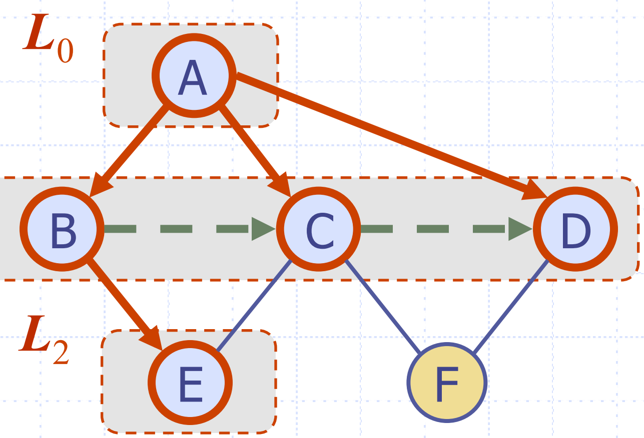
Presentation for use with the textbook Data Structures and Algorithms in Java, 6th edition, by M. T. Goodrich, R. Tamassia, and M. H. Goldwasser, Wiley, 2014

Breadth-First Search



Breadth-First Search

- Breadth-first search (BFS) is a general technique for traversing a graph
- A BFS traversal of a graph G
  - Visits all the vertices and edges of G
  - Determines whether G is connected
  - Computes the connected components of G
  - Computes a spanning forest of G
- BFS on a graph with n vertices and m edges takes $O(n + m)$ time
- BFS can be further extended to solve other graph problems
  - Find and report a path with the minimum number of edges between two given vertices
  - Find a simple cycle, if there is one

BFS Algorithm

- The algorithm uses a mechanism for setting and getting "labels" of vertices and edges

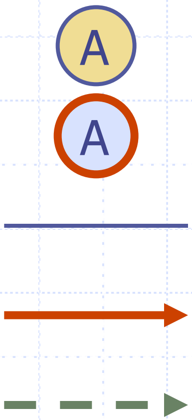
```
Algorithm BFS(G)
    Input graph G
    Output labeling of the edges
        and partition of the
        vertices of G
  for all u ∈ G.vertices()
    setLabel(u, UNEXPLORED)
  for all e ∈ G.edges()
    setLabel(e, UNEXPLORED)
  for all v ∈ G.vertices()
    if getLabel(v) = UNEXPLORED
      BFS(G, v)
```

```
Algorithm BFS(G, s)
  L_0 ← new empty sequence
  L_0.addLast(s)
  setLabel(s, VISITED)
  i ← 0
  while ¬L_i.isEmpty()
    L_{i+1} ← new empty sequence
    for all v ∈ L_i.elements()
      for all e ∈ G.incidentEdges(v)
        if getLabel(e) = UNEXPLORED
          w ← opposite(v,e)
          if getLabel(w) = UNEXPLORED
            setLabel(e, DISCOVERY)
            setLabel(w, VISITED)
            L_{i+1}.addLast(w)
          else
            setLabel(e, CROSS)
    i ← i + 1
```

