

## 1. Detailed computational results for the B&P procedure based on BDDF

In the following tables (Table 1 to 6) we include the detailed computational results of our B&P procedure based on BDDF for each tested instance. An instance is solved to guaranteed optimality if and only if  $LB = UB$ . The information provided in the different columns is as follows:

- $\#id$  = instance ID number
- $UB$  = best found upper bound (best solution)
- $LB\ root$  = lower bound in the root node of the B&B search tree
- $LB$  = best found (global) lower bound
- $\#iter$  = total number of CG iterations across all nodes in the search tree
- $\#iter\ root$  = number of CG iterations in the root node
- $\#nodes$  = number of nodes in the B&B search tree (excluding the root node)
- $time\ LP$  = CPU time (in seconds) for solving the LPs in all nodes
- $time\ LP\ root$  = CPU time (in seconds) for solving the LP in the root node
- $time$  = total CPU time (in seconds)

**Table 1** B&P results per instance for  $m = 2$  and  $n = 40$ 

<i>#id</i>	<i>UB</i>	<i>LB</i>	<i>root</i>	<i>LB</i>	<i>#iter</i>	<i>#iter</i>	<i>root</i>	<i>#nodes</i>	<i>time LP</i>	<i>time LP</i>	<i>root</i>	<i>time</i>
1	606		584	606	484		201	2	3.16		0.65	4.40
6	3,886		3,875	3,886	345		220	2	1.71		0.72	3.83
11	9,617		9,592	9,617	303		237	1	1.13		0.66	2.04
16	38,356		38,277	38,356	4,293		336	10	26.77		1.07	69.35
21	41,048		41,048	41,048	477		477	0	1.22		1.22	1.23
26	87		87	87	151		151	0	0.49		0.49	0.49
31	3,812		3,758	3,812	976		225	3	10.07		1.05	13.76
36	10,713		10,660	10,713	7,232		329	29	31.65		1.42	64.10
41	30,802		30,798	30,802	429		413	1	2.15		1.81	4.22
46	34,146		34,146	34,146	565		565	0	1.49		1.49	1.50
51	0		0	0	1		1	0	0.04		0.04	0.04
56	1,279		1,272	1,279	307		211	1	1.56		0.71	1.96
61	11,488		11,302	11,488	8,546		338	18	52.81		1.29	110.45
66	35,279		35,130	35,279	13,731		364	35	89.80		2.19	229.61
71	47,952		47,935	47,952	739		480	2	5.12		1.93	14.62
76	0		0	0	1		1	0	0.02		0.02	0.03
81	571		451	571	1,785		288	8	19.95		1.69	27.62
86	6,048		5,996	6,048	1,216		407	3	6.05		2.49	11.43
91	26,075		26,075	26,075	482		482	0	2.37		2.37	2.38
96	66,116		66,110	66,116	662		513	1	2.97		2.30	4.61
101	0		0	0	1		1	0	0.02		0.02	0.03
106	0		0	0	1		1	0	0.03		0.03	0.03
111	17,936		17,897	17,936	2,069		400	7	11.66		2.14	31.09
116	25,870		25,764	25,870	4,007		366	8	21.32		2.05	51.32
121	64,516		64,507	64,516	573		460	2	3.30		2.26	7.00

**Table 2** B&P results per instance for  $m = 4$  and  $n = 40$ 

<i>#id</i>	<i>UB</i>	<i>LB</i>	<i>root</i>	<i>LB</i>	<i>#iter</i>	<i>#iter</i>	<i>root</i>	<i>#nodes</i>	<i>time LP</i>	<i>time LP</i>	<i>root</i>	<i>time</i>
1	439		438	439	186		140	1	0.25		0.17	0.36
6	2,374		2,372	2,374	220		194	1	0.67		0.41	1.18
11	5,737		5,735	5,737	208		189	1	0.42		0.27	0.78
16	21,493		21,484	21,493	1,442		227	17	6.04		0.97	13.41
21	22,793		22,793	22,793	318		318	0	0.46		0.46	0.46
26	88		88	88	113		113	0	0.21		0.21	0.21
31	2,525		2,496	2,525	342		157	1	1.32		0.41	2.05
36	6,420		6,355	6,420	1,044		214	4	5.29		0.43	10.20
41	17,685		17,633	17,685	615		193	2	4.31		1.07	8.97
46	19,124		19,124	19,124	268		268	0	0.75		0.75	0.76
51	0		0	0	1		1	0	0.01		0.01	0.01
56	826		798	826	588		121	5	2.75		0.23	4.42
61	7,357		7,315	7,357	2,738		214	37	15.30		1.28	39.04
66	20,251		20,247	20,251	317		303	1	1.39		1.21	1.97
71	26,740		26,740	26,740	381		381	0	0.99		0.99	1.00
76	0		0	0	1		1	0	0.01		0.01	0.01
81	564		540	564	578		171	8	2.04		0.35	3.79
86	4,725		4,719	4,725	279		216	2	1.42		0.89	2.28
91	15,569		15,557	15,569	392		254	3	2.30		0.82	4.83
96	36,266		36,266	36,266	330		330	0	0.77		0.77	0.77
101	0		0	0	1		1	0	0.01		0.01	0.02
106	0		0	0	1		1	0	0.01		0.01	0.01
111	11,263		11,212	11,263	926		146	5	4.39		0.90	8.08
116	15,566		15,539	15,566	1,113		252	3	4.99		0.74	9.00
121	35,751		35,739	35,751	897		301	6	4.96		0.98	11.29

