Method 2
The general linear model

The General Linear Model

BSc Program in Cognitive Science
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Conceptual fundations and history of the GLM

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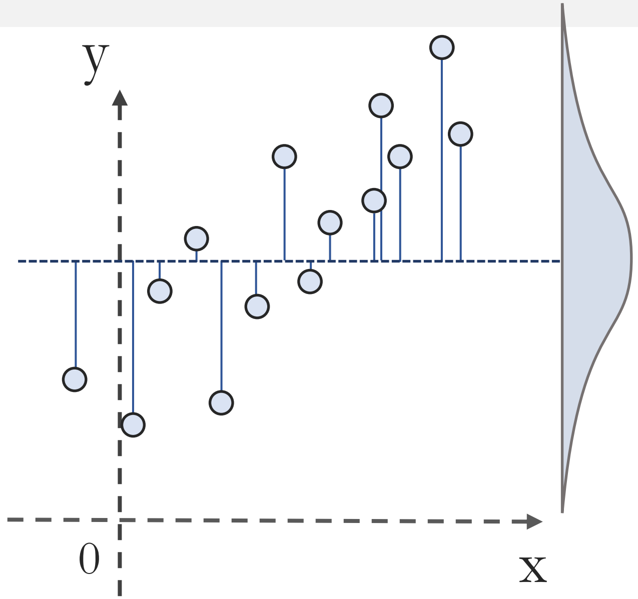
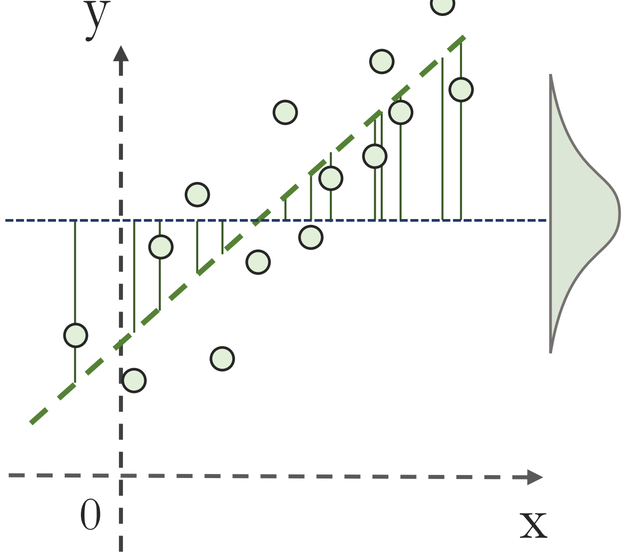
Where I messed up

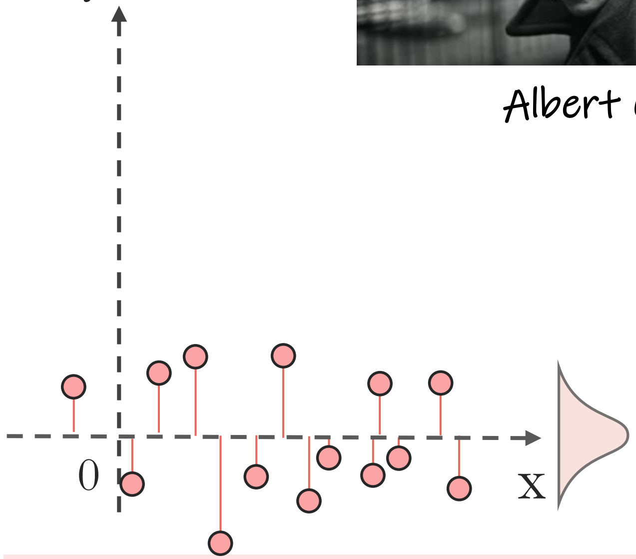
« Mal nommer un objet, c'est ajouter au malheur de ce monde. »

"To call things by incorrect names is to add to the world's misery."

Albert Camus







$$TSS = \sum_{i=1}^{n} (y_i - \bar{y})^2 \quad = \quad ESS = \sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2 \quad + \quad RSS = \sum_{i=1}^{n} (\hat{\varepsilon}_i)^2 = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

Total Sum of Squares (TSS, or SST)

