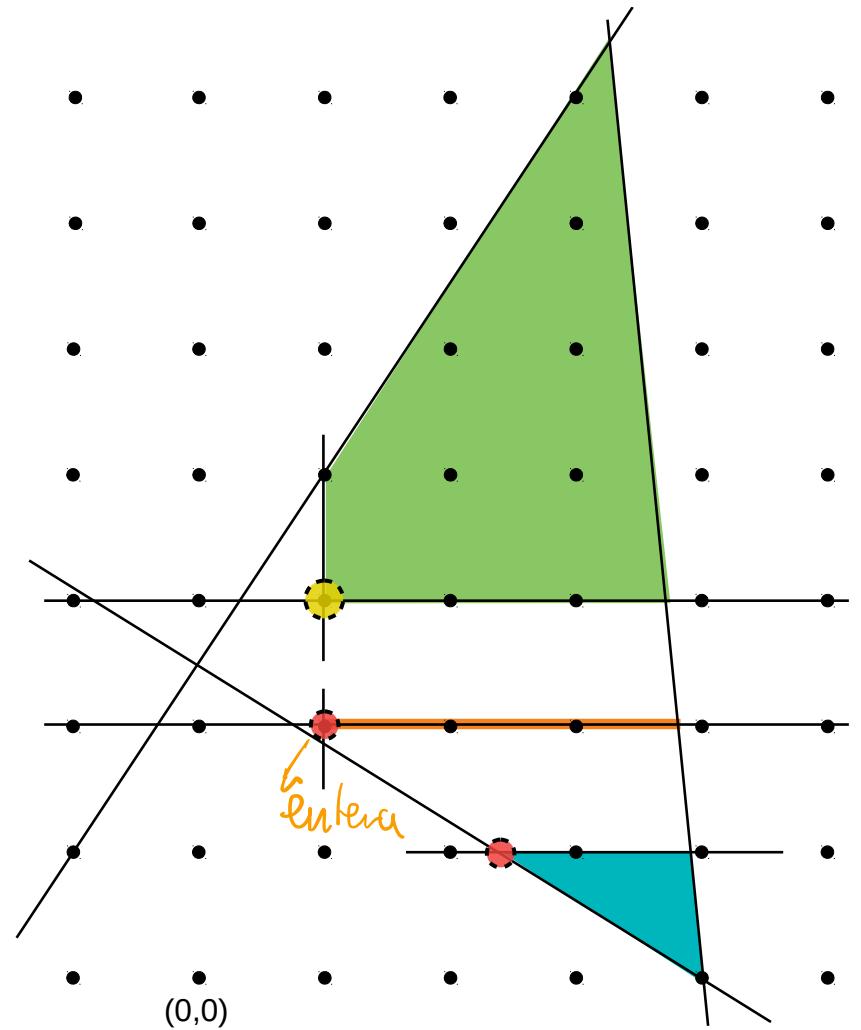
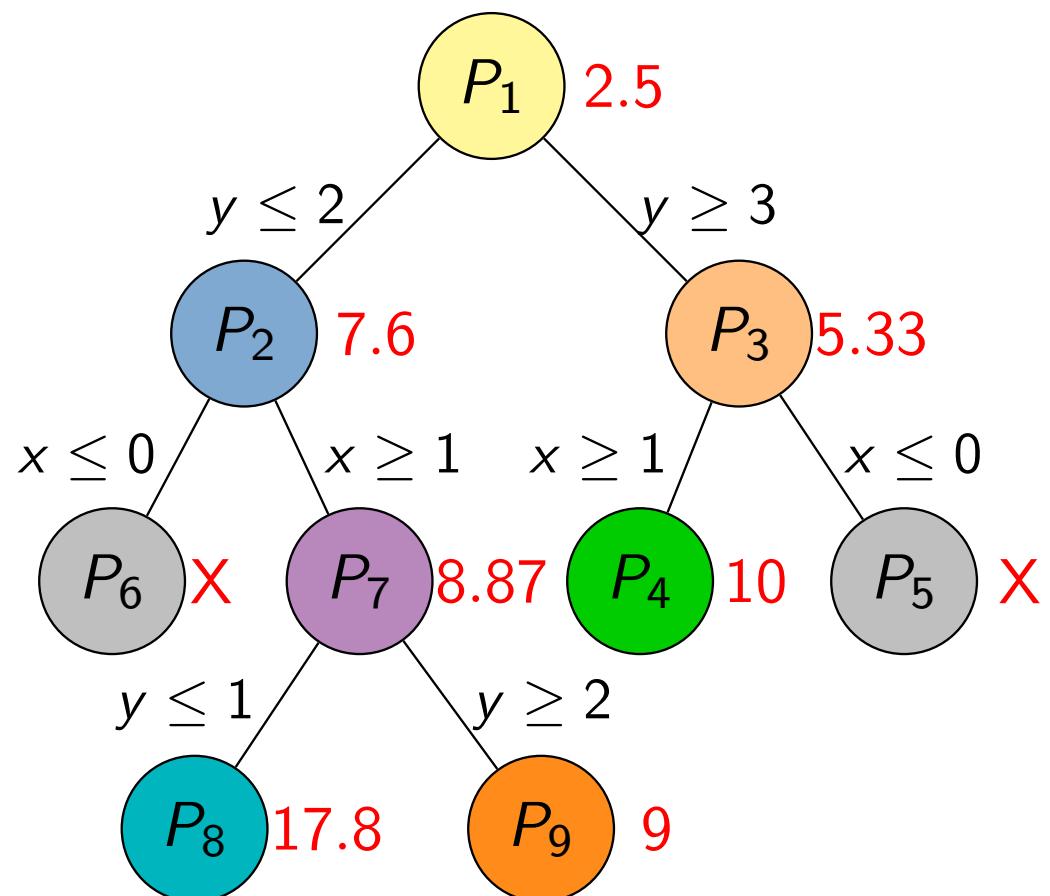
$10 \geq \text{Value P.E.} \geq 8.87$



