Log Linear Models for Text Classification

Mausam

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Introduction

- So far we've looked at "generative models"
  - Naive Bayes
- But there is now much use of conditional or discriminative probabilistic models in NLP, Speech, IR (and ML generally)
- Because:
  - They give high accuracy performance
  - They make it easy to incorporate lots of linguistically important features
  - They allow automatic building of language independent, retargetable NLP modules

Joint vs. Conditional Models

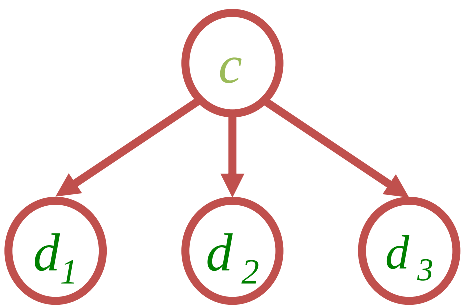
- We have some data {(*d*, *c*)} of paired observations *d* and hidden classes *c*.
- Joint (generative) models place probabilities over both observed data and the hidden stuff (generate the observed data from hidden stuff):
  - All the classic Stat-NLP models:
    - *n*-gram models, Naive Bayes classifiers, hidden Markov models, probabilistic context-free grammars, IBM machine translation alignment models

Joint vs. Conditional Models

- Discriminative (conditional) models take the data as given, and put a probability over hidden structure given the data:
    - Logistic regression, conditional loglinear or maximum entropy models, conditional random fields
    - Also, SVMs, (averaged) perceptron, etc. are discriminative classifiers (but not directly probabilistic)

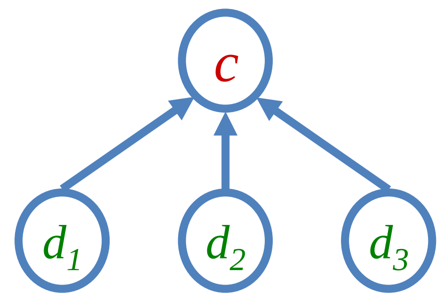
Bayes Net/Graphical Models

- Bayes net diagrams draw circles for random variables, and lines for direct dependencies
- Some variables are observed; some are hidden
- Each node is a little classifier (conditional probability table) based on incoming arcs



Naive Bayes

Generative



Logistic Regression

Discriminative

Conditional vs. Joint Likelihood

- A *joint* model gives probabilities $P(d,c)$ and tries to maximize this joint likelihood.
  - It turns out to be trivial to choose weights: just relative frequencies.
- A *conditional* model gives probabilities $P(c|d)$. It takes the data as given and models only the conditional probability of the class.
  - We seek to maximize conditional likelihood.
  - Harder to do (as we'll see…)
  - More closely related to classification error.

Text Categorization with Word Features

Data BUSINESS: Stocks hit a yearly low …
Label: BUSINESS Features {…, stocks, hit, a, yearly, low, …}

(Zhang and Oles 2001)

- Features are presence of each word in a document and the document class (they do feature selection to use reliable indicator words)
- Tests on classic Reuters data set (and others)
  - Naïve Bayes: 77.0% F_1
  - Logistic regression: 86.4%
  - Support vector machine: 86.5%

Case Study: Word Senses

- Words have multiple distinct meanings, or senses:
  - Plant: living plant, manufacturing plant, …
  - Title: name of a work, ownership document, form of address, material at the start of a film, …
- Many levels of sense distinctions
  - Homonymy: totally unrelated meanings (river bank, money bank)
  - Polysemy: related meanings (star in sky, star on tv)
  - Systematic polysemy: productive meaning extensions (metonymy such as organizations to their buildings) or metaphor
  - Sense distinctions can be extremely subtle (or not)
- Granularity of senses needed depends a lot on the task
- Why is it important to model word senses?
  - Translation, parsing, information retrieval?

Word Sense Disambiguation

- Example: living plant vs. manufacturing plant
- How do we tell these senses apart?
  - “context”

The manufacturing plant which had previously sustained the town’s economy shut down after an extended labor strike.

  - Maybe it’s just text categorization
  - Each word sense represents a class
  - Run a naive-bayes classifier?
- Bag-of-words classification works OK for noun senses
  - 90% on classic, shockingly easy examples (line, interest, star)
  - 80% on senseval-1 nouns
  - 70% on senseval-1 verbs

Verb WSD

- Why are verbs harder?
  - Verbal senses less topical
  - More sensitive to structure, argument choice

- Verb Example: "Serve"
  - [function] The tree stump serves as a table
  - [enable] The scandal served to increase his popularity
  - [dish] We serve meals for the homeless
  - [enlist] She served her country
  - [jail] He served six years for embezzlement
  - [tennis] It was Agassi's turn to serve
  - [legal] He was served by the sheriff

Better Features

- There are smarter features:
  - Argument selectional preference:
    - serve NP[meals] vs. serve NP[papers] vs. serve NP[country]
  - Subcategorization:
    - [function] serve PP[as]
    - [enable] serve VP[to]
    - [tennis] serve <intransitive>
    - [food] serve NP {PP[to]}

- Other constraints (Yarowsky 95)
  - One-sense-per-discourse (only true for broad topical distinctions)
  - One-sense-per-collocation (pretty reliable when it kicks in: manufacturing plant, flowering plant)

Complex Features with NB?

- Example: Washington County jail served 11,166 meals last month - a figure that translates to feeding some 120 people three times daily for 31 days.
- So we have a decision to make based on a set of cues:
  - context:jail, context:county, context:feeding, context:meals, …
  - subcat:NP, direct-object-head:meals
- Not clear how build a generative derivation for these:
  - Choose topic, then decide on having a transitive usage, then pick "meals" to be the object's head, then generate other words?
  - Hard to make this work (though maybe possible)
  - No real reason to try

A Discriminative Approach

- View WSD as a discrimination task, directly estimate:

  P(sense | context:jail, context:county,
  context:feeding, context:meals, …
  subcat:NP, direct-object-head:meals, ….)

- Have to estimate multinomial (over senses) where there are a huge number of things to condition on

- Many feature-based classification techniques out there
  - Log-linear models extremely popular in the NLP community!

