BACKGROUND

The prime decomposition theorem states that a compact, connected, orientable 3-manifold can be decomposed into a connected sum of prime manifolds [1], and that this is unique up to insertion or deletion of copies of S^3 [2]. The connected sum of two spaces is made by removing a ball from each and identifying their bounding spheres.

The torus decomposition theorem states that there is a minimal collection of disjointly embedded incompressible tori such that cutting along the edge of each yields components that are each either atoroidal or Seifert-fibered. It also states that this collection is unique up to isomorphism.

The geometrization theorem states that any 3-manifold can be decomposed canonically into submanifolds which each have one of the following eight geometries: $S^3, \mathbb{E}^3, \mathbb{H}^3, S^2 \times \mathbb{R}, \mathbb{H}^2 \times \mathbb{R}, \tilde{SL}(2, \mathbb{R})$, Nil geometry, and Sol geometry. This is done by decomposing along the spheres given by the prime decomposition theorem and tori given by the torus decomposition theorem. [3] [5] [4]

My program will likely never support torus decomposition, so it most likely will never be able to show every manifold.

My program will allow you to smoothly create a connected sum of several geometries. "Smoothness" means that the boundaries that are identified have the same curvatures in each surface. If they are not smoothly identified, then they will appear to have different curvatures from each side. As a result, a geodesic that barely intersects the border can have a very different path than one that barely misses.

There has been previous work in visualizing S^3 and $\mathbb{H}^3$ and quotient spaces thereof. In particular, Jeff Weeks has written a program that can view compact quotient spaces of $S^3, \mathbb{E}^3$, and $\mathbb{H}^3$. [6]

My program will allow non-compact spaces, and it will allow different spaces to be glued together.

Corvin Zahn made a computer generated video illustrating a wormhole. He used a solution to Einstein's field equations that had previously been found, and simulated the camera moving through this wormhole. He did not go into specifics about the program used. He detailed what kind of manifold was used, but not how he used it. I have emailed him with questions regarding it, but he is yet to respond. [7]

References

[1] H. Kneser. Geschlossen flächen in dreidimensionalen mannigfaltigkeiten. *Jahresbericht der Deutschen Mathematiker Vereinigung*, 38:248–260, 1929.

[2] J. Milnor. A unique decomposition theorem for 3-manifolds. *American Journal of Mathematics*, 84(1):1–7, January 1962.
[3] Grisha Perelman. The entropy formula for the ricci flow and its geometric applications. http://arxiv.org/abs/math.DG/0211159, November 2002.
[4] Grisha Perelman. Finite extinction time for the solutions to the ricci flow on certain three-manifolds. http://arxiv.org/abs/math.DG/0307245, July 2003.
[5] Grisha Perelman. Ricci flow with surgery on three-manifolds. http://arxiv.org/abs/math.DG/0303109, March 2003.
[6] Jeff Weeks. Curved spaces. http://www.geometrygames.org/CurvedSpaces/index.html.
[7] Corvin Zahn. Flight through a wormhole. http://www.spacetimetravel.org/wurmlochflug/wurmlochflug.html, March 2008.