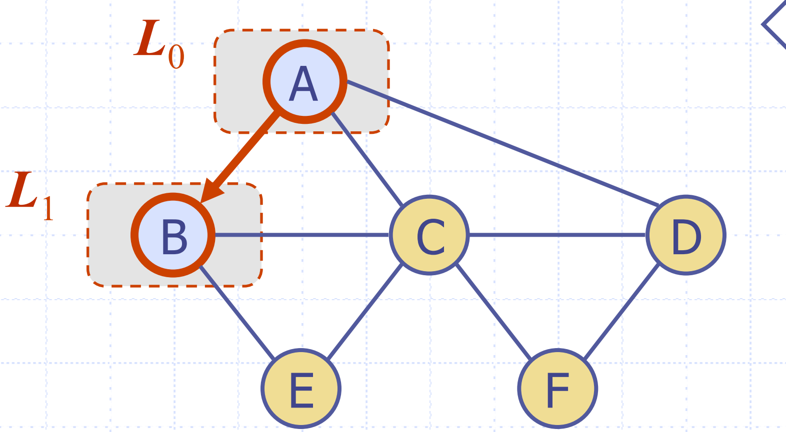
Java Implementation

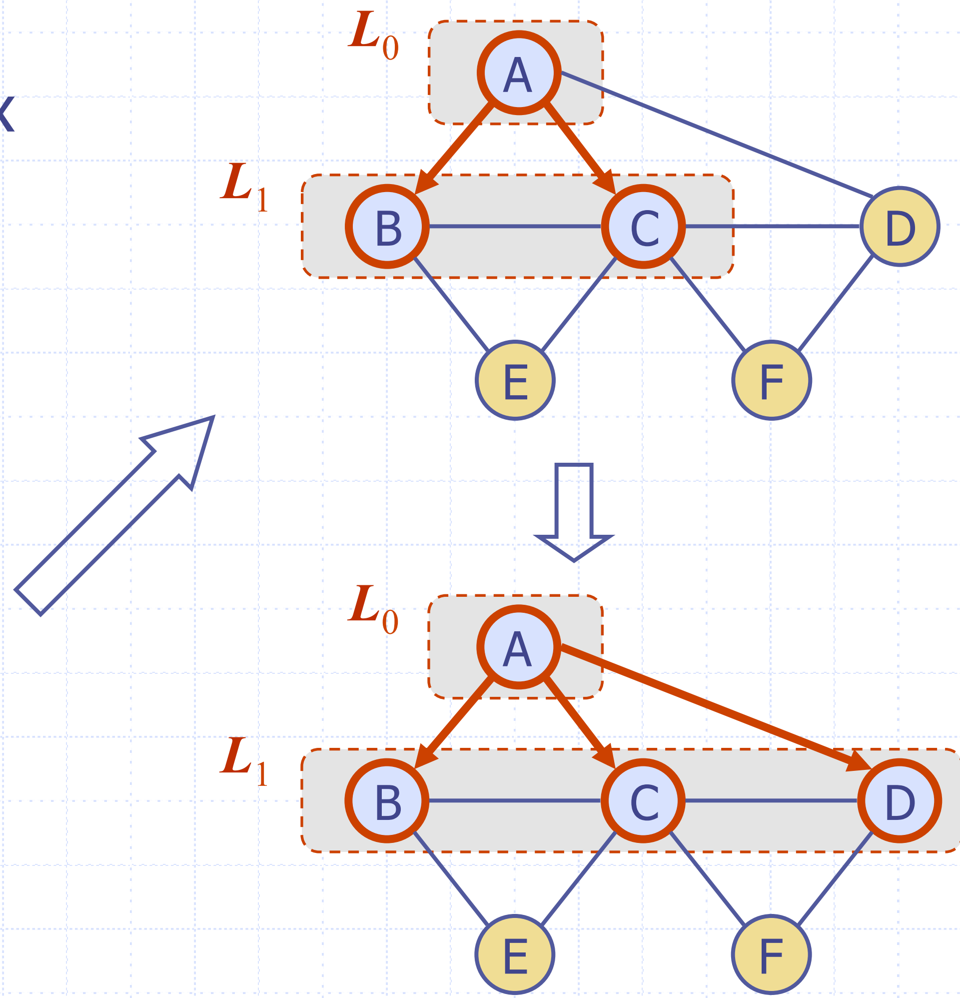
```
/** Performs breadth-first search of Graph g starting at Vertex u. */
public static <V,E> void BFS(Graph<V,E> g, Vertex<V> s,
                Set<Vertex<V>> known, Map<Vertex<V>,Edge<E>> forest) {
  PositionalList<Vertex<V>> level = new LinkedPositionalList<>( );
  known.add(s);
  level.addLast(s);                         // first level includes only s
  while (!level.isEmpty( )) {
    PositionalList<Vertex<V>> nextLevel = new LinkedPositionalList<>( );
    for (Vertex<V> u : level)
      for (Edge<E> e : g.outgoingEdges(u)) {
        Vertex<V> v = g.opposite(u, e);
        if (!known.contains(v)) {
          known.add(v);
          forest.put(v, e);                 // e is the tree edge that discovered v
          nextLevel.addLast(v);             // v will be further considered in next pass
        }
      }
    level = nextLevel;                      // relabel 'next' level to become the current
  }
}
```