Feature Representations

Washington County jail served 11,166 meals last month - a figure that translates to feeding some 120 people three times daily for 31 days.

context:jail = 1
context:county = 1
context:feeding = 1
context:game = 0
…
local-context:jail = 1
local-context:meals = 1
…
subcat:NP = 1
subcat:PP = 0
…
object-head:meals = 1
object-head:ball = 0

- Features are indicator functions which count the occurrences of certain patterns in the input
- *We will have different feature values for every pair of input x and class y*

Features

- In NLP uses, usually a feature specifies
  1. an indicator function – a yes/no boolean matching function – of properties of the input and
  2. a particular class

$$\phi_i(x,y) \equiv [\Phi(x) \wedge y = y_j]$$ [Value is 0 or 1]

- Each feature picks out a data subset and suggests a label for it

Example of Features

- context:jail & served:functional
- context:jail & served:dish
- …

- subcat:NP & served:functional
- subcat:NP & served:dish
- …

Feature-Based Linear Classifiers

- Linear classifiers at classification time:
  - Linear function from feature sets $\{\phi_i\}$ to classes $\{y\}$.
  - Assign a weight w_i to each feature ϕ_i.
  - We consider each class for an observed datum x
  - For a pair (x,y), features vote with their weights:
    - vote(y) = $\Sigma w_i \phi_i(x,y)$
    - Choose the class y which maximizes $\Sigma w_i \phi_i(x,y)$
  - We need probabilistic semantics to this method.
    - Log linear classifiers

Exponential Models
(log-linear, maxent, Logistic, Gibbs)

- Model: use the scores as probabilities:

$$p(y|x;w) = \frac{\exp\left(w \cdot \phi(x,y)\right)}{\sum_{y'} \exp\left(w \cdot \phi(x,y')\right)}$$

Make positive

Normalize

- Learning: maximize the (log) conditional likelihood of training data $\{(x_i, y_i)\}_{i=1}^n$

$$L(w) = \sum_{i=1}^{n} \log p(y_i|x_i;w) \qquad w^* = \arg\max_w L(w)$$

  - Prediction: output argmax$_y$ p(y|x;w)

Feature-Based Linear Classifiers

- Exponential (log-linear, maxent, logistic, Gibbs) models:
  - Given this model form, we will choose parameters $\{w_i\}$ that *maximize the conditional likelihood* of the data according to this model.
  - We construct not only classifications, but probability distributions over classifications.
    - There are other (good!) ways of discriminating classes – SVMs, boosting, even perceptrons – but these methods are not as trivial to interpret as distributions over classes.

Derivative of Log-linear Model

$$p(y|x;w) = \frac{\exp\left(w \cdot \phi(x,y)\right)}{\sum_{y'} \exp\left(w \cdot \phi(x,y')\right)}$$

- Unfortunately, argmax_w L(w) doesn't have a close formed solution
- We will have to differentiate and use gradient ascent

$$L(w) = \sum_{i=1}^{n} \log p(y_i|x_i;w)$$

$$L(w) = \sum_{i=1}^{n} \left(w \cdot \phi(x_i, y_i) - \log \sum_{y} \exp(w \cdot \phi(x_i, y)) \right)$$

$$\frac{\partial}{\partial w_j} L(w) = \sum_{i=1}^{n} \left(\phi_j(x_i, y_i) - \sum_{y} p(y|x_i;w)\phi_j(x_i, y) \right)$$

Total count of feature j in correct candidates

Expected count of feature j in predicted candidates

Proof
(Conditional Likelihood Derivative)

- Recall

$$\boxed{p(y|x;w) = \frac{\exp\left(w \cdot \phi(x,y)\right)}{\sum_{y'} \exp\left(w \cdot \phi(x,y')\right)}}$$

$$P(Y \mid X, w) = \prod_{(x,y)\in D} p(y \mid x, w)$$

- We can separate this into two components:

$$\log P(Y \mid X, w) = \sum_{(x,y)\in D} \log \exp \sum_i w_i \phi_i(x,y) \quad - \sum_{(x,y)\in D} \log \sum_{y'} \exp \sum_i w_i \phi_i(x,y')$$

- The derivative is the difference between the derivatives of each component

$$\log P(Y \mid X, w) = N(w) \ - \ D(w)$$

Proof: Numerator

$$\frac{\partial N(w)}{\partial w_i} = \frac{\partial \sum_{(x,y)\in D} \log \exp \sum_i w_i \phi_i(x,y)}{\partial w_i} = \frac{\partial \sum_{(x,y)\in D} \sum_i w_i \phi_i(x,y)}{\partial w_i}$$

$$= \sum_{(x,y)\in D} \frac{\partial \sum_i w_i \phi_i(x,y)}{\partial w_i}$$

$$= \sum_{(x,y)\in D} \phi_i(x,y)$$

Derivative of the numerator is: the empirical count(ϕ_i, y)

Proof: Denominator

$$\frac{\partial D(w)}{\partial w_i} = \frac{\partial \sum_{(x,y)\in D} \log \sum_{y'} \exp \sum_i w_i \phi_i(x,y')}{\partial w_i}$$

$$= \sum_{(x,y)\in D} \frac{1}{\sum_{y''} \exp \sum_i w_i \phi_i(x,y'')} \frac{\partial \sum_{y'} \exp \sum_i w_i \phi_i(x,y')}{\partial w_i}$$

$$= \sum_{(x,y)\in D} \frac{1}{\sum_{y''} \exp \sum_i w_i \phi_i(x,y'')} \sum_{y'} \frac{\exp \sum_i w_i \phi_i(x,y')}{1} \frac{\partial \sum_i w_i \phi_i(x,y')}{\partial w_i}$$

$$= \sum_{(x,y)\in D} \sum_{y'} \frac{\exp \sum_i w_i \phi_i(x,y')}{\sum_{y''} \exp \sum_i w_i \phi_i(x,y'')} \frac{\partial \sum_i w_i \phi_i(x,y')}{\partial w_i}$$

$$= \sum_{(x,y)\in D} \sum_{y'} P(y' \mid x, w)\, \phi_i(x,y')$$

= predicted count(ϕ_i, w)

