

算法设计与分析第二次上机报告

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- Assignment: realize the given problems.
- note: The code listed in the passage is python. Python version: Python 2.7.10

Problem 1

- 1.1 Problem Description:
 - Matrix-chain product. The following are some instances
 - $\langle 3, 5, 2, 1, 10 \rangle$
 - $\langle 2, 7, 3, 6, 10 \rangle$
 - $\langle 10, 3, 15, 12, 7, 2 \rangle$
 - $\langle 7, 2, 4, 15, 20, 5 \rangle$
- 1.2 How to solve it?
 - I solved this problem with Dynamic Programming. There are two strategy: from top to down and from down to top.
 - The dynamic programming formula:

```
m[i,j] = 0, when i = j;
m[i,j] = min{m[i,k]+m[k+1,j]+Pi-1*Pk*Pj, when i < j;
```

- from top to down: i use the recursive strategy.
- from down to top: i use two two-dimension array to store the 'k', by which the optimal substructure can be represented.
- Code lists 1 (recursive, top-down):

```
def recursive_matrix_chain(p,i,j):
    if i == j:
        return 0
    for k in range(i,j):
        q = recursive_matrix_chain(p,i,k)+recursive_matrix_chain(p,k+1,j)+
        p[i-1]*p[k]*p[j]
        if q < m[i][j]:
            m[i][j] = q
            s[i][j] = k
    return m[i][j]
```

- Code lists 2 (down-top):

```

def matrix_chain(p):
    n = len(p) - 1
    m = [[0]*(n+1) for i in range(n+1)]
    s = [[0]*(n+1) for i in range(n+1)]
    for l in range(2,n+1):
        for i in range(1,n-l+2):
            j = i + l - 1
            m[i][j] = sys.maxint
            for k in range(i,j):
                q = m[i][k]+m[k+1][j]+p[i-1]*p[k]*p[j]
                if q < m[i][j]:
                    m[i][j] = q
                    s[i][j] = k
    return m,s

def print_optimal_parens(s,i,j):
    if i == j:
        print 'A%d'%(i),
    else:
        print '(',
        print_optimal_parens(s,i,s[i][j])
        print_optimal_parens(s,s[i][j]+1,j)
        print ')',

```

- 1.3 Result:

```

Documents/algorithms/hw2
▶ python matrix_chain.py
( ( A1 A2 ) ( ( A3 A4 ) ( A5 A6 ) ) )

[3, 5, 2, 1, 10]
( ( A1 ( A2 A3 ) ) A4 )

[2, 7, 3, 6, 10]
( ( ( A1 A2 ) A3 ) A4 )

[10, 3, 15, 12, 7, 2]
( A1 ( A2 ( A3 ( A4 A5 ) ) ) )

[7, 2, 4, 15, 20, 5]
( A1 ( ( ( A2 A3 ) A4 ) A5 ) )

[5, 10, 3, 12, 5, 60, 6]
( ( A1 A2 ) ( ( A3 A4 ) ( A5 A6 ) ) )

```

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Problem 2

- 2.1 Problem Description:

- Longest Common Subsequence (LCS). The following are some instances.
- X: xzyzzyx
- Y: zxyyzxz
- X: ALLAAQANKESSESFISRLLAIVAD
- Y: KLQKKLAETEKRCTLLAAQANKENSNESFISRLLAIVAG

- 2.2 How to solve it?

- I solved this problem with Dynamic Programming. There are two strategies: from top to bottom and from bottom to top.
- The dynamic programming formula:

```
c[i,j] = 0, when i = 0 or j = 0;  
c[i,j] = c[i-1,j-1]+1, when i,j>0 and xi = yj  
c[i,j] = max(c[i,j-1],c[i-1,j]), when i,j>0 and xi != yj
```

- Code lists 1(recursive,top-down):

```
def RECURSIVE_LCS(x,y):  
    if (len(x) == 0 or len(y) == 0):  
        return 0  
    else:  
        a = x[0]  
        b = y[0]  
        if (a == b):  
            listc.append(a)  
            return RECURSIVE_LCS(x[1:],y[1:]) + 1  
        else:  
            return MAX_SE(RECURSIVE_LCS(x[1:],y),RECURSIVE_LCS(x,y[1:]))  
  
def MAX_SE(a,b):  
    if(a >= b):  
        return a  
    else:  
        return b
```