Example

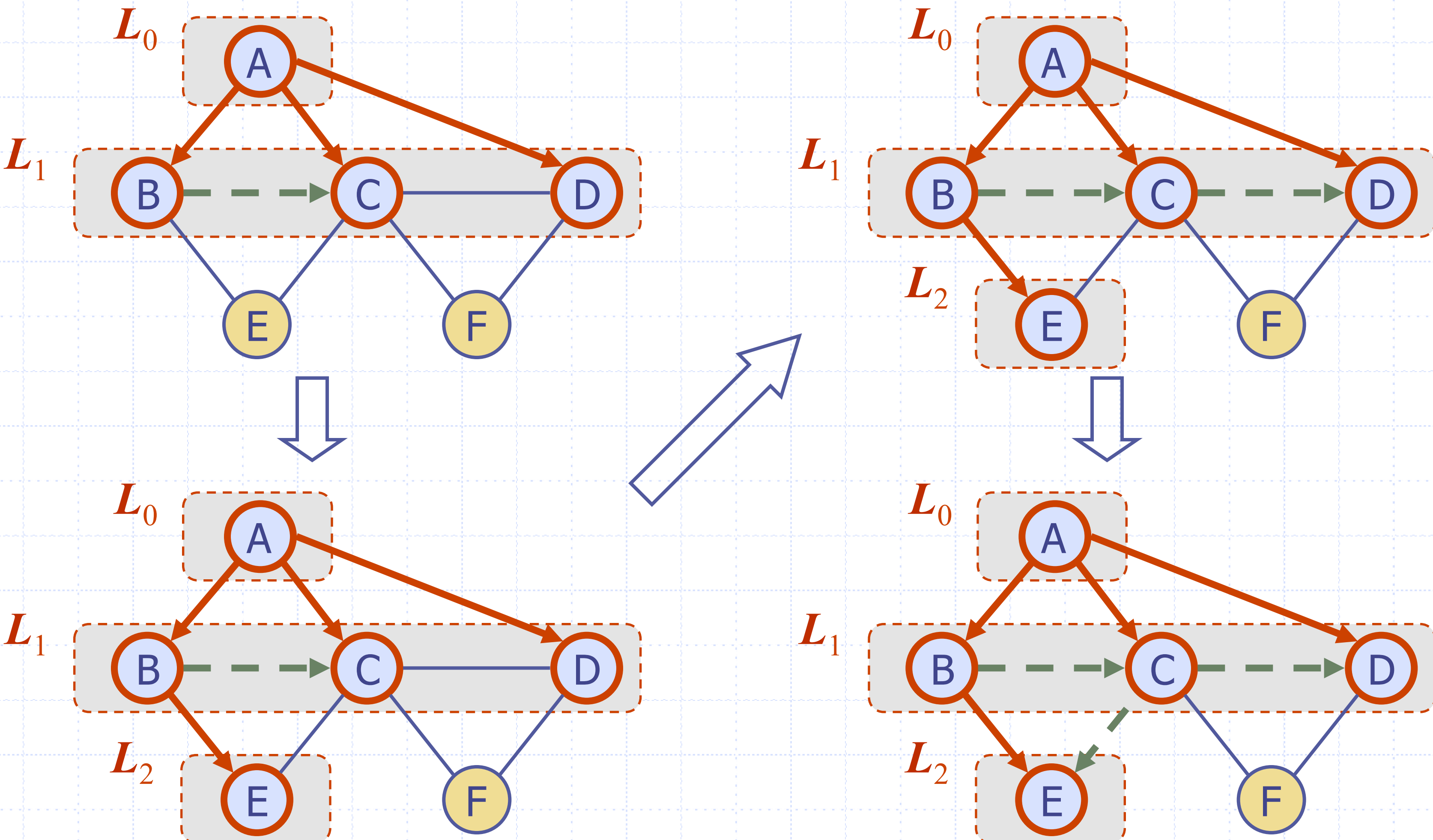


unexplored vertex
visited vertex
unexplored edge