**Table 3** B&P results per instance for  $m = 2$  and  $n = 50$ 

<i>#id</i>	<i>UB</i>	<i>LB</i>	<i>root</i>	<i>LB</i>	<i>#iter</i>	<i>#iter</i>	<i>root</i>	<i>#nodes</i>	<i>time LP</i>	<i>time LP</i>	<i>root</i>	<i>time</i>
1	1,268	1,232	1,268	935	220	3	15.37	1.49	19.68			
6	14,272	14,261	14,272	627	445	2	5.51	2.42	10.55			
11	23,028	23,000	23,028	685	427	1	4.59	1.84	8.69			
16	46,072	46,011	46,072	8,638	501	20	50.85	2.67	159.61			
21	111,069	111,067	111,069	721	626	1	4.11	3.50	7.76			
26	26	26	26	244	244	0	1.51	1.51	1.52			
31	5,378	5,289	5,378	1,692	367	6	34.63	2.21	49.74			
36	18,956	18,891	18,956	1,574	441	4	12.40	2.93	27.69			
41	38,058	37,952	38,058	2,584	524	5	23.06	3.57	62.93			
46	82,105	82,084	82,105	3,617	600	4	18.41	3.62	51.26			
51	0	0	0	1	1	0	0.04	0.04	0.04			
56	761	730	761	622	242	2	8.54	1.72	11.76			
61	13,682	13,582	13,682	3,238	402	5	27.69	2.94	49.80			
66	40,907	40,907	40,907	781	781	0	4.11	4.11	4.12			
71	78,532	78,532	78,532	554	554	0	3.28	3.28	3.29			
76	0	0	0	1	1	0	0.05	0.05	0.06			
81	542	538	542	575	434	1	5.24	3.59	6.30			
86	12,557	12,267	12,557	22,169	547	44	99.21	4.31	287.87			
91	47,349	47,293	47,349	3,846	475	10	28.22	3.36	86.36			
96	92,822	92,801	92,822	1,258	595	5	9.33	4.01	26.22			
101	0	0	0	1	1	0	0.04	0.04	0.05			
106	0	0	0	1	1	0	0.05	0.05	0.06			
111	15,564	15,543	15,564	1,009	517	3	5.06	3.83	8.47			
116	19,608	19,520	19,608	10,005	495	19	48.65	3.99	133.97			
121	41,696	41,696	41,696	800	800	0	3.36	3.36	3.36			

**Table 4** B&P results per instance for  $m = 4$  and  $n = 50$ 

<i>#id</i>	<i>UB</i>	<i>LB</i>	<i>root</i>	<i>LB</i>	<i>#iter</i>	<i>#iter</i>	<i>root</i>	<i>#nodes</i>	<i>time LP</i>	<i>time LP</i>	<i>root</i>	<i>time</i>
1	785		777	785	342		173	3	2.15		0.64	4.04
6	8,317		8,298	8,317	481		298	2	2.51		0.73	4.69
11	12,879		12,871	12,879	404		292	2	3.73		1.13	8.02
16	25,376		25,376	25,376	338		338	0	1.51		1.51	1.51
21	59,440		59,440	59,440	450		450	0	1.39		1.39	1.39
26	54		54	54	182		182	0	0.48		0.48	0.48
31	3,061		3,061	3,061	243		243	0	0.74		0.74	0.75
36	10,796		10,794	10,796	288		274	1	1.64		1.27	2.48
41	21,806		21,783	21,806	1,865		370	4	13.01		1.67	28.24
46	44,455		44,452	44,455	448		344	2	4.09		2.77	6.52
51	0		0	0	1		1	0	0.01		0.01	0.01
56	570		538	570	689		180	4	3.57		0.45	5.42
61	7,898		7,850	7,898	8,417		263	37	74.56		2.80	146.32
66	23,138		23,138	23,138	495		495	0	2.81		2.81	2.83
71	42,645		42,625	42,645	2,652		361	9	23.32		2.22	56.51
76	0		0	0	1		1	0	0.02		0.02	0.03
81	495		478	495	282		219	1	1.47		0.77	1.66
86	8,369		8,328	8,369	1,444		321	3	11.76		2.65	24.22
91	26,551		26,546	26,551	463		360	1	4.48		2.90	7.37
96	50,326		50,312	50,326	1,006		321	5	14.07		2.24	36.07
101	0		0	0	1		1	0	0.02		0.02	0.02
106	0		0	0	1		1	0	0.02		0.02	0.02
111	10,069		10,047	10,069	955		313	7	12.21		2.90	29.11
116	11,552		11,519	11,552	1,655		268	7	18.29		2.82	43.54
121	23,792		23,768	23,792	1,371		293	4	11.39		1.97	24.50