- Code list 2(bottom-top):

```

def lcs_len(a, b):
    n = len(a)
    m = len(b)

    l = [[0] * (m + 1) for i in range(n + 1)]
    direct = [[0] * m for i in range(n)]

    for i in range(n + 1)[1:]:
        for j in range(m + 1)[1:]:
            if a[i - 1] == b[j - 1]:
                l[i][j] = l[i - 1][j - 1] + 1
            elif l[i][j - 1] > l[i - 1][j]:
                l[i][j] = l[i][j - 1]
                direct[i - 1][j - 1] = -1
            else:
                l[i][j] = l[i - 1][j]
                direct[i - 1][j - 1] = 1

    return l, direct

def get_lcs(direct, a, i, j):
    lcs = []
    get_lcs_inner(direct, a, i, j, lcs)
    return lcs

def get_lcs_inner(direct, a, i, j, lcs):
    if i < 0 or j < 0:
        return

    if direct[i][j] == 0:
        get_lcs_inner(direct, a, i - 1, j - 1, lcs)
        lcs.append(a[i])

    elif direct[i][j] == 1:
        get_lcs_inner(direct, a, i - 1, j, lcs)
    else:
        get_lcs_inner(direct, a, i, j - 1, lcs)

```

• 2.3 Result:

- from down to top: i can get the Longest Common Subsequence.

```

Documents/algorithms/hw2
▶ python LCS.py
the longest number is: 4
xyzz
the longest number is: 23
ALLAAQANKESSEFISRLLAIVA

```

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- from top to down:i can get the longest number with the recursive method.

```
Documents/algorithms/hw2
▶ python RECURSIVE_LCS.py
4
```

-

Problem 3

- 3.1 Problem Description:
 - Max Sum. The following are some instances:
 - (-2, 11, -4, 13, -5, -2)
- 3.2 How to solve it?
 - I use a temp number to store the largest currently number.With an easy loop and comparison,i can get max sum.This strategy is from down to top.
 - Code list:

```
def max_sum(p):
    sum = p[0]
    result = p[0]
    start = 0
    for i in range(1,len(p)-1):
        if sum > 0:
            sum += p[i]
        else:
            sum = p[i]
            start = i
        if sum > result:
            result = sum
            end = i
    return result,start,end
```

- 3.3 Result:

```
Documents/algorithms/hw2
▶ python max_sum.py
20
[11, -4, 13]
```

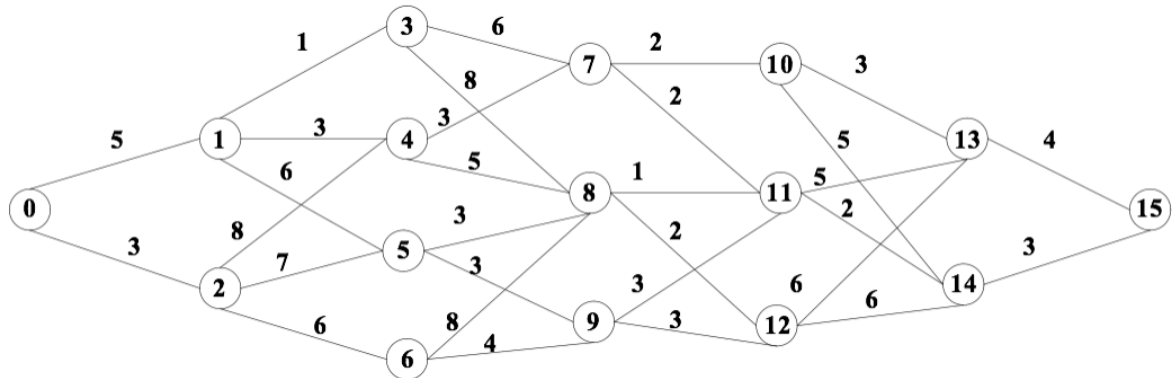
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Problem 4

- 4.1 Problem Description:
 - Shortest path in multistage graphs. Find the shortest path from 0 to 15 for the following graph. A

multistage graph is a graph

- (1) $G=(V,E)$ with V partitioned into $K \geq 2$ disjoint subsets such that if (a,b) is in E , then a is in V_i , and b is in V_{i+1} for some subsets in the partition;
- (2) $|V_1| = |V_K| = 1$.