discovery edge
cross edge

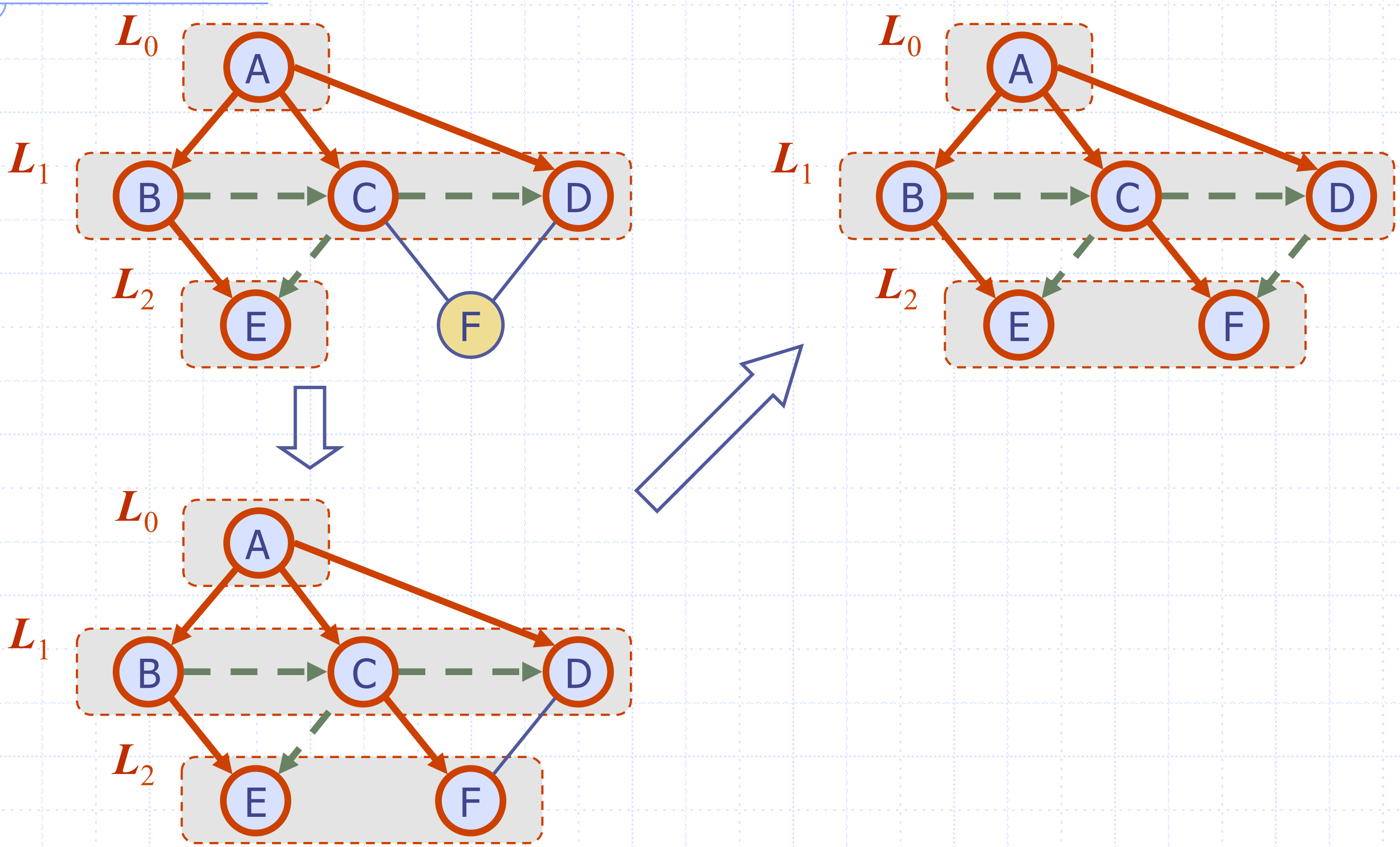




Example (cont.)



