# Design Document

## Project: Softened Barnes-Hut N-Body simulation

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Status: proposed

### 1. Overview

This document proposes modifications to the existing direct sum N-body simulation through incorporating Plummer softening in the gravitational force model and the Barnes-Hut approximation. The goal of this upgrade is to improve numerical stability and allow for bodies to clump, enabling more realistic simulations of colliding planetary systems.

### 2. Assumptions

The document assumes that the reader is familiar with the current physical model used by the simulation, including classical Newtonian gravity and basic numerical integration techniques. As such, appropriate technical terminology will be used.

The intended audience includes research collaborators, physics students using the simulation, and faculty advisors reviewing architectural changes.

### 3. Purpose

The purpose of this document is to seek approval for the proposed modifications before implementation. It aims to ensure agreement on:

- The necessity of the force model changes.
- The tradeoffs involved.
- The chosen strategy for improving numerical stability.

### 4. Problem Statement

In discrete N-Body simulations:

- Gravitational force scales as $1/r^2$.
- As $r \rightarrow 0$, forces approach infinity.
- Numerical integrators cannot resolve extremely large acceleration within finite timesteps.
- The current direct-sum algorithm scales as $O(n^2)$.

- No collision model is currently implemented

While these issues can be partially mitigated by decreasing the simulation timestep, doing so significantly increases computational cost and does not resolve the underlying instability caused by close range interactions.

As the number of bodies increases, the simulation slows dramatically due to quadratic scaling, and the absence of collision handling causes objects to pass through or explosively repel one another. These limitations prevent realistic modeling of gravitational clumping and colliding planetary systems.

## 5. Goals

The proposed goals for this project are to:

- Reduce computational complexity from O($n^2$) to O($n\ log\ n$) using the Barnes-Hut algorithm.
- Improve energy stability in extended simulations.
- Eliminate explosive acceleration at small separations.
- Maintain acceptable accuracy relative to the current model.

## 6. Non-Goals

While not primary objectives, the proposed solution also introduces additional functionality:

- Adaptive timestep control.
- Eliminating I/O based data output.
- Multithreaded capabilities.
- A built in OpenGL-based animation library (Anim)

## 7. Proposed Design

### 7.1 Force Model Change

Replace classical force law:

$$F = \frac{Gm_1m_2}{r^2}\hat{r}$$

With Plummer-softened force:

$$F = \frac{Gm_1m_2}{(r^2+\epsilon^2)^{3/2}}\bar{r}$$

Where:

- $\epsilon$ = softening parameter and $\epsilon \ll r$

This model caps the gravitational force and eliminates the singularity at close distances.

### 7.2 Integration Impact

The project will continue using the second-order symplectic Leapfrog integrator, as it provides strong long-term energy conservation compared to non-symplectic methods.

### 7.3 Barnes-Hut Interaction

To overcome the O($n^2$) computational costs of the direct-sum force algorithm, the project uses the Barnes-Hut algorithm, which introduces a controlled approximation to improve performance.

- For each time step an octree is constructed to represent the spatial distribution of objects.
- Groups of distant objects are approximated as a single mass, significantly reducing the number of force calculations required.

### 7.4 Collision

Since objects continuously experience gravity, a collision model is required to prevent unphysical overlap. The proposed approach uses a mass-spring-damper model to simulate non-elastic collisions.

The method enables bodies to clump together while dissipating energy in a controlled manner creating more realistic planetary interactions.

### 7.5 Anim

A major bottleneck in the previous simulation model was I/O. For large n-body simulations with powerful enough hardware the primary bottleneck was I/O operations.

The addition of a built-in OpenGL animation library allows for in-simulation rendering, ensuring computation remains the dominant performance factor.

## 8. Tradeoffs

### 8.1 Benefits

- The Barnes-Hut approximation makes 1000+ body simulations possible.
- Collision allows realistic gravitational clumping.
- Softening eliminates numerical instability at close distances.

- Reduced computation allows for smaller time steps and improved accuracy.
- Integrated animation removes the I/O bottleneck.

### 8.2 Costs

- The Barnes-Hut reduces accuracy relative to direct-sum methods.
- Development of Anim library is expensive and introduces platform specific dependencies.
- The increased complexity makes verification of energy conservation more costly.

## 9. Risks

- Oversoftening may suppress gravitational clumping and needs to be selected carefully.
- Undersoftening may fail to resolve instability.
- Adding additional collision physics may result in energy conservation issues.

## 10. Validation Plan

- Stress tests high-density clusters over long runtimes ensuring stability.
- Comparing total energy drift of the original model to the new model.
- Validating the performance of the new model compared to the original model for similar scenarios.