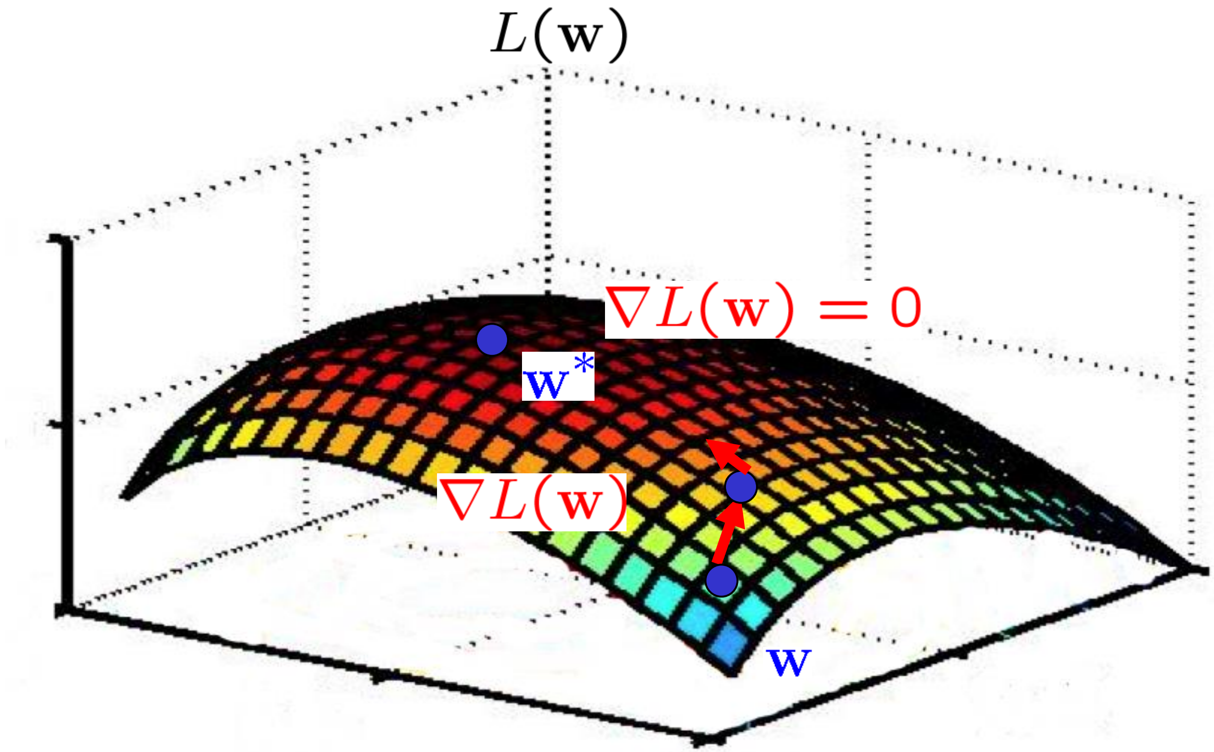
Proof (concluded)

$$\frac{\partial \log P(Y \mid X, w)}{\partial w_i} = \text{actual count}(\phi_i, Y) - \text{predicted count}(\phi_i, w)$$

- The optimum parameters are the ones for which each feature's predicted expectation equals its empirical expectation. The optimum distribution is:
  - Always unique (but parameters may not be unique)
  - Always exists (if feature counts are from actual data).
- These models are also called maximum entropy models because we find the model having maximum entropy and satisfying the constraints:

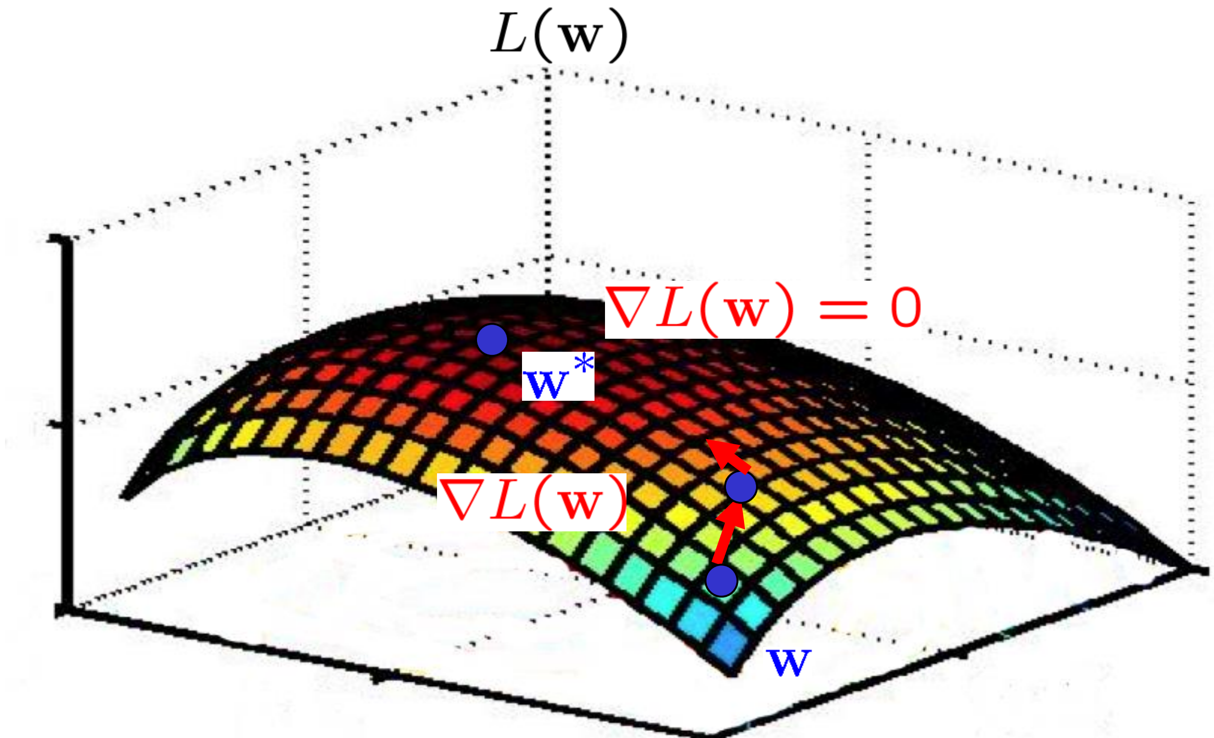
$$E_p(\phi_i) = E_{\tilde{p}}(\phi_i), \forall i$$