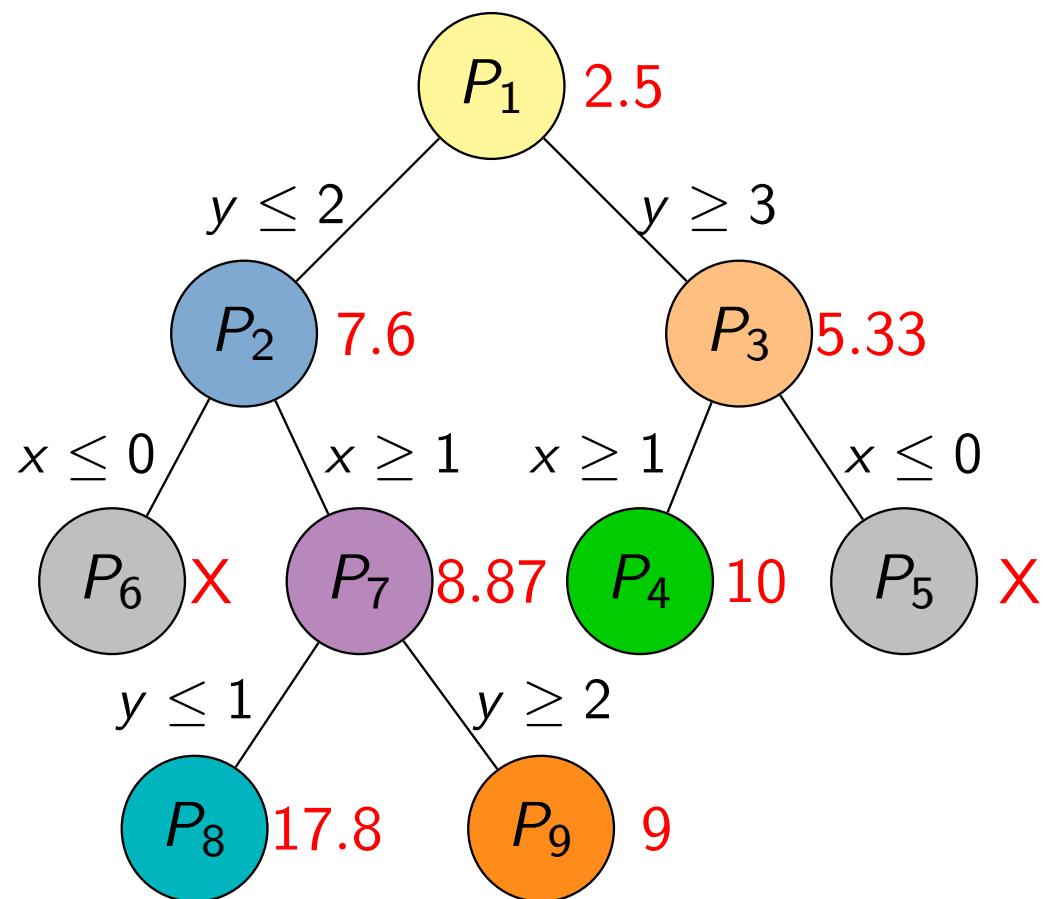
Branch and Bound



Branch and Bound

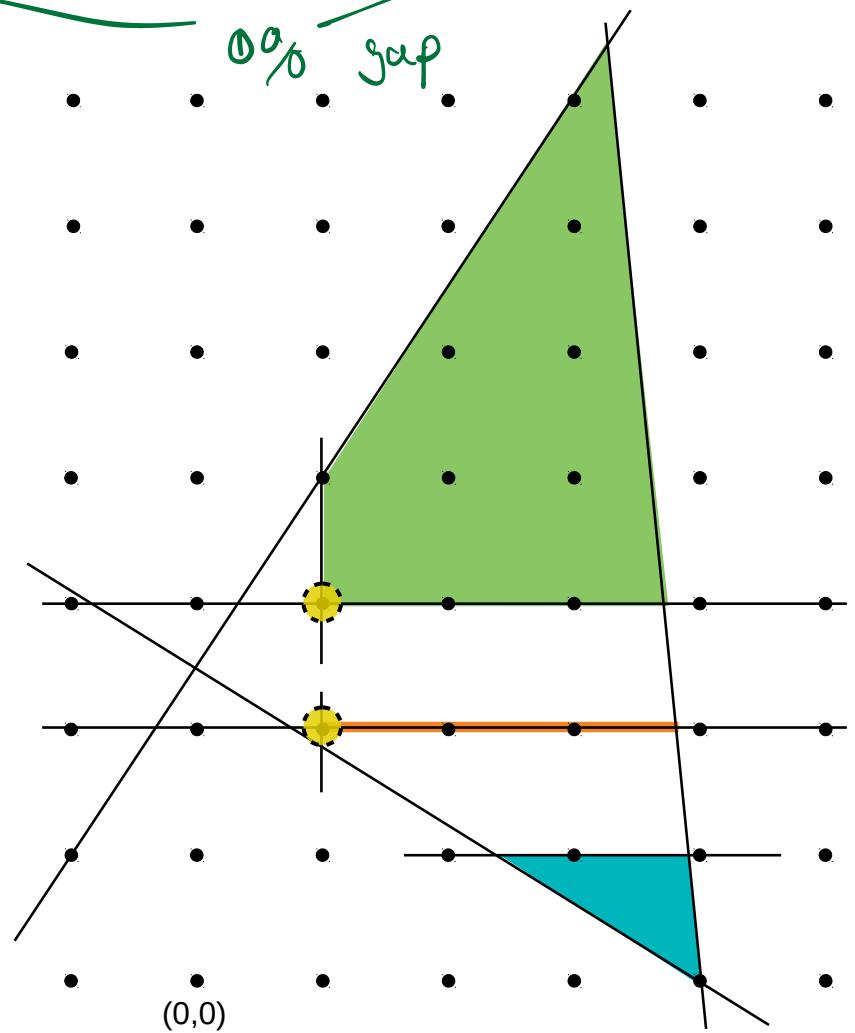