**Table 5** B&P results per instance for  $m = 2$  and  $n = 100$ 

<i>#id</i>	<i>UB</i>	<i>LB</i>	<i>root</i>	<i>LB</i>	<i>#iter</i>	<i>#iter</i>	<i>root</i>	<i>#nodes</i>	<i>time LP</i>	<i>time LP</i>	<i>root</i>	<i>time</i>
1	3,339	3,314	3,339	2,999	549	5	104.77	11.60	156.18			
6	30,665	30,644	30,665	9,784	840	13	403.82	21.12	855.91			
11	93,894	93,894	93,894	1,134	1,134	0	17.40	17.40	17.42			
16	209,100	209,057	209,100	16,470	1,414	8	206.91	31.48	816.28			
21	457,836	457,814	457,836	16,402	1,873	10	248.53	28.16	1,096.78			
26	92	92	92	391	391	0	8.43	8.43	8.45			
31	12,729	12,725	12,729	940	724	1	27.71	19.02	43.98			
36	56,671	56,574	56,671	15,383	1,056	17	495.50	29.59	1,435.73			
41	237,964	237,770	237,906	40,421	1,395	24	834.62	36.01	5,086.75			
46	422,831	422,804	422,831	41,848	1,612	25	666.84	36.06	3,511.15			
51	0	0	0	1	1	0	0.14	0.14	0.16			
56	5,047	4,983	5,047	2,309	683	4	57.09	15.15	91.75			
61	45,573	45,411	45,473	56,982	1,033	34	1,275.14	19.90	5,021.28			
66	126,513	126,405	126,430	71,940	1,126	45	1,003.64	25.51	5,070.91			
71	327,305	327,300	327,305	2,650	1,905	2	60.02	48.01	154.00			
76	0	0	0	1	1	0	0.14	0.14	0.16			
81	908	791	908	3,935	1,523	5	110.89	50.68	146.54			
86	36,686	36,206	36,209	163,328	1,360	297	1,318.04	46.00	4,981.33			
91	129,929	129,588	129,802	52,557	1,259	24	945.45	38.14	5,021.95			
96	254,194	254,140	254,145	42,980	1,539	23	608.86	32.32	5,128.69			
101	0	0	0	1	1	0	0.24	0.24	0.28			
106	0	0	0	1	1	0	0.23	0.23	0.26			
111	84,220	84,003	84,055	53,064	1,468	26	909.90	58.55	5,075.99			
116	191,205	191,085	191,107	77,643	1,465	41	832.52	36.03	5,039.19			
121	242,019	241,954	241,957	67,337	1,706	36	808.76	54.49	5,021.20			

**Table 6** B&P results per instance for  $m = 4$  and  $n = 100$ 

<i>#id</i>	<i>UB</i>	<i>LB</i>	<i>root</i>	<i>LB</i>	<i>#iter</i>	<i>#iter</i>	<i>root</i>	<i>#nodes</i>	<i>time LP</i>	<i>time LP</i>	<i>root</i>	<i>time</i>
1	2,001	1,990	2,001	708	393	3	14.54	4.03	26.54			
6	16,893	16,893	16,893	631	631	0	7.54	7.54	7.56			
11	50,234	50,196	50,216	101,050	710	393	1,677.41	9.06	4,943.34			
16	110,219	110,104	110,219	34,870	713	57	971.12	7.52	2,905.22			
21	237,392	237,389	237,392	1,731	1,146	1	30.26	20.16	51.67			
26	141	141	141	378	378	0	5.32	5.32	5.34			
31	7,130	7,080	7,130	8,102	473	23	442.34	5.81	712.41			
36	30,791	30,773	30,791	1,053	696	1	24.25	9.66	45.38			
41	126,185	126,130	126,185	10,125	690	22	254.39	14.51	914.88			
46	219,536	219,526	219,536	3,448	1,017	4	59.95	17.04	190.07			
51	0	0	0	1	1	0	0.10	0.10	0.11			
56	3,076	3,021	3,076	2,873	522	7	64.60	8.18	94.04			
61	24,863	24,805	24,825	134,520	742	415	1,586.59	7.92	4,976.25			
66	67,967	67,947	67,967	39,896	840	119	673.85	17.34	2,242.08			
71	170,694	170,674	170,675	70,736	1,109	104	1,233.12	24.32	5,012.55			
76	0	0	0	1	1	0	0.10	0.10	0.11			
81	819	754	819	1,417	844	3	24.33	14.35	28.48			
86	21,282	21,202	21,256	75,623	636	136	1,406.58	21.54	5,025.99			
91	70,606	70,582	70,606	35,852	919	74	615.14	15.18	2,357.56			
96	133,587	133,571	133,587	24,286	995	42	231.11	26.19	751.82			
101	0	0	0	1	1	0	0.13	0.13	0.15			
106	0	0	0	1	1	0	0.10	0.10	0.12			
111	46,705	46,616	46,658	56,541	776	67	1,223.29	11.38	5,016.42			
116	101,571	101,514	101,517	50,825	1,004	86	1,281.40	29.08	5,028.39			
121	127,618	127,593	127,618	30,859	1,018	60	521.97	23.93	1,866.95			