Unconstrained Optimization



- Basic idea: move uphill from current guess
- Gradient ascent / descent follows the gradient incrementally
- At local optimum, derivative vector is zero
- Will converge if step sizes are small enough, but not efficient
- All we need is to be able to evaluate the function and its derivative

Unconstrained Optimization



- For convex functions, a local optimum will be global
- Basic gradient ascent isn't very efficient, but there are simple enhancements which take into account previous gradients: conjugate gradient, L-BFGS
- There are special-purpose optimization techniques for maxent, like iterative scaling, but they aren't better

What About Overfitting?

- For Naïve Bayes, we were worried about zero counts in MLE estimates
  - Can that happen here?
- Regularization (smoothing) for Log-linear models
  - Instead, we worry about large feature weights
  - Add a regularization term to the likelihood to push weights towards zero

$$L(w) = \sum_{i=1}^{n} \log p(y_i | x_i; w) - \frac{\lambda}{2} ||w||^2$$

Derivative for Regularized Maximum Entropy

- Unfortunately, $\text{argmax}_w L(w)$ still doesn't have a close formed solution
- We will have to differentiate and use gradient ascent

$$L(w) = \sum_{i=1}^{n} \left(w \cdot \phi(x_i, y_i) - \log \sum_{y} \exp(w \cdot \phi(x_i, y)) \right) - \frac{\lambda}{2} ||w||^2$$

$$\frac{\partial}{\partial w_j} L(w) = \sum_{i=1}^{n} \left(\phi_j(x_i, y_i) - \sum_{y} p(y|x_i; w) \phi_j(x_i, y) \right) - \lambda w_j$$

Total count of feature j in correct candidates

Expected count of feature j in predicted candidates

Big weights are bad

L1 and L2 Regularization

L2 Regularization for Log-linear models

- Instead, we worry about large feature weights
- Add a regularization term to the likelihood to push weights towards zero

$$L(w) = \sum_{i=1}^{n} \log p(y_i | x_i; w) - \frac{\lambda}{2} ||w||^2$$

Regularization Constant

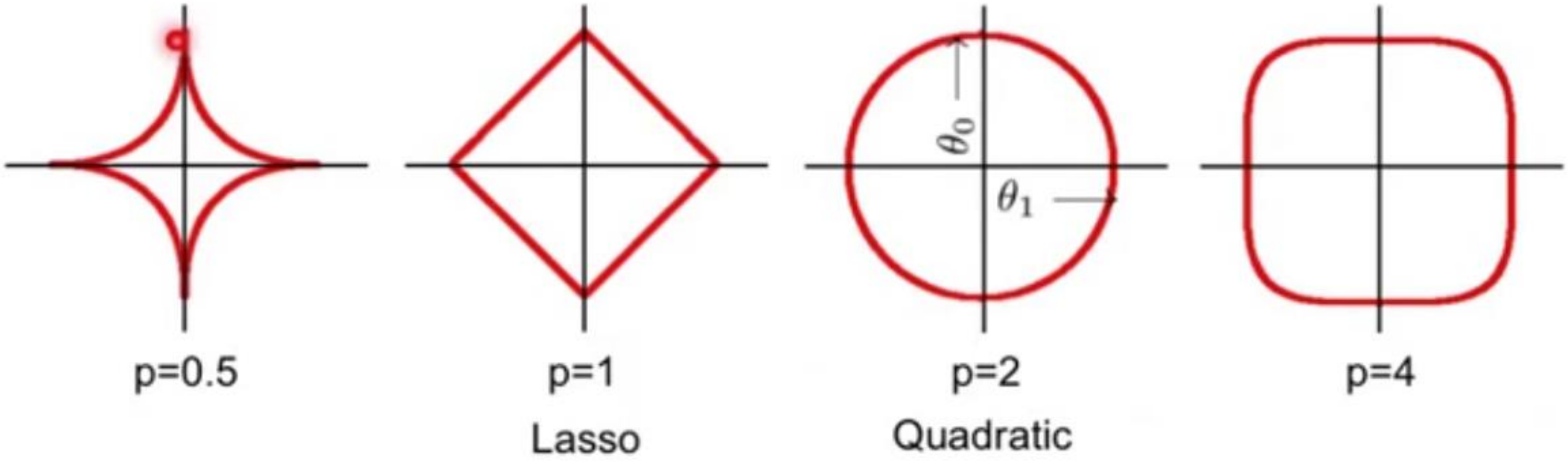
L1 Regularization for Log-linear models

- Instead, we worry about number of active features
- Add a regularization term to the likelihood to push weights to zero