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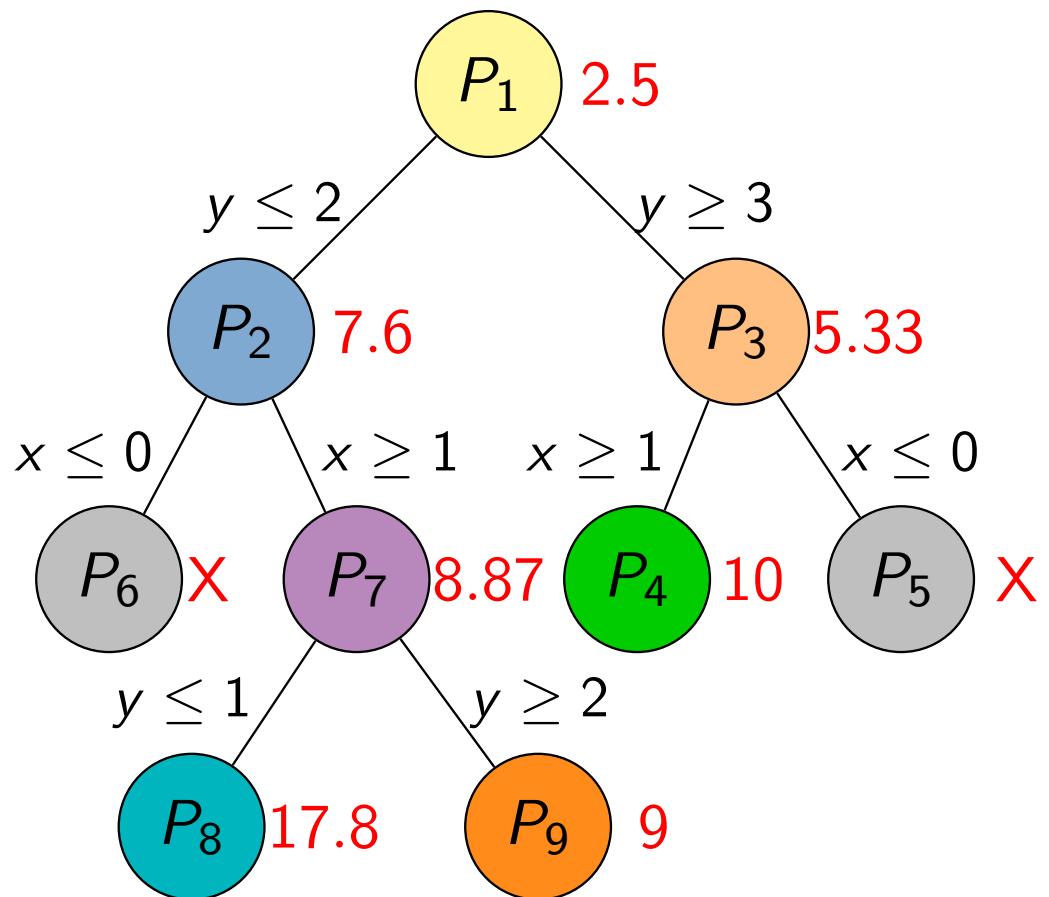