Example (cont.)



Properties

Notation

G_s: connected component of s

Property 1

$BFS(G, s)$ visits all the vertices and edges of G_s

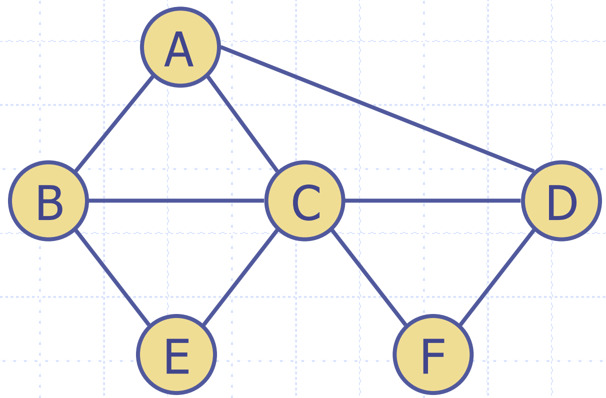
Property 2

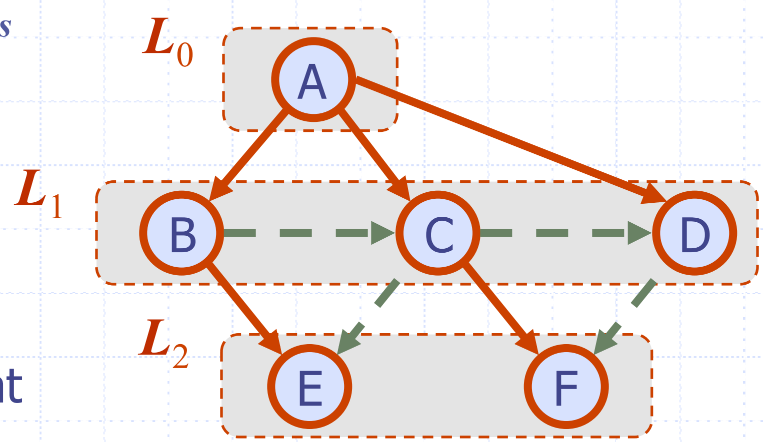
The discovery edges labeled by $BFS(G, s)$ form a spanning tree T_s of G_s

Property 3

For each vertex v in L_i

- The path of T_s from s to v has i edges
- Every path from s to v in G_s has at least i edges





Analysis

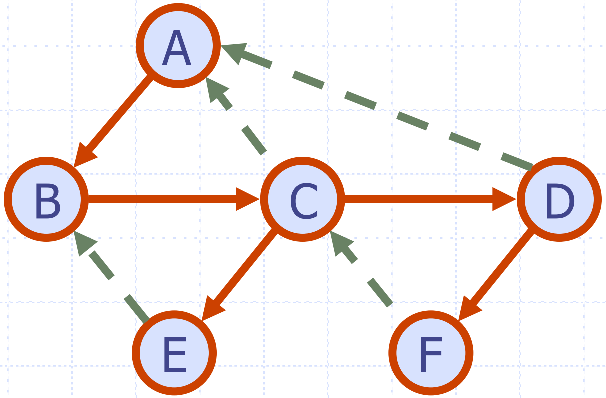
- Setting/getting a vertex/edge label takes $O(1)$ time
- Each vertex is labeled twice
  - once as UNEXPLORED
  - once as VISITED
- Each edge is labeled twice
  - once as UNEXPLORED
  - once as DISCOVERY or CROSS
- Each vertex is inserted once into a sequence L_i
- Method incidentEdges is called once for each vertex
- BFS runs in $O(n + m)$ time provided the graph is represented by the adjacency list structure
  - Recall that $\Sigma_v \deg(v) = 2m$

