$y_0 = a\sin(x_0 - \theta_0), y_1 = a\sin(x_1 - \theta_0)$
$\frac{y_1}{y_0} = \frac{\sin(x_1-\theta_0)}{\sin(x_0-\theta_0)}$
???
$(x', y', z') = (\cos y \cos x, \cos y \sin x, \sin y)$
???

Instead of mapping to $\mathbb{R}^3$, as we did for other surfaces, we can simply embed into $\mathbb{R}^4$.

Given points $\mathbf{x}, \mathbf{y}$,
They are almost always not equal or antipodal.
Let $\mathbf{y}'$ be a renormalized form of $\mathbf{y} - \langle \mathbf{x}, \mathbf{y} \rangle$.
The geodesic is $\mathbf{x}\cos\theta + \mathbf{y}'\sin\theta$.
The direction is $\mathbf{y}'$.
The distance is $\arccos \langle \mathbf{x}, \mathbf{y} \rangle$.
The vector is $\arccos \langle \mathbf{x}, \mathbf{y} \rangle \, \mathbf{y}'$.
If we're dealing with a 2-sphere:
We have a rotation of $\arccos \langle \mathbf{x}, \mathbf{y} \rangle$ around the axis $\mathbf{x} \times \mathbf{y}'$.
This gives the quaternion $\langle \mathbf{x}, \mathbf{y} \rangle \, (\mathbf{x} \times \mathbf{y}')$.
We then convert this to a rotation matrix with:

$$Q = \begin{bmatrix} 1 - 2y^2 - 2z^2 & 2xy - 2zw & 2xz + 2yw \\ 2xy + 2zw & 1 - 2x^2 - 2z^2 & 2yz - 2xw \\ 2xz - 2yw & 2yz + 2xw & 1 - 2x^2 - 2y^2 \end{bmatrix}$$

???
If we're not dealing with a 2-sphere, it gets more complex.
Let $\mathbf{u}, \mathbf{v}$ be unit vectors that are linearly independant with $\mathbf{x}, \mathbf{y}$. You can pick them at random since this will almost certainly be true,
???
Given a point $\mathbf{x}$ and a vector $\mathbf{v}$,
$\mathbf{y} = \mathbf{x}\cos|\mathbf{v}| + \frac{\mathbf{v}}{|\mathbf{v}|}\sin|\mathbf{v}|$
$\mathbf{y}' = \frac{\mathbf{v}}{|\mathbf{v}|}$
The rotation matrix can be found as before.