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• 4.2 How to solve it?

- The strategy i used:from top to down ,recursive
- The dynamic programming formula:

```
s[0,j] = 0 ,when j = 0;
s[0,j] = s[0,k] + s[k,j],when j!=0
```

- the data structure:

```
graphs = {(0,1):(1,5),(0,2):(1,3),(1,3):(2,1),(1,4):(2,3),(1,5):(2,6),(2,4):
(2,8),(2,5):(2,7),(2,6):(2,6),(3,7):(3,6),(3,8):(3,8),(4,7):(3,3),(4,8):
(3,5),(5,8):(3,3),(5,9):(3,3),(6,8):(3,8),(6,9):(3,4),(7,10):(4,2),(7,11):
(4,2),(8,11):(4,1),(8,12):(4,2),(9,11):(4,3),(9,12):(4,3),(10,13):(5,3),(1
0,14):(5,5),(11,13):(5,5),(11,14):(5,2),(12,13):(5,6),(12,14):(5,6),(13,15
):(6,4),(14,15):(6,3)}
```

- code list(recursive,top-down):

```
def RECURSIVE_SHORTEST_GRAPHS(graphs,j):
    if j == 0:
        return 0
    m = sys.maxint
    for k in range(j):
        if (graphs.has_key((k,j))):
            p = RECURSIVE_SHORTEST_GRAPHS(graphs,k)+graphs[(k,j)][1]
            if p < m:
                m = p
                s[j] = (k,j)
    return m
```

- 4.3 Result:

```
Documents/algorithms/hw2
▶ python multistage_graphs.py
the least cost of the path is: 18
the path is: 15 14 11 7 4 1 0
```

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Problem 5

- 5.1 Problem Description:
 - Longest Common Substring. The following are some instances.
 - X: xzyzzyx
 - Y: zxyyzxz
 - X:MAEEEEVAKLEKHLMLLRQEYVKLQKKLAETEKRCALLAAQANKESSESFISRLLAIVAD
 - Y:MAEEEEVAKLEKHLMLLRQEYVKLQKKLAETEKRCTLLAAQANKENS NESFISRLLAIVAG

- 5.2 How to solve it?

- The dynamic programming formula:

```
m[i,j] = m[i-1,j-1] + 1,when i = j;
m[i,j] = 0,when i != j;
```

- code list 1(recursive,top-down)

```
def RECURSIVE_LCStr(x,y):
    if (len(x) == 0 or len(y) == 0):
        return 0
    else:
        a = x[0]
        b = y[0]
        if (a == b):
            return RECURSIVE_LCStr(x[1:],y[1:])+1
        else:
            return 0
```

- code list 2(down-top)


```

def LCStr(a,b):
    n = len(a)
    m = len(b)

    c = [[0] * (m + 1) for i in range(n + 1)]
    for i in range(n):
        for j in range(m):
            if (a[i] == b[j]):
                c[i][j] = c[i-1][j-1] + 1
            else:
                c[i][j] = 0
    return c

def get_max(c):
    coordinate = []
    max = 0
    for i in range(len(a)+1):
        for j in range(len(b)+1):
            if c[i][j] > max:
                max = c[i][j]
            else:
                pass
    for i in range(len(a)+1):
        for j in range(len(b)+1):
            if c[i][j] == max:
                coordinate.append((i,j))
    return max,coordinate

def get_lss(c,a,x,max):
    temp = []
    while max > 0:
        temp.append(a[x])
        x -= 1
        max -= 1
    lss = temp[::-1]
    return lss

```

- 5.3 Result:

- from down to top:i can get most common string.

```

Documents/algorithms/hw2
▶ python LCStr.py
[(2, 2), (6, 5)]
efg
abc

```

- from top to down:i can get the largest common number with the recursive method.

Documents/algorithms/hw2

▶ python RECURSIVE_LCStr.py

('LSS Length is:{0}', 2)

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