$q \geq \text{valor P.E} \geq q$

0% gap

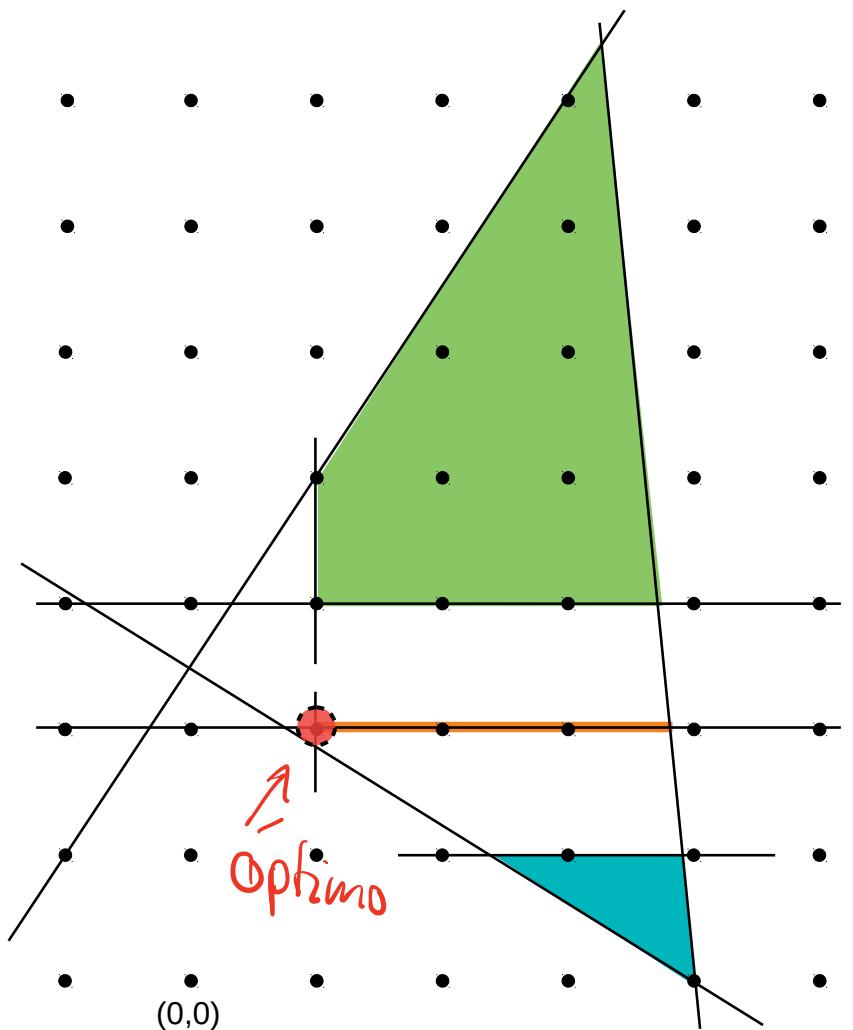


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3 motivos para no subdividir

- Infactibilidad (P_5 y P_6)
 - Sol. entera (P_4)
 - Cota peor que mejor
“cota dual”