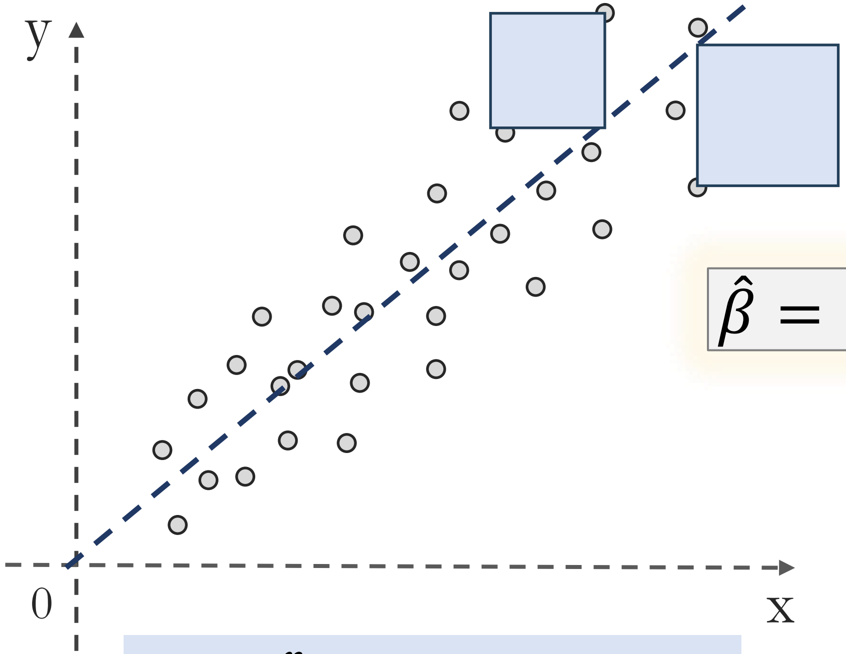
Explained Sum of Squares (ESS)
or Model Sum of Squares
or Sum of Squares due to Regression (SSR)

Residual Sum of Squares (RSS)
or Sum of Squared Residuals (SSR)
or Sum of Squared Estimate of Errors (SSE)

Residuals	Errors
$r_i = y_i - (\hat{\alpha} + \hat{\beta} xi))$	$\epsilon_i = y_i - (\alpha + \beta xi))$

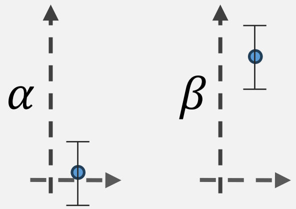
Towards Bayesian Data Analysis

Ordinary Least Squares



$$\min_{\alpha,\beta} \sum_{i=1}^{n} \big(y_i - (\alpha + \beta x_i)\big)^2$$

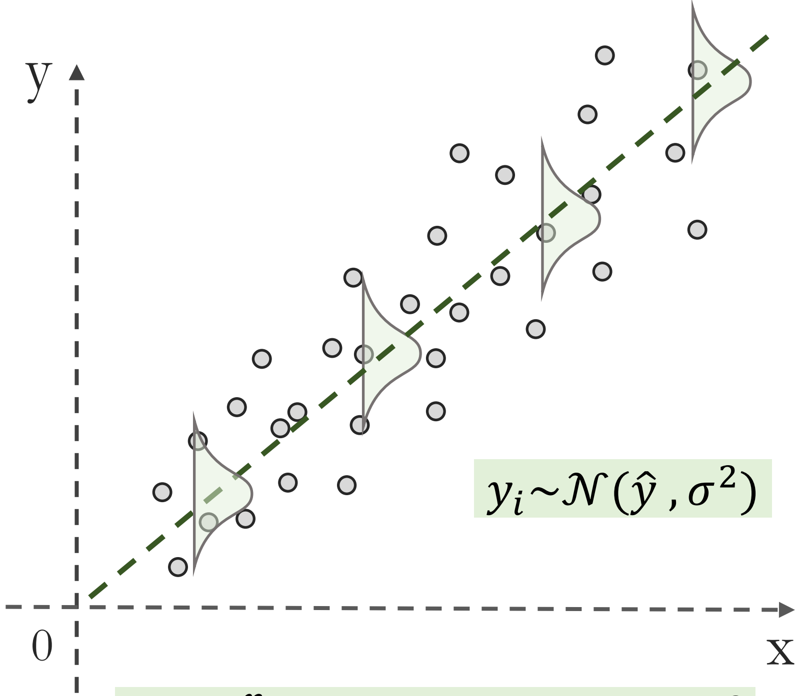
Adding a standard error



$$\hat{\beta} = (X^t X)^{-1} X^t y$$

If the errors from the linear model are independent and normally distributed, then the least squares estimate is also the maximum likelihood estimate.

Maximum Likelihood Estimate



$$y_i \sim \mathcal{N}(\hat{y}, \sigma^2)$$

$$\max_{\alpha,\beta} \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y_i - (\alpha + \beta x_i)}{\sigma}\right)^2}$$

$$\rightarrow \max_{\alpha,\beta} \sum_{i=1}^{n} - \ln\left(\sigma\sqrt{2\pi}\right) - \frac{1}{2}\left(\frac{y_i - (\alpha + \beta x_i)}{\sigma}\right)^2$$

Propagation of uncertainty

$$P(\beta|y) \propto P(y|\beta)P(\beta)$$

Likelihood Prior

$$P(\beta|y) = \frac{P(y|\beta)P(\beta)}{P(y)}$$

Marginal probability

$$\int P(y|\beta)P(\beta)d\beta$$

Bayesian inference helps us:

- Propagate uncertainties across parameters using probability and simulations
- Add prior information about expected values