$$L(w) = \sum_{i=1}^{n} \log p(y_i | x_i; w) - \lambda ||w||$$

L_p Norms for Regularization

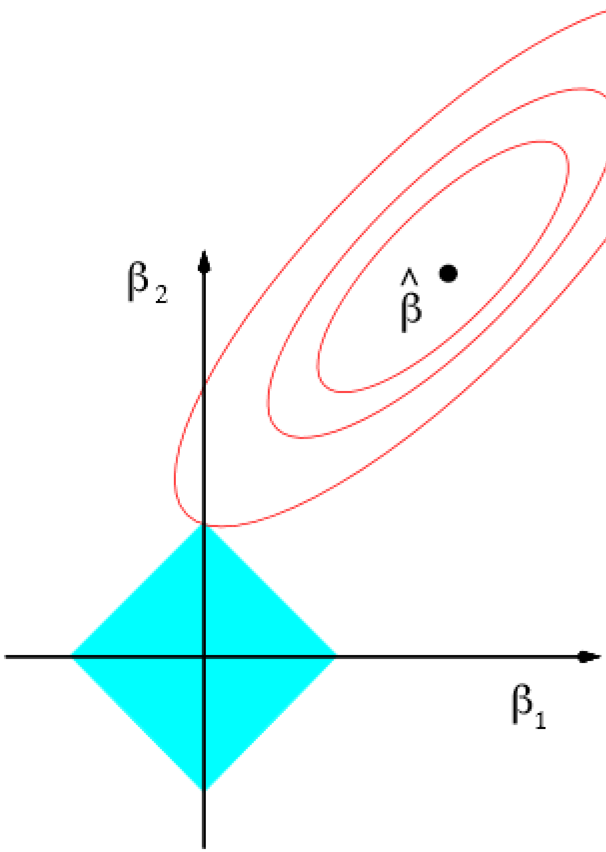
Isosurfaces: $||\theta||_p$ = constant

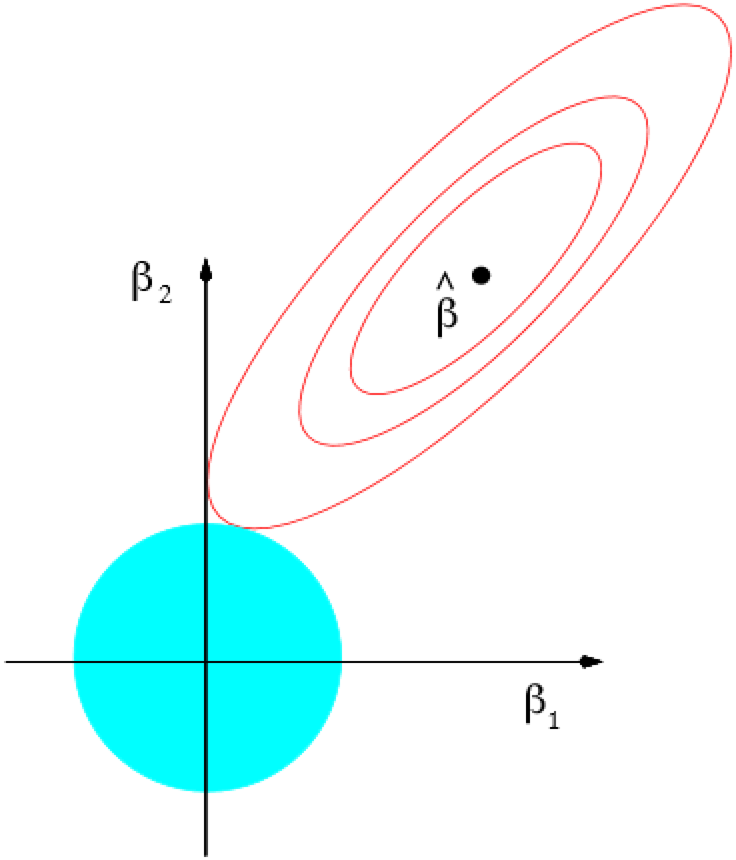


L_0 = limit as $p \to 0$: "number of nonzero weights", a natural notion of complexity

L1 vs L2

- Optimizing L1 harder
  - Discontinuous objective function
  - Subgradient descent versus gradient descent





How to pick weights?

- Goal: choose "best" vector w given training data
  - For now, we mean "best for classification"
- The ideal: the weights which have greatest test set accuracy / F1 / whatever
  - But, don't have the test set
  - Must compute weights from training set
- Maybe we want weights which give best training set accuracy?
  - May not (does not) generalize to test set
  - Easy to overfit
- Use devset

Gradient Descent & Large Training Data

repeat

$$w^{(t+1)} \leftarrow w^{(t)} + \eta \frac{\partial L}{\partial w}$$

$$w_j^{(t+1)} \leftarrow w_j^{(t)} + \eta \left[\frac{1}{N} \sum_{i=1}^{N} \left(\phi_j(x_i, y_i) - \sum_y p(y \mid x_i, w^{(t)}) \phi_j(x_i, y) \right) - \lambda w_j^{(t)} \right]$$

until convergence

Prohibitive for large datasets

Stochastic Gradient Descent

repeat

$$\cancel{w_j^{(t+1)} \leftarrow w_j^{(t)} + \eta\left[\frac{1}{N}\sum_{i=1}^{N}\left(\phi_j(x_i, y_i) - \sum_y p(y \mid x_i, w^{(t)})\phi_j(x_i, y)\right) - \lambda w_j^{(t)}\right]}$$

until convergence

Use gradient at current point as approx. for avg gradient!

repeat

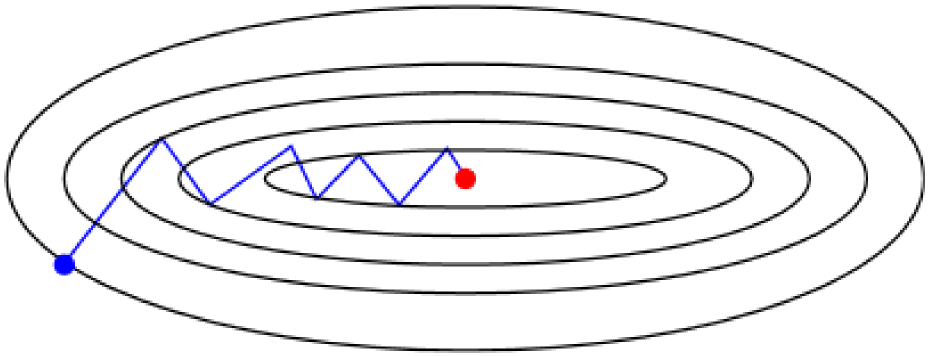
$$w_j^{(t+1)} \leftarrow w_j^{(t)} + \eta^{(t)}\left[\phi_j(x_i, y_i) - \sum_y p(y \mid x_i, w^{(t)})\phi_j(x_i, y) - \lambda w_j^{(t)}\right]$$