El problema de la mochila

Consideremos la siguiente instancia del problema de la mochila:

$$\begin{aligned} \text{máx } & 8x_1 + 6x_2 + 7x_3 + 5x_4 \\ \text{s.a. } & 4x_1 + 3x_2 + 5x_3 + 8x_4 \leq 10 \\ & x \in \{0, 1\}^4 \end{aligned}$$

Se puede probar que la solución a la **relajación lineal** se obtiene de la siguiente forma: se ordenan los elementos de acuerdo a

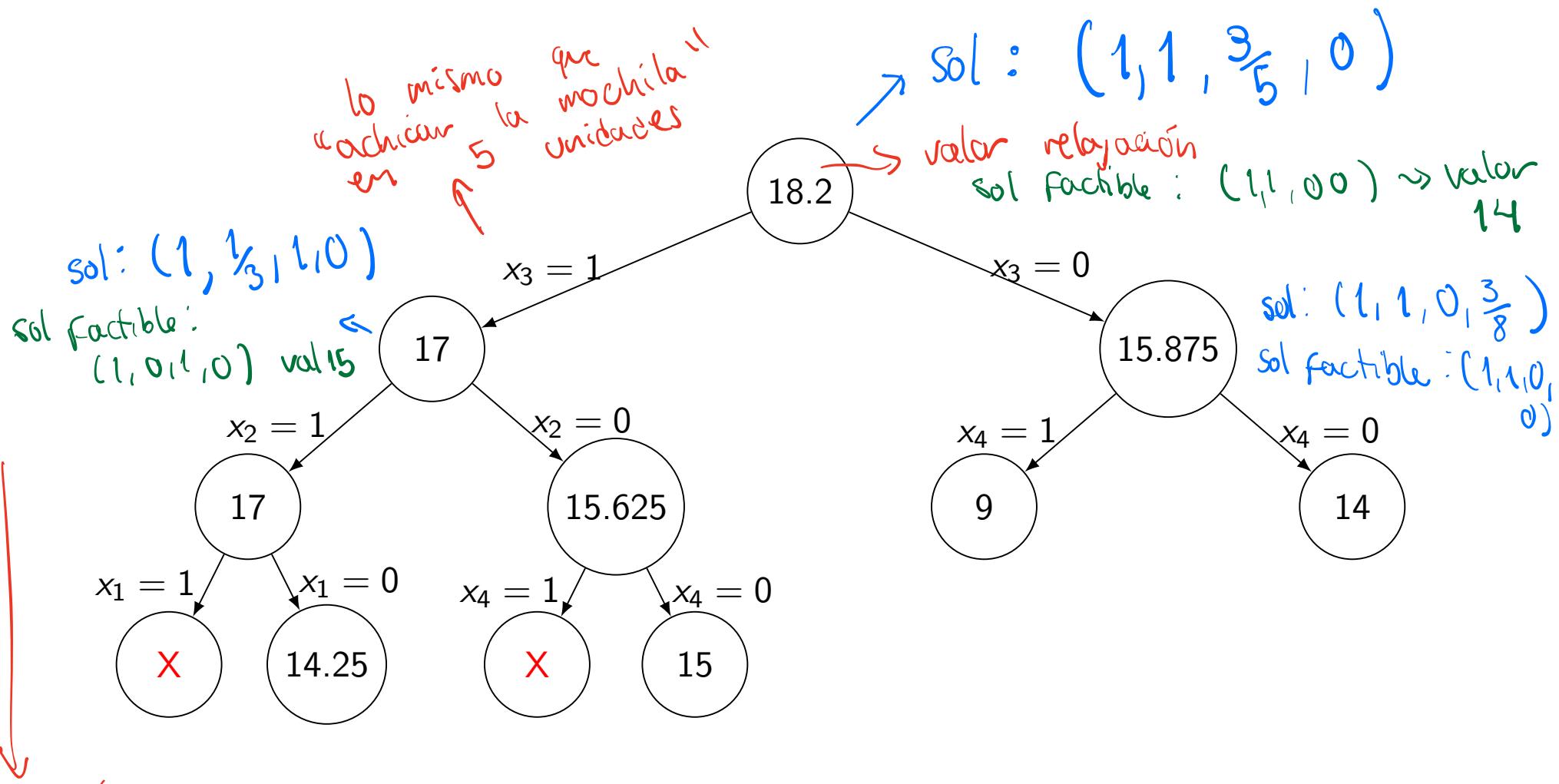
valor
peso

de mayor a menor, y luego se llena la mochila en ese orden. El último ítem podría ser incluido de manera fraccionaria.

En nuestro caso ya están ordenados así. Resolvamos usando **Branch and Bound**.



Ejecución de B&B



Review a man