Applications

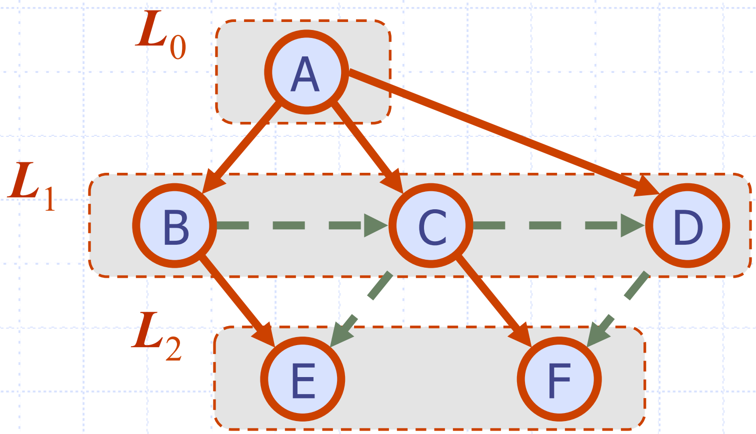
- Using the template method pattern, we can specialize the BFS traversal of a graph G to solve the following problems in $O(n + m)$ time
  - Compute the connected components of G
  - Compute a spanning forest of G
  - Find a simple cycle in G, or report that G is a forest
  - Given two vertices of G, find a path in G between them with the minimum number of edges, or report that no such path exists

DFS vs. BFS

Applications	DFS	BFS
Spanning forest, connected components, paths, cycles	√	√
Shortest paths		√
Biconnected components	√	



DFS



BFS

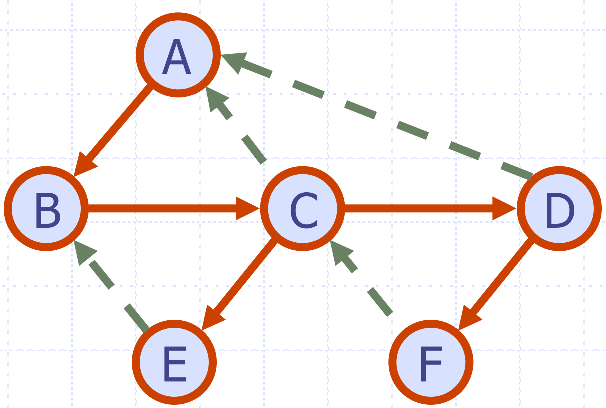
DFS vs. BFS (cont.)

Back edge (v,w)

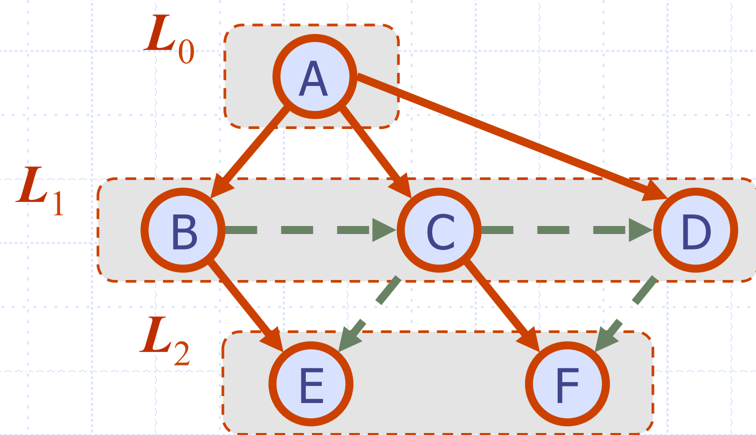
- w is an ancestor of v in the tree of discovery edges

Cross edge (v,w)

- w is in the same level as v or in the next level



DFS



BFS