until convergence

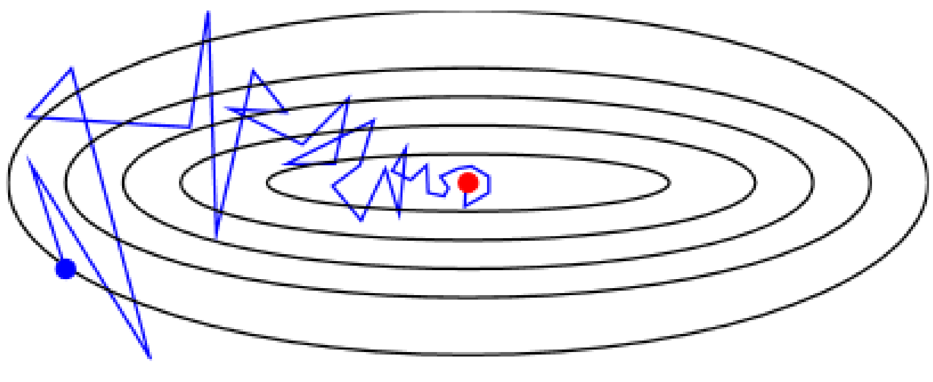
Reduce learning rate slowly (e.g., as η/t)

SGD vs. GD

- Deterministic gradient method [Cauchy, 1847]:

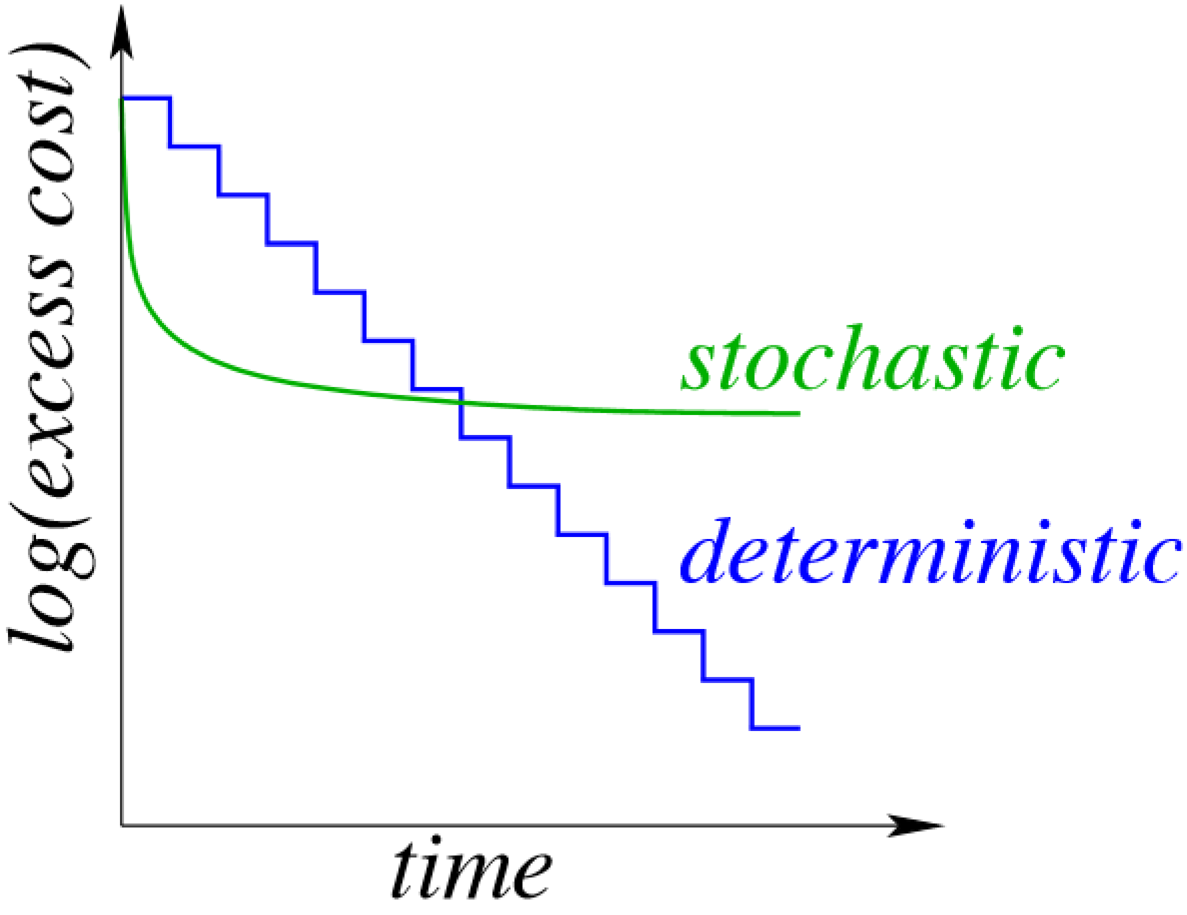


- Stochastic gradient method [Robbins & Monro, 1951]:



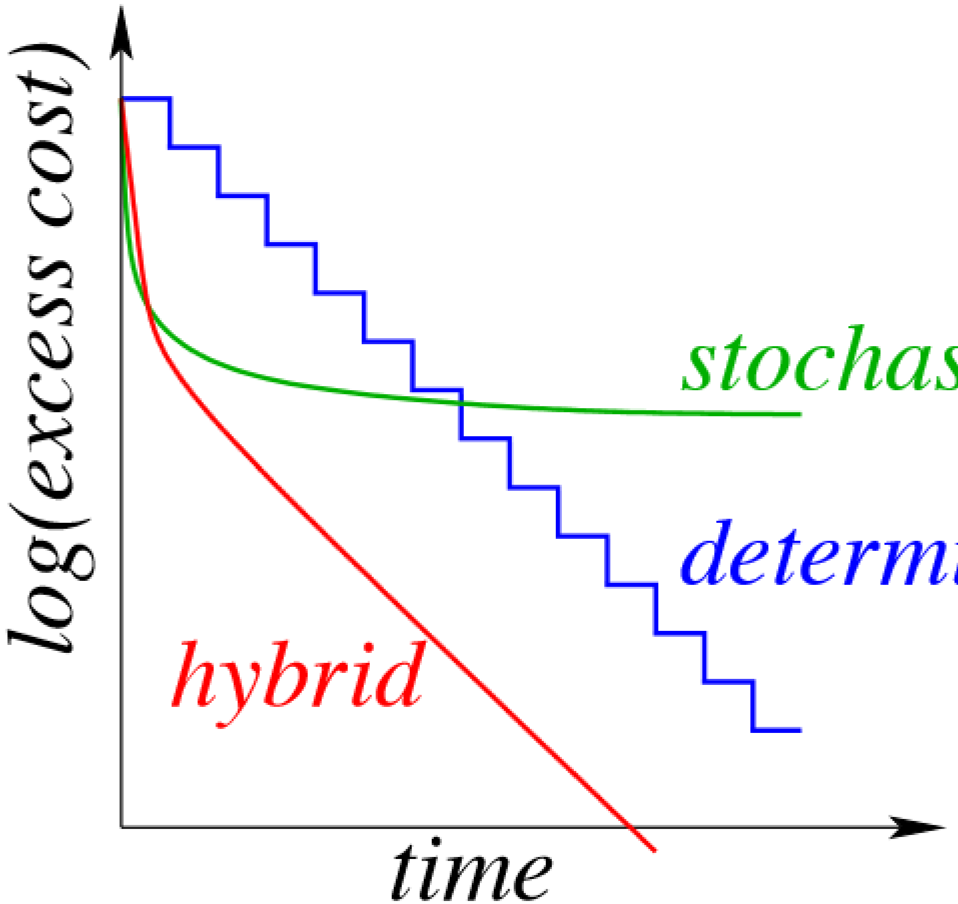
Convergence rates

- GD: O(1/t^2), SGD: O(1/sqrt(t))



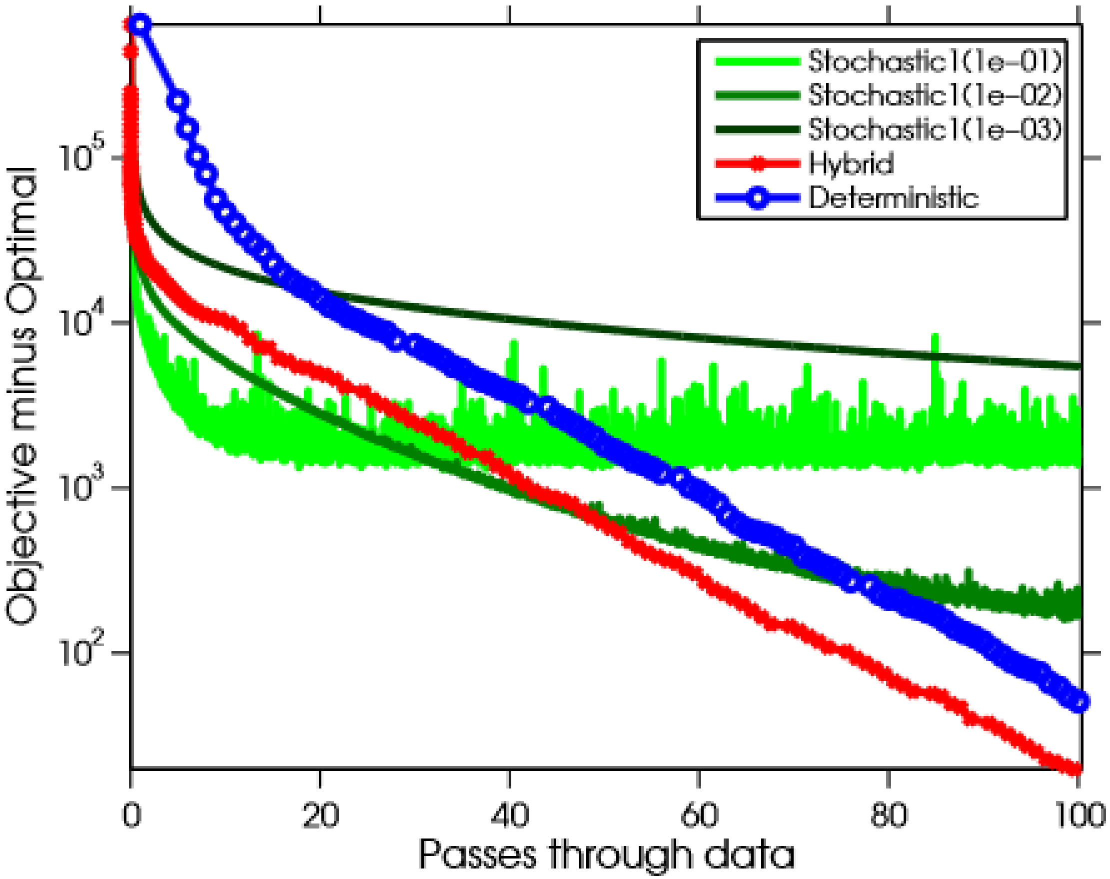
Stochastic will be superior for low-accuracy/time situations.

Hybrid Approaches



Hybrid #1: Batch

- Batch Gradient

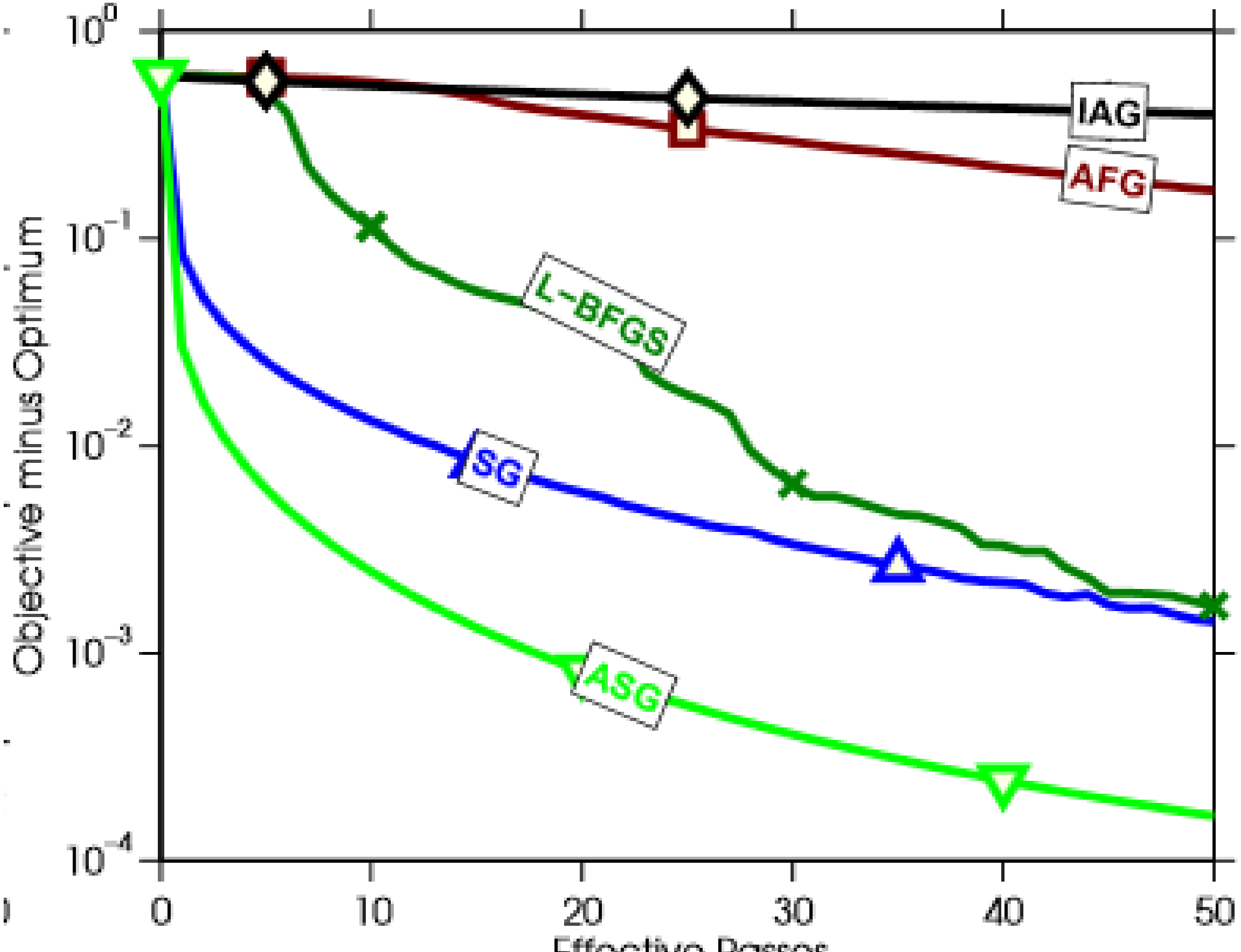


Hybrid #2: Stochastic Avg Gradient

- [Schmidt 2013]
- Use average gradient over all data points
  - Choose a datapt randomly (xi, yi)
  - Compute gradient at (xi,yi)
  - Recompute a new average gradient
    - Replace the prev gradient for (xi, yi) by the new one
  - Do the weight updates

- Assumes gradients of non-selected examples don’t change
- Better theoretical and practical convergence

Stochastic Avg Gradient



Word Sense Disambiguation Results

[Suarez and Palomar, 2002]

- With clever features, small variations on simple log-linear models did very well in an word sense competition:

Figure 1: List of types of features

- **0**: ambiguous-word shape
- ***s***: words at positions ±1, ±2, ±3
- ***p***: POS-tags of words at positions ±1, ±2, ±3
- ***b***: lemmas of collocations at positions $(-2,-1)$, $(-1,+1)$, $(+1,+2)$
- ***c***: collocations at positions $(-2,-1)$, $(-1,+1)$, $(+1,+2)$
- ***k***m: lemmas of nouns at any position in context, occurring at least $m\%$ times with a sense
- ***r***: grammatical relation of the ambiguous word
- ***d***: the word that the ambiguous word depends on
- ***L***: lemmas of content-words at positions ±1, ±2, ±3 ("relaxed" definition)
- ***W***: content-words at positions ±1, ±2, ±3 ("relaxed" definition)
- ***S*, *B*, *C*, *P*, and *D***: "relaxed" versions

Table 5: Comparing with SENSEVAL-2 systems

ALL		Nouns		Verbs		Adjectives	
0.713	jhu(R)	0.702	jhu(R)	0.643	jhu(R)	0.802	jhu(R)
0.684	**vME+SM**	0.702	**vME+SM**	0.609	jhu	0.774	**vME**
0.682	jhu	0.683	**MEbfs.pos**	0.595	css244	0.772	**MEbfs.pos**
0.677	**MEbfs.pos**	0.681	jhu	0.584	umd-sst	0.772	css244
0.676	**vME**	0.678	**vME**	0.583	**vME**	0.771	**MEbfs**
0.670	css244	0.661	**MEbfs**	0.583	**MEbfs.pos**	0.764	jhu
0.667	**MEbfs**	0.652	css244	0.583	**MEfix**	0.756	**MEfix**
0.658	**MEfix**	0.646	**MEfix**	0.580	**MEbfs**	0.725	duluth 8
0.627	umd-sst	0.621	duluth 8	0.515	duluth 10	0.712	duluth 10
0.617	duluth 8	0.612	duluth Z	0.513	duluth 8	0.706	duluth 7
0.610	duluth 10	0.611	duluth 10	0.511	ua	0.703	umd-sst
0.595	duluth Z	0.603	umd-sst	0.498	duluth 7	0.689	duluth 6
0.595	duluth 7	0.592	duluth 6	0.490	duluth Z	0.689	duluth Z
0.582	duluth 6	0.590	duluth 7	0.478	duluth X	0.687	ua
0.578	duluth X	0.586	duluth X	0.477	duluth 9	0.678	duluth X
0.560	duluth 9	0.557	duluth 9	0.474	duluth 6	0.655	duluth 9
0.548	ua	0.514	duluth Y	0.431	duluth Y	0.637	duluth Y
0.524	duluth Y	0.464	ua				

- The winning system is a famous semi-supervised learning approach by Yarowsky
- The other systems include many different approaches: Naïve Bayes